कक्षा XI-XII के लिए

गणित

की पाठ्यपुस्तक

पुस्तक I

सम्पादक

मनमोहन सिंह अरोरा

सहायक सम्पादक

अजीत कौर चिलाना

राजेन्द्र पी० गुप्ता

सतीश शिराली

विद्ययाऽमृतमश्नुते

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NCERT

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प्रकाशन विभाग में श्री विनोद कुमार पंडित, सचिव, राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, श्री अरविन्द मार्ग, नई दिल्ली 110016 द्वारा प्रकाशित तथा आरसी प्रेस, पहाड़गंज, नई दिल्ली 110055 में मुद्रित ।

प्राक्कथन

'कक्षा XI—XII के लिए गणित की पाठ्यपुस्तक' की श्रृंखला में इस पुस्तक I को देख कर मुझे प्रसन्नता है। इस पुस्तक में पाठ्य सामग्री के प्रस्तुतीकरण से, जो स्पष्ट और चित्त आकर्षक है, मैं बहुत प्रभावित हूँ। विभिन्न विषयों का विकास प्रारम्भिक संकल्पनाओं से लेकर उस स्तर तक सुचारु रूप से किया गया है जो 16 + से 18 + की आयु वर्ग के विद्यार्थियों से अपेक्षित है। इस प्रक्रम में वास्तविक जीवन की स्थितियों को दक्षतापूर्वक प्रयुक्त किया गया है। पुस्तक के सम्पादन कार्य की ओर विशेष ध्यान दिया गया है। विषय वस्तु में स्वाभाविक प्रवाह है तथा विद्यार्थियों द्वारा पिछली कक्षाओं में अध्ययन की गई सामग्री से निरन्तरता बनाए रखी गई है। इसी उद्देश्य को ध्यान में रखते हुए उपयुक्त स्थानों पर पुनरावलोकन अनुच्छेद तथा प्रश्नावलियाँ रखी गई हैं।

इस पुस्तक का प्रथम प्रारूप राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद् के विज्ञान एवं गणित शिक्षा विभाग में गणित एकक द्वारा तैयार किया गया। इस सम्बन्ध में मानकों तथा लेखन प्रयुक्तियों का विकास प्रो० मनमोहन सिंह अरोरा के कुशल मार्गदर्शन में किया गया। विशेष रूप से जिन व्यक्तियों ने इस पुस्तक के प्रथम प्रारूप को तैयार करने में सहायता प्रदान की वे हैं विज्ञान एवं गणित शिक्षा विभाग के श्री आर० सी० सक्सेना, डा० एस० सी० दास, डा० राम अवतार, श्री जी० डी० ढल, डा० एस० के० सिंह गौतम, डा० के० सी० मदान और डा० ए० आर० साहू तथा पाठ्यपुस्तक विभाग के डा० वी० पी० गुप्ता।

प्रथम प्रारूप का आलोचनात्मक विवेचन अध्यापकों, विषय विशेषज्ञों, अध्यापक प्रशिक्षकों, राज्य शिक्षा संस्थानों एवं राज्य विज्ञान शिक्षा संस्थानों के प्रतिनिधियों तथा गणित के उपयोगकर्ताओं के एक समुदाय द्वारा अक्टूबर 1977 में कुरुक्षेत्र विश्वविद्यालय के गणित विभाग में इसी उद्देश्य से आयोजित एक कार्य शिविर में किया गया तथा इसके सुधार के लिए सुझाव दिए गए। इस पुस्तक को इस रूप में लाने में उनके महत्त्वपूर्ण योगदान के लिए मैं उन सबका अत्यन्त आभारी हूँ।

पुस्तक का अन्तिम लेखन तथा विषय-सम्पादन प्रो० मनमोहन सिंह अरोरा ने किया। इस कार्य में दिल्ली विश्वविद्यालय के गणित विभाग की डा० (कु०) अजीत कौर चिलाना, पंजाब विश्वविद्यालय के गणित के उच्चतर अध्ययन-केन्द्र के डा० सतीश शिराली तथा विज्ञान एवं गणित शिक्षा विभाग के डा० आर० पी० गुप्ता ने बहुमूल्य सहायता प्रदान की। हिंदी संस्करण की देखरेख श्री महेन्द्र शंकर ने की जिसमें डा० के० सी० मदान तथा श्री ईश्वर चन्द्र ने आवश्यक सहायता प्रदान की। विभिन्न एककों के लेखकों के अतिरिक्त पुस्तक के उत्तर डा० एस० सी० दास, श्री आर० एस० कोठारी, डा० (श्रीमती) सूरजा कुमारी तथा कु० नीलम पसरीचा ने प्रदान किए।

मैं इन सबका तथा विशेष रूप से प्रो० मनमोहन सिंह अरोरा का आभारी हूँ जिन्होंने अपने व्यस्त कार्यक्रम के साथ-साथ इस पुस्तक के सम्पादन का कठिन कार्य किया। उनके परिश्रम तथा उत्साहपूर्ण नेतृत्व के बिना इतने अल्प समय में यह कार्य पूर्ण नहीं हो सकता था।

इस पुस्तक के उपयोगकर्ताओं की प्रतिक्रियाएँ प्राप्त करके परिषद् अति प्रसन्न होगी तथा इसमें और अधिक सुधार हेतु दिये गए सुझावों का स्वागत करेगी।

शिब के० मित्र
निदेशक
राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्

नई दिल्ली
मई 1978

प्रस्तावना

यह पुस्तक 'कक्षा XI—XII के लिए पाठ्यपुस्तक' शृंखला का प्रथम भाग है। एक प्रमुख प्रश्न जो हमारे सम्मुख आया वह यह था कि 'क्या उपयोगकर्ताओं के विभिन्न वर्गों के लिए अर्थात् जैविकी, भौतिकी, अर्थशास्त्र, इत्यादि के विद्यार्थियों के लिए भिन्न-भिन्न गणित होना चाहिए अथवा एक ही कोर (core) पाठ्यक्रम कक्षा XI—XII के सभी विद्यार्थियों की माँग को पूर्ण कर देगा?' अनेक शिक्षकों तथा विषय विशेषज्ञों से प्राप्त समालोचनाएँ इस पक्ष में थीं कि इस स्तर पर गणित का एक ही पाठ्यक्रम होना चाहिए जिनमें अनुप्रयोगों के विभिन्न क्षेत्रों से स्थितियों का समावेश किया जाए।

अत: विभिन्न एककों की प्रथम पाठ्य सामग्री तैयार करना प्रारम्भ करने से पहले परिषद् के गणित एकक के सदस्यों द्वारा एक विस्तृत रूपरेखा तैयार की गई। इस योजना में वास्तविक जीवन की स्थितियों के चयन तथा उन्हें समावेश करने के स्थानों का इस अवस्था के लिए इष्टतम स्तर तक विकास के द्वारा किसी विषय का समावेश करने के दृष्टिकोण से बच्चे की पूर्व-आकांक्षाओं, उसकी योग्यता के अपेक्षित स्तर, उसकी ग्राह्य क्षमता तथा अन्य सम्बन्धित बातों को समुचित महत्व दिया गया। अत: विस्तृत योजना तथा गहन विचार-धारा इस पुस्तक के आधार हैं।

प्रथम प्रारूप परिषद् में मेरे साथियों द्वारा तैयार किया गया तथा इसका पर्यवेक्षण कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र के गणित विभाग में इसी उद्देश्य से आयोजित एक कार्यशिविर में किया गया जिसमें अनेक अनुभवी अध्यापकों, विषय-विशेषज्ञों, इत्यादि ने भाग लिया। मैं कार्यशिविर में भाग लेने वाले सभी व्यक्तियों का आभारी हूँ और साथ ही अपने साथियों का भी जिन्होंने इस कार्य को इस स्तर तक लाने में प्रसन्नतापूर्वक अपना पूर्ण योग दिया। मैं कुछ व्यक्तियों का ज़िक्र किए बिना नहीं रह सकता जिन्होंने इस पुस्तक के कुछ एककों को प्रथम प्रारूप प्रदान किया। वे हैं—विज्ञान एवं गणित शिक्षा विभाग के श्री आर० सी० सक्सेना, डा० एस० सी० दास, डा० राम अवतार, श्री जी० डी० ढल, डा० एस० के० सिंह गौतम, डा० के० सी० मदान और डा० ए० आर० साहू तथा पाठ्य पुस्तक विभाग के डा० वी० पी० गुप्ता।

इसके उपरान्त मैंने दिल्ली विश्वविद्यालय के गणित विभाग की डा० (कु०) अजीत कौर चिलाना, पंजाब विश्वविद्यालय के गणित के उच्चतर अध्ययन-केन्द्र के डा० सतीश शिराली तथा विज्ञान एवं गणित शिक्षा विभाग के डा० आर० पी० गुप्ता की बहुमूल्य सहायता से पाठ्य सामग्री का सम्पादन तथा पुनर्लेखन किया। डा० एस० सी० दास, श्री आर० एस० कोठारी, डा० (श्रीमती) सूरजा कुमारी तथा कु० नीलम पसरीचा ने इस पुस्तक के कुछ एककों के उत्तर प्रदान किए। श्री महेन्द्र शंकर ने डा० के० सी० मदान तथा श्री ईश्वर चन्द्र की सहायता से हिन्दी संस्करण का सम्पादन कार्य किया। मुझे इनमें से प्रत्येक का हार्दिक आभार प्रकट करने में अति प्रसन्नता है।

इस पुस्तक की कुछ मुख्य विशेषताओं की ओर ध्यान दिलाना समयोचित ही होगा ।

1. इस पुस्तक की पद्धति वही है जो परिषद् की IX तथा X कक्षाओं की पुस्तकों की है । प्राय: आकर्षक स्वान्वेषण तर्कों द्वारा तथा कहीं-कहीं दृढ़ता को खोकर, संकल्पनाओं तथा परिणामों के समझाने पर बल दिया गया है ।
2. किसी संकल्पना का परिचय कराते समय उपयुक्त उदाहरण तथा स्थिति से सम्बन्धित अभिप्रेरण प्रदान किये गए हैं ।
3. पुस्तक में अत्यधिक हल किये गए उदाहरणों का समावेश किया गया है ताकि विद्यार्थी किसी सकल्पना के अनुप्रयोग के विस्तृत क्षेत्र को जान सके ।
4. पुस्तक को विभिन्न स्वतन्त्र एककों में विभाजित किया गया है । प्रत्येक एकक के प्रारम्भ में इसमें समाहित सामग्री के विषय में संक्षेप में बताया गया है तथा अंत में मुख्य संकल्पनाओं की सूची दी गई है ताकि विद्यार्थी अविलम्ब ही किसी एकक विशेष में पढ़ी हुई सामग्री का पुनरावलोकन कर सके ।
5. प्रत्येक एकक के अंत में अग्रिम अध्ययन हेतु सुझाव दिये गए हैं ताकि योग्य विद्यार्थी अपने ज्ञान की परिधि बढ़ा सकें ।
6. प्रत्येक एकक में अनेक प्रश्न देने का उद्देश्य अध्यापक के लिए प्रश्नों का एक संग्रह प्रदान करना है जिससे वह विभिन्न योग्यताओं के विद्यार्थियों की माँग पर्याप्त रूप से पूर्ण कर सके ।
7. यह दर्शाने के लिए कि अगले अनुछेच्द अथवा एकक का अध्ययन करने से पहले विद्यार्थी को किन संकल्पनाओ को भलीभांति याद रखना है पुनरावलोकन अनुच्छेदों, एककों तथा प्रश्नावलियों का समावेश किया गया है ।
8. उपयुक्त स्थानों पर विविध प्रश्नावलियों का समावेश किया गया है ताकि उपयुक्त अन्तरालों पर विद्यार्थी पाठ्य सामग्री को दोहरा सके तथा उस विषय में प्रवीणता प्राप्त कर सके ।
9. विद्यार्थी की गणितीय विचारधारा को उत्तेजित करने तथा उसके विचारों को स्पष्ट करने के लिए अनेक स्थानों पर "क्यों ?" लिख दिया गया है । अत: अध्यापक से अपेक्षित है कि वह स्वयं उत्तर प्रदान करने की अपेक्षा विद्यार्थी को उत्तर देने के लिए प्रोत्साहित करे ।
10. कठिन प्रश्नों तथा अनुच्छेदों को तारांकित किया गया है ।
11. ऐतिहासिक विवरणों का समावेश किया गया है ताकि विद्यार्थी किसी विशेष संकल्पना के विकास के प्रक्रम में झांक सके तथा यह अनुमान लगा सके कि एक अन्वेषक तथा गणितज्ञ की मस्तिष्क की क्रियाएँ कैसी होती हैं ।

पाठ्य पुस्तक किसी अध्यापन-अध्ययन प्रक्रिया में एक आवश्यक उपकरण होती है । इस प्रक्रिया में एक अध्यापक के कार्य के महत्व को बढ़ा चढ़ा कर नहीं कहा जा सकता । यहाँ यह याद दिलाना अधिक आवश्यक नहीं है कि अपने विषय को जितना सम्भव हो सके रुचिपूर्ण तथा जीवंत बनाना, विद्यार्थियों में रुचि उत्पन्न करना तथा इसे बनाए रखना तथा उसे निरन्तर विस्तीर्ण सीमाओं की ओर ले जाना अध्यापक का पावन कर्त्तव्य है । मुझे विश्वास है कि अध्यापक इस पुस्तक में निहित विचारधारा को समझेंगे एवं इससे सहमत होंगे तथा इस बात का प्रयास करेंगे कि विद्यार्थियों का विषय में अनुराग हो तथा वे इसमें उच्च प्रवीणता प्राप्त करें ।

अंत में मैं विद्यार्थी के लिए कुछ शब्द अवश्य कहूँगा।

आप गणित की इस पुस्तक का अध्ययन करने जा रहे हैं। यदि आप आरम्भ में ही अध्ययन की 'अच्छी' आदतें डाल लें तो आपको गणित का अध्ययन अति आकर्षक एवं लाभकारी लगेगा। अध्ययन की कु अच्छी आदतें निम्न हैं:

1. गणित को केवल करके ही सीखा जा सकता है। अपनी पाठ्यपुस्तक को केवल पढ़िए ही नहीं। आप सदा एक कागज और पेंसिल लेनी चाहिए और फिर पुस्तक का अध्ययन करना चाहिए।

2. पाठ्यपुस्तक में जहाँ कहीं भी आपके सम्मुख "क्यों?" आ जाए तो आपको इसका उत्तर खोजने का प्रयत्न करना चाहिए।

3. किसी एक प्रश्न पर बहुत अधिक समय न लगाएँ। यह सदा अच्छा रहता है कि पहले हम अगले प्रश्न चले जाएँ और बाद में शुद्ध मस्तिष्क के साथ उस प्रश्न पर वापिस आ जाएं जिसमें हम कठिनाई अनु कर रहे थे।

4. मानव मस्तिष्क कुछ सीमित सामग्री का ही संग्रह कर सकता है। जिस किसी वस्तु का अधिक प्रयोग न होता वह संग्रह से निकाल दी जाती है। अतः यह अच्छा रहेगा यदि आप प्रत्येक एकक के मुख्य पदि की एक संक्षिप्त सूची बना लें तथा बीच-बीच में उन्हें दोहराते रहें।

हमारे अत्यधिक प्रयत्नों के होते हुए भी यह सम्भव है कि पुस्तक में कुछ अप्रत्याशित त्रुटियाँ रह हों। यदि किसी त्रुटि की ओर हमारा ध्यान दिलाया गया तो परिषद् इसके लिए अति कृतज्ञ होगी त पुस्तक के आगे के संस्करणों में सुधार के लिए दिए गए सुझावों का स्वागत करेगी।

नई दिल्ली
मई 1978

मनमोहन सिंह अरोरा
सम्पादक

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद् निम्नलिखित व्यक्तियों के प्रति आभारी है जिन्होंने अक्तूबर, 1977 में गणित विभाग कुरुक्षेत्र विश्वविद्यालय में आयोजित एक कार्यशिविर में इस पाठ्यपुस्तक के लिए तैयार किये गए प्रारूप के पर्यवेक्षण में महत्वपूर्ण योगदान दिया :

1. डा० मनमोहन सिंह अरोरा
   राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली

2. डा० राम अवतार
   राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली

3. श्री एन० बी० बद्रीनारायन
   क्षेत्रीय शिक्षा महाविद्यालय, मैसूर

4. श्री एस० जी० बाजवा
   ए० ई० सी० स्कूल, थाना

5. डा० प्रकाश चन्द्र
   कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

6. डा० एस० डी० चोपड़ा
   कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

7. श्री सुकुमार चोपड़ा
   कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

8. श्रीमती एस० देवासुन्दरम्
   सेंट जेवियर्स स्कूल, दिल्ली

9. श्री जी० डी० ढल
   राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली

10. डा० एस० के० सिंह गौतम
    राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली

11. डा० ए० के० गायन
    इंडियन इन्स्टीट्यूट आफ टैक्नोलाजी, खड़गपुर

12. डा० एम० एल० गोगना
    कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

13. कु० अंजु गोयल
कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

14. श्री सत्यपाल गुलाटी
ए० आर० एस० डी० हायर सेकेंडरी स्कूल, दिल्ली

15. डा० आर० पी० गुप्ता
राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली

16. डा० वी० पी० गुप्ता
राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली

17. डा० इक्तदार हुसैन
अलीगढ़ मुस्लिम विश्वविद्यालय, अलीगढ़

18. कु० प्रेम सुधा शास्त्री
कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

19. श्री जी० आर० कक्कड़
केन्द्रीय विद्यालय, चंडीगढ़

20. श्री जे० डी० कालरा
केन्द्रीय विद्यालय, एच० एम० टी०, पिंजौर

21. श्री के० पी० कन्सल
एयर फोर्स सैंट्रल स्कूल, दिल्ली कैंट, दिल्ली

22. डा० रविन्द्र कुमार
रामजस कालेज, दिल्ली

23. डा० (श्रीमती) सूरजा कुमारी
राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली

24. श्रीमती कंचन मानकतला
गार्गी कालेज, नई दिल्ली

25. डा० आर० एन० मुखर्जी
बी० आई० टी० एस०, पिलानी

26. श्री के० एस० मूर्ति
केन्द्रीय विद्यालय, सिकन्दराबाद

27. श्री बी० नायडू
राजकीय कालेज, मलिया, हैदराबाद

28. श्री शान्ति नारायण
भूतपूर्व डीन ऑफ कालेजिस, दिल्ली विश्वविद्यालय, दिल्ली

29. कु० उमेश परमार
कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

30. डा० आई० बी० एस० पासी
कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

31. श्रीमती राजरानी
बी० एन० रस्तोगी राजकीय कन्या उच्चतर माध्यमिक विद्यालय, दिल्ली

32. श्री बी० एस० पाण्डु रंगा राव
मैसूर विश्वविद्यालय, मैसूर

33. श्री एस० एल० सैनी
कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र

34. कु० कमला साहनी
ब्रिटिश स्कूल, चाणक्यपुरी, नई दिल्ली

35. श्री आई० डी० शर्मा
केन्द्रीय विद्यालय, आई० आई० टी० कैम्पस, नई दिल्ली

36. श्री ओ० पी० श्योरन
केन्द्रीय विद्यालय, आई० एन० ए० कालोनी, नई दिल्ली

37. डा० सतीश शिराली
पंजाब विश्वविद्यालय, चंडीगढ़

38. डा० बी० वी० सिंगबाल
टाटा इन्स्टीट्यूट ऑफ फंडामैंटल रिसर्च, बम्बई

39. श्री बलबीर सिंह
राजपूताना राइफल्स हीरोज मैमोरियल हायर सैकेंडरी स्कूल, दिल्ली कैंट, दिल्ली

40. श्री तरलोचन सिंह
एस० एस० खालसा हायर सैकेंडरी स्कूल, नई दिल्ली

41. श्री पी० के० तिवारी
केन्द्रीय विद्यालय, हिंडन, गाजियाबाद

42. श्री ओ० एन० तिवारी
भास्करानन्द इन्टर कालेज, नर्वल, कानपुर

43. श्री यशपाल वर्मा
नवयुग स्कूल, नई दिल्ली

विषय-सूची

एकक 1

संख्या - निकाय

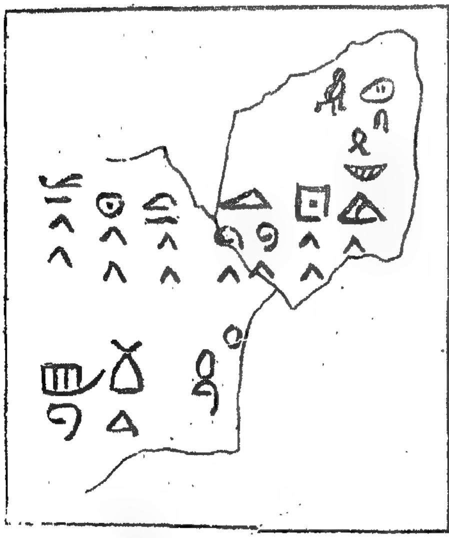
इस एकक में हम प्राकृत संख्याओं (धनपूर्णांकों), पूर्णांकों तथा परिमेय संख्याओं, इत्यादि का पुनरावलोकन करेंगे। प्राकृत संख्याओं का पूर्णांकों में, पूर्णांकों का परिमेय संख्याओं में तथा परिमेय संख्याओं का वास्तविक संख्याओं में विस्तार करने का अभिप्रेरण किया गया है। परिमेय तथा अपरिमेय संख्याओं को दशमलवों के रूप में व्यक्त करने पर विचार किया गया है। अन्त में वास्तविक संख्याओं के गुणधर्मों को दोहराया गया है।

1.1 भूमिका

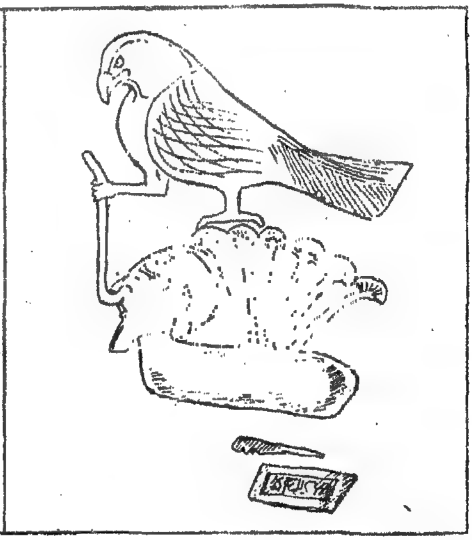
प्राकृत संख्याओं, पूर्णांकों तथा परिमेय संख्याओं, इत्यादि से हम पहले ही परिचित हैं। प्रस्तुत एकक में हम विभिन्न संख्या-निकायों (number-systems) को दोहरायेंगे तथा इनके विषय में और अधिक जानकारी प्राप्त करेंगे।

ऐतिहासिक तौर पर हम जानते हैं कि गणना का प्रारम्भ आदि मानव से हुआ था। अपनी वस्तुओं की गणना के लिए मनुष्य ने अपनी उंगलियों, छोटे पत्थरों, घोंघों, इत्यादि का प्रयोग किया अथवा लकड़ी पर चिन्ह बनाकर या कुछ अन्य देसी विधियों के प्रयोग से गणना की। फिर मनुष्य ने समूहों में रहना तथा मिलना-जुलना आरम्भ किया और इस प्रकार उसे संख्याओं के लिए चिन्हों की आवश्यकता पड़ी ताकि वह दूसरों से सुचारू रूप से विचार विनिमय कर सके। यद्यपि इतिहासकार इस विषय पर एकमत नहीं हैं कि संख्यांकों (numerals) का प्रारम्भ कहाँ और कब हुआ तथापि प्राय: विश्वास किया जाता है कि संख्याओं का प्रयोग लगभग उतना पुराना है जितनी स्वयं मानव सभ्यता।

यदि हम आज से लगभग 5000 से 6000 वर्ष पहले के इतिहास पर दृष्टि डालें तो ज्ञात होगा कि उस समय मिस्रवासी तथा मेसोपोटामिया सभ्यता के निवासी संख्यांकों का विस्तृत प्रयोग करते थे। भिन्न-भिन्न



लगभग 3400 ई० पू० में मिट्टी के बर्तन के टुकड़े से प्रारम्भिक मिस्री संख्यांक



लगभग 500 ई० पू० में गूढ़ाक्षरों में लिखित (पवित्र) लिखावट। इसका तात्पर्य है कि "बाज राजा हरपून झील की भूमि से 6000 बन्दी बनाकर ले गया"।

सभ्यताओं में भिन्न-भिन्न प्रकार के संख्यांकों का विकास हुआ और इन संख्यांकों में भी समय के साथ-साथ परिवर्तन हुए। उदाहरणतया, मिस्रवासी आरम्भ में संख्याओं के लिए संकेतों के रूप में चित्रों का प्रयोग करते थे। यूनानी तथा हिब्रू लोग संख्याओं के लिए अपनी वर्णमाला के विभिन्न अक्षरों का प्रयोग करते थे। लगभग तृतीय शताब्दी के भारतीय शिला लेखों से ज्ञात होता है कि पुरातन भारतवासी अपनी संख्यांक प्रणाली में बहुत से संकेतों का प्रयोग किया करते थे।

ा भारत में हुआ था। आठवीं शताब्दी के अन्त में भारतीय खगोलविद्या तथा गणित बग़दाद के क पहुँचा। ऐसा माना जाता है कि कंक नामक एक भारतीय विद्वान, ब्रह्मगुप्त के **'ब्रह्मस्फुट सिद्धान्त'** अल-मंसूर के दरबार में पहुँचा। बाद में इसका अनुवाद अरबी भाषा में हुआ। इस प्रकार भारतीय प्रणाली अरबों तक पहुँची। प्रसिद्ध अरब गणितज्ञ **अल-ख्वारिज़्मी**, जो खगोल-शास्त्र की अनेक के भी लेखक हैं, ने इस अनुवाद के आधार पर अपना बहुत सा काम किया। **अल-ख्वारिज़्मी** अपनी ा **"अल-जबर"** (algebra) के लिए अत्यधिक प्रसिद्ध हैं। उन्होंने अंकगणित पर भी लिखा। उनकी ों का अनुवाद 'लेटिन' भाषा में हुआ और इस प्रकार हिन्दू-संख्यांक यूरोप में पहुँचे और हिन्दू-अरब कहलाए।

दस अंकों 0, 1, 2, 3, ..., 9 तथा स्थानीय मान के सिद्धान्त के उपयोग से बनी इस संख्यांक-प्रणाली **लव प्रणाली (decimal system)** कहते हैं। आधार-10 के अतिरिक्त अन्य आधार वाली भी थीं और आज भी प्रचलित हैं। उदाहरणार्थ, बेबीलोन के निवासी **सेक्साजेसीमल प्रणाली simal system)** का प्रयोग करते थे जिसका आधार 60 होता है। जब हम घंटों, मिनटों तथा में समय नापते हैं तो हम आधार-60 का प्रयोग करते हैं यद्यपि हम इसमें स्थानीय मान के सिद्धान्त का ीं करते। आज के युग में कम्प्यूटर **आधार-2 अथवा द्वि-आधार प्रणाली (binary system)** का रते हैं जिसमें केवल दो अंक 0 तथा 1 का प्रयोग होता है। परन्तु द्वि-आधार प्रणाली आज का कार नहीं है। यहाँ तक कि आदि मानव भी एक प्रकार की द्वि-आधार प्रणाली का प्रयोग करते थे। देयों तक, विभिन्न गणितज्ञों ने जिन्होंने संख्याओं का अध्ययन किया, इन्हें कई रहस्यमय तथा धार्मिक। 17वीं शताब्दी में महान जर्मन गणितज्ञ गोटफ्रिड विलहेल्म लेबनिज (1646-1716) जोकि एक धार्मिक ा, ने ईश्वर को अंक 1 तथा शून्य को अंक 0 से प्रदर्शित किया। उनका विश्वास था कि जिस प्रकार शून्य से ब्रह्मांड बनाया ठीक उसी प्रकार वे 0 तथा 1 इन दो अंकों से किसी भी संख्या को व्यक्त ा है।

ाकृत संख्याएँ तथा पूर्णांक (Natural Numbers and Integers)

गणन संख्याएँ 1,2,3, ... **प्राकृत संख्याएँ** कहलाती हैं। आपको याद होगा कि यदि हम किन्हीं संख्याओं का योग करें अथवा किन्हीं दो प्राकृत संख्याओं का गुणन करें तो हमें सदैव एक प्राकृत प्त होती है। अतः हम कहते हैं कि प्राकृत **संख्याएँ योग तथा गुणन की संक्रियाओं के** ा **(closed)** हैं। परन्तु यदि हम किसी प्राकृत संख्या को किसी अन्य प्राकृत संख्या में से घटाएँ तो

मान प्रणाली तथा संख्यांक 0 का आविष्कार अन्य संख्यांकों के पश्चात् हुआ था। यद्यपि इस तथ्य -भिन्न मत हैं कि स्थानीय मान प्रणाली, जिसमें शून्य के लिए भी संकेत दिया गया, का आविष्कार ा फिर भी प्रायः यह माना जाता है कि ऐसा करने की पहल हिन्दुओं ने की थी।

क्या होगा ? स्पष्ट है कि हमें सदा एक प्राकृत संख्या प्राप्त नहीं होगी। अतः यह आवश्यक है कि हम प्राकृत संख्याओं के निकाय का पूर्णांकों के निकाय में विस्तार करें। संख्याएँ ..., —3, —2, —1, 0, 1, 2, 3,... पूर्णांक कहलाती हैं। यह सुगमतापूर्वक देखा जा सकता है कि यदि हम किन्हो दो पूर्णांकों का योग करें, किन्हीं दो पूर्णांकों का अन्तर लें अथवा किन्हीं दो पूर्णांकों का परस्पर गुणन करें तो हमें सदैव एक पूर्णांक ही प्राप्त होगा। परन्तु यदि किसी पूर्णांक को किसी अन्य पूर्णांक से भाग दें तो भागफल क्या होगा? निश्चित है कि हमें सदा एक पूर्णांक प्राप्त नहीं होगा। अतः हम कहते हैं कि पूर्णांक विभाजन की संक्रिया के लिए संवृत नहीं होते। हम देखते हैं कि प्राकृत संख्याएँ भी विभाजन की संक्रिया के लिए संवृत नहीं होतीं। इसलिए यह आवश्यक है कि हम इस संख्या-निकाय का और अधिक विस्तार करें जैसा कि अगले अनुच्छेद में किया गया है।

1.4 परिमेय संख्याएँ (Rational Numbers)

अब हम आपका ऐसी संख्याओं से परिचय कराते हैं जो $\frac{p}{q}$ की तरह की हैं जहाँ p तथा q दोनों पूर्णांक हैं तथा $q \neq 0$ है। इस प्रकार की संख्याएँ परिमेय संख्याएँ कहलाती हैं। हम नीचे परिमेय संख्या की परिभाषा को दोहराते हैं।

परिमेय संख्या वह संख्या होती है जो $\frac{p}{q}$ के रूप में लिखी जा सके, जहाँ p तथा q दोनों पूर्णांक हैं तथा $q \neq 0$ है। यदि q एक प्राकृत संख्या है तथा p और q में 1 के अतिरिक्त कोई अन्य उभयनिष्ठ गुणनखंड नहीं है तो हम कहते हैं कि **परिमेय संख्या $\frac{p}{q}$ निम्नतम पदों (lowest terms) में है।** कभी-कभी हम यह भी कहते हैं कि **p और q निम्नतम पदों में हैं।**

कोई परिमेय संख्या या तो धनात्मक (positive), या ऋणात्मक (negative) या शून्य हो सकती है। यह आसानी से देखा जा सकता है कि प्रत्येक पूर्णांक एक परिमेय संख्या होती है। (क्यों ?) हम नीचे यह दिखाएँगे कि **परिमेय संख्याएँ योग, व्यवकलन, गुणन तथा विभाजन (शून्य के अतिरिक्त) की संक्रियाओं के लिए संवृत होती हैं।**

उपपत्ति: आइए, दो परिमेय संख्याएँ a तथा b लें। मान लीजिए, $a = \frac{p}{q}$ तथा $b = \frac{r}{s}$, जहाँ p, q, r तथा s सभी पूर्णांक हैं और $q \neq 0$, $s \neq 0$ हैं।

अब, $a + b = \frac{p}{q} + \frac{r}{s} = \frac{ps}{qs} + \frac{qr}{qs}$

अर्थात्, $a + b = \frac{ps + qr}{qs}$

परन्तु हम जानते हैं कि किन्ही दो पूर्णांकों का योग तथा किन्ही दो पूर्णांकों का गुणनफल भी एक पूर्णांक होता है। अतः ps, qr, qs तथा $ps+qr$ पूर्णांक हैं और $qs \neq 0$ क्योंकि $q \neq 0$, $s \neq 0$ है। इस प्रकार

$a+b$ दो पूर्णांकों का भागफल है जिसका हर (denominator) शून्य नहीं है। अतः $a+b$ एक परिमेय संख्या है।

अब, $ab=\frac{p}{q}\left(\frac{r}{s}\right)$

अर्थात्, $ab=\frac{pr}{qs}$

पुनः, pr और qs पूर्णांक हैं तथा $qs\neq0$ है। अतः ab भी एक परिमेय संख्या है।

['दो परिमेय संख्याओं का अन्तर तथा भागफल (जहाँ हर शून्य नहीं है) भी एक परिमेय संख्या होती है' सिद्ध करना पाठक के अभ्यासार्थ छोड़ दिया गया है।]

हम परिमेय संख्याओं के योग तथा गुणन के गुणधर्मों को नीचे दोहराते हैं:

यदि a, b, c परिमेय संख्याएँ हों, तो

1.	$a+b=b+a$	[योग क्रमविनिमेय (commutative) है।]
2.	$a+(b+c)=(a+b)+c$	[योग सहचारी (associative) है।]
3.	$a+0=a$	[योज्य तत्समक (additive identity): 0]
4.	$a+(-a)=0$	[योज्य प्रतिलोम (additive inverse): $-a$]
5.	$ab=ba$	[गुणन क्रमविनिमेय है।]
6.	$a(bc)=(ab)c$	[गुणन सहचारी है।]
7.	$a.1=a$	[गुणनात्मक तत्समक (multiplicative identity): 1]
8.	$a\left(\frac{1}{a}\right)=1,\ a\neq0$	[गुणनात्मक प्रतिलोम (multiplicative inverse): $\frac{1}{a}$]
9.	$a(b+c)=ab+ac$	[गुणन योग पर वितरणात्मक (distributive) है।]

1.5 सांत तथा असांत दशमलव (Terminating and Non-terminating Decimals)

परिमेय संख्याओं को ऐसे दशमलवों में व्यक्त किया जा सकता है जो **सांत** (terminating) अर्थात् परिमित (finite) अथवा **असांत** (non-terminating) अर्थात् अपरिमित* (infinite) होते हैं। उदाहरणार्थ, $\frac{3}{5}=0.6,\ -\frac{7}{64}=-0.109375$ तथा $\frac{637}{250}=2.548$ उन परिमेय संख्याओं के उदाहरण हैं जो सांत दशमलव में व्यक्त की जा सकती हैं।

$\frac{5}{6}=0.8333\ldots,\ \frac{3}{11}=0.272727\ldots$ तथा $\frac{6}{7}=0.857142857142\ldots$ उन परिमेय संख्याओं के

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*अपरिमित दशमलव के परिशुद्ध अर्थ के लिए उच्चस्तरीय संकल्पनाओं (concepts) की आवश्यकता है जो इस पुस्तक की सीमा के बाहर हैं।

उदाहरण हैं जो असांत दशमलव में व्यक्त की जा सकती हैं। इन असांत दशमलवों में हम देखते हैं कि कुछ अंकों के समूह की पुनरावृत्ति होती है। अतः इस प्रकार के दशमलवों को **आवर्ती दशमलव (recurring or periodic decimals)** कहा जाता है। एक आवर्ती दशमलव के आवर्ती भाग के ऊपर रेखा लगाने का प्रचलन है। उदाहरणार्थ, $\frac{5}{6}$ तथा $\frac{6}{7}$ को हम इस प्रकार लिखते हैं :

$$\frac{5}{6} = 0.8\overline{3} \text{ तथा } \frac{6}{7} = 0.\overline{857142}, \text{ इत्यादि।}$$

यह सिद्ध* किया जा सकता है कि **प्रत्येक परिमेय संख्या एक दशमलव में व्यक्त की जा सकती है जो या तो सांत अथवा असांत आवर्ती होता है।**

जब कोई परिमेय संख्या दशमलव में व्यक्त की जानी होती है तो हम कैसे बता सकते हैं कि यह दशमलव सांत होगा अथवा असांत आवर्ती होगा। परिमेय संख्याओं $\frac{5}{6}$, $\frac{3}{11}$ तथा $\frac{6}{7}$ के दशमलव निरूपणों का पुनः अध्ययन कीजिए।

यह सिद्ध* किया जा सकता है कि निम्नतम पदों की कोई परिमेय संख्या सांत दशमलव में तभी तथा केवल तभी व्यक्त की जा सकती है यदि उसके हर में 2 तथा 5 के अतिरिक्त कोई अन्य अभाज्य (prime) गुणनखण्ड न हो। इस प्रकार, यदि किसी परिमेय संख्या के हर में 2 तथा 5 के अतिरिक्त कोई अन्य अभाज्य गुणनखण्ड है तो उस संख्या का प्रसार (expansion) असांत आवर्ती होगा।

आपको याद होगा कि एक सांत दशमलव दो पूर्णांकों के भागफल के रूप में लिखा जा सकता है। अतः **प्रत्येक सांत दशमलव एक परिमेय संख्या के रूप में लिखा जा सकता है।**

उदाहरण 1 : 0.0625 को परिमेय संख्या के रूप में लिखिए।

हल : $0.0625 = \frac{625}{10000}$

यदि हम चाहें तो $\frac{625}{10000}$ को निम्नतम पदों में लिख सकते हैं। हम देखते हैं कि

$$\frac{625}{10000} = \frac{1}{16}$$

आइए, अब उदाहरणार्थ, एक **असांत आवर्ती दशमलव** $0.\overline{3}$ लें। **क्या हम एक ऐसी परिमेय संख्या ज्ञात कर सकते हैं जिसे यदि दशमलव में व्यक्त किया जाए तो प्रसार $0.\overline{3}$ हो ?**

आइए, दी गई दशमलव संख्या को, उदाहरणार्थ, x से व्यक्त करें।

तब, $x = 0.3333...$ (1)

हम जानते हैं कि यदि किसी दशमलव को 10 या 10 की किसी घात (power) से गुणा किया जाए तो परिणामस्वरूप दशमलव बिन्दु दाईं ओर उपयुक्त स्थान पर स्थानान्तरित हो जाएगा।

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*उपपत्ति इस पुस्तक की सीमा के बाहर है।

(1) के दोनों पक्षों को 10 से गुणा करने पर हम निम्न प्राप्त करते हैं :

$$10x = 3.3333\ldots \qquad (2)$$

अब यदि (1) को (2) में से घटाएँ तो हम देखते हैं कि दशमलव का आवर्ती भाग समाप्त हो जाता है।

इस प्रकार, $9x = 3$

अतः $x = \frac{1}{3}$, जो अभीष्ट परिमेय संख्या है।

आइए, एक अन्य उदाहरण लें।

उदाहरण 2: एक परिमेय संख्या ज्ञात कीजिए, जिसे यदि दशमलव में व्यक्त किया जाए तो प्रसार $0.23\overline{4}$ प्राप्त हो।

हल : मान लीजिए, $x = 0.23444\ldots \qquad (1)$

x को केवल 10 या 10 की घात से गुणा करके तथा (1) को परिणाम में से घटाने पर हमें कुछ अधिक सहायता नहीं मिलेगी। (क्यों ?) इसलिए हम निम्न प्रकार से आगे बढ़ते हैं :

(1) के दोनों पक्षों को 10^2 से गुणा करके हम निम्न प्राप्त करते हैं :

$$100\,x = 23.444\ldots \qquad (2)$$

हम देखते हैं कि (2) में आवर्ती भाग दशमलव बिन्दु के पश्चात है। अब हम उदाहरण 1 की स्थिति में पहुँच गए हैं।

(2) के दोनों पक्षों को 10 से गुणा करके हम निम्न प्राप्त करते हैं :

$$1000x = 234.444\ldots \qquad (3)$$

(3) में से (2) को घटाने पर,

$$900x = 211$$

अतः $x = \frac{211}{900}$, जो अभीष्ट परिमेय संख्या है।

उदाहरण 3: वह परिमेय संख्या ज्ञात कीजिए जिसे यदि दशमलव में व्यक्त किया जाए तो प्रसार $4.6\overline{732}$ प्राप्त हो।

हल : मान लीजिए $x = 4.6732732\ldots \qquad (1)$

पुनः, सर्वप्रथम हम दशमलव बिन्दु के बाद केवल आवर्ती भाग ही रखना चाहेंगे। आइए (1) के दोनों पक्षों को 10 से गुणा करें। हमें निम्न प्राप्त होता है :

$$10x = 46.732732\ldots \qquad (2)$$

अब हम उदाहरण 1 की स्थिति में आ गए हैं। हम (2) के दोनों पक्षों को 10^3 से गुणा करते हैं और निम्न प्राप्त करते हैं :

$$10^4x = 46732.732732\ldots \qquad (3)$$

(3) में से (2) को घटाने पर,

$$9990x = 46686$$

इस प्रकार, $x = \frac{46686}{9990} = \frac{7781}{1665}$, जो कि वांछित परिमेय संख्या है।

1.6 परिमेय संख्याओं में क्रम-सम्बन्ध (Ordering Among Rationals)

आइए अब परिमेय संख्याओं में क्रम-सम्बन्ध का अध्ययन करें। हम किसी संख्या के किसी अन्य संख्या से छोटी (या बड़ी) होने को दर्शाने के लिए संकेत $<$ (या $>$) प्रयुक्त करते हैं। उदाहरणार्थ, $-2<\frac{4}{3}$, $\frac{3}{2}<5$, $0>-2$, $\frac{23}{41}<\frac{18}{23}$, इत्यादि।

अब यह स्पष्ट है कि यदि a तथा b परिमेय संख्याएँ हैं तो या तो $a=b$ या $a<b$ या $a>b$।

दूसरे शब्दों में, यदि दो संख्याएँ दी हों तो उपरोक्त तीन अवस्थाओं में से एक तथा केवल एक अवस्था ही सम्भव है। स्पष्ट कारणों से हम इसे त्रिविकल्पता अभिगृहीत (trichotomy axiom) कहते हैं।

अब हम $<$ के निम्न गुणधर्मों को दोहराते हैं :

यदि a, b तथा c परिमेय संख्याएँ हैं और

1. $a < b$ तथा $b < c$ तो $a < c$ [$<$ की संक्रामिता (transitivity of $<$)]
2. $a < b$ तो $a + c < b + c$ [योज्य गुणधर्म]
3. $a < b$ तथा $c > 0$ तो $ac < bc$ [गुणन गुणधर्म]

निश्चय ही गुणधर्म 2 तथा 3 बताते हैं कि हम किसी असमिका (inequality) के दोनों पक्षों में बिना उसकी दिशा परिवर्तन किए, कोई परिमेय संख्या जोड़ सकते हैं अथवा दोनों पक्षों को किसी धनात्मक परिमेय संख्या से गुणा कर सकते हैं। हम इन गुणधर्मों का प्रयोग यह सिद्ध करने में करेंगे कि किन्हीं दो भिन्न परिमेय संख्याओं के बीच हम सदैव एक परिमेय संख्या प्राप्त कर सकते हैं।

उपपत्ति : किन्ही दो भिन्न परिमेय संख्याओं पर विचार कीजिए। त्रिविकल्पता अभिगृहीत के अनुसार इनमें से एक दूसरी से छोटी होगी। मान लीजिए हम छोटी संख्या को a तथा बड़ी संख्या को b से व्यक्त करते हैं।

अर्थात्, $a < b$

तब, $a + a < b + a$ (गुणधर्म 2)

या, $a < \frac{a+b}{2}$ (i) (गुणधर्म 3)

पुनः, $a < b$

तब, [illegible]

या, [illegible] (ii)

(i), [illegible]

अब क्योंकि हम जानते हैं कि परिमेय संख्याएँ योग तथा गुणन की संक्रियाओं के लिए संवृत होती हैं, अतः $\frac{a+b}{2}$ एक परिमेय संख्या है।

इस प्रकार यदि कोई दो परिमेय संख्याएँ दी हों तो हम दोनों के बीच सदैव एक अन्य परिमेय संख्या प्राप्त कर सकते हैं। ऐसी एक संख्या दी हुई दोनों संख्याओं का मध्यमान (average) ही है। यदि उपरोक्त विधि का बार-बार प्रयोग किया जाए तो हम सुगमतापूर्वक देख सकते हैं कि किन्हीं दो दी हुई परिमेय संख्याओं के बीच बहुत सी अन्य परिमेय संख्याएँ प्राप्त की जा सकती हैं। अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : $\frac{2}{5}$ तथा $\frac{3}{4}$ के बीच कोई तीन परिमेय संख्याएँ ज्ञात कीजिए।

हल :

ऐसी एक संख्या दी गई दोनों संख्याओं का मध्यमान अर्थात्

$\frac{1}{2}\left(\frac{2}{5}+\frac{3}{4}\right)$ या $\frac{23}{40}$ है।

अब, $\frac{2}{5} < \frac{23}{40} < \frac{3}{4}$

अभी हमें दो परिमेय संख्याएँ और ज्ञात करनी हैं। ऐसी एक संख्या $\frac{2}{5}$ तथा $\frac{23}{40}$ के बीच तथा अन्य $\frac{23}{40}$ तथा $\frac{3}{4}$ के बीच प्राप्त की जा सकती है।

$\frac{2}{5}$ तथा $\frac{23}{40}$ के बीच, उनका मध्यमान अर्थात्

$\frac{1}{2}\left(\frac{2}{5}+\frac{23}{40}\right)$ या $\frac{39}{80}$ एक परिमेय संख्या है।

अर्थात्, $\frac{2}{5} < \frac{39}{80} < \frac{23}{40}$

इसी प्रकार, $\frac{23}{40}$ तथा $\frac{3}{4}$ के बीच $\frac{1}{2}\left(\frac{23}{40}+\frac{3}{4}\right)$ या $\frac{53}{80}$ एक परिमेय संख्या है।

अर्थात्, $\frac{23}{40} < \frac{53}{80} < \frac{3}{4}$

इस प्रकार, हमने $\frac{2}{5}$ तथा $\frac{3}{4}$ के बीच में तीन परिमेय संख्याएँ $\frac{39}{80}, \frac{23}{40}$ तथा $\frac{53}{80}$ ज्ञात कर ली हैं।

***उदाहरण 2**: सिद्ध कीजिए कि यदि a, b, c तीन परिमेय संख्याएँ हैं और $a < b$ तथा < 0, तो $ac > bc$ है।

हल : हमें दिया गया है कि $c < 0$

आइए, दोनों ओर $(-c)$ जोड़ें । इस प्रकार,

$c+(-c) < 0+(-c)$ (गुणधर्म 2)

अर्थात्, $0 < (-c)$

या, $(-c) > 0$

अब क्योंकि, $a < b$ तथा $(-c) > 0$,

अतः, $a(-c) < b(-c)$ (गुणधर्म 3)

अर्थात्, $-ac < -bc$

आइए, दोनों ओर $ac + bc$ जोड़ें । इस प्रकार,

$ac+bc-ac < ac+bc-bc$ (गुणधर्म 2)

अर्थात्, $bc < ac$

या, $ac > bc$, जो इष्ट परिणाम है ।

दूसरे शब्दों में, यदि हम किसी असमिका के दोनों पक्षों को एक ऋणात्मक परिमेय संख्या से गुणा करें तो असमिका की दिशा विपरीत हो जाती है ।

उदाहरण 3 : यदि a और b ऐसी परिमेय संख्याएँ हैं कि $a < b$ है तो

$a < a+\frac{b-a}{n} < b$ होगा,

जहाँ n, 1 से बड़ी कोई प्राकृत संख्या है ।

हल : हम यह सिद्ध करना चाहते हैं कि

$$a < a+\frac{b-a}{n} < b$$

यह सिद्ध करना निम्न सिद्ध करने के समान है :

(i) $a < a+\frac{b-a}{n}$

(ii) $a+\frac{b-a}{n} < b$

हमें दिया हुआ है कि $a < b$

इसलिए, $0 < b-a$ (1)

हम (1) के दोनों पक्षों को n से भाग देते हैं और निम्न प्राप्त करते हैं :

$0 < \frac{b-a}{n}$ (2)

(2) के दोनों पक्षों में 'a' जोड़ने पर हमें (i) में दिया वाँछित परिणाम प्राप्त हो जाता है ।

(ii) में हम दिखाना चाहते हैं कि

$\frac{b-a}{n} < b-a$

या, $n(b-a)-(b-a) > 0$

या, $(n-1)(b-a) > 0$

अब हम (1) के दोनों पक्षों को $(n-1)$ से गुणा करते हैं। [हम देखते हैं कि क्योंकि $n > 1$ है इसलिए $(n-1) > 0$ है।] हमें निम्न प्राप्त होता है :

$$0 \times (n-1) < (b-a)(n-1)$$

अर्थात्, $0 < (b-a)(n-1)$, जो कि निस्सन्देह वाँछित परिणाम है।

हम इस उदाहरण के परिणाम का प्रयोग दो दी हुई परिमेय संख्याओं के बीच में चाहें जितनी परिमेय संख्याएँ ज्ञात करने के लिए कर सकते हैं। यह सम्भवतः औसत की विधि से सरल है। आइए उदाहरण 1 की संख्याएँ लें और इस विधि से उनके बीच में तीन परिमेय संख्याएँ ज्ञात करें। हमें निम्न संख्याएँ प्राप्त होती हैं :

$$\frac{2}{5}+\frac{1}{2}\left(\frac{3}{4}-\frac{2}{5}\right)=\frac{23}{40};\quad \frac{2}{5}+\frac{1}{3}\left(\frac{3}{4}-\frac{2}{5}\right)=\frac{31}{60};\text{ तथा}$$

$$\frac{2}{5}+\frac{1}{4}\left(\frac{3}{4}-\frac{2}{5}\right)=\frac{39}{80}।$$

प्रश्नावली 1.1

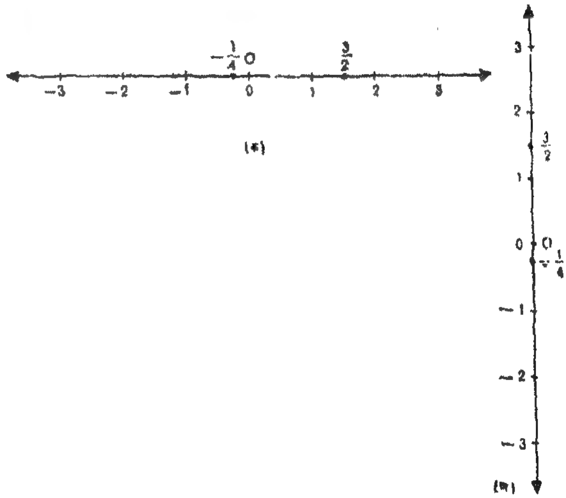
1. सिद्ध कीजिए कि किन्हीं दो परिमेय संख्याओं का अन्तर भी एक परिमेय संख्या होती है।
2. सिद्ध कीजिए कि किन्हीं दो परिमेय संख्याओं का भागफल (हर शून्य के अतिरिक्त) भी एक परिमेय संख्या होती है।
3. निम्न परिमेय संख्याओं को सांत अथवा असांत आवर्ती दशमलव के रूप में लिखिए :

   (i) $\frac{31}{32}$ (ii) $\frac{86}{125}$ (iii) $\frac{7}{13}$ (iv) $\frac{16}{17}$

   [पाठक पहले यह निश्चित कर लें कि उपरोक्त में से कौन सी संख्याओं का प्रसार सांत होगा और कौन सी संख्याओं का असांत आवर्ती।]
4. एक ऐसी परिमेय संख्या ज्ञात कीजिए, जिसे यदि दशमलव में व्यक्त किया जाए तो प्रसार 2.376 प्राप्त हो।
5. एक ऐसी परिमेय संख्या ज्ञात कीजिए, जो यदि दशमलव में व्यक्त की जाए तो प्रसार $22.378\overline{2378}$ प्राप्त हो।
6. $\frac{1}{3}$ तथा $\frac{1}{2}$ के बीच दो परिमेय संख्याएँ ज्ञात कीजिए।
7. $-\frac{5}{6}$ तथा $\frac{3}{8}$ के बीच तीन परिमेय संख्याएँ ज्ञात कीजिए।

1.7 परिमेय संख्याओं का ज्यामितीय निरूपण (Geometrical Representation of Rational Numbers)

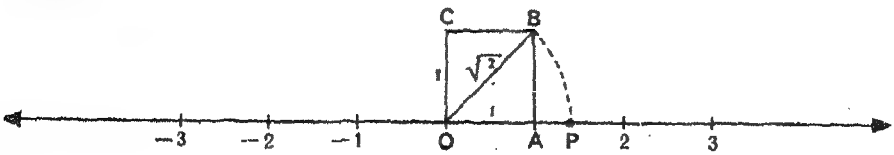
हम यहाँ विस्तार से नहीं बताएँगे कि एक परिमेय संख्या को संख्या रेखा पर कैसे निरूपित किया जाता है। परन्तु हम यह बताना चाहेंगे कि हम एक प्रचलित रीति से संख्या रेखा को क्षैतिज रेखा (horizontal line) के रूप में खींचते आए हैं जैसा कि आकृति 1.1 (क) में दिखाया गया है। परन्तु गणितीय दृष्टि से यह आवश्यक नहीं है। हम संख्या रेखा को ऊर्ध्वाधर अर्थात् खड़ी रेखा (vertical line) के रूप में भी खींच सकते हैं जैसा कि आकृति 1.1 (ख) में दिखाया गया है। कोई स्वेच्छ बिन्दु (arbitrary point), 0 प्रस्थान बिन्दु (starting point) अर्थात् मूलबिन्दु (origin) के रूप में लिया जाता है। एक स्वेच्छ दूरी (arbitrary length) को मात्रक (unit) चुना जाता है और संख्या रेखा पर मूलबिन्दु के ऊपर तथा नीचे चुनी हुई मात्रक के पदों में समान दूरी पर चिन्ह लगाए जाते हैं। इस प्रकार मूलबिन्दु संख्या 0 को निरूपित करता है। धनात्मक संख्याओं को मूलबिन्दु के ऊपर तथा ऋणात्मक संख्याओं को मूलबिन्दु के नीचे निरूपित करने का प्रचलन है।



आकृति 1.1

परिमेय संख्याओं को संख्या रेखा पर निरूपित करने के पश्चात् हम स्वयं से निम्न (स्वाभाविक) प्रश्न पूछते हैं: क्या संख्या रेखा पर ऐसे बिन्दु हैं जो किसी परिमेय संख्या को निरूपित नहीं करते? हम कहते हैं कि वस्तुतः ऐसे बिन्दु हैं। अब हम पहले एक ऐसा ही बिन्दु प्राप्त करने का प्रयत्न करेंगे।

आइए, मात्रक दूरी OA पर एक वर्ग OABC की रचना करें जैसा कि आकृति 1.2 में दिखाया गया है। OB इस वर्ग का एक विकर्ण है।



आकृति 1.2

पाइथैगोरस प्रमेय द्वारा,

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{2}$$

O को केन्द्र तथा OB को त्रिज्या (radius) लेकर हम एक चाप खींचते हैं जो संख्या रेखा को बिन्दु P पर काटता है जैसा कि आकृति में दिखाया गया है। स्पष्ट रूप से $OP = OB = \sqrt{2}$। इस प्रकार बिन्दु P संख्या $\sqrt{2}$ को निरूपित करता है। अब हम सिद्ध करेंगे कि $\sqrt{2}$ परिमेय संख्या नहीं है।

उपपत्ति : उपपत्ति की जो विधि हम अपनाएँगे उसे विरोध (contradiction) की विधि कहा जाता है। अर्थात्, पहले हम यह कल्पना कर लेते हैं कि $\sqrt{2}$ एक परिमेय संख्या है और फिर हम अपनी कल्पना के विरोध पर पहुँचते हैं। इस विधि को असंगति प्रदर्शन (reductio ad absurdum) की विधि भी कहते हैं।

हम मान लेते हैं कि $\sqrt{2}$ एक परिमेय संख्या है। अतः $\sqrt{2}$ को $\frac{p}{q}$ के रूप में व्यक्त करना सम्भव है जहाँ p तथा q पूर्णांक हैं और $q \neq 0$ है। आइए, यह भी मान लें कि p तथा q निम्नतम पदों में हैं।

अब, $\sqrt{2} = \frac{p}{q}$

इस प्रकार, $2 = \frac{p^2}{q^2}$

अथवा, $p^2 = 2q^2$ (1)

अतः p^2 एक सम पूर्णांक है। (क्यों ?)

इसलिए* p भी एक सम पूर्णांक है।

अतः हम p को निम्न रूप से लिख सकते हैं :

$p = 2r$, जहाँ r एक पूर्णांक है।

(1) में प्रतिस्थापन (substitution) करके हम निम्न प्राप्त करते हैं :

$4r^2 = 2q^2$

या, $q^2 = 2r^2$

पुनः उपरोक्त तर्क से ही हम यह निष्कर्ष निकाल सकते हैं कि q एक सम पूर्णांक है। इस प्रकार p तथा q दोनों ही सम पूर्णांक हैं। और इसलिए इनमें एक उभयनिष्ठ गुणनखंड 2 होना चाहिए। परन्तु यह हमारी इस कल्पना, कि p तथा q निम्नतम पदों में हैं, का विरोध करता है। अतः $\sqrt{2}$ परिमेय संख्या नहीं है।

इस प्रकार हमने संख्या रेखा पर एक बिन्दु प्राप्त कर लिया है जो किसी परिमेय संख्या को निरूपित नहीं करता।

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* "यदि p^2 एक सम पूर्णांक है तो p भी सम पूर्णांक है" स्वयं इसे भी सिद्ध करने की आवश्यकता है। निश्चय ही इस कथन की अपेक्षा इसकी प्रतिस्थिति (contrapositive) सिद्ध करना अधिक आसान है अर्थात् "यदि p सम पूर्णांक नहीं है तो p^2 भी सम पूर्णांक नहीं है"।

यदि p सम पूर्णांक नहीं है तो यह $p = 2k+1$ के रूप की संख्या है जहाँ k एक पूर्णांक है।

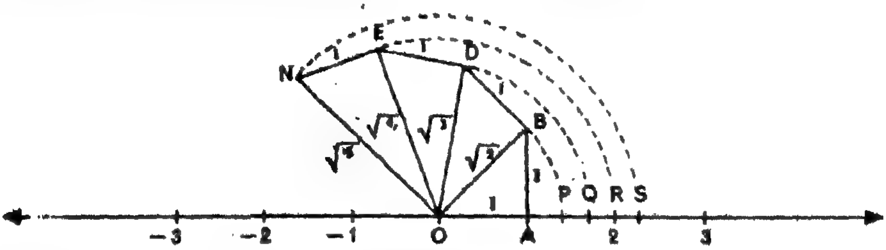
इस प्रकार, $p^2 = (2k+1)^2 = 2(2k^2+2k)+1$

अब, $2k^2+2k$ एक पूर्णांक है। (क्यों ?)

अतः p^2, उदाहरणार्थ, $p^2 = 2m+1$ के रूप की संख्या है जहाँ m एक पूर्णांक है। इससे स्पष्ट है कि p^2 सम पूर्णांक नहीं है।

यह तथ्य कि संख्या रेखा पर ऐसे अन्य बिन्दु भी हैं जो किसी परिमेय संख्या को निरूपित नहीं करते, निम्न विधि से दिखाया जा सकता है :

वर्ग OABC के विकर्ण OB पर हम एक समकोण त्रिभुज OBD की रचना करते हैं जिसका समकोण B पर है और BD=OA (मात्रक दूरी) है जैसा कि आकृति 1.3 में दिखाया गया है।



आकृति 1.3

पुनः, पाइथेगोरस प्रमेय से,

$$OD = \sqrt{OB^2 + BD^2} = \sqrt{3}$$

O को केन्द्र और OD को त्रिज्या लेकर हम एक चाप खींचते हैं जो संख्या रेखा को बिन्दु Q पर काटता है जैसा कि आकृति में दिखाया गया है। स्पष्ट रूप से, $OQ = OD = \sqrt{3}$। इस प्रकार, बिन्दु Q संख्या $\sqrt{3}$ को निरूपित करता है। अब यह सिद्ध करना कि $\sqrt{3}$ परिमेय संख्या नहीं है पाठक के अभ्यासार्थ छोड़ दिया गया है।

यदि हम इस विधि की पुनरावृत्ति करें तो हम पायेंगे कि संख्या रेखा पर ऐसे और भी बिन्दु हैं जो किसी परिमेय संख्या को निरूपित नहीं करते। [निस्संदेह, यह स्पष्ट है कि हम कुछ ऐसे बिन्दु भी प्राप्त करते हैं जो $\sqrt{4}$ जैसी परिमेय संख्याओं को निरूपित करते हैं।]

1.8 अपरिमेय संख्याएँ (Irrational Numbers)

हमने पिछले अनुच्छेद में देखा कि संख्या रेखा पर ऐसे बिन्दु हैं जो किसी परिमेय संख्या को निरूपित नहीं करते। इससे संख्या-निकाय के विस्तार की आवश्यकता है जिसमें ऐसी संख्याएँ भी सम्मिलित हों जो परिमेय नहीं हैं जैसे कि $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $-\sqrt{6}$, $\sqrt{7}$, इत्यादि। ऐसी संख्याएं अपरिमेय संख्याएं कहलाती हैं। इस प्रकार, अपरिमेय संख्या वह संख्या होती है जो $\frac{p}{q}$ के रूप में व्यक्त नहीं की जा सकती, जहाँ p तथा q दोनों पूर्णांक हों और $q \neq 0$ हो।

अपरिमेय संख्या का एक अन्य उदाहरण संख्या* π है जो एक वृत्त की परिधि का उसके व्यास से अनुपात (ratio) होता है। यह अनुपात एक स्थिरांक (constant) होता है और वृत्त की त्रिज्या पर निर्भर नहीं

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* π को अपरिमेय संख्या सिद्ध करना इस पुस्तक की सीमा के बाहर है।

होता। निश्चित रूप से बेबीलोन तथा मिस्र के लोग यह जानते थे कि एक वृत्त की परिधि का उसके व्यास से अनुपात एक स्थिरांक है, यद्यपि इस अनुपात को व्यक्त करने के लिए उन्होंने संकेत π का प्रयोग नहीं किया। लगभग 2000 ई० पू० बेबीलोन के लोग π के मान के लिए $3\frac{1}{8}$ और मिस्रवासी $4\left(\frac{64}{81}\right)$ का प्रयोग करते थे। **आर्यभट्ट** (जन्म 476 ई०) ने 499 ई० में अपनी प्रसिद्ध कृति **'आर्यभट्टीयम्'** में संकेत किया है कि π का मान 3.1416 होता है। आपको याद होगा कि हम π का मान $\frac{22}{7}$ प्रयोग करते आए हैं। परन्तु इसका यह अर्थ नहीं है कि π एक परिमेय संख्या है। $\frac{22}{7}$ केवल π के मान के सन्निकट एक संख्या है।

यह जानना अत्यन्त रुचिकर है कि तीव्रगति के इलैक्ट्रॉनिक कम्प्यूटरों के आगमन से, फरवरी 1967 ई० में पैरिस के परमाणु ऊर्जा आयोग में काम करने वाले दो महानुभावों ने 5 लाख दशमलव स्थानों तक π का मान निकाला था।

क्या दो अपरिमेय संख्याओं का योग भी अपरिमेय है? स्पष्ट है, सदा ऐसा नहीं होता। उदाहरणार्थ, दो अपरिमेय संख्याएँ $\sqrt{2}$ तथा $-\sqrt{2}$ लीजिए। इन संख्याओं का योग 0 है। इस प्रकार **अपरिमेय संख्याएँ योग की संक्रिया के लिए संवृत नहीं हैं।**

[पाठक यह सिद्ध करे कि **अपरिमेय संख्याएँ व्यवकलन, गुणन तथा विभाजन की संक्रियाओं के लिए संवृत नहीं हैं।**]

उदाहरण 1: सिद्ध कीजिए कि $3-\sqrt{2}$ अपरिमेय है।

हल: हम इसे असंगति प्रदर्शन की विधि से हल कर सकते हैं। हम मान लेते हैं कि $3-\sqrt{2}$ एक परिमेय संख्या है। हम इसे, उदाहरणार्थ, r से व्यक्त करते हैं।

तब, $$r = 3-\sqrt{2}$$
अर्थात्, $$\sqrt{2} = 3 - r$$

परन्तु $3 - r$ परिमेय संख्या है। (क्यों?)
जिससे यह निष्कर्ष निकलता है कि $\sqrt{2}$ भी परिमेय संख्या है। परन्तु, निश्चय ही यह असत्य है।
इस प्रकार हमें अपनी ही कल्पना का विरोध प्राप्त हो गया।
अत: $3-\sqrt{2}$ अपरिमेय है।

उदाहरण 2: सिद्ध कीजिए $\sqrt{2}+\sqrt{3}$ अपरिमेय है।

हल: हम कल्पना करते हैं कि $\sqrt{2}+\sqrt{3}$ परिमेय संख्या है। हम इसे, उदाहरणार्थ, r से व्यक्त करते हैं।

तब, $$r = \sqrt{2}+\sqrt{3}$$
अत: $$r^2 = 5 + 2\sqrt{6}$$
अर्थात्, $$\sqrt{6} = \frac{r^2-5}{2}$$

परन्तु $\frac{r^2 - 5}{2}$ परिमेय है। (क्यों?) जिससे यह निष्कर्ष निकलता है कि $\sqrt{6}$ भी परिमेय है।

वस्तुत: यह असत्य है।

[पाठक यह सिद्ध करें कि $\sqrt{6}$ अपरिमेय है।]

हम पुन: विरोध की स्थिति में पहुँच गए हैं।

अत: $\sqrt{2} + \sqrt{3}$ अपरिमेय है।

1.9 दशमलवों से अपरिमेय संख्याओं का सन्निकटन (Approximation of Irrational Numbers by Decimals)

अपरिमेय संख्या $\sqrt{2}$ पर विचार कीजिए। हम जानते हैं कि $1^2 = 1$ तथा $2^2 = 4$ है क्योंकि दी हुई अपरिमेय संख्या का वर्ग 2 है, अत: यह स्पष्ट है कि

$$1^2 < (\sqrt{2})^2 < 2^2$$

अर्थात्, $1 < \sqrt{2} < 2$

इस प्रकार $\sqrt{2}$, 1 तथा 2 के बीच की एक संख्या है। परन्तु यह संख्या $\sqrt{2}$ का एक अपरिष्कृत (crude) सन्निकटन है।

आइए, इससे उत्तम सन्निकटन प्राप्त करने का प्रयत्न करें। हम देखते हैं कि

$$(1.1)^2 = 1.21$$
$$(1.2)^2 = 1.44$$
$$(1.3)^2 = 1.69$$
$$(1.4)^2 = 1.96$$
$$(1.5)^2 = 2.25$$

इस प्रकार, $(1.4)^2 < (\sqrt{2})^2 < (1.5)^2$

अर्थात्, $1.4 < \sqrt{2} < 1.5$

हम पुन: देखते हैं कि

$$(1.41)^2 = 1.9881$$
$$(1.42)^2 = 2.0164$$

इस प्रकार, $(1.41)^2 < (\sqrt{2})^2 < (1.42)^2$

अर्थात् : $1.41 < \sqrt{2} < 1.42$

इसी विधि का प्रयोग करते हुए हम संख्या $\sqrt{2}$ का अधिक उत्तम सन्निकटन प्राप्त कर सकते हैं।

इस प्रकार,

$$1.414 < \sqrt{2} < 1.415$$
$$1.4142 < \sqrt{2} < 1.4143$$
$$1.41421 < \sqrt{2} < 1.41422$$
$$1.414213 < \sqrt{2} < 1.414214$$
$$1.4142135 < \sqrt{2} < 1.4142136, \text{ इत्यादि।}$$

हम देखते हैं कि हम संख्या $\sqrt{2}$ के उत्तरोत्तर सन्निकटनों के दशमलवों में कोई आवर्ती भाग प्राप्त नहीं करते हैं। वास्तव में कुछ लेखक, परिमेय संख्याओं को असांत* आवर्ती दशमलव के रूप में तथा अपरिमेय संख्याओं को असांत अनावर्ती (non-recurring) दशमलव के रूप में परिभाषित करते हैं।

1.10 वास्तविक संख्याएँ (Real Numbers)

परिमेय तथा अपरिमेय संख्याएँ मिलकर वास्तविक संख्याएँ कहलाती हैं। वास्तविक संख्याओं में योग, व्यवकलन, गुणन तथा विभाजन की संक्रियाएँ संभव हैं। इन संक्रियाओं का गहन विवरण देना इस पुस्तक की सीमा के बाहर है। आपको याद होगा कि वास्तविक संख्याओं में योग तथा गुणन की संक्रियाओं के वे सभी गुणधर्म (1-9) होते हैं जो कि अनुच्छेद 1.4 में दिए जा चुके हैं। इसके अतिरिक्त वास्तविक संख्याओं में भी क्रम-सम्बन्ध** होता है। त्रिविकल्पता अभिगृहीत भी इन संख्याओं में सत्य होता है। इनके अतिरिक्त, यदि a, b, c कोई तीन वास्तविक संख्याएँ हैं और

1. $a < b$ तथा $b < c$ तो $a < c$ ($<$ की संक्रामिता)
2. $a < b$ तो $a+c < b+c$ (योज्य गुणधर्म)
3. $a < b$ तथा $c>0$ तो $ac < bc$ (गुणन गुणधर्म)

निश्चित रूप से गुणधर्म 2 तथा 3 हमें बताते हैं कि हम किसी असमिका की दिशा में परिवर्तन किए बिना उसके दोनों पक्षों में कोई भी वास्तविक संख्या जोड़ सकते हैं अथवा दोनों पक्षों को किसी भी धनात्मक वास्तविक संख्या से गुणा कर सकते हैं।

यह सिद्ध करना कि "जब हम किसी असमिका के दोनों पक्षों को किसी ऋणात्मक वास्तविक संख्या से गुणा करते हैं तो असमिका की दिशा विपरीत हो जाती है" पाठक के अभ्यासार्थ छोड़ दिया गया है।

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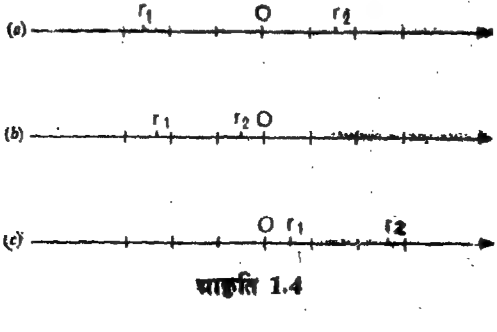
* एक सांत दशमलव को दशमलव के अन्त में शून्य लगाकर असांत आवर्ती दशमलव के रूप में भी लिखा जा सकता है। उदाहरणार्थ, $0.25=0.2500\ldots$

** हम यह दिखाएँगे कि हम दो वास्तविक संख्याओं को संख्या रेखा पर निरूपित करके उनकी तुलना कैसे करते हैं। मान लीजिए r_1 तथा r_2 दो वास्तविक संख्याएँ हैं। यदि संख्या r_2 को निरूपित करने वाला बिन्दु संख्या r_1 को निरूपित करने वाले बिन्दु के दाईं ओर है तो हम कहते हैं कि $r_1 < r_2$। (देखिए आकृति 1.4)



आकृति 1.4

संख्या रेखा का प्रत्येक बिन्दु एक वास्तविक संख्या को निरूपित करता है और विलोमतः प्रत्येक वास्तविक संख्या को संख्या रेखा के किसी एक बिन्दु से निरूपित किया जा सकता है। साथ ही, किसी परिमेय संख्या को असांत आवर्ती दशमलव के रूप में तथा किसी अपरिमेय संख्या को असांत अनावर्ती दशमलव के रूप में व्यक्त किया जा सकता है। क्योंकि परिमेय तथा अपरिमेय संख्याएँ मिलकर वास्तविक संख्याएँ कहलाती हैं, अतः निष्कर्ष निकलता है कि **कोई भी वास्तविक संख्या एक असांत दशमलव के रूप में व्यक्त की जा सकती है।**

वास्तविक संख्याओं का एक अति महत्वपूर्ण गुणधर्म है "पूर्णता" (completeness) जिसका वर्णन हम यहाँ नहीं करेंगे।

प्रश्नावली 1.2

1. संख्या रेखा पर निम्न संख्याओं का निरूपण करने वाले बिन्दुओं को अंकित कीजिए :
   $-4, 3.5, -2.7, 1.9, \frac{4}{9}, 4\frac{1}{8}$
2. संख्या रेखा पर एक मात्रक दूरी लीजिए तथा संख्याओं $2\sqrt{2}$ तथा $-2\sqrt{2}$ का निरूपण करने वाले बिन्दुओं को अंकित कीजिए।
3. क्या $\frac{\sqrt{20}}{\sqrt{45}}$ परिमेय संख्या है ? क्या $\left(\sqrt{7}+\frac{1}{\sqrt{7}}\right)^2$ परिमेय संख्या है ?
4. क्या 1.01001000100001... अपरिमेय है ? क्यों ?
5. यदि p^2, 3 का पूर्णांक गुणज (integer multiple) हो तो सिद्ध कीजिए कि p भी 3 का पूर्णांक गुणज होगा।
   [संकेत : यदि p, 3 का पूर्णांक गुणज नहीं है तो यह $3k+1$ या $3k+2$ के रूप की संख्या है जहाँ k एक पूर्णांक है।]
6. सिद्ध कीजिए कि $\sqrt{3}$ एक अपरिमेय संख्या है।
7. सिद्ध कीजिए कि $\sqrt{6}$ एक अपरिमेय संख्या है।
8. कोई दो ऐसी अपरिमेय संख्याएँ बताइए जिनका अन्तर एक परिमेय संख्या है।
9. कोई दो ऐसी अपरिमेय संख्याएँ बताइए जिनका अन्तर एक अपरिमेय संख्या है।
10. कोई दो ऐसी अपरिमेय संख्याएँ बताइए जिनका गुणनफल एक परिमेय संख्या है।
11. कोई दो ऐसी अपरिमेय संख्याएँ बताइए जिनका गुणनफल एक अपरिमेय संख्या है।
12. सिद्ध कीजिए कि संख्याएँ $5\sqrt{3}$ तथा $5+\sqrt{3}$ अपरिमेय संख्याएँ हैं।
13. सिद्ध कीजिए कि $\sqrt{3}-\sqrt{2}$ एक अपरिमेय संख्या है।

14. सिद्ध कीजिए कि द्विघात समीकरण $x^2 - x - 3 = 0$ के दोनों मूल (roots) अपरिमेय हैं।

15. दो संख्याओं 6.556 तथा 6.557 में से कौन सी संख्या $\sqrt{43}$ का उत्तम (निकटतम) सन्निकटन है ?

16. ऐसी दो संख्याएँ a तथा b निर्धारित कीजिए कि $a < \sqrt{10} < b$ तथा $b - a < 0.1$ ।

17. सिद्ध कीजिए कि किन्हीं दो वास्तविक संख्याओं के बीच हम सदैव एक अन्य वास्तविक संख्या प्राप्त कर सकते हैं।

*18. यदि a, b तथा c वास्तविक संख्याएँ हों और $a < b$ तथा $c < 0$, तो सिद्ध कीजिए कि $ac > bc$ ।

1.11 मुख्य संकल्पनाएँ (Key Concepts)

प्राकृत संख्याएँ	विरोध (असंगति प्रदर्शन) द्वारा उपपत्ति
पूर्णांक	प्रतिस्थिति
परिमेय संख्याएँ	सांत दशमलव
अपरिमेय संख्याएँ	असांत आवर्ती दशमलव
वास्तविक संख्याएँ	असांत अनावर्ती दशमलव
संवृत गुणधर्म (closure property)	त्रिविकल्पता अभिगृहीत
वास्तविक संख्याओं में क्रम-सम्बन्ध	

1.12 अग्रिम अध्ययन हेतु सुझाव (Suggestions for Further Reading)

परिमेय तथा अपरिमेय संख्याओं के उत्तम वर्णन के लिए पाठक निम्न पुस्तक का अध्ययन करें :

[1] I. Niven : **Numbers : Rational and Irrational**, The L. W. Singer Company, New York (U.S.A.), 1961.

कुछ उच्चस्तरीय अध्ययन के लिए निम्न पुस्तक है :

[2] R. Courant and H. Robbins : **What is Mathematics ?** Oxford University Press, New York (U.S.A.), 1963.

एक अन्य पुस्तक, जिसमें केवल परिमेय संख्याओं का वर्णन है, निम्न है :

[3] **The Rational Numbers (Booklet 6)**, National Council of Teachers of Mathematics, Washington D. C. (U.S.A.), 1964.

हिन्दू गणितज्ञों के योगदान के अध्ययन हेतु निम्न पुस्तकें हैं :

[4] C. N. Srinivasiengar : **The History of Ancient Indian Mathematics**, The World University Press Pvt. Ltd., Calcutta (India), 1967.

[5] B. B. Datta and A. N. Singh : **History of Hindu Mathematics**, Vols. I and II, Asia Publishing House, Bombay (India), 1962.

संस्कृति के विकास के साथ-साथ संख्या की संकल्पना के विकास के सुन्दर विवरण के लिए निम्न पुस्तक है :

[6] R. L. Wilder : **Evolution of Mathematical Concepts : An Elementary Study**, John Wiley and Sons Inc., New York (U.S.A.), 1968.

गणित के इतिहास की एक महत्वपूर्ण पुस्तक निम्न है :

[7] D. E. Smith : **History of Mathematics**, Vols. I and II, Ginn and Company, Boston (U.S.A.), 1923 and 1925.

एकक II

सम्मिश्र संख्याएँ

वास्तविक संख्याओं के निकाय का सम्मिश्र संख्याओं में विस्तार किया गया है और सम्मिश्र संख्याओं के बीजगणित का विकास किया गया है।

2.1 भूमिका

एकक I में हमने प्राकृत संख्याओं का वास्तविक संख्याओं में विस्तार किया था। वैकल्पिक रूप से सभी समीकरणों को हल करने की आकांक्षा का प्रयोग विभिन्न संख्या निकायों के विकास के अभिप्रेरण के लिए भी किया जा सकता है। उदाहरणार्थ, समीकरण $x+2=2$ या व्यापक रूप से समीकरण $x+b=b$ का हल प्राकृत संख्याओं के समुच्चय में प्राप्त करने पर विचार कीजिए। स्पष्ट रूप से, ऐसी कोई प्राकृत संख्या x नहीं है जिसे 2 में जोड़ने से हमें संख्या 2 प्राप्त हो। अत: हमें प्राकृत संख्या-निकाय का पूर्ण संख्याओं (whole numbers) में विस्तार करने की आवश्यकता है।

अब समीकरण $x+11=7$ अथवा व्यापक रूप से समीकरण $x+c=d$ $(c>d)$ के पूर्ण संख्याओं में हल करने के बारे में आप क्या सोचते हैं? पुन: पूर्ण संख्याओं में इस का हल प्राप्त नहीं हो सकता। अत: हमें पूर्ण संख्याओं के निकाय का पूर्णांकों में विस्तार करने की आवश्यकता है।

अब, उदाहरणार्थ, $5x=3$ जैसे समीकरणों के हल की आकांक्षा से संख्या-निकाय को परिमेय संख्याओं में विस्तार करने की अभिप्रेरणा प्राप्त होती है।

पुन: यदि हम वास्तविक संख्याओं को भी सम्मिलित कर लें तो एक द्विघात समीकरण, उदाहरणार्थ, $x^2=3$ का हल भी सम्भव है।

अन्त में, आपको याद होगा कि वास्तविक संख्याएँ भी कुछ द्विघात समीकरणों, उदाहरणार्थ, $x^2=-4$ अर्थात् ऐसे द्विघात समीकरण जिनमें विविक्तकर (discriminant) ऋणात्मक है, के हल प्राप्त करने में अपर्याप्त हैं।

यद्यपि यूनान के लोगों ने यह जान लिया था कि वास्तविक संख्याओं में ऋणात्मक संख्याओं का वर्गमूल नहीं होता, तथापि जैन गणितज्ञ महावीर ने सर्वप्रथम स्पष्ट रूप से यह कठिनता व्यक्त की थी। 850 ई० में

लिखी गई अपनी कृति गणित सार संग्रह में उन्होंने कहा है कि "जैसा कि स्वाभाविक है कि कोई ऋणात्मक राशि एक वर्ग नहीं होती अतः ऐसी राशि का वर्गमूल भी नहीं हो सकता"। एक अन्य हिन्दू गणितज्ञ भास्कर ने भी 1150 ई० में रचित अपनी कृति बीजगणित में लिखा है कि "किसी ऋणात्मक राशि का वर्गमूल नहीं होता क्योंकि यह एक वर्ग नहीं होती"।

इस प्रकार हमें वास्तविक संख्याओं के निकाय का एक ऐसे निकाय में विस्तार करने की आवश्यकता है जिसमें ऋणात्मक संख्याओं के भी वर्गमूल सम्मिलित हों तथा जिसमें सभी सामान्य संक्रियाएँ तथा उनके गुणधर्म सत्य हों।

2.2 वास्तविक संख्याओं से सम्मिश्र संख्याओं की ओर

हमने देखा है कि ऋणात्मक संख्याओं के वास्तविक वर्गमूल नहीं होते। अतः हम वास्तविक संख्याओं के निकाय का एक ऐसे निकाय में विस्तार करना चाहते हैं जिसमें हम ऋणात्मक संख्याओं के वर्गमूलों को भी ज्ञात कर सकें।

विशिष्ट रूप से संख्या -1 का कोई वर्गमूल नहीं है। हम $\sqrt{-1}$ को एक नई संख्या के संकेत के रूप में सम्मिलित करते हैं जिसे हम i से भी व्यक्त करेंगे।

अब एक ऐसे संख्या-निकाय पर विचार कीजिए जिसमें वास्तविक संख्याएँ तथा संख्या i सम्मिलित हों। आइए, इस निकाय में संख्या i का स्वयं से गुणन निम्न रूप से परिभाषित करें :

$$\sqrt{-1}.\sqrt{-1} = -1$$

या, $\quad i \,.\, i = -1$

या, $\quad i^2 = -1$

अब, संख्या -1 का वर्गमूल i है। (हम $\sqrt{-1}$ को -1 का वर्गमूल पढ़ते हैं।) संख्या $\sqrt{-1}$ के लिए यह संकेत i सर्वप्रथम स्विट्ज़रलैंड के गणितज्ञ लियोनार्ड ऑयलर (1707-1783) ने 1748 ई० में दिया था।

अन्त में, हम इस नये निकाय का विस्तार इस प्रकार करते हैं कि इसमें अन्य दो प्रकार की संख्याएँ सम्मिलित हों अर्थात् एक yi जो संख्याओं y तथा i का गुणनफल है तथा दूसरी $(x+yi)$ जो दो संख्याओं x तथा yi का योग है, जहाँ x तथा y वास्तविक संख्याएँ हैं। इस संख्या-निकाय को हम C से व्यक्त करते हैं। दूसरे शब्दों में, C में सभी संख्याएँ $(x+yi)$ सम्मिलित हैं, जहाँ x तथा y वास्तविक संख्याएँ हैं। संख्या $(x+yi)$ सम्मिश्र संख्या (complex number) कहलाती है। संख्याएँ $3+5i$, $\sqrt{2}-7i$ तथा $-\frac{1}{2}+i$ सभी सम्मिश्र संख्याओं के उदाहरण हैं। हम संख्या $(x+i0)$ को वास्तविक संख्या x तथा संख्या $(0+yi)$ को संख्या yi ही समझते हैं। संख्या x सम्मिश्र संख्या $(x+yi)$ का वास्तविक भाग (real part) तथा संख्या y काल्पनिक भाग (imaginary part) कहलाता है। उदाहरणार्थ, संख्या 3 सम्मिश्र संख्या $3+5i$ का वास्तविक भाग है तथा संख्या 5 काल्पनिक भाग है। इसी प्रकार संख्या $-\frac{1}{2}$ सम्मिश्र संख्या $-\frac{1}{2}+i$ का वास्तविक भाग है तथा संख्या 1 काल्पनिक भाग है, इत्यादि। पुनः यदि $y \neq 0$ हो, तो

संख्या $(x+yi)$ एक **अवास्तविक सम्मिश्र संख्या (non-real complex number)** कहलाती है* । संख्याएँ $1-2i$, $4i$ तथा $\frac{3}{2}+\frac{1}{2}i$ अवास्तविक सम्मिश्र संख्याओं के उदाहरण हैं । यदि $x=0$ तथा $y \neq 0$ हो तो संख्या $(x+yi)$ **विशुद्ध काल्पनिक संख्या (pure imaginary number)** कहलाती है । संख्याएँ $4i$, $0-5i$ तथा $\frac{3}{7}i$, इत्यादि विशुद्ध काल्पनिक संख्याओं के उदाहरण हैं । हम देखते हैं कि अवास्तविक सम्मिश्र संख्याओं में विशुद्ध काल्पनिक संख्याएँ भी सम्मिलित हैं । पुनः **सम्मिश्र संख्याओं के निकाय में वास्तविक संख्याओं का निकाय भी सम्मिलित है ।**

1637 ई० में अपनी सुप्रसिद्ध कृति **"ला ज्योमैटरि" (ज्यामिती)** में "real (वास्तविक)" तथा "imaginary (काल्पनिक)" नाम देने का श्रेय फ्रांसीसी गणितज्ञ **रेने-दकार्त (1596-1650)** को है । 17 वीं तथा 18वीं शताब्दी के अधिकतर गणितज्ञ संख्या $a+b\sqrt{-1}$ को एक काल्पनिक राशि कहते रहे । प्रतिभाशाली जर्मन गणितज्ञ **कार्ल फ्रेडरिक गौस** (1777-1855) ने 1832 ई० में संख्याओं $b\sqrt{-1}$ तथा $a+b\sqrt{-1}$ को भिन्न नाम दिए । उन्होंने संख्या $a+b\sqrt{-1}$ को "complex number **(सम्मिश्र संख्या)**" कहा तथा यही नाम तब से प्रयुक्त होता आया है ।

17 वीं शताब्दी के गणितज्ञ संख्याओं, उदाहरणार्थ, i, $-3i$, $2+5i$, इत्यादि को कोई व्यावहारिक अर्थ नहीं दे सके । इन संख्याओं से वे अशान्त हुए तथा इन्हें "अविश्वसनीय" समझा गया । सम्भवतः इन्हीं कारणों से इन संख्याओं को काल्पनिक कहा गया और दुर्भाग्यवश यही मिथ्या नाम आज तक चलता आया है ।

हम कहते हैं कि **दो सम्मिश्र संख्याएँ तभी तथा केवल तभी समान होती हैं यदि उनके वास्तविक भाग तथा काल्पनिक भाग परस्पर समान हों ।** दूसरे शब्दों में, **यदि a+bi तथा c+di सम्मिश्र संख्याएँ हैं तो a+bi=c+di तभी तथा केवल तभी होगा जबकि a=c तथा b=d हो ।**

आइए कुछ उदाहरण लें ।

उदाहरण 1: यदि $a+bi=2-3i$ तो

$a=2$ तथा $b=-3$ होगा ।

उदाहरण 2: यदि संख्याएँ $2x+3yi$ तथा $2+9i$ एक ही सम्मिश्र संख्या निरूपित करती हों तो x तथा y ज्ञात कीजिए ।

हल: हम x तथा y ज्ञात करना चाहते हैं ताकि

$2x+3yi=2+9i$ हो ।

दो सम्मिश्र संख्याओं की समता की परिभाषा से हम निम्न प्राप्त करते हैं :

$2x=2$, अतः $x=1$

तथा $3y=9$, अतः $y=3$

---

* यद्यपि कुछ लेखक ऐसी संख्या को **काल्पनिक संख्या** कहते हैं परन्तु हम इसी शब्द के प्रयोग को प्राथमिकता देंगे ।

प्रश्नावली 2.1

निम्नलिखित सम्मिश्र संख्याओं के वास्तविक तथा काल्पनिक भाग लिखिए :

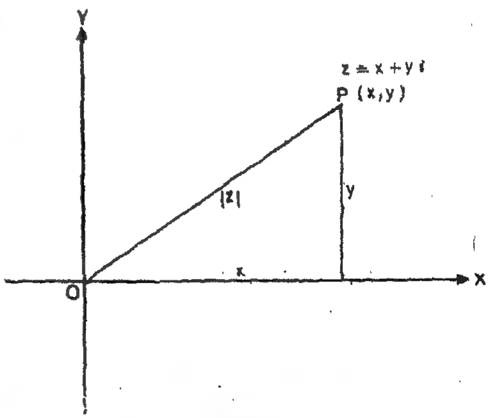
1. $7 + 3i$ 2. $\sqrt{3} + \frac{\sqrt{2}}{7}\, i$
3. $(-5)i + 3$ 4. $0 + 7i$
5. $\sqrt{5} + 0i$ 6. $2 + \sqrt{2}\, i$

x तथा y का मान ज्ञात कीजिए, यदि

7. $3x + (2x - y)i = 6 - 3i$
8. $(3y - 2) + (5 - 4x)i = 0$
9. $3x + 5yi = 5i$
10. $\left(\frac{3}{\sqrt{2}}x - 2\right) + 5\sqrt{2}\, yi = \sqrt{2}$
11. $\left(\frac{1}{2}x - \frac{1}{3}y\right) + \frac{2}{3}yi = -2 + 4i$

2.3 सम्मिश्र संख्याओं का ज्यामितीय निरूपण (Geometrical Representation of Complex Numbers)

किसी सम्मिश्र संख्या $x + yi$ का ज्यामितीय निरूपण करने के लिए हम XOY तल (plane) में एक समकोणीय अक्ष पद्धति (rectangular system of axes) OX तथा OY लेते हैं। तल का कोई बिन्दु अद्वितीय रूप से (uniquely) निर्धारित किया जा सकता है यदि हमें उस बिन्दु का x-निर्देशांक (x-coordinate) अर्थात् भुज (abscissa), उदाहरणार्थ, x तथा y-निर्देशांक (y-coordinate) अर्थात् कोटि (ordinate), उदाहरणार्थ, y ज्ञात हो। दूसरे शब्दों में, यदि हमें वास्तविक संख्याओं का क्रमित-युग्म (ordered pair) (x, y) ज्ञात है तो बिन्दु अद्वितीय रूप से निर्धारित किया जा सकता है। विलोमतः तल के प्रत्येक बिन्दु P के तदनुरूपी वास्तविक संख्याओं का एक अद्वितीय क्रमित युग्म (x, y) प्राप्त किया जा सकता है।



आकृति 2.1

इस प्रकार हम देखते हैं कि वास्तविक संख्याओं के एक क्रमित-युग्म (x, y) द्वारा सम्मिश्र संख्या $(x+yi)$ अद्वितीय रूप से निर्धारित की जा सकती है। अतः

प्रत्येक सम्मिश्र संख्या $(x + yi)$ तल XOY के एक अद्वितीय बिन्दु P (x, y) से निरूपित की जा सकती है। (देखिए आकृति 2.1) स्पष्ट रूप से, संख्याओं $(x + 0i)$ के तदनुरूपी बिन्दु x-अक्ष पर तथा संख्याओं $(0 + yi)$ के तदनुरूपी बिन्दु y-अक्ष पर होंगे। जब हम सम्मिश्र संख्याओं का निरूपण एक तल में करते हैं तो हम इस तल को कभी-कभी सम्मिश्र तल (complex plane) कहते हैं।

प्रायः एक सम्मिश्र संख्या को z से व्यक्त किया जाता है। इसे हम $z = x + yi$ लिखते हैं। दूरी OP को सम्मिश्र संख्या $(x+yi)$ का मापांक (modulus) कहा जाता है जिसे प्रायः $|z|$ से व्यक्त किया जाता है। स्पष्ट रूप से,

$$|z| = \sqrt{x^2+y^2}$$

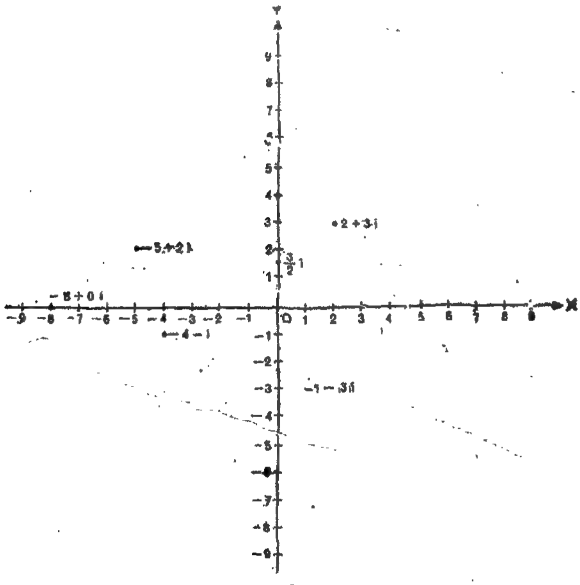
संख्या $\sqrt{x^2+y^2}$ के लिए "माड्यूलस" नाम सन् 1821 में महान फ्रांसीसी गणितज्ञ आगस्टीन-लुइस कोशी (1789-1857) ने दिया था। यह जानना अत्यन्त रुचिकर है कि यद्यपि कोशी ने विद्यालय में इंजीनियरी का अध्ययन किया था तथापि उन्होंने अपना जीवन गणित को ही समर्पित किया। उन्होंने गणित पर प्रचुर मात्रा में लिखा और उसकी विभिन्न शाखाओं में 700 से अधिक निबन्ध प्रकाशित किए।

किसी सम्मिश्र संख्या के तल में ज्यामितीय निरूपण के विषय में कुछ कार्य ब्रिटिश गणितज्ञ जॉन वालिस (1616-1703) ने भी किया। परन्तु वस्तुतः जर्मन गणितज्ञ गौस ने 1831 ई० में सम्मिश्र संख्या $(x + yi)$ को (सम्मिश्र) तल के एक बिन्दु से निरूपित किया। वास्तव में उन्होंने सम्मिश्र संख्याओं के योग तथा गुणन का ज्यामितीय रूप से भी वर्णन किया।

आकृति 2.2 में हम निम्न संख्याओं को आलेखित (plot) करते हैं:

$$2 + 3i, \quad -5 + 2i, \quad -4 - i$$

$$1 - 3i, \quad \frac{3}{2}i, \quad -8 + 0i$$



आकृति 2.2

प्रश्नावली 2.2

निम्नलिखित सम्मिश्र संख्याओं को   तल में निरूपित कीजिए :

1. $1+2i$ 2. $-9+3i$
3. $-3-\frac{1}{2}i$ 4. $5-7i$
5. $0-7i$ 6. $2i$
7. $\frac{3}{2}+5i$ 8. $0+0i$
9. 1 10. i

वे सम्मिश्र संख्याएँ लिखिए जो   तल में निम्नलिखित बिन्दुओं से निरूपित होती हैं :

11. $(0,2)$ 12. $(1,-2)$
13. $(2,3)$ 14. $(3,0)$
15. $(4,-1)$ 16. $\left(-\frac{1}{2}, -\frac{1}{3}\right)$
17. क्या सम्मिश्र   तल के कोई दो भिन्न बिन्दु किसी एक ही सम्मिश्र संख्या को निरूपित कर सकते हैं ? सकारण उत्तर दीजिए।

2.4 सम्मिश्र संख्याओं का बीजगणित (Algebra of Complex Numbers)

अब हम सम्मिश्र संख्याओं को जोड़ना, घटाना, गुणन तथा भाग करना सीखेंगे। योग और व्यवकलन के लिए हम सम्मिश्र संख्याओं को   में बहुपद (polynomials) मान सकते हैं। आइए, देखें कैसे।

2.4.1 योग

मान लीजिए हम $2+3i$ तथा $5+7i$ को जोड़ना चाहते हैं। यदि हम इन्हें i में बहुपद समझें तो हमें निम्न प्राप्त होता है :

$$(2+3i)+(5+7i) = 7+10i$$

$$\boxed{\begin{array}{r} 2+3i \\ 5+7i \\ \hline 7+10i \end{array}}$$

हम उपर्युक्त को सम्मिश्र संख्याओं का योग परिभाषित करने के लिए अभिप्रेरण के रूप में प्रयोग करते हैं।

यदि $z_1=a+bi$ तथा $z_2=c+di$ सम्मिश्र संख्याएँ हों तो z_1 तथा z_2 के योग को $(a+c)+(b+d)i$ से परिभाषित करते हैं। हम इस योग को z_1+z_2 से व्यक्त करते हैं तथा इसे इस प्रकार लिखते हैं :

$$z_1+z_2=(a+c)+(b+d)i$$

हम देखते हैं कि $x+yi=(x+0i)+(0+yi)$ है। इस प्रकार यह वास्तविक संख्या x तथा काल्पनिक संख्या yi का योग है।

आइए कुछ उदाहरण लें।

उदाहरण 1 : $(3+4i)$ तथा $(-2+5i)$ का योग ज्ञात कीजिए।

हल : $(3+4i)+(-2+5i)=(3-2)+(4+5)i$

$$=1+9i$$

उदाहरण 2 : $-2i$ तथा $\frac{3}{2}-4i$ का योग ज्ञात कीजिए।

हल : हम $-2i$ को $0-2i$ के रूप में लिख सकते हैं।

इस प्रकार, $(0-2i)+\left(\frac{3}{2}-4i\right)=\left(0+\frac{3}{2}\right)+(-2-4)i$

$$=\frac{3}{2}-6i$$

यह सिद्ध करना सरल है कि सम्मिश्र संख्याओं का योग क्रम-विनिमेय होता है। क्योंकि यदि $z_1=a+bi$ तथा $z_2=c+di$ सम्मिश्र संख्याएँ हैं, तो

$$z_1+z_2=(a+c)+(b+d)i$$

तथा

$$z_2+z_1=(c+a)+(d+b)i$$

परन्तु हम जानते हैं कि वास्तविक संख्याओं का योग क्रम-विनिमेय होता है। इस प्रकार, $a+c=c+a$ तथा $b+d=d+b$। इससे सिद्ध होता है कि सम्मिश्र संख्याओं का योग क्रम-विनिमेय होता है।

यह सिद्ध करना कि सम्मिश्र संख्याओं का योग सहचारी होता है, पाठक के अभ्यासार्थ छोड़ दिया गया है।

अब हम एक उदाहरण लेते हैं।

उदाहरण 3 : $\frac{1}{2}+\frac{5}{2}i$, $-\frac{3}{2}i$ तथा $-\frac{5}{2}-i$ का योग ज्ञात कीजिए।

हल : हम योग के साहचर्य नियम का प्रयोग करते हैं।

इस प्रकार, अभीष्ट योग $=\left[\left(\frac{1}{2}+\frac{5}{2}i\right)+\left(0-\frac{3}{2}i\right)\right]+\left(-\frac{5}{2}-i\right)$

$$=\left(\frac{1}{2}+i\right)+\left(-\frac{5}{2}-i\right)=-2+0i=-2$$

स्पष्ट रूप से, $0=0+0i$ योज्य तत्समक है। क्योंकि, यदि $(a+bi)$ एक सम्मिश्र संख्या है तो

$$(a+bi)+(0+0i)=(a+bi)$$

2.4.2 व्यवकलन

मान लीजिए, हम $(3-2i)$ को $(5-7i)$ में से घटाना चाहते हैं। यदि हम इन संख्याओं को i में बहुपद समझें तो आपको याद होगा कि हमें $3-2i$ के चिन्ह बदलकर फिर योग करना चाहिए। अत:

$$(5-7i) - (3-2i) = (5-3) + (-7+2)i$$
$$= 2-5i$$

$$\begin{array}{|c|} 5-7i \\ +3-2i \\ -\quad + \\ \hline 2-5i \end{array}$$

हम उपर्युक्त को सम्मिश्र संख्याओं का व्यवकलन परिभाषित करने के लिए अभिप्रेरण के रूप में प्रयोग करते हैं।

यदि $z_1 = a+bi$ तथा $z_2 = c+di$ दो सम्मिश्र संख्याएँ हों तो z_2 को z_1 में से घटाने का तात्पर्य है $z_1 + (-z_2)$ प्राप्त करना, जहाँ $(-z_2)$ इस प्रकार की सम्मिश्र संख्या है कि $z_2 + (-z_2) = 0+0i$।

$(-z_2)$ संख्या z_2 का योज्य प्रतिलोम कहलाता है तथा इसे z_2 के चिन्ह बदलकर प्राप्त किया जाता है। हम $z_1 - z_2$ को इस प्रकार लिखते हैं :

$$z_1 - z_2 = z_1 + (-z_2) = (a-c) + (b-d)i$$

आइए कुछ उदाहरण लें।

उदाहरण 1 : $(-3+4i)$ का योज्य प्रतिलोम ज्ञात कीजिए।

हल : यदि $z_1 = -3+4i$ तो योज्य प्रतिलोम $(-z_1)$, z_1 के चिन्ह बदलकर प्राप्त किया जा सकता है।

इस प्रकार, $(-z_1) = 3-4i$

उदाहरण 2 : $(-5-7i)$ को $(3-6i)$ में से घटाइए।

हल : $(3-6i) - (-5-7i) = 8+i$

$$\begin{array}{|c|} 3-6i \\ -5-7i \\ +\quad + \\ \hline 8+i \end{array}$$

हम देखते हैं कि सम्मिश्र संख्याओं का निकाय योग तथा व्यवकलन की संक्रियाओं के लिए संवृत होता है। इसी अनुच्छेद में कहीं और हम देखेंगे कि यह निकाय गुणन तथा विभाजन की संक्रियाओं के लिए भी संवृत है।

प्रश्नावली 2.3

निम्नलिखित संक्रियाएँ कीजिए तथा परिणाम को $x + yi$ के रूप में लिखिए :

1. $(-3-2i) + (-6+3i)$
2. $\left(\frac{1}{2} + \frac{7}{2}i\right) - \left(4 + \frac{5}{2}i\right)$
3. $(7-2i) - (3+2i) + (7+8i)$
4. $-(-1+i) + 4i - 5$
5. $(\sqrt{3} - 2i) + \sqrt{3} - (-2-7i)$
6. $(1-i) - (-1-6i)$

7. $(1-2i)-i + (4-7i)-2i + (5i+3)$
8. सिद्ध कीजिए कि सम्मिश्र संख्याओं का योग सहचारी होता है। दूसरे शब्दों में, यदि z_1, z_2 तथा z_3 सम्मिश्र संख्याएँ हैं तो
$$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$$
9. एक उदाहरण द्वारा दिखाइए कि सम्मिश्र संख्याओं का व्यवकलन क्रम-विनिमेय नहीं होता।
10. सिद्ध कीजिए कि $|z| = |-z|$

2.4.3 गुणन

मान लीजिए कि हम $z_1 = (-3+5i)$ तथा $z_2 = (3-6i)$ का गुणन करना चाहते हैं। पुनः, यदि हम इन्हें i में इस गुणधर्म के साथ बहुपद मानें कि हम गुणनफल में i^2 को -1 से प्रतिस्थापित कर सकते हैं, तो हम निम्न प्राप्त करते हैं :

$$z_1 \cdot z_2 = -9 + 33i - 30i^2$$

परन्तु, $i^2 = -1$

अतः, $z_1 \cdot z_2 = -9 + 33i + 30$

या, $z_1 \cdot z_2 = 21 + 33i$

$$\begin{array}{r} -3+5i \\ 3-6i \\ \hline -9+15i \\ +18i-30i^2 \\ \hline -9+33i-30i^2 \\ \hline \end{array}$$

पुनः हम उपर्युक्त को सम्मिश्र संख्याओं का गुणन परिभाषित करने के लिए अभिप्रेरण के रूप में प्रयोग करते हैं।

यदि $z_1 = a + bi$ तथा $z_2 = c + di$ सम्मिश्र संख्याएँ हों, तो z_1 तथा z_2 के गुणनफल को $(ac - bd) + (ad + bc)i$ से परिभाषित करते हैं। इस गुणनफल को $z_1 . z_2$ (अथवा केवल $z_1 z_2$) से व्यक्त करते हैं तथा इसे निम्न प्रकार से लिखते हैं :

$$z_1 z_2 = (ac - bd) + (ad + bc)i$$

$$\begin{array}{r} c+di \\ a+bi \\ \hline ac+adi \\ +bci+bdi^2 \\ \hline (ac-bd)+(ad+bc)i \\ \hline \end{array}$$

हम देखते हैं कि $yi = (y + 0i)\ (0 + 1i)$ है और इस प्रकार यह वास्तविक संख्या y तथा काल्पनिक संख्या i का गुणनफल है।

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1: $3 + 4i$ को $4 + 5i$ से गुणा कीजिए।

हल : $(3+4i)\ (4+5i) = 12 + 15i + 16i + 20i^2$
$$= -8 + 31i$$

उदाहरण 2: $(2 + \sqrt{3}\,i)^2$ को सरल कीजिए।

हल : $(2 + \sqrt{3}\,i)^2 = 4 + 2\ (2)\ (\sqrt{3}\,i) + 3i^2$
$$= 4 - 3 + 4\sqrt{3}\,i$$
$$= 1 + 4\sqrt{3}\,i$$

टिप्पणी : यह सरलता से देखा जा सकता है कि

$i^3 = i^2(i) = -i$

$i^4 = (i^2)\ (i^2) = 1$

$i^5 = (i^4)\ (i) = i$

$i^6 = (i^4)\ (i^2) = -1$, इत्यादि ।

हम देखते हैं कि किसी धनात्मक पूर्णांक k के लिए i^k का मान या तो -1 या $+1$ या i या $-i$ होता है। हम k के किसी बड़े मान के लिए, i^k का मान उसे i^2 (या उसकी घात) तथा i के गुणा के रूप में व्यक्त करके सुगमता से निकाल सकते हैं। उदाहरणार्थ,

$i^{23} = i^{22}.\ i = (i^2)^{11}\ i = (-1)^{11}\ i = -i$

$i^{42} = (i^2)^{21} = -1$

$i^{64} = (i^2)^{32} = 1$

$i^{97} = (i^2)^{48}\ (i) = i$

अब हम यह दिखाते हैं कि सम्मिश्र संख्याओं का गुणन क्रमविनिमेय होता है। क्योंकि, यदि $z_1 = a+bi$ तथा $z_2 = c+di$ दो सम्मिश्र संख्याएँ हों, तो

$z_1\ z_2 = (a+bi)\ (c+di) = (ac-bd) + (ad+bc)\ i$

तथा $z_2\ z_1 = (c+di)\ (a+bi) = (ca-db) + (da+cb)\ i$

परन्तु a, b, c तथा d वास्तविक संख्याएँ हैं,

अतः $ac-bd = ca-db$

तथा $ad+bc = da+cb$

इससे सिद्ध होता है कि सम्मिश्र संख्याओं का गुणन क्रमविनिमेय होता है। इस प्रकार, हम yi को iy भी लिख सकते हैं।

यह सिद्ध करना कि सम्मिश्र संख्याओं का गुणन सहचारी होता है, पाठक के अभ्यासार्थ छोड़ दिया है। अब हम एक उदाहरण लेते हैं।

उदाहरण 3 : $(2+i)^3$ को सरल कीजिए।

हल : $(2+i)^3 = 2^3 + 3(2)^2\ i + 3(2)i^2 + i^3$

$= 8 + 12\ i + 6\ i^2 + i^2 \cdot i$

$= 8 + 12\ i - 6 - 1\ (i)$

$= 2 + 11\ i$

स्पष्ट रूप से, $1+0i$ (अर्थात् 1) गुणनात्मक तत्समक है, क्योंकि यदि $a+bi$ एक सम्मिश्र संख्या है, तो

$$(a+bi)\ (1+0i) = a+bi$$

यह सिद्ध करना भी पाठक के अभ्यासार्थ छोड़ दिया गया है कि सम्मिश्र संख्याओं का गुणन योग पर वितरणात्मक (distributive) है। दूसरे शब्दों में, यदि z_1, z_2 तथा z_3 सम्मिश्र संख्याएँ हैं, तो

$$z_1\ (z_2+z_3) = z_1\ z_2 + z_1\ z_3$$

2.4.4 ऋणात्मक वास्तविक संख्याओं के वर्गमूल (Square Roots of Negative Reals)

आपको याद होगा कि हमने अनुच्छेद 2.2 में संख्या i की स्वयं से गुणा की परिभाषा $i^2 = -1$ के रूप में की थी और हमने कहा था कि -1 का वर्गमूल i होता है।

अब, $(-i)(-i)$ के बारे में आप क्या सोचते हैं ? गुणन की परिभाषा के अनुसार,

$$(-i)(-i) = 0 + i^2 = -1$$

इस प्रकार, $-i$ भी संख्या -1 का वर्गमूल है।

अतः हम इस निष्कर्ष पर पहुंचते हैं कि संख्या -1 के दो वर्गमूल i तथा $-i$ हैं।

आइए, अब हम एक धनात्मक संख्या P पर विचार करें। हम देखते हैं कि

$$(i\sqrt{P})^2 = i^2 P = -P \qquad (1)$$

तथा $(-i\sqrt{P})^2 = i^2 P = -P \qquad (2)$

इस प्रकार एक ऋणात्मक संख्या $-P$ के दो वर्गमूल, $i\sqrt{P}$ तथा $-i\sqrt{P}$ हैं। यह तथ्य कि केवल यही दो वर्गमूल हैं पाद टिप्पणी* (foot note) में सिद्ध किया गया है।

यदि $P>0$, तो ऋणात्मक संख्या $-P$ के दो वर्गमूल $i\sqrt{P}$ तथा $-i\sqrt{P}$ होते हैं। कभी-कभी हम इन्हें संयुक्त रूप से $\pm i\sqrt{P}$ लिखते हैं। विशिष्ट रूप से संख्या -1 के दो वर्गमूल, $+i$ तथा $-i$ अथवा (संयुक्त रूप में लिखने पर) $\pm i$ हैं। पुनः आगे हम केवल $i\sqrt{P}$ को ही $(-P)$ का वर्गमूल कहेंगे तथा इसे इस प्रकार पढ़ेंगे : "$\sqrt{-P}$ अर्थात् $i\sqrt{P}$ संख्या $(-P)$ का वर्गमूल है"। दूसरे शब्दों में, जब कभी भी हमारे सामने संकेत $\sqrt{-P}$ आए हमें उसके स्थान पर $i\sqrt{P}$ लिख लेना चाहिए।

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* मान लीजिए $-P$ का एक वर्गमूल $a+bi$ है। तब परिभाषा से,

$$(a+bi)^2 = -P$$

अर्थात्, $a^2+2abi-b^2 = -P \qquad (1)$

(1) से हमें निम्न प्राप्त होता है :

$$\text{तथा} \quad \left.\begin{aligned} a^2-b^2 &= -P \\ 2ab &= 0 \end{aligned}\right\} \qquad (2)$$

चूंकि $2ab = 0$ है, अतः या तो $a = 0$ या $b = 0$ या दोनों 0 के बराबर होंगे। यह स्पष्ट है कि a और b दोनों एक साथ शून्य नहीं हो सकते।

यदि $b = 0$ तो $a^2 = -P$ जो कि संभव नहीं है। (क्यों ?)

इस प्रकार, b शून्य नहीं हो सकता।

यदि $a = 0$ तो $-b^2 = -P$

अर्थात् $b = \pm\sqrt{P}$

इस प्रकार, $0 \pm \sqrt{P}\, i$ अर्थात् $\pm\sqrt{P}\, i$, $-P$ के दो वर्गमूल हैं। यह तथ्य कि केवल यही दो वर्गमूल हैं, इससे स्पष्ट है कि (2) का कोई अन्य हल नहीं है।

आइए कुछ उदाहरण लें।

उदाहरण 1 : निम्नलिखित संख्याओं के वर्गमूल ज्ञात कीजिए।

$$(\text{क}) -36 \quad (\text{ख}) -\frac{16}{25}$$

हल :

(क) -36 के वर्गमूल $\pm\sqrt{-36} = \pm 6i$ हैं।

(ख) $-\frac{16}{25}$ के वर्गमूल $\pm\sqrt{-\frac{16}{25}} = \pm\frac{4}{5}i$ हैं।

उदाहरण 2 : $\sqrt{-9}\ \sqrt{-121}$ का मान ज्ञात कीजिए।

हल : $\sqrt{-9} = i\sqrt{9}$

$\sqrt{-121} = i\sqrt{121}$

इस प्रकार, $\sqrt{-9}\ \sqrt{-121} = i^2\sqrt{9}\ \sqrt{121} = i^2\sqrt{1089} = -33$

टिप्पणी : उपरोक्त गुणा में करणियों (radicals) के अन्दर की संख्याओं का गुणन करना तथा फिर वर्गमूल निकालना गलत होगा। आइए देखें कि ऐसा करने पर क्या होगा। इस प्रकार हम निम्न प्राप्त करेंगे :

$$\sqrt{-9}\ \sqrt{-121} = \sqrt{(-9)(-121)} = \sqrt{1089} = 33$$

जो वस्तुतः सही उत्तर नहीं है। हमने कहाँ गलती की? हमने नियम $\sqrt{a}\ \sqrt{b} = \sqrt{ab}$ का प्रयोग गलत रूप से किया। यह नियम तभी और केवल तभी सत्य होता है जब a तथा b में से कम से कम एक धनात्मक अथवा शून्य हो।

अतः इस प्रकार की भ्रामक स्थिति से बचने के लिए, हम पुनः इस बात पर बल देते हैं कि जब कभी भी हमारे सामने चिन्ह $\sqrt{-P}$ आए हमें उसके स्थान पर $i\sqrt{P}$ लिख लेना चाहिए।

उदाहरण 3 : $\sqrt{-\frac{49}{25}}\ \sqrt{-\frac{1}{9}}$ का मान ज्ञात कीजिए।

हल : $\sqrt{-\frac{49}{25}}\ \sqrt{-\frac{1}{9}} = i\sqrt{\frac{49}{25}}\ i\sqrt{\frac{1}{9}} = i^2\sqrt{\frac{49}{225}} = -\frac{7}{15}$

प्रश्नावली 2.4

निम्नलिखित संख्याओं को $x + yi$ के रूप में लिखिए :

1. i^{20} 2. $8i^{8} + 6i^{16} - 12i^{19}$

3. $i^{14} + (2 - 6i)^{3}$

निम्नलिखित संक्रियाएँ कीजिए तथा परिणाम को $a + bi$ के रूप में लिखिए :

4. $(3 - 7i)\ (3 + 5i)$ 5. $(-5 + 3i)\ (8 - 7i)$

6. $(\sqrt{2} - \sqrt{3}\,i)^2$ 7. $-2i\ (5i^3)$

8. $(-2 + 3i)\ (3 - 5i)$ 9. $\left(-4 - \frac{3}{2}i\right)\left(-4 + \frac{3}{2}i\right)$

10. $(2i+5)^2$ 11. $\left(\frac{1}{2} - \frac{3}{2}i\right)^2$

12. $(2+7i)^3$ 13. $\left(-3 - \frac{1}{2}i\right)^3$

14. $(\sqrt{5} + 7i)\ (\sqrt{5} - 7i)^2$

15. यदि z_1, z_2 तथा z_3 सम्मिश्र संख्याएँ हैं, तो सिद्ध कीजिए कि

$$z_1(z_2\, z_3) = (z_1\, z_2)\, z_3$$

अर्थात् सम्मिश्र संख्याओं का गुणन सहचारी होता है ।

16. यदि $z_1 = -3i$, $z_2 = 3 + 4i$ तथा $z_3 = 2 - 3i$, तो जाँच कीजिए कि

$$z_1\,(z_2 + z_3) = z_1\, z_2 + z_1\, z_3$$

17. यदि z_1, z_2 तथा z_3 सम्मिश्र संख्याएँ हैं, तो सिद्ध कीजिए कि

$$z_1\,(z_2 + z_3) = z_1\, z_2 + z_1\, z_3$$

अर्थात् सम्मिश्र संख्याओं का गुणन योग पर वितरणात्मक होता है ।

निम्नलिखित संख्याओं के वर्गमूल ज्ञात कीजिए :

18. $\frac{-36}{121}$ 19. -144

20. -31 21. $\frac{-180}{225}$

मान निकालिए :

22. $2\sqrt{-9}\ \sqrt{-16}$ 23. $\sqrt{-4}\ \sqrt{\frac{-25}{36}}$

24. $\sqrt{-49}\ (2 + \sqrt{-9})$

25. $\sqrt{-4}\ (1 - \sqrt{-169})$

26. यदि z एक सम्मिश्र संख्या है, तो सिद्ध कीजिए कि

$$|\,z^2\,| = |\,z\,|^2$$

2.4.5 सम्मिश्र संख्या का गुणनात्मक प्रतिलोम (Multiplicative Inverse of a Complex Number)

आपको याद होगा कि किसी वास्तविक संख्या p को किसी अन्य वास्तविक संख्या q (शून्य के अतिरिक्त) से भाग करने का अर्थ है p को $\frac{1}{q}$ से गुणा करना जहाँ $\frac{1}{q}$ संख्या q का व्युत्क्रम अर्थात् गुणनात्मक प्रतिलोम है। हम दो सम्मिश्र संख्याओं के विभाजन के लिए भी इसी प्रकार की परिभाषा देना चाहेंगे। अतः आइए, एक शून्येतर (non-zero) सम्मिश्र संख्या का गुणनात्मक प्रतिलोम प्राप्त करना सीखें।

मान लीजिए, $a + bi$ एक शून्येतर सम्मिश्र संख्या है। सम्मिश्र संख्याओं का गुणनात्मक तत्समक $1 + 0i$ है।

हम एक सम्मिश्र संख्या $x + yi$ इस प्रकार की प्राप्त करना चाहते हैं कि

$$(a + bi)(x + yi) = 1 + 0i \tag{1}$$

समीकरण (1) को संतुष्ट करने वाली सम्मिश्र संख्या $x + yi$ को हम शून्येतर सम्मिश्र संख्या $a + bi$ का गुणनात्मक प्रतिलोम कहते हैं तथा इसे $\frac{1}{a + bi}$ से व्यक्त करते हैं।

अब समीकरण (1) तभी सत्य होगा जबकि

$$\left.\begin{aligned} ax - by &= 1 \\ \text{तथा} \quad bx + ay &= 0 \end{aligned}\right\} \tag{2}$$

चूँकि $a + bi \neq 0$ है, अतः यह स्पष्ट है कि $a^2 + b^2 \neq 0$। अतः हम समीकरण (2) को युगपत समीकरण के रूप में हल कर सकते हैं तथा x और y के निम्न (अद्वितीय) मान प्राप्त करते हैं :

$$x = \frac{a}{a^2 + b^2}, \quad y = \frac{-b}{a^2 + b^2}$$

[पाठक को चाहिए कि वह यह जाँच करे कि x और y के ये मान वस्तुतः (1) को संतुष्ट करते हैं।]

इस प्रकार, $\frac{a}{a^2 + b^2} + \frac{(-b)}{a^2 + b^2} i$, शून्येतर सम्मिश्र संख्या $a + bi$ का गुणनात्मक प्रतिलोम है।

आइए कुछ उदाहरण लें।

उदाहरण 1 : $3 + 2i$ का गुणनात्मक प्रतिलोम ज्ञात कीजिए।

हल : गुणनात्मक प्रतिलोम $\frac{3}{3^2 + 2^2} + \frac{(-2)}{3^2 + 2^2} i$

अर्थात् $\frac{3}{13} - \frac{2}{13} i$ है।

उदाहरण 2 : $9 - 12i$ का गुणनात्मक प्रतिलोम ज्ञात कीजिए।

हल : गुणनात्मक प्रतिलोम $\frac{9}{9^2+12^2}+\frac{-(-12)}{9^2+12^2}i$

अर्थात् $\frac{9}{225}+\frac{12}{225}i$ है।

2.4.6 विभाजन

अब हम दो सम्मिश्र संख्याओं z_1 तथा $z_2, (z_2 \neq 0)$ के विभाजन को केवल z_1 से z_2 के गुणनात्मक प्रतिलोम के गुणन के रूप में परिभाषित कर सकते हैं। हम इसे इस प्रकार से लिखते हैं :

$$z_1 \div z_2 \quad \text{अर्थात्} \quad \frac{z_1}{z_2}=z_1\left(\frac{1}{z_2}\right)$$

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : $(2+3i)\div(1+2i)$ ज्ञात कीजिए तथा परिणाम को $x+yi$ के रूप में लिखिए।

हल : हम पहले $1+2i$ का गुणनात्मक प्रतिलोम ज्ञात करते हैं।

हम देखते हैं कि

$$\frac{1}{1+2i}=\frac{1}{1^2+2^2}+\frac{(-2)}{1^2+2^2}i=\frac{1}{5}-\frac{2}{5}i$$

इस प्रकार, $(2+3i)\div(1+2i)=(2+3i)\left(\frac{1}{5}-\frac{2}{5}i\right)$

$$=\frac{2}{5}-\frac{4}{5}i+\frac{3}{5}i-\frac{6}{5}i^2$$

$$=\frac{8}{5}+\left(-\frac{1}{5}\right)i$$

उदाहरण 2 : $\frac{5+2i}{-1+\sqrt{3}i}$ को $a+bi$ के रूप में लिखिए।

हल : $\frac{5+2i}{-1+\sqrt{3}i}=(5+2i)\left(\frac{1}{-1+\sqrt{3}i}\right)$

$$=(5+2i)\left(\frac{-1}{1+(\sqrt{3})^2}+\frac{-\sqrt{3}}{1+(\sqrt{3})^2}i\right)$$

$$=(5+2i)\left(-\frac{1}{4}-\frac{\sqrt{3}}{4}i\right)$$

$$=-\frac{5}{4}-\frac{5\sqrt{3}}{4}i-\frac{2}{4}i-\frac{2\sqrt{3}}{4}i^2$$

इस प्रकार, $\frac{5+2i}{-1+\sqrt{3}i}=\frac{-5+2\sqrt{3}}{4}+\frac{-(5\sqrt{3}+2)}{4}i$

आपको याद होगा कि किसी धनात्मक पूर्णांक k के लिए i^k या तो -1 या $+1$ या i या $-i$ होता है। यह स्पष्ट है कि किसी ऋणात्मक पूर्णांक k के लिए भी i^k या तो -1 या $+1$ या i या $-i$ होता है। उदाहरणार्थ,

$$i^{-23}=\frac{1}{i^{23}}=\frac{1}{(i^2)^{11}i}=\frac{1}{-i}=i$$

$$i^{-48}=\frac{1}{i^{48}}=\frac{1}{(i^2)^{24}}=1$$

2.4.7 यद्यपि हमने सम्मिश्र संख्याओं के बीजगणित की बात की है तथापि हमने उनके धनात्मक अथवा ऋणात्मक होने के विषय में कुछ नहीं कहा। हमने ऐसा जानबूझकर किया है। यह सिद्ध किया जा सकता है कि सम्मिश्र संख्याओं के समुच्चय में क्रम-सम्बन्ध को परिभाषित करना तथा साथ ही क्रम-सम्बन्धों द्वारा परिचित गुणधर्मों की संतुष्टि (satisfy) कराना असम्भव है। अतः हम एक सम्मिश्र संख्या के किसी अन्य सम्मिश्र संख्या से बड़ी अथवा छोटी होने के विषय में कुछ नहीं कहते।

2.5 सम्मिश्र संख्या का संयुग्मी (Conjugate of a Complex Number)

सम्मिश्र संख्याएँ $a+bi$ तथा $a-bi$ एक दूसरे के संयुग्मी (conjugates) कहलाते हैं। इस प्रकार $8+3i$ संख्या $8-3i$ का संयुग्मी है, $2i$ संख्या $-2i$ का संयुग्मी है, $-2i$ संख्या $2i$ का संयुग्मी है, $-3+\frac{2}{5}i$ संख्या $-3-\frac{2}{5}i$ का संयुग्मी है, इत्यादि। किसी सम्मिश्र संख्या z के संयुग्मी को $\bar{z}$ से व्यक्त किया जाता है।

हम देखते हैं कि किसी वास्तविक संख्या का संयुग्मी वह संख्या स्वयं ही होती है।

पुनः, किसी सम्मिश्र संख्या के संयुग्मी का संयुग्मी स्वयं वही सम्मिश्र संख्या होती है। अर्थात् $\overline{(\bar{z})}=z$।

क्योंकि यदि $z=a+bi$, तो $\bar{z}=a-bi$

अतः, $\overline{(\bar{z})}=a+bi$।

अब हम एक उदाहरण लेते हैं।

उदाहरण 1 : यदि $z=a+bi$, तो सिद्ध कीजिए कि $z\bar{z}=|z|^2$।

हल : क्योंकि $z=a+bi$, अतः $\bar{z}=a-bi$

इस प्रकार, $z\bar{z}=(a+bi)(a-bi)=a^2+b^2=|z|^2$

आपको याद होगा कि सम्मिश्र संख्या $z=a+bi$ का गुणनात्मक प्रतिलोम $\frac{1}{z}$ निम्न है :

$$\frac{1}{z}=\frac{a}{a^2+b^2}+\frac{(-b)i}{a^2+b^2}=\frac{\bar{z}}{|z|^2}=\frac{\bar{z}}{z\bar{z}}$$

इस प्रकार, किसी शून्येतर सम्मिश्र संख्या z का गुणनात्मक प्रतिलोम ज्ञात करने के लिए हम $\bar{z}$ ज्ञात करते हैं और इसे $|z|^2$ या $z\bar{z}$ से विभाजित करते हैं।

आइए कुछ और उदाहरण लें।

उदाहरण 2 : $3+2i$ का गुणनात्मक प्रतिलोम ज्ञात कीजिए। [अनुच्छेद 2.4.5 का उदाहरण 1 भी देखें।]

हल : मान लीजिए $z=3+2i$, तब $\overline{z} = 3-2i$

साथ ही, $|z|^2 = 9+4 = 13$

इस प्रकार, $3+2i$ का गुणनात्मक प्रतिलोम

$\frac{3-2i}{13}$ अर्थात् $\frac{3}{13}-\frac{2}{13}i$ है।

उदाहरण 3 : $9-12i$ का गुणनात्मक प्रतिलोम ज्ञात कीजिए। [अनुच्छेद 2.4.5 का उदाहरण 2 भी देखें।]

हल : चूँकि $z= 9-12i$, अतः $\overline{z} = 9+12i$

साथ ही, $|z|^2 = 9^2+12^2=225$

इस प्रकार, $9-12i$ का गुणनात्मक प्रतिलोम

$\frac{9+12i}{225}$ अर्थात् $\frac{9}{225}+\frac{12}{225}i$ है।

अब हम संयुग्मियों के कुछ रुचिपूर्ण गुणधर्मों को सिद्ध करते हैं।

I. **दो सम्मिश्र संख्याओं के योग का संयुग्मी उनके संयुग्मियों का योग होता है। दूसरे शब्दों में, यदि z_1 तथा z_2 सम्मिश्र संख्याएँ हैं, तो**

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

मान लीजिए, $z_1= a+bi$ तथा $z_2= c+di$

तब, $z_1+z_2 = (a+c) + (b+d)i$

अब, $\overline{z_1}= a-bi$, $\overline{z_2} = c-di$

तथा $\overline{z_1+z_2} = (a+c)-(b+d)i$

साथ ही, $\overline{z_1} + \overline{z_2} = (a-bi) + (c-di) = (a+c)-(b+d)i$

इस प्रकार, $\overline{z_1+z_2} = \overline{z_1} + \overline{z_2}$

II. **दो सम्मिश्र संख्याओं के गुणनफल का संयुग्मी उनके संयुग्मियों का गुणनफल होता है। दूसरे शब्दों में, यदि z_1 तथा z_2 सम्मिश्र संख्याएँ हैं, तो**

$$\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$$

III. दो सम्मिश्र संख्याओं के भागफल (शून्येतर हर) का संयुग्मी उनके संयुग्मियों का भागफल होता है। दूसरे शब्दों में, यदि z_1 तथा z_2 सम्मिश्र संख्याएँ हैं, तो

$$\overline{\left[\frac{z_1}{z_2}\right]} = \frac{\overline{z_1}}{\overline{z_2}}$$

गुणधर्मों II तथा III को सिद्ध करना पाठक के अभ्यासार्थ छोड़ दिया गया है।

उदाहरण 4: सम्मिश्र संख्याओं $(3+4i)$ तथा $(5-12i)$ के योग, अन्तर, गुणनफल, संयुग्मियों, गुणनात्मक प्रतिलोमों तथा भागफल को ज्ञात कीजिए।

हल : योग $= (3+4i) + (5-12i) = 8-8i$

अन्तर $= (3+4i) - (5-12i) = (3-5) + (4+12)i$

$= -2+16i$

गुणनफल $= (3+4i)(5-12i) = (15+48) + (-36+20)i$

$= 63-16i$

$(3+4i)$ का संयुग्मी $(3-4i)$ है जबकि $(5-12i)$ का संयुग्मी $(5+12i)$ है।

$(3+4i)$ का गुणनात्मक प्रतिलोम

$$\frac{3}{3^2+4^2} + \frac{(-4)}{3^2+4^2}\, i \quad \text{अर्थात्} \quad \frac{3}{25} - \frac{4}{25} i \text{ है।}$$

$(5-12i)$ का गुणनात्मक प्रतिलोम निम्न है :

$$\frac{1}{5-12i} = \frac{5+12i}{(5-12i)(5+12i)} = \frac{5+12i}{25-144i^2}$$

$$= \frac{5}{169} + \frac{12}{169} i$$

$$\text{भागफल} = \frac{3+4i}{5-12i} = \frac{(3+4i)(5+12i)}{(5-12i)(5+12i)}$$

$$= \frac{(15+48i^2) + (36+20)i}{25-144i^2}$$

$$= \frac{-33}{169} + \frac{56}{169} i$$

प्रश्नावली 2.5

निम्नलिखित संख्याओं के संयुग्मी ज्ञात कीजिए :

1. $3-7i$
2. $-\sqrt{5} + 3i$
3. $i\sqrt{-9}+7i$
4. $(6+5i)^2$
5. $6i-3$

निम्नलिखित में प्रत्येक संख्या का गुणनात्मक प्रतिलोम ज्ञात कीजिए :

6. $3-4i$

7. $\sqrt{3}+7i$

8. $6-\sqrt{3}\,i$

9. $-i$

10. $4-\sqrt{-9}$

निम्न संक्रियाएँ कीजिए तथा परिणाम को $a+bi$ के रूप में लिखिए :

11. $\dfrac{2-3i}{5-4i}$

12. $\dfrac{2+3i}{-5-4i}$

13. $\dfrac{3+i}{2-i}$

14. $\dfrac{3+2i}{4-3i}$

15. $\dfrac{-3}{2-3i}$

16. $\dfrac{2-\sqrt{-25}}{1-\sqrt{-16}}$

17. $\dfrac{-2}{4i}$

18. $\dfrac{12}{i^3}$

19. $\dfrac{3i-5}{-2-i}$

20. यदि z_1 तथा z_2 सम्मिश्र संख्याएँ हैं, तो सिद्ध कीजिए कि

(i) $\overline{z_1 \cdot z_2}=\overline{z_1} \cdot \overline{z_2}$

(ii) $\overline{\left(\dfrac{z_1}{z_2}\right)}=\dfrac{\overline{z_1}}{\overline{z_2}}$

21. सिद्ध कीजिए कि

(i) $z=\overline{z}$ तभी तथा केवल तभी होगा जब z एक वास्तविक संख्या हो ।

(ii) $z=-\overline{z}$ तभी तथा केवल तभी होगा जब z एक विशुद्ध काल्पनिक संख्या हो ।

22. सिद्ध कीजिए कि किसी सम्मिश्र संख्या तथा उसके संयुग्मी का योग एक वास्तविक संख्या होती है ।

23. सिद्ध कीजिए कि किसी सम्मिश्र संख्या तथा उसके संयुग्मी का गुणनफल एक वास्तविक संख्या होती है ।

2.6 मुख्य संकल्पनाएँ

सम्मिश्र संख्या	ऋणात्मक वास्तविक संख्याओं के वर्गमूल
सम्मिश्र संख्या का मापांक	व्युत्क्रम अथवा गुणनात्मक प्रतिलोम
सम्मिश्र तल	सम्मिश्र संख्या का संयुग्मी
सम्मिश्र संख्याओं का	
योग	
अन्तर	
गुणन	
विभाजन	

2.7 अग्रिम अध्ययन हेतु सुझाव

सम्मिश्र संख्याओं के उत्तम वर्णन के लिए निम्न पुस्तक देखें :

[1] W. Ledermann: **Complex Numbers,** Routledge and Kegan Paul Ltd., London (U.K.), 1962.

सम्मिश्र संख्याओं का वास्तविक संख्याओं के क्रमित-युग्मों के रूप में सुन्दर वर्णन निम्न पुस्तक में उपलब्ध है :

[2] R. Spreckelmeyer: **The Complex Numbers,** D. C. Heath & Co., Boston (U.S.A.), 1965.

अनुच्छेद 1.12 में दी गई पुस्तकों [4], [5] तथा [7] के अतिरिक्त सम्मिश्र संख्याओं के इतिहास के उत्तम वर्णन के लिए पाठक निम्न पुस्तक देखें :

[3] M. Kline: **Mathematical Thought from Ancient to Modern Times,** Oxford University Press, New York (U.S.A.), 1972.

एकक III

द्विघात समीकरण और असमीकरण

द्विघात समीकरणों, जिनके गुणांक (coefficients) वास्तविक हैं तथा विविक्तकर ऋणात्मक हैं, के हल प्राप्त किए गए हैं। मूलों (roots) तथा गुणांकों के सम्बन्धों का उल्लेख किया गया है। द्विघात समीकरणों, जिनके गुणांक सम्मिश्र हैं, का अध्ययन किया गया है। अन्त में द्विघात असमीकरणों के हलों तथा उनके आलेखीय निरूपण के विषय में बताया गया है।

3.1 भूमिका

अब जबकि हम ऋणात्मक वास्तविक संख्याओं के वर्गमूल ज्ञात करना सीख गए हैं तो हमारे लिए द्विघात समीकरणों, $ax^2 + bx + c = 0$, $a \neq 0$, जहाँ a, b तथा c वास्तविक संख्याएँ हैं और विविक्तकर D अर्थात् $b^2 - 4ac$ ऋणात्मक है, के हल ज्ञात करना सम्भव होना चाहिए। परन्तु ऐसा करने से पहले आइए द्विघात समीकरणों के हल सम्बन्धी कुछ महत्वपूर्ण तथ्यों का जिनका अध्ययन हमने पिछली कक्षाओं में किया था, पुनरावलोकन करें।

3.2 पुनरावलोकन*

द्विघात समीकरण : $ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in R$,

जहाँ R वास्तविक संख्याओं के सम्मुच्चय को व्यक्त करता है।

विविक्तकर : $D = b^2 - 4ac$

द्विघात सूत्र (Quadratic Formula) : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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*यदि पाठक द्विघात समीकरणों पर पिछली कक्षाओं में किए गए कार्य से परिचित है तो वह अविच्छिन्नता को खोए बिना सीधे ही अनुच्छेद 3.3 पर जा सकता है।

इस सूत्र का प्रयोग तभी किया जाता है जब कि $D \geqslant 0$ हो।

मूलों का योग : $-\dfrac{b}{a}$

मूलों का गुणनफल : $\dfrac{c}{a}$

प्रश्नावली 3.1
(पुनरावलोकन हेतु)

निम्नलिखित समीकरणों को x, y अथवा z के लिए हल कीजिए :

1. $3x^2+10x+7=0$
2. $2x^2-9x+10=0$
3. $\sqrt{3}\,y^2+10y-8\sqrt{3}=0$
4. $2z^2-10=z$
5. $8x-4x^2=1$
6. $2y^2+0.3y=0.35$
7. $16x^2+1=8x$
8. $10ax^2-6x+15ax=9,\ (a\neq0)$
9. $\dfrac{4}{y}-3=\dfrac{5}{2y+3}$
10. $\dfrac{1}{3-2x}+\dfrac{1}{5+2x}=\dfrac{1}{2}$
11. $3^x+3^{-x}-2=0$
12. $y^4-13y^2+36=0$
13. $x^{-2}-12=-x^{-1}$
14. $\left(\dfrac{2x+3}{x+1}\right)+6\left(\dfrac{x+1}{2x+3}\right)=7$
15. $\sqrt{3y}-2y+3=0$
16. $3x^2+x+6=0$
17. $\sqrt{\dfrac{x}{1-x}}+\sqrt{\dfrac{1-x}{x}}=\dfrac{13}{6}$
18. $4y^2=8y+39$
19. $\sqrt{3y+1}+1=\sqrt{y}$
20. $\dfrac{3}{7}x^2+\dfrac{1}{3}x+6=1$
21. $\sqrt{2x+1}+\sqrt{3x+2}=\sqrt{5x+3}$
22. एक आयताकार खेत की लम्बाई उसकी चौड़ाई से 10 मीटर अधिक है। यदि खेत का क्षेत्रफल 144 वर्ग मीटर है तो उसकी लम्बाई तथा चौड़ाई ज्ञात कीजिए।
23. एक संख्या तथा उसके व्युत्क्रम (reciprocal) का योग $\dfrac{41}{20}$ है। वह संख्या ज्ञात कीजिए।
24. दो संख्याएँ इस प्रकार की हैं कि उनका योग 54 तथा गुणनफल 629 है। संख्याएँ ज्ञात कीजिए।

25. जंगल के अन्दर कुल संख्या के $\frac{1}{8}$ के वर्ग के बराबर बानर (apes) उत्साह के साथ खेल रहे हैं। शेष 12 बानर पहाड़ी पर हैं। चारों ओर की पहाड़ियों से अपनी किलकारियों की प्रतिध्वनि से उन्हें खीज उत्पन्न होती है। बानरों की कुल संख्या कितनी है?
[स्रोत : भास्कर की कृति लीलावती]

26. ऊँटों के झुंड का एक-चौथाई जंगल में देखा गया। झुंड के वर्गमूल का दुगुना पहाड़ों की ढलानों पर गया हुआ था। पाँच ऊँटों के तिगुने ऊँट नदी के किनारे पर थे। उन ऊँटों की संख्या कितनी थी?
[स्रोत : महावीर की कृति गणित सार संग्रह]

निम्नलिखित में से प्रत्येक द्विघात समीकरण के मूल निर्धारित किए बिना उनके योग तथा गुणनफल ज्ञात कीजिए :

27. $4x^2=4x+37$

28. $4\sqrt{3}x^2+9x=3\sqrt{3}$

29. $7y^2-16y+4=0$

30. $\frac{3}{5}z^2-\frac{13}{5}-2z=0$

31. $\frac{1}{3}y^2+\frac{1}{4}y=0$

32. $\sqrt{3}x^2-\frac{11}{2}x+\frac{3\sqrt{3}}{4}=0$

द्विघात समीकरण बनाइए जिनके मूल निम्नलिखित हैं :

33. $-3, -4$

34. $-2, \frac{1}{2}$

35. $3-\sqrt{3}, 3+\sqrt{3}$

36. $7, -7$

37. $\frac{3}{4}, -2$

38. $4\sqrt{5}, -4\sqrt{5}$

39. $-\frac{3}{7}, -\frac{2}{3}$

40. $2+3\sqrt{2}, 2-3\sqrt{2}$

3.3 वास्तविक गुणांकों वाले द्विघात समीकरणों के मूल—द्विघात सूत्र (Roots of Quadratic Equations with Real Coefficients—The Quadratic Formula)

मान लीजिए, $ax^2+bx+c=0, a\neq 0, a, b, c \in R$ एक द्विघात समीकरण है। हम इस समीकरण के मूल अथवा हल प्राप्त करना चाहते हैं, विशिष्ट रूप से उस स्थिति में जबकि $D=b^2-4ac<0$। हम किस प्रकार आगे बढ़ें?

पहले की तरह, हम दिए हुए समीकरण को इस प्रकार लिखते हैं :

$$ax^2+bx=-c$$

अर्थात्, $$x^2+\frac{b}{a}x=-\frac{c}{a} \tag{1}$$

(1) के दोनों पक्षों में हम $\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2$ जोड़ते हैं (क्यों?) तथा निम्न प्राप्त करते हैं :

$$\left(x+\frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$$

अर्थात्, $$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2} \qquad (2)$$

यदि $b^2 - 4ac \geqslant 0$ तो हम इस स्थिति में (2) के दोनों पक्षों का वर्गमूल ज्ञात करके निम्न द्विघात सूत्र प्राप्त करते हैं :

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad (3)$$

तथा इस स्थिति में मूल निम्नलिखित हैं :

$$\frac{-b+\sqrt{b^2-4ac}}{2a} = r_1 \text{ (मान लीजिए)}; \quad \frac{-b-\sqrt{b^2-4ac}}{2a} = r_2 \text{ (मान लीजिए)}$$

यदि $b^2 - 4ac < 0$, तो आपको याद होगा कि इसके दो वर्गमूल $\pm i\sqrt{-(b^2-4ac)}$ हैं। इस प्रकार, $\frac{b^2-4ac}{4a^2}$ के दो वर्गमूल $\frac{\pm i\sqrt{-(b^2-4ac)}}{2a}$ होंगे।

अतः (2) से,

$$x + \frac{b}{2a} = \frac{\pm i\sqrt{-(b^2-4ac)}}{2a}$$

जिससे हमें $b^2 - 4ac < 0$ की स्थिति के लिए निम्न द्विघात सूत्र प्राप्त होता है :

$$x = \frac{-b \pm i\sqrt{-(b^2-4ac)}}{2a}$$

अतः दोनों मूल निम्नलिखित हैं :

$$\frac{-b+i\sqrt{-(b^2-4ac)}}{2a} = r_1 \text{ (मान लीजिए)}; \quad \frac{-b-i\sqrt{-(b^2-4ac)}}{2a} = r_2 \text{ (मान लीजिए)}।$$

आपको याद होगा कि $b^2 - 4ac$ द्विघात समीकरण का विविक्तकर (discriminant) कहलाता है तथा इसे प्राय: D से व्यक्त किया जाता है।

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1: $x^2 + 4x + 20 = 0$ के हल ज्ञात कीजिए।

हल : हम देखते हैं कि

$$b^2 - 4ac = 16 - 80 = -64 < 0$$

इस प्रकार, $$x = \frac{-4 \pm i\sqrt{64}}{2} = \frac{-4 \pm 8i}{2}$$

अर्थात्, $x = -2 \pm 4i$

अतः दिए हुए समीकरण के हल $-2+4i$ तथा $-2-4i$ हैं।

हम देखते हैं कि दोनों मूल एक दूसरे के संयुग्मी हैं।

उदाहरण 2 : $\frac{3}{7}x^2-\frac{5}{2}x+7=0$ के मूल ज्ञात कीजिए।

हल : पुनः, $b^2-4ac=\frac{25}{4}-12=-\frac{23}{4}<0$

इस प्रकार,
$$x=\frac{\frac{5}{2}\pm i\sqrt{\frac{23}{4}}}{\frac{6}{7}}=\frac{\frac{5}{2}\pm\frac{\sqrt{23}}{2}i}{\frac{6}{7}}$$

अर्थात्,
$$x=\frac{1}{12}(35\pm 7\sqrt{23}\,i)$$

अतः दिए हुए समीकरण के मूल

$$\frac{1}{12}(35+7\sqrt{23}\,i) \text{ तथा } \frac{1}{12}(35-7\sqrt{23}\,i) \text{ हैं।}$$

पुनः, हम देखते हैं कि दोनों मूल एक दूसरे के संयुग्मी हैं।

3.4 द्विघात समीकरणों के मूलों की प्रकृति (Nature of the Roots of Quadratic Equations)

हम पहले ही जानते हैं कि किसी द्विघात समीकरण के मूलों को प्राप्त किए बिना ही हम केवल D $(D\geqslant 0)$ का मान प्राप्त करके उसके मूलों की प्रकृति (nature) के विषय में बता सकते हैं। जब $D<0$ है तो हम देखते हैं कि मूल अवास्तविक सम्मिश्र हैं तथा साथ ही वे एक दूसरे के संयुग्मी हैं। हम उपरोक्त तथ्यों को संक्षेप में नीचे एक सारणी के रूप में दे रहे हैं :

द्विघात समीकरण : $ax^2+bx+c=0,\ a\neq 0,\ a, b, c\in R$

विविक्तकर : $D=b^2-4ac$

D का मान	द्विघात सूत्र	मूलों की प्रकृति
$D>0$	$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	वास्तविक तथा असमान
$D=0$	$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	वास्तविक तथा समान
$D<0$	$\frac{-b\pm i\sqrt{-(b^2-4ac)}}{2a}$	अवास्तविक सम्मिश्र तथा परस्पर संयुग्मी

आइए एक उदाहरण लें।

उदाहरण 1: निम्नलिखित द्विघात समीकरणों के मूल निर्धारित किए बिना उनकी प्रकृति बताइए :

(i) $2y^2-2\sqrt{6}\,y+3=0$

(ii) $\sqrt{3}\,y^2-3y+y^2-3=\sqrt{3}$

(iii) $12x^2-10\sqrt{2}\,x+7=0$

हल : (i) $2y^2-2\sqrt{6}\,y+3=0$

$$D=b^2-4ac=(2\sqrt{6})^2-4\,(2)\,(3)=0$$

इस प्रकार, मूल वास्तविक तथा समान हैं।

(ii) $\sqrt{3}\,y^2-3y+y^2-3=\sqrt{3}$

अर्थात्, $(\sqrt{3}+1)\,y^2-3y-3-\sqrt{3}=0$

यहाँ, $a=\sqrt{3}+1,\ b=-3,\ c=-3-\sqrt{3}$ हैं।

इस प्रकार, $D=b^2-4ac=9-4(\sqrt{3}+1)(-3-\sqrt{3})=9+4\sqrt{3}\,(\sqrt{3}+1)^2$

अर्थात्, $D>0$

अतः, मूल वास्तविक तथा असमान हैं।

(iii) $12x^2-10\sqrt{2}\,x+7=0$

$$D=(10\sqrt{2})^2-4\,(12)(7)=200-336<0$$

अतः, मूल अवास्तविक सम्मिश्र तथा परस्पर संयुग्मी हैं।

3.5 द्विघात समीकरण के मूलों तथा गुणांकों में सम्बन्ध (Relation between Roots and Coefficients of a Quadratic Equation)

मान लीजिए, हम द्विघात् समीकरण $ax^2+bx+c=0,\ a\neq 0,\ a, b, c \in R$, के दो मूलों को r_1 तथा r_2 से व्यक्त करते हैं। हम पहले देख चुके हैं कि यदि $D=b^2-4ac\geqslant 0$, तो

$$r_1+r_2=-\frac{b}{a}=-\frac{x \text{ का गुणांक}}{x^2 \text{ का गुणांक}}$$

तथा $$r_1r_2=\frac{c}{a}=\frac{\text{स्थिरांक पद}}{x^2 \text{ का गुणांक}}$$

अब हम $D<0$ की स्थिति में मूलों का योग तथा गुणनफल ज्ञात करते हैं।

जब $D<0$, तो समीकरण के मूल निम्न हैं :

$$\frac{-b+i\sqrt{-D}}{2a}\ (=r_1) \text{ तथा } \frac{-b-i\sqrt{-D}}{2a}\ (=r_2)$$

अब, $$r_1+r_2=\frac{-b+i\sqrt{-D}-b-i\sqrt{-D}}{2a}=-\frac{b}{a}$$

तथा $r_1 r_2 = \dfrac{(-b+i\sqrt{-D})(-b-i\sqrt{-D})}{4a^2} = \dfrac{b^2-i^2(\sqrt{-D})^2}{4a^2}$

$$= \frac{b^2+(-D)}{4a^2} = \frac{b^2-(b^2-4ac)}{4a^2} = \frac{c}{a}$$

इस प्रकार, b^2-4ac के सभी मानों के लिए

मूलों का योग $= r_1+r_2 = \dfrac{-b}{a} = -\dfrac{x \text{ का गुणांक}}{x^2 \text{ का गुणांक}}$

मूलों का गुणनफल $= r_1 r_2 = \dfrac{c}{a} = \dfrac{\text{स्थिरांक पद}}{x^2 \text{ का गुणांक}}$

उपरोक्त से हम देखते हैं कि किसी द्विघात समीकरण के मूल निर्धारित किए बिना उनका योग तथा गुणनफल प्राप्त करना सम्भव है।

उदाहरणार्थ, अनुच्छेद 3.3 के उदाहरण 1 में समीकरण $x^2+4x+20=0$ है।

मूलों का योग $= -\dfrac{4}{1} = -4$

तथा मूलों का गुणनफल $= \dfrac{20}{1} = 20$

आइए, जाँच कर के देखें कि वस्तुतः ऐसा है या नहीं। इस समीकरण के प्राप्त किए गए मूल $-2+4i$ तथा $-2-4i$ थे।

अतः, योग $= (-2+4i)+(-2-4i) = -4$

गुणनफल $= (-2+4i)(-2-4i) = (-2)^2-(4i)^2 = 20$

अब, आइए अनुच्छेद 3.3 का उदाहरण 2 देखें। दिया हुआ समीकरण

$\dfrac{3}{7}x^2 - \dfrac{5}{2}x + 7 = 0$ है।

मूलों का योग $= -\dfrac{\frac{-5}{2}}{\frac{3}{7}} = \dfrac{35}{6}$

तथा मूलों का गुणनफल $= \dfrac{7}{\frac{3}{7}} = \dfrac{49}{3}$

[इस समीकरण के प्राप्त किए गए मूल $\dfrac{1}{12}(35 \pm 7\sqrt{23}\,i)$ थे।

पाठक जाँच करके देखें कि इन संख्याओं का योग $\dfrac{35}{6}$ तथा गुणनफल $\dfrac{49}{3}$ है।]

हम कुछ और उदाहरण लेते हैं।

उदाहरण 1 : यदि α, β समीकरण $ax^2+bx+c=0$ के मूल हैं तो निम्न के मान ज्ञात कीजिए :

(i) $\alpha^2+\beta^2$ (ii) $\alpha^3+\beta^3$ (iii) $\alpha^4+\beta^4+\alpha^2\beta^2$ (iv) $\dfrac{\alpha+\beta}{\alpha^{-1}+\beta^{-1}}$

हल : क्योंकि α, β समीकरण $ax^2+bx+c=0$ के मूल हैं,

अतः, $\alpha+\beta=\dfrac{-b}{a}$ तथा $\alpha\beta=\dfrac{c}{a}$

(i) अब, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$$=\left(\frac{-b}{a}\right)^2-2\left(\frac{c}{a}\right)=\frac{b^2}{a^2}-\frac{2c}{a}=\frac{b^2-2ac}{a^2}$$

(ii) $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$$=\left(-\frac{b}{a}\right)^3-\frac{3c}{a}\left(-\frac{b}{a}\right)$$

$$=-\frac{b^3}{a^3}+\frac{3bc}{a^2}=\frac{3abc-b^3}{a^3}$$

(iii) $\alpha^4+\beta^4+\alpha^2\beta^2=(\alpha^2+\beta^2)^2-\alpha^2\beta^2$

$$=[(\alpha+\beta)^2-2\alpha\beta]^2-(\alpha\beta)^2$$

$$=\left[\left(-\frac{b}{a}\right)^2-\frac{2c}{a}\right]^2-\frac{c^2}{a^2}$$

$$=\left(\frac{b^2-2ac}{a^2}\right)^2-\frac{c^2}{a^2}$$

$$=\frac{b^4+3a^2c^2-4ab^2c}{a^4}$$

(iv) $\dfrac{\alpha+\beta}{\alpha^{-1}+\beta^{-1}}=\dfrac{\alpha+\beta}{\dfrac{1}{\alpha}+\dfrac{1}{\beta}}=\alpha\beta=\dfrac{c}{a}$

उदाहरण 2 : द्विघात समीकरण $x^2+kx+12=0$ के मूल r_1 तथा r_2 ऐसे हैं कि $r_1-r_2=1$ है। k का मान निर्धारित कीजिए।

हल : $x^2+kx+12=0$

क्योंकि r_1 तथा r_2 इस समीकरण के मूल हैं,

अतः, $r_1+r_2=-k$

तथा $r_1r_2=12$

अब, $(r_1-r_2)^2=(r_1+r_2)^2-4r_1r_2$

अर्थात्, $1 = k^2 - 48$

इस प्रकार, $k^2 - 49 = 0$, अतः $k = \pm 7$

उदाहरण 3: दिए हुए समीकरण $x^2 + px + q = 0$ में p तथा q का मान निर्धारित कीजिए ताकि समीकरण के मूल p तथा q हों।

हल: मान लीजिए, α तथा β समीकरण $x^2 + px + q = 0$ के मूल हैं।

तब, $\alpha + \beta = -p$ (1)

तथा $\alpha\beta = q$ (2)

यदि p तथा q मूल हैं तो उनका योग $p+q$ तथा गुणनफल pq होगा।

(1) तथा (2) में प्रतिस्थापन करने पर हम निम्न प्राप्त करते हैं :

$$\left.\begin{aligned} p+q &= -p \\ \text{तथा} \quad pq &= q \end{aligned}\right\} \quad (3)$$

यदि $q \neq 0$ तो $p = 1$ तथा $q = -2p$, अतः $q = -2$

यदि $q = 0$, तो $p.0 = 0$ तथा $0 = -2p$, अतः $p = 0$

अतः दी हुई समस्या के दो हल $p = 0, q = 0$ तथा $p = 1, q = -2$ हैं।

प्रश्नावली 3.2

निम्नलिखित समीकरणों के मूल ज्ञात कीजिए :

1. $x^2 - 6x + 13 = 0$
2. $y^2 + 16y + 73 = 0$
3. $9x^2 - 12x + 5 = 0$
4. $x^2 - 2\sqrt{5}x + 14 = 0$
5. $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$
6. $\frac{3}{2}x^2 + 4\sqrt{3}x + 4 = 0$
7. $19x^2 - 6\sqrt{2}x + 2 = 0$
8. $2\sqrt{3}y^2 - 7y + 6\sqrt{3} = 0$
9. $(y+1)(y-3) + 7 = 0$
10. $\left(\frac{7x^2+1}{x^2-1}\right) - 4\left(\frac{x^2-1}{7x^2+1}\right) + 3 = 0$

निम्नलिखित द्विघात समीकरणों के मूल निर्धारित किए बिना उनकी प्रकृति बताइए :

11. $5x^2-6x+7=0$

12. $4\sqrt{3}x^2-9x+3\sqrt{3}=0$

13. $\frac{3}{4}x^2-\frac{2}{3}x+1=0$

14. $\frac{4}{9}x^2-x+\frac{9}{16}=0$

15. $(x-2a)(x-2b)=4ab$

16. $y^2+y+5=0$

k के किन मानों के लिए निम्नलिखित में से प्रत्येक समीकरण के मूल वास्तविक तथा समान होंगे ?

17. $kx^2-12x+9=0$

18. $9x^2+kx+1=0$

19. $y^2+10y+k=0$

*20. $k\ (k>0)$ का मान निर्धारित कीजिए ताकि दोनों समीकरणों, $x^2+k\,x+64=0$ तथा $x^2-8\,x+k=0$ के मूल वास्तविक हों।

निम्नलिखित में से प्रत्येक समीकरण के मूल निर्धारित किए बिना उसके मूलों के योग तथा गुणनफल लिखिए :

21. $x^2=1$

22. $4\sqrt{3}\,x^2-7\,x+\sqrt{3}=0$

23. $-\frac{2}{3}x^2-\frac{4}{15}x+9=0$

यदि α तथा β समीकरण $a\,x^2+b\,x+c=0$, $a\neq0$ के मूल हों, तो निम्नलिखित के मान ज्ञात कीजिए :

24. $\alpha^2-\beta^2$

25. $\frac{1}{\alpha}+\frac{1}{\beta}$

26. $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

27. $\alpha^4+\beta^4$

28. $\alpha^6+\beta^6$

29. द्विघात समीकरण $5\,x^2-p\,x+1=0$ के मूल r_1 तथा r_2 ऐसे हैं कि $r_1-r_2=1$ है। p का मान ज्ञात कीजिए।

30. यदि α तथा β समीकरण $x^2-3ax+a^2=0$ के मूल हों तथा $\alpha^2+\beta^2=\frac{7}{4}$ हो तो a का मान निर्धारित कीजिए।

3.6 द्विघात समीकरणों (वास्तविक गुणांक) का बनाना [Forming Quadratic Equations (Real coefficients)]

दिए हुए द्विघात समीकरण के मूल निर्धारित करना हमने सीख लिया है। पिछली कक्षाओं में हमने यह भी देखा था कि यदि मूल दिए हों तो द्विघात समीकरण कैसे बनाई जाती है। यदि α तथा β दिए हुए मूल हों तो हम तुरन्त अभीष्ट समीकरण* निम्न रूप से लिखते हैं :

$$(x-\alpha)(x-\beta)=0$$

अर्थात्, $x^2-(\alpha+\beta)\,x+\alpha\,\beta=0$

अथवा, x^2-(मूलों का योग) $x+$(मूलों का गुणनफल) $=0$

हमने α तथा β के वास्तविक मानों के लिए ऐसा किया था। अनुच्छेद 3.5 में हमने यह भी देखा है कि जब α तथा β सम्मिश्र तथा अवास्तविक हों तो वे परस्पर संयुग्मी होते हैं। मूलों के योग तथा गुणनफल के सूत्र यहाँ भी लागू होते हैं। इस प्रकार यदि मूल दिए हों तो द्विघात समीकरण लिखने की विधि इस स्थिति में भी सत्य होगी।

हम कैसे जानते हैं कि इस प्रकार प्राप्त किए गए द्विघात समीकरण के गुणांक वास्तविक होंगे? आपको याद होगा कि यदि α तथा β सम्मिश्र एवं परस्पर संयुग्मी हों तो वस्तुतः उनके योग तथा गुणनफल वास्तविक होते हैं।

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : वास्तविक गुणांकों का द्विघात समीकरण प्राप्त कीजिए जिसका एक मूल $1+3\,i$ हो।

हल : दूसरा मूल अवश्य ही $1-3\,i$ होना चाहिए। (क्यों ?)

इस प्रकार, मूलों का योग $=(1+3\,i)+(1-3\,i)=2$

तथा, मूलों का गुणनफल $=(1+3\,i)\,(1-3\,i)=10$

अतः, अभीष्ट समीकरण $x^2-2\,x+10=0$ है।

उदाहरण 2 : द्विघात समीकरण ज्ञात कीजिए जिसके मूल

$$\frac{6}{5}\pm\frac{1}{3}i \quad \text{हैं।}$$

हल : मूलों का योग $=\left(\frac{6}{5}+\frac{1}{3}i\right)+\left(\frac{6}{5}-\frac{1}{3}i\right)=\frac{12}{5}$

मूलों का गुणनफल $=\left(\frac{6}{5}+\frac{1}{3}i\right)\left(\frac{6}{5}-\frac{1}{3}i\right)=\frac{349}{225}$

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*वास्तव में, $k\,(x-\alpha)\,(x-\beta)=0$, जहाँ k एक शून्येतर वास्तविक संख्या है, के मूल भी α और β हैं। परन्तु हम इसे $(x-\alpha)\,(x-\beta)=0$ के समान ही मानेंगे और इसी कारण हम एक ही द्विघात समीकरण की बात करते हैं।

इस प्रकार, अभीष्ट समीकरण $x^2-\frac{12}{5}x+\frac{349}{225}=0$

अर्थात् $225x^2-540x+349=0$ है।

उदाहरण 3: यदि α तथा β समीकरण $3x^2-2x+5=0$ के मूल हों, तो वह समीकरण ज्ञात कीजिए जिसके मूल $\frac{\alpha}{\beta}$ तथा $\frac{\beta}{\alpha}$ हैं।

हल : क्योंकि α तथा β समीकरण $3x^2-2x+5=0$ के मूल हैं,

अतः, $\alpha+\beta=\frac{2}{3}$

तथा $\alpha\beta=\frac{5}{3}$

क्योंकि $\alpha\beta=\frac{5}{3}$ है, अतः स्पष्ट है कि न तो α तथा न ही β शून्य हो सकता है।

अब, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=-\frac{26}{15}$

तथा $\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha}=1$

अतः अभीष्ट समीकरण $x^2+\frac{26}{15}x+1=0$

अर्थात् $15x^2+26x+15=0$ है।

उदाहरण 4: यदि α तथा β द्विघात समीकरण $ax^2+bx+c=0$ के मूल हों, तो सिद्ध कीजिए कि 2α तथा 2β समीकरण $ax^2+2bx+4c=0$ के मूल होंगे।

हल : क्योंकि α तथा β समीकरण $ax^2+bx+c=0$ के मूल हैं,

अतः, $\alpha+\beta=-\frac{b}{a}$

तथा $\alpha\beta=\frac{c}{a}$

अब, $2\alpha+2\beta=2(\alpha+\beta)=-\frac{2b}{a}$

तथा $2\alpha(2\beta)=4\alpha\beta=\frac{4c}{a}$

अतः, अभीष्ट समीकरण $x^2+\frac{2b}{a}x+\frac{4c}{a}=0$

अर्थात् $ax^2+2bx+4c=0$ है।

प्रश्नावली 3.3

वास्तविक गुणांकों के वे द्विघात समीकरण ज्ञात कीजिए जिनके मूल निम्न हैं :

1. $\frac{3}{5}, \frac{-4}{3}$
2. $7-3i$
3. $1+\sqrt{3}, 1-\sqrt{3}$
4. $-7+8i$
5. $3, 1+\sqrt{2}$
6. $4 \frac{1}{4}$
7. $\frac{m-n}{m+n}, -\frac{m+n}{m-n}$
8. $\pm i(a-b)$

*9. $\frac{1}{10-\sqrt{72}}, \frac{1}{10+6\sqrt{2}}$

10. यदि α तथा β द्विघात समीकरण $ax^2+bx+c=0$ के मूल (शून्येतर), हैं तो वे समीकरण ज्ञात कीजिए जिनके मूल निम्न हैं :

    (i) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

    (ii) $\frac{1}{\alpha}, \frac{1}{\beta}$

11. यदि α तथा β द्विघात समीकरण $ax^2+bx+c=0$ के मूल हैं तो सिद्ध कीजिए कि α^3 तथा β^3 समीकरण $a^3x^2+(b^3-3abc)x+c^3=0$ के मूल हैं।

3.7 सम्मिश्र संख्याओं के वर्गमूल (Square Roots of Complex Numbers)

हम देख चुके हैं कि प्रत्येक (शून्येतर) वास्तविक संख्या के दो वर्गमूल होते हैं। आइए, अब देखें कि क्या हम सम्मिश्र संख्याओं के वर्गमूल ज्ञात कर सकते हैं। उदाहरणार्थ, सम्मिश्र संख्या $-48-14i$ पर विचार कीजिए।

मान लीजिए $a+bi$ संख्या $-48-14i$ का एक वर्गमूल है।

तब, $(a+bi)^2 = -48-14i$ (1)

(1) के दोनों पक्षों के वास्तविक तथा काल्पनिक भागों को बराबर करने पर हम निम्न प्राप्त करते हैं:

$$a^2-b^2 = -48 \quad (2)$$

तथा, $$2ab = -14 \quad (3)$$

हम यह भी जानते हैं कि (1) के दोनों पक्षों की संख्याओं के मापांक समान होने चाहिए।

अर्थात्, $|(a+bi)^2| = |-48-14i|$

परन्तु, $|(a+bi)^2| = |(a+bi)|^2 = a^2+b^2$ [प्रश्न 18, प्रश्नावली 2.2]

इस प्रकार, $a^2+b^2 = \sqrt{48^2+14^2} = 50$ (4)*

(2) तथा (4) से,

$a^2 = 1$, अत: $a = \pm 1$

(3) में प्रतिस्थापित करने पर,

$b = \mp 7$

[पाठक को चाहिए कि वह जाँच करे कि $(1-7i)^2 = -48-14i$ तथा $(-1+7i)^2 = -48-14i$ है।]

इस प्रकार, सम्मिश्र संख्या $-48-14i$ के दो वर्गमूल $1-7i$ तथा $-1+7i$ हैं।

हम उपरोक्त विधि से सिद्ध कर सकते हैं कि **प्रत्येक शून्येतर सम्मिश्र संख्या के ठीक दो वर्गमूल होते हैं। यदि हम इनमें से एक को w (w सम्मिश्र) से व्यक्त करें तो दूसरा $-w$ होगा। [इसकी उपपत्ति पाठक के अभ्यासार्थ छोड़ दी गई है।]**

अब हम एक अन्य उदाहरण पर विचार करते हैं।

उदाहरण : $2+3i$ का वर्गमूल ज्ञात कीजिए।

हल : मान लीजिए $x+yi$ एक वर्गमूल है।

तब, $(x+yi)^2 = 2+3i$ (1)

वास्तविक तथा काल्पनिक भागों को बराबर करने पर,

$x^2-y^2 = 2$ (2)

तथा $2xy = 3$ (3)

पुन: $|(x+yi)^2| = |2+3i|$

अर्थात्, $x^2+y^2 = \sqrt{13}$ (4)

(2) तथा (4) से हम देखते हैं कि

$2x^2 = 2+\sqrt{13}$

अत:, $x = \pm \sqrt{\dfrac{2+\sqrt{13}}{2}}$

(3) के दोनों पक्षों का वर्ग करने पर तथा x^2 का मान प्रतिस्थापित करने पर हम निम्न प्राप्त करते हैं :

$$y^2 = \frac{9}{\dfrac{4(2+\sqrt{13})}{2}} = \frac{9}{2}\left[\frac{\sqrt{13}-2}{(\sqrt{13}+2)(\sqrt{13}-2)}\right] = \frac{\sqrt{13}-2}{2}$$

जिससे $y = \pm \sqrt{\dfrac{\sqrt{13}-2}{2}}$

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* (4) नया समीकरण नहीं है। यह (2) और (3) से भी प्राप्त किया जा सकता है, क्योंकि $(a^2+b^2)^2 = (a^2-b^2)^2 + 4a^2b^2$

अतः सम्मिश्र संख्या $2+3i$ के दो वर्गमूल

$$\sqrt{\frac{\sqrt{13}+2}{2}}+\sqrt{\frac{\sqrt{13}-2}{2}}\,i \text{ तथा } -\sqrt{\frac{\sqrt{13}+2}{2}}-\sqrt{\frac{\sqrt{13}-2}{2}}\,i \text{ हैं।}$$

प्रश्नावली 3.4

निम्नलिखित सम्मिश्र संख्याओं के वर्गमूल ज्ञात कीजिए :

1. $-4-3i$
2. $5-12i$
3. $-2i$
4. $7+24i$
5. $-21-20i$
6. $-2+2\sqrt{3}\,i$
7. $9-40i$
8. i
9. $-i$

3.8 सम्मिश्र गुणांकों के द्विघात समीकरणों के मूल (Roots of Quadratic Equations with Complex Coefficients)

आइए, अब द्विघात समीकरण $a x^2+b x+c=0, a\neq 0, a, b, c \in C$ पर विचार करें। हम इस समीकरण के मूल (अर्थात् हल) प्राप्त करना चाहते हैं।

दिए हुए समीकरण को हम इस प्रकार लिखते हैं :

$$a x^2+b x=-c$$

अर्थात्, $$x^2+\frac{b}{a}x=-\frac{c}{a} \qquad (1)$$

(1) के दोनों पक्षों में हम $\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2$ जोड़ते हैं (क्यों ?) तथा निम्न प्राप्त करते हैं :

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2} \qquad (2)$$

क्योंकि सम्मिश्र संख्याएँ योग, व्यवकलन, गुणन तथा विभाजन की संक्रियाओं के लिए संवृत होती हैं, अतः $\frac{b^2-4ac}{4a^2}$ एक सम्मिश्र संख्या है। हम पुनः b^2-4ac को विविक्तकर कहते हैं तथा इसे D से व्यक्त करते हैं।

यदि $D=b^2-4ac=0$, तो (2) का दक्षिण पक्ष $0+0i$ है तथा इसका वर्गमूल $0+0i$ है। इस स्थिति में हम पुनरावृत्त मूल (**repeated root**)

$$x=-\frac{b}{2a} \text{ प्राप्त करते हैं।}$$

यदि $D=b^2-4ac\neq 0$, तो $\frac{b^2-4ac}{4a^2}$ के दो वर्गमूल हैं। (क्यों ?) यदि हम b^2-4ac के दो वर्गमूलों को α तथा $-\alpha$ से व्यक्त करें, तो $\frac{b^2-4ac}{4a^2}$ के दो वर्गमूल $\frac{\alpha}{2a}$ तथा $-\frac{\alpha}{2a}$ होंगे।

अतः (2) से,

$$x+\frac{b}{2a}=\pm\frac{\alpha}{2a}$$

अथवा, $x=\frac{-b\pm\alpha}{2a}$

अर्थात्, $x=\frac{-b+\alpha}{2a}$, $\frac{-b-\alpha}{2a}$

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1: $x^2-(5-i)x+(18+i)=0$ के मूल ज्ञात कीजिए।

हल: आइए, सबसे पहले $D=b^2-4ac$ ज्ञात करें।

$$D=b^2-4ac=(5-i)^2-4(18+i)$$
$$=-48-14i$$

हम देखते हैं कि $D\neq 0$। हम अनुच्छेद 3.7 में देख चुके हैं कि $-48-14i$ के दो वर्गमूल $1-7i$ तथा $-1+7i$ हैं।

अतः, $x=\frac{-b+(1-7i)}{2a}$ तथा $x=\frac{-b+(-1+7i)}{2a}$

अब, $-b=(5-i)$ तथा $2a=2$

इस प्रकार, $x=3-4i$ तथा $2+3i$ है।

दूसरे शब्दों में, अभीष्ट हल $3-4i$ तथा $2+3i$ हैं।

आइए, जाँच करके देखें।

क्या $(3-4i)^2-(5-i)(3-4i)+(18+i)=0+0i$ है ? आइए, देखें।

वाम पक्ष $=9-24i-16-15+23i+4+18+i=0+0i$

अतः, वस्तुतः $3-4i$ एक हल है।

[पाठक के लिए यह अभ्यासार्थ छोड़ दिया गया है कि वह जाँच करे कि $2+3i$ भी एक हल है।]

उदाहरण 2: $x^2-3ix+4=0$ को C पर हल कीजिए।

हल: $D=b^2-4ac=-9-16=-25$

D एक ऋणात्मक वास्तविक संख्या है। इसके दो वर्गमूल $5i$ और $-5i$ हैं।

इस प्रकार, $x=\frac{3i+5i}{2}$ तथा $\frac{3i-5i}{2}$ है।

अतः, अभीष्ट हल $4i$ तथा $-i$ हैं।

[यह जाँच करके देखना कि वस्तुतः $4i$ तथा $-i$ दो हल हैं—पाठक के अभ्यासार्थ छोड़ दिया गया है।

प्रश्नावली 3.5

निम्नलिखित समीकरणों को C पर हल कीजिए :

1. $x^2-ix+6=0$
2. $ix^2-4x-4i=0$
3. $x^2-(7-i)x+(18-i)=0$
4. $2x^2-(3+7i)x+(9i-3)=0$
5. $x^2-(3\sqrt{2}+2i)x+6\sqrt{2}i=0$
6. $y^2-y+1+i=0$
7. $ix^2-3x-2i=0$
8. $(3x-i)^2+4=0$
9. $x^2=-2i$
10. $x^2=7+24i$
11. $y^2=-21-20i$

3.9 संख्या-निकाय का और विस्तार ?

हम देख चुके हैं कि हमें वास्तविक संख्याओं के निकाय का सम्मिश्र संख्याओं में विस्तार करने की आवश्यकता पड़ी ताकि हम $x^2=-3$ जैसे समीकरणों के हल प्राप्त करने में समर्थ हो सकें। हमने देखा कि यह निकाय सम्मिश्र गुणांकों के द्विघात समीकरणों को हल करने में भी पर्याप्त सिद्ध हुआ। यदि हमें उच्चतर घात, उदाहरणार्थ, त्रिघात (घात-3) (cubic), चतुर्थघात (घात-4) (biquadratic), इत्यादि समीकरणों के हल प्राप्त करने हों तो क्या हमें संख्या-निकाय का और अधिक विस्तार करने की आवश्यकता होगी ? नहीं। यह सिद्ध किया जा सकता है कि सम्मिश्र संख्याओं का निकाय वास्तविक अथवा सम्मिश्र गुणांकों के किसी भी घात के सभी बहुपद समीकरणों (polynomial equations) को हल करने के लिए पर्याप्त है। परन्तु इस कथन को सिद्ध करना इस पुस्तक की सीमा के बाहर है।

3.10 ऐतिहासिक दृष्टिकोण

द्विघात समीकरणों की धारणा वस्तुतः अति पुरातन है। निश्चय ही लगभग 4000 वर्ष पूर्व बेबीलोन के निवासी द्विघात समीकरणों के विषय में जानते थे। लगभग 1600 ई० पू० के शिलालेख, जिन्हें 'येल टेबलेटस' (Yale Tablets) कहा जाता है, उपलब्ध हैं जिनमें द्विघात समीकरणों से सम्बन्धित अनेकों बिना हल की हुई समस्याएँ अंकित हैं।

यूनान के लोगों ने भी द्विघात समीकरणों का प्रयोग ज्यामितीय समस्याओं के हल करने में किया। सुप्रसिद्ध गणितज्ञ यूक्लिड (जन्म लगभग 365 ई० पू०) ने अपनी कृतियों में अनेकों द्विघात समीकरण दिए हैं।

द्विघात समीकरणों में हिन्दू गणितज्ञों का योगदान भी अति महत्वपूर्ण तथा विस्तृत है। यह कहा जाता है कि हिन्दुओं ने लगभग 500 ई० पू० में 'शुल्ब-सूत्र' के समय में अपने पूजा-स्थलों का निर्माण समीकरण $ax^2+bx-c=0$ के हलों के आधार पर किया था। आर्यभट्ट (जन्म 476 ई०) ने उस गुणोत्तर श्रेणी (geometric series) जिसमें द्विघात समीकरणों के हल सम्बद्ध हैं, के योग का नियम दिया है। ब्रह्मगुप्त (जन्म 598 ई०) ने उस द्विघात समीकरण का नियम दिया है जो द्विघात सूत्र की तरह ही है। लगभग 850 ई० में महावीर ने एक समस्या प्रस्तुत की थी जिसमें एक द्विघात समीकरण तथा उसके हल का प्रयोग सम्बद्ध था। लगभग 1025 ई० में हिन्दू गणितज्ञ श्रीधर ने सर्वप्रथम द्विघात समीकरण के हल के लिए नियम दिया था जो 'हिन्दू-नियम', (Hindu Rule) कहलाता है। यही नियम आजकल वर्ग पूर्ण करने की विधि (method of completing the square) कहलाता है।

अरब गणितज्ञ अल-ख्वारिज़िमी ने लगभग 805 ई० में द्विघात समीकरण को हल करने की दो व्यापक विधियों का वर्णन किया है। वह दोनों विधियाँ यूनानियों की कृतियों पर आधारित हैं। लगभग 1100 ई० में उमर-खय्याम ने भी द्विघात समीकरण को हल करने का नियम दिया था।

यह जानना अत्यन्त रुचिकर है कि बेबीलोन तथा मिस्र के लोगों के समय से लेकर लगभग 1550 ई० तक जिन समीकरणों के हल किए जाते थे उनके सभी के संख्यात्मक गुणांक (numerical coefficients) थे। यद्यपि बहुत से गणितज्ञों ने जान लिया था कि संख्यात्मक गुणांकों के एक समुच्चय की विधि दूसरे समुच्चयों के लिए भी लागू होगी परन्तु उन्होंने इसे व्यापक रूप से सिद्ध नहीं किया। बीजीय समीकरणों (algebraic equations) में अक्षर संख्याओं (literal numbers) का व्यापक गुणांकों के रूप में प्रयोग करने की सक्षम तथा दूर-गामी धारणा का श्रेय महान फ्रांसीसी गणितज्ञ फ्रेंकोइस-वीटा (1540-1603 ई०) को जाता है। इससे भी अधिक आश्चर्यजनक बात यह है कि वीटा एक वकील थे जो फ्रांस के राजाओं के लिए काम करते थे। गणित उनके लिए केवल एक शौक था। अपने इस शौक के लिए उन्हें कितना अधिक 'परिश्रम' करना पड़ा होगा ?

अरबों के पश्चात् लगभग 800 वर्षों तक द्विघात समीकरणों के सम्बन्ध में कुछ महत्वपूर्ण कार्य नहीं हुआ। गुणनखण्डों के प्रयोग से द्विघात समीकरण के विषय में पहला महत्वपूर्ण कार्य थामस हैरियट (1560-1621 ई०) की कृतियों में मिलता है। आधुनिक समय में द्विघात समीकरणों में योगदान के लिए जो व्यक्ति विशेष उल्लेख के अधिकारी हैं उनमें स्विटज़रलैंड के गणितज्ञ लियोनार्ड ऑयलर (1707-1783), फ्रांस के गणितज्ञ ई० देबौत (1730-1783) तथा ब्रिटिश गणितज्ञ जे० जे० सिलवैस्टर (1814-1897) हैं।

3.11 द्विघात असमीकरण (Quadratic Inequations)

अब हम वास्तविक गुणांकों के एक चर में द्विघात असमीकरणों के हल ज्ञात करना सीखेंगे। एक चर में द्विघात असमीकरण निम्न प्रकार का एक व्यंजक (expression) होता है :

$$ax^2+bx+c\leqslant 0 \text{ (अथवा} <0) \qquad (1)$$

$$ax^2+bx+c\geqslant 0 \text{ (अथवा} >0) \qquad (2)$$

जहाँ a, b तथा c कोई वास्तविक संख्याएँ हैं तथा $a\neq 0$ है।

x के वे मान जो दिए हुए असमीकरण को संतुष्ट करते हैं, असमीकरण के हल कहलाते हैं। हल ज्ञात करने की दो विधियाँ हैं :

विधि I : तदनुरूपी बहुपद के गुणनखंड करके—यह बीजगणित की विधि कहलाती है।

विधि II : इस विधि में असमीकरण का आलेख खींचा जाता है। यह आलेखीय विधि कहलाती है।

उदाहरण 1 : निम्न असमीकरण के हल ज्ञात कीजिए :

$$x^2+3x-18\geqslant 0$$

हल :

विधि 1 : बीजगणित की विधि

$x^2+3x-18\geqslant 0$ (दिया हुआ है।)

अर्थात् $(x-3)(x+6)\geqslant 0$

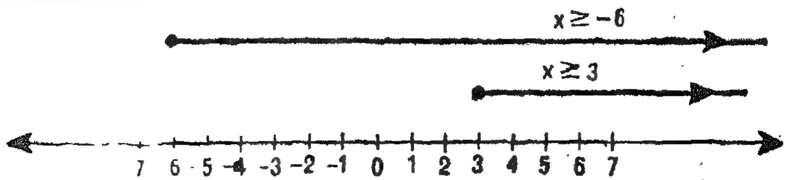
अब यह गुणनफल दो ही संभावनाओं में ऋणेतर हो सकता है।

स्थिति 1 : $(x-3)\geqslant 0$ तथा $(x+6)\geqslant 0$

अर्थात्, $x\geqslant 3$ तथा $x\geqslant -6$ (1)

हम देखते हैं कि (1) के दोनों असमीकरणों को एक साथ संतुष्ट होने के लिए यह आवश्यक है कि $x\geqslant 3$ हो। (क्यों ?)

[पाठक को चाहिए कि वह संख्या रेखा पर $x\geqslant 3$ तथा $x\geqslant -6$ आलेखित कर उनका प्रतिच्छेद ज्ञात करे। (देखिए आकृति 3.1)]



आकृति 3.1

स्पष्ट है, प्रतिच्छेद $x\geqslant 3$ है।

अतः स्थिति 1 के अनुसार वे सभी x जो $x\geqslant 3$ को संतुष्ट करते हैं असमीकरण के हल होंगे।

स्थिति 2 : $(x-3)\leqslant 0$ तथा $(x+6)\leqslant 0$ (2)

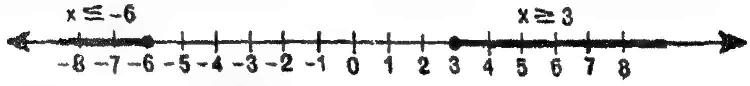
अर्थात्, $x\leqslant 3$ तथा $x\leqslant -6$

यह स्पष्ट है कि यदि $x\leqslant -6$ हो तो (2) के दोनों असमीकरण संतुष्ट हो जाते हैं।

अतः स्थिति 2 के अनुसार वे सभी x जो $x\leqslant -6$ को संतुष्ट करते हैं, असमीकरण के हल होंगे।

दोनों स्थितियों से हमें $x^2+3x-18\geqslant 0$ के सभी हल प्राप्त होते हैं। इन हलों में वे सभी x हैं जिनके लिए $x\leqslant -6$ या $x\geqslant 3$ है।

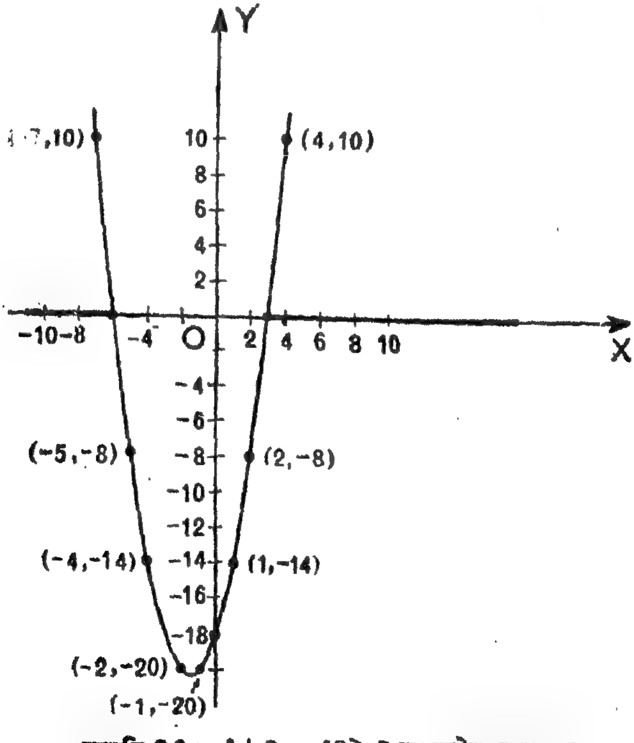
हम संख्या रेखा पर इन हलों को निम्न रूप से निरूपित करते हैं। (देखिए आकृति 3.2)



आकृति 3.2 : $x^2+3x-18\geqslant 0$ के हल

विधि II : आलेखीय विधि

संगत द्विघात फलन $f(x)=x^2+3x-18$ है। हम यह पहले सीख चुके हैं कि इस प्रकार के फलन का आलेख किस प्रकार खींचा जाता है। (देखिए आकृति 3.3) अब हमें x के वे मान चाहिए जिनके लिए



आकृति 3.3 : $x^2+3x-18\geqslant 0$ का आलेख तथा हल

$x^2+3x-18\geqslant 0$ है अर्थात्, $f(x)\geqslant 0$ अर्थात् $y\geqslant 0$ है। दूसरे शब्दों में, हम x के वे मान खोजते हैं जिनके लिए वक्र x-अक्ष पर या उससे ऊपर हो।

स्पष्ट है, x के ये मान 3 (3 सहित) के दाईं ओर या —6 (—6 सहित) के बाईं ओर होंगे।

अतः हलों में वे सभी x सम्मिलित होंगे जिनके लिए $x\leqslant -6$ या $x\geqslant 3$ हो।

उदाहरण 2 : निम्नलिखित असमीकरण के हल ज्ञात कीजिए :

$$15x^2+4x-4\leqslant 0$$

हल :

विधि I : $15x^2+4x-4\leqslant 0$ (दिया हुआ है)

अर्थात्, $(5x-2)(3x+2)\leqslant 0$ (क्यों ?)

अब यह गुणनफल धनेतर (non-positive) होने के लिए दो संभावनाएँ हो सकती हैं :

स्थिति 1 : $(5x-2)\geqslant 0$ तथा $(3x+2)\leqslant 0$

अर्थात्, $x\geqslant \frac{2}{5}$ तथा $x\leqslant -\frac{2}{3}$ (1)

निस्संदेह x का ऐसा कोई मान नहीं है जो (1) के दोनों असमीकरणों को एक साथ संतुष्ट करे। (क्यों ?)

अतः स्थिति 1 से कोई हल प्राप्त नहीं होता।

स्थिति 2 : $(5x-2)\leqslant 0$ तथा $(3x+2)\geqslant 0$

अर्थात्, $x\leqslant \frac{2}{5}$ तथा $x\geqslant -\frac{2}{3}$

या, $-\frac{2}{3}\leqslant x\leqslant \frac{2}{5}$

अतः हलों में वे सब x सम्मिलित हैं जिनके लिए

$$-\frac{2}{3}\leqslant x\leqslant \frac{2}{5}$$

हम आलेख द्वारा इन हलों को निम्न रूप से निरूपित करते हैं। (देखिए आकृति 3.4)

आकृति 3.4 : $15x^2+4x-4\leqslant 0$ के हल

विधि II : पाठक को तदनुरूपी फलन $f(x)=15x^2+4x-4$ का आलेख खींचना चाहिए तथा x के वे मान ज्ञात करने चाहिए जिनके लिए $f(x)\leqslant 0$ अर्थात् $y\leqslant 0$ हो। दूसरे शब्दों में, हम x के वे मान खोजते हैं जिनके लिए वक्र x—अक्ष पर या उससे नीचे हो।

प्रश्नावली 3.6

1. निम्न असमीकरणों के हल ज्ञात कीजिए। आलेखी विधि का प्रयोग कीजिए।

    (i) $15x^2+4x-4\geqslant 0$

    (ii) $x^2-7x+6>0$

    (iii) $4-x^2<0$

    (iv) $9-x^2\geqslant 0$

    (v) $x^2-2x+1\geqslant 0$

2. निम्न असमीकरणों के हल ज्ञात कीजिए। बीजगणित की विधि का प्रयोग कीजिए। साथ ही संख्या रेखा पर हलों को निरूपित कीजिए।

    (i) $x^2-2x+1<0$

    (ii) $x^2-4x-21\geqslant 0$

    (iii) $2-3x-2x^2\geqslant 0$

    (iv) $x^2+x-12\leqslant 0$

3.12 मुख्य संकल्पनाएँ

द्विघात सूत्र ($b^2-4ac<0$ की स्थिति के लिए)	सम्मिश्र गुणांकों के द्विघात समीकरण
सम्मिश्र संख्याओं के वर्गमूल	द्विघात असमीकरण

3.13 अग्रिम अध्ययन हेतु सुझाव

द्विघात समीकरणों तथा उच्च घातीय समीकरणों के सिद्धांतों का वर्णन निम्न पुस्तक में है :

[1] H.S. Hall and S.R. Knight : **Higher Algebra**, (4th edition) Macmillan and Company Ltd., London (U.K.), 1950.

रुचिपूर्ण ऐतिहासिक टिप्पणी सहित द्विघात समीकरणों के सुन्दर वर्णन के लिए निम्न पुस्तक देखें :

[2] Morris Kline: **Mathematics, A Cultural Approach**, Addison – Wesley Publishing Company, Inc., Massachusetts (U.S.A.), 1963.

हिन्दू गणितज्ञों का योगदान अनुच्छेद 1.12 में दी गई पुस्तकों [4] तथा [5] में उपलब्ध है। गणित के इतिहास की एक महत्वपूर्ण पुस्तक भी अनुच्छेद 1.12 में [7] में दी गई है।

विविध प्रश्नावली I

(एकक I, II तथा III पर)

1. निश्चित कीजिए कि निम्नलिखित में से कौन से कथन सत्य हैं :

   (*i*) प्रत्येक प्राकृत संख्या एक परिमेय संख्या होती है ।

   (*ii*) ऐसी दो प्राकृत संख्याएं हैं जिनका गुणनफल शून्य है ।

   (*iii*) किन्हीं दो विषम प्राकृत संख्याओं का योग विषम होता है ।

   (*iv*) विषम पूर्णांक व्यवकलन की संक्रिया के लिए संवृत नहीं होते ।

   (*v*) किन्हीं दो भिन्न पूर्णांकों के बीच हम सदैव एक अन्य पूर्णांक ज्ञात कर सकते हैं ।

   (*vi*) प्रत्येक दशमलव एक परिमेय संख्या को निरूपित करता है ।

   (*vii*) संख्या रेखा पर दो भिन्न बिन्दु एक ही वास्तविक संख्या को निरूपित नहीं कर सकते ।

   (*viii*) 0 एक परिमेय संख्या है ।

   (*ix*) परिमेय संख्याएँ गुणन की संक्रिया के लिए संवृत होती हैं ।

   (*x*) अपरिमेय संख्याएँ गुणन की संक्रिया के लिए संवृत होती हैं ।

2. क्या $2\sqrt{7} \times 3\sqrt{7}$ परिमेय संख्या है ? क्या $\dfrac{5\sqrt{\pi}}{11\sqrt{\pi}}$ परिमेय संख्या है ?

   क्या $(3+\sqrt{7})(4-\sqrt{13})(3-\sqrt{7})(4+\sqrt{13})$ परिमेय संख्या है ?

3. निम्नलिखित परिमेय संख्याओं को सांत अथवा असांत आवर्ती दशमलवों में व्यक्त कीजिए :

   (*i*) $\dfrac{34}{11}$ (*ii*) $\dfrac{2}{9}$

   (*iii*) $\dfrac{13}{40}$ (*iv*) $\dfrac{21}{37}$

   (*v*) $\dfrac{1}{7}$ (*vi*) $\dfrac{11}{90}$

4. वे परिमेय संख्याएँ ज्ञात कीजिए जिन्हें यदि दशमलवों में व्यक्त किया जाए तो निम्न प्रसार प्राप्त हो:

   (*i*) $5.6666\ldots$ (*ii*) $2.\overline{27}$

   (*iii*) $8.\overline{181}$ (*iv*) $0.23\overline{564}$

   (*v*) $0.2373\overline{9}$ (*vi*) $0.\overline{2374}$

5. -2 तथा $\frac{7}{3}$ के बीच तीन परिमेय संख्याएँ ज्ञात कीजिए।

6. यदि p^2, 5 का पूर्णांक गुणज है तो सिद्ध कीजिए कि p भी 5 का पूर्णांक गुणज है।
[संकेत : यदि p, 5 का पूर्णांक गुणज नहीं है तो यह $5k+1$ या $5k+2$ या $5k+3$ या $5k+4$ के रूप की संख्या है, जहाँ k एक पूर्णांक है।]

7. सिद्ध कीजिए कि $\sqrt{5}$ एक अपरिमेय संख्या है।
[संकेत : उपरोक्त प्रश्न 6 का प्रयोग कीजिए।]

8. सिद्ध कीजिए कि द्विघात समीकरण $2x^2-x-\frac{3}{2}=0$ के मूल अपरिमेय हैं।

9. सिद्ध कीजिए कि $2+\sqrt{5}, 2-\sqrt{5}, 2\sqrt{5}$ तथा $\frac{2}{\sqrt{5}}$ अपरिमेय संख्याएँ हैं।

10. निम्नलिखित संख्याओं को आरोही क्रम में लिखिए :
$\sqrt{3}+\sqrt{5}, \sqrt{2}+\sqrt{6}, \sqrt{8}, \sqrt{11}-\sqrt{3}$

11 a तथा b को निर्धारित कीजिए ताकि $a<\sqrt{6}<b$ तथा $b-a<0.01$ हो।

12. क्या 0. 12112111211112 . . . अपरिमेय है ? क्यों ?

13. x तथा y का मान ज्ञात कीजिए यदि

(*i*) $2x+3yi-4=9i$ (*ii*) $x+4yi=ix+y+3$

(*iii*) $(x-yi)(3+5i)=-6+24i$

14. निम्नलिखित सम्मिश्र संख्याओं को तल में निरूपित कीजिए :

(*i*) $-12+8i$ (*ii*) $-3i$

(*ii*) $3i$ (*iv*) $-12-8i$

(*v*) $\frac{5}{2}$

15. तल में निम्नलिखित बिन्दुओं द्वारा निरूपित सम्मिश्र संख्याएँ लिखिए :

(*i*) $(0,0)$ (*ii*) $(0,-1)$

(*iii*) $(-1,0)$ (*iv*) $\left(-\frac{1}{3}, \frac{1}{5}\right)$

(*v*) $\left(\sqrt{2}, \frac{3}{4}\right)$ (*vi*) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}}\right)$

16. निम्नलिखित संक्रियाएँ कीजिए तथा परिणाम को $a+bi$ के रूप में व्यक्त कीजिए :

(i) $(8+3\sqrt{8}\,i)+(5+2\sqrt{2}\,i)-(5+\sqrt{2}\,i)$

(ii) $(3-4i)+2i-(8+7i)$

(iii) $(4+3i)-(3+4i)$

(iv) $(7-2i)-(3+4i)-(-10+2i)$

(v) $(2-\sqrt{3}\,i)(2+\sqrt{3}\,i)+6+4i$

(vi) $(5+4i)(-5-4i)+25+3i$

(vii) $(2i^3+3i^2-1)(4+5i)$

(viii) $(2+3i)(2-3i)(1+i)^2$

(ix) $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2$

(x) $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3$

(xi) $\dfrac{13+2i}{1+\sqrt{2}\,i}$

(xii) $\dfrac{\sqrt{2}+\sqrt{3}\,i}{-7-i}$

(xiii) $\dfrac{(2-8i)(7+8i)}{1+i}$

(xiv) $\dfrac{(1-i)(2-i)(3-i)}{1+i}$

(xv) $\dfrac{(4+5i)^2}{(2+3i)^2}$

(xvi) $\dfrac{1}{i^7}$

(xvii) $\dfrac{1}{(-2+i)(1+3i)}$

(xviii) $i^{20}+(1-2i)^3$

(xix) $\dfrac{-i-2}{(1+\sqrt{3}\,i)(1-\sqrt{3}\,i)}$

17. सिद्ध कीजिए कि $1+i^2+i^4+i^6=0$

18. सिद्ध कीजिए कि $1+i^{10}+i^{20}+i^{30}$ एक वास्तविक संख्या है।

19. निम्नलिखित संख्याओं के व्युत्क्रम ज्ञात कीजिए :

(i) $7+\sqrt{7}\,i$ (ii) $i-5$

20. यदि z एक सम्मिश्र संख्या है तो सिद्ध कीजिए कि $z+\bar{z}$ तथा $z\bar{z}$ सदैव वास्तविक संख्याएँ हैं।

21. निम्नलिखित में से प्रत्येक संख्या के सम्मिश्र संयुग्मी ज्ञात कीजिए :

(i) $(3-7i)^2$ (ii) $(\sqrt{7}\,i+2)^2$

(iii) $\dfrac{(8-3i)(6-i)}{2-2i}$

22. निम्नलिखित संख्याओं के वर्गमूल ज्ञात कीजिए :

(i) $\frac{-441}{625}$ (ii) -32

(iii) $\frac{-8}{729}$

23. निम्नलिखित का मान ज्ञात कीजिए :

(i) $-3\sqrt{-9}-2\sqrt{-16}+5\sqrt{-25}$

(ii) $(5+\sqrt{-36})(6+3\sqrt{-81})$

(iii) $(-1-\sqrt{-12})+\left(4+\frac{\sqrt{-25}}{2}\right)+\left(6-\frac{\sqrt{-36}}{3}\right)$

24. निम्नलिखित समीकरणों के मूल ज्ञात कीजिए :

(i) $y+14=4y^2$

(ii) $\sqrt{2}\,x^2-3x+3\sqrt{2}=0$

(iii) $6x^2-x+4=0$

(iv) $x(x+7)+14=0$

(v) $3x^{-2}+7x^{-1}+5=0$

(vi) $(x+1)(x+2)(x+3)(x+4)=120$

25. निम्नलिखित द्विघात समीकरणों के मूल निर्धारित किए बिना उनकी प्रकृति बताइए :

(i) $x^2-x+1=0$ (ii) $7x^2+6x+3=0$

(iii) $6\sqrt{3}\,x^2-4x+\sqrt{3}=0$ (iv) $\sqrt{2}\,y^2-\sqrt{3}+\sqrt{3}\,y=0$

(v) $\frac{7}{8}x^2-x+\frac{8}{7}=0$

26. k के किन मानों के लिए निम्नलिखित में से प्रत्येक समीकरण के मूल वास्तविक तथा समान होंगे ?

(i) $3x^2-kx+k=0$

(ii) $5y^2-20y+(k-1)=0$

(iii) $kx^2+kx+1=-4x^2-x$

27. निम्नलिखित समीकरणों के मूल निर्धारित किए बिना उनके मूलों का योग तथा गुणनफल ज्ञात कीजिए :

(i) $x^2=-3$ (ii) $\frac{1}{3}x^2+\frac{5}{3}x-\frac{2}{3}=0$

(iii) $\sqrt{5}\,y^2-2\sqrt{5}\,y=5$

28. यदि a , b तथा c वास्तविक हों तो सिद्ध कीजिए कि
$a^2+b^2+c^2-ab-bc-ca=0$ तभी तथा केवल तभी होगा जबकि $a=b=c$ हो।
[संकेत : 'यदि . . .तो' . . . भाग को सिद्ध करने के लिए वाम पक्ष को 'a' में द्विघात समीकरण समझिए।]

29. सिद्ध कीजिए कि समीकरण
$(x-a)\ (x-b)+(x-b)\ (x-c)+(x-c)\ (x-a)=0$ के मूल तभी तथा केवल तभी समान होंगे यदि $a=b=c$ हो।
[संकेत : उपरोक्त प्रश्न 28 का प्रयोग कीजिए।]

30. समीकरण $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{c}$ के मूलों का योग शून्य है। सिद्ध कीजिए कि मूलों का गुणनफल $-\frac{1}{2}(a^2+b^2)$ है।

31. यदि समीकरण $k\ (x-1)^2=5x-7$ का एक मूल दूसरे का दुगुना हो, तो k का मान निर्धारित कीजिए।

32. समीकरण $ay^2+by+c=0$ के मूलों का अनुपात $p : q$ है। सिद्ध कीजिए कि $ac\ (p+q)^2=b^2pq$ है।

33. यदि द्विघात समीकरण $ax^2+bx+c=0$ का एक मूल दूसरे का वर्ग है, तो सिद्ध कीजिए कि

$$b^3+a^2c+ac^2=3abc$$

34. यदि α और β समीकरण $x^2-2x+3=0$ के मूल हों तो ऐसे समीकरण ज्ञात कीजिए जिसके मूल $\alpha+2$ तथा $\beta+2$ हों।

35. यदि α और β समीकरण $2x^2-5x+7=0$ के मूल हों तो ऐसी समीकरण ज्ञात कीजिए जिसके मूल $2\alpha+3\beta$ तथा $3\alpha+2\beta$ हों।

36. वास्तविक गुणांकों के द्विघात समीकरण ज्ञात कीजिए जिनके मूल निम्न हैं :

(i) $\sqrt{2}+3, \sqrt{2}-3$ (ii) $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

(iii) $1-i$ (iv) $\frac{3}{4}, \frac{4}{3}$

(v) $\sqrt{3}, -\sqrt{3}+1$

37. निम्नलिखित सम्मिश्र संख्याओं के वर्गमूल ज्ञात कीजिए :

(i) $2i$ (ii) $8i-15$

(iii) $-3+5i$ (iv) $8+6i$

(v) $8-6i$ (vi) $-8+6i$

(vii) $-8-6i$

38. निम्नलिखित समीकरणों को C पर हल कीजिए :

(i) $y^2-(2+i)y+(-1+7i)=0$

(ii) $2x^2+ix^2-2i=(5-i)x-2$

(iii) $x^2=(3-2i)x+(5i-5)$

*39. निम्नलिखित सर्वसमिका (identity) की जाँच कीजिए :

$x^4+4=(x-1-i)(x-1+i)(x+1+i)(x+1-i)$

40. यदि $x=1+\frac{1}{2}i$ हो, तो $3x^2+5x-8$ का मान ज्ञात कीजिए।

41. आलेखी विधि द्वारा निम्न असमीकरणों को हल कीजिए :

(i) $4-(x-1)^2>0$ (ii) $2x^2-4x+1\leqslant 0$

(iii) $-x^2+2x-3\geqslant 0$ (iv) $-2x^2+9x+18\leqslant 0$

42. बीजगणित की विधि द्वारा निम्न असमीकरणों को हल कीजिए। हलों को संख्या रेखा प भी निरूपित कीजिए।

(i) $x^2+x+\frac{1}{4}\geqslant 0$ (ii) $x^2-x-20\leqslant 0$

(iii) $(x-2)^2+1<0$

घातांक

यह एक पुनरावलोकन एकक है। यह मान लिया गया है कि पाठक घातांकों (exponents) के मूल नियमों से पहले से ही परिचित हैं तथा सरल घातांकीय समीकरणों (exponential equations) को हल करना जानते हैं। हम नीचे पुनरावलोकन हेतु एक प्रश्नावली दे रहे हैं ताकि आपने जो अब तक पढ़ा है उसे आप दोहरा सकें। यदि आपको किसी संकल्पना के विषय में कोई कठिनाई हो तो पिछली कक्षाओं में पढ़ी गई सामग्री का अध्ययन करें।

1. यदि 'a' एक शून्येतर वास्तविक संख्या है तथा m एक प्राकृत संख्या, तो a^m को निम्न रूप से परिभाषित करते हैं :

$$a^m = \underbrace{a \times a \times \ldots \times a}_{(m \text{ बार})}$$

यहाँ a को आधार (base); m को a का घातांक (exponent) तथा a^m को a की घात m (m^{th} power) कहा जाता है।

हम परिभाषित करते हैं कि $a^0 = 1$

परिकलित कीजिए :

$$3^4;\ (-2)^3;\ (\tfrac{1}{4})^5,\ (2\sqrt{2})^5;\ \left(-\frac{2}{5}\right)^3$$

[घातांकीय संकेतन (exponential notation) का प्रयोग सर्वप्रथम डायोफेन्टस (जन्म लगभग 250 ई०) ने किया था। डायोफेन्टस के जीवन के विषय में कुछ अधिक ज्ञात नहीं है। केवल इतना ही ज्ञात है कि सम्भवतः वे यूनानवासी थे। यूनान में एकत्रित की गई सामग्री में एक बीजगणितीय समस्या* है

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* पाठक निम्न पुस्तक का अध्ययन करें :

Morris Kline : **Mathematical Thought from Ancient to Modern Times**, Oxford University Press, New York (U.S.A.), 1972, Page 139.

जिससे डायोफैन्टस के जीवन के विषय में निम्न तथ्य ज्ञात हुए हैं :

उनका बाल्यकाल उनके जीवन के $\frac{1}{6}$ भाग में समाप्त हुआ; उनकी दाढ़ी इससे $\frac{1}{12}$ भाग के बाद बढ़ी; इससे $\frac{1}{7}$ भाग के बाद उनका विवाह हुआ तथा 5 वर्ष बाद उनके पुत्र का जन्म हुआ। पुत्र पिता की आधी आयु पर्यन्त जीवित रहा और पिता की मृत्यु पुत्र से 4 वर्ष पश्चात् हुई।

क्या आप डायोफैन्टस की आयु ज्ञात कर सकते हैं ? उत्तर 84 वर्ष है।]

2. (ऋणेत्तर पूर्णांकीय) घातांकों के निम्न नियम आपको याद होंगे :

मान लीजिए a तथा b शून्येतर वास्तविक संख्याओं को व्यक्त करते हैं और m तथा n ऋणेतर पूर्णांकों को व्यक्त करते हैं। तब,

I. $a^m a^n = a^{m+n}$

II. $\dfrac{a^m}{a^n} = \begin{cases} a^{m-n}, \text{ यदि } m > n \\ \dfrac{1}{a^{n-m}}, \text{ यदि } m \leqslant n \end{cases}$

III. $(a^m)^n = a^{mn}$

IV. $(a\,b)^m = a^m b^m$

V. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

घातांकों के नियमों का प्रयोग करके निम्नलिखित संक्रियाएँ कीजिए :

(i) $(-3)^0$; -2^6 ; $(-2)^6$

(ii) $(4^2)(4^3)$; $(-2)^3(-2)^5$; $(\frac{1}{3})^4(\frac{1}{3})^2$

(iii) $\dfrac{\left(\frac{5}{2}\right)^6}{\left(\frac{5}{2}\right)^3}$; $\dfrac{y^2}{y^8}$; $\dfrac{x^3}{x^5}$; $\dfrac{x^8}{x^8}$; $\dfrac{x^0}{x^2}$

(iv) $[(-\frac{1}{2})^2]^3$; $(a^2)^3$; $(c^3)^0$; $(2b^3)^3$

(v) $(-a^3b^2)^4$; $(-\frac{3}{2}x^2y)^3$; $(x^2y)(-3xy^3)(\frac{5}{3}xy)$

(vi) $\left(\dfrac{x^2}{y^3}\right)^2$; $\left(\dfrac{2a^3}{b^4}\right)^4$; $\left(\dfrac{-3x^2y^2}{2ab^2}\right)^5$

(vii) $\dfrac{2}{7}a^3b^2\left(-\dfrac{49}{2}a^6b^3\right)$; $\dfrac{3a^m(5b^n)^2}{(a^{2m})^2}$

3. अब हम एक शून्येतर वास्तविक संख्या 'a' की ऋणात्मक पूर्णांकीय घातों (negative integral powers) को परिभाषित करते हैं। हम इस की परिभाषा इस प्रकार से चाहेंगे कि नियम I – V यहाँ भी लागू हों। उदाहरणार्थ, यदि m एक धनात्मक पूर्णांक है तो हम a^{-m} को इस प्रकार परिभाषित करना चाहेंगे कि नियम I यहाँ भी लागू रहे।

अर्थात्, $a^m a^{-m} = a^{m-m} = a^0 = 1$

इससे a^{-m} की निम्न परिभाषा का सुझाव मिलता है :

यदि a एक शून्येतर वास्तविक संख्या है तथा m एक धनात्मक पूर्णांक है, तो

$$a^{-m} = \frac{1}{a^m}$$

हम कहते हैं कि a^{-m} तथा a^m एक दूसरे के **व्युत्क्रम** हैं। इस परिभाषा के साथ अब हम (**पूर्णांकीय**) घातांकों के निम्न नियम प्राप्त करते हैं :

यदि a और b शून्येतर वास्तविक संख्याओं तथा m और n पूर्णांकों को व्यक्त करते हों, तो

I. $a^m a^n = a^{m+n}$

II. $\frac{a^m}{a^n} = a^{m-n}$

III. $(a^m)^n = a^{mn}$

IV. $(ab)^m = a^m b^m$

V. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(क) परिकलित कीजिए :

5^{-3} ; $(-3)^{-2}$; $(\sqrt{7})^{-4}$; $(\frac{3}{5})^{-2}$

(ख) घातांकों के नियमों का प्रयोग करके निम्नलिखित संक्रियाएँ कीजिए। आपके उत्तर में कोई ऋणात्मक घातांक नहीं आना चाहिए।

(i) $4^{-3}4^6$; $(\frac{1}{3})^{-2}(\frac{1}{2})^{-3}$; $6^{-2}6^{-1}\ 6^0$

(ii) $\frac{11^{-3}}{11^{-5}}$; $\frac{a^2}{a^5}$

(iii) $(\frac{3}{4})^{-1}$; $(4^{-2})^3$; $(2^{-3})^{-3}$; $[(\frac{1}{2})^2]^{-1}$

(iv) $(2^{-1}3^{-2})^3$; $(5^{-2}4^4)^{-2}$; $(6^{-1}4^3)^{-1}$

(v) $(\frac{5}{3})^{-2}$; $(\frac{2^{-2}}{7})^{-1}$; $(3^{-2})^{-4}\ (2^{-3})^{-2}$

(vi) $\frac{a^{-1}m^{-2}}{a^{-2}m^{-1}}$; $\frac{2^{-1}m^{-4}n^{-1}}{3^{-1}m^{-2}n^2}$

4. अब हम एक धनात्मक वास्तविक संख्या की **परिमेय घातों (rational powers)** को परिभाषित करते हैं। पुनः यह परिभाषा इस प्रकार की होनी चाहिए कि नियम I-V यहाँ भी लागू रहें। अब यदि a एक धनात्मक वास्तविक संख्या है तथा n एक प्राकृत संख्या है तो हमें $a^{\frac{1}{n}}$ को किस प्रकार परिभाषित करना चाहिए ? यह परिभाषा ऐसी होनी चाहिए कि

$$\underbrace{a^{\frac{1}{n}} \times a^{\frac{1}{n}} \times \cdots \times a^{\frac{1}{n}}}_{n \text{ बार}} = a^{\frac{1}{n}+\frac{1}{n}+\cdots+\frac{1}{n}} = a^{\frac{n}{n}} = a$$

दूसरे शब्दों में, $a^{\frac{1}{n}}$ की nवीं घात a होनी चाहिये जो कि यह कहने के समान है कि $a^{\frac{1}{n}}$, a का nवाँ मूल*, जिसे $\sqrt[n]{a}$ लिखा जाता है, होना चाहिए। इससे $a^{\frac{1}{n}}$ की निम्न परिभाषा का सुझाव मिलता है :

यदि a एक धनात्मक वास्तविक संख्या है तथा n एक प्राकृत संख्या है, तो

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

अब हम a की परिमेय** घात को परिभाषित करते हैं।

यदि a एक धनात्मक वास्तविक संख्या है, m एक पूर्णांक है तथा n एक प्राकृत संख्या है, तो

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

अब हम दो महत्वपूर्ण परिणाम सिद्ध करते हैं।

(i) यदि a एक धनात्मक वास्तविक संख्या, m एक पूर्णांक तथा n और p प्राकृत संख्याएँ हैं, तो

$$\sqrt[np]{a^{mp}} = \sqrt[n]{a^m}$$

क्योंकि यदि हम यह मान लें कि $\sqrt[np]{a^{mp}} = c$, तो

$$a^{mp} = c^{np}$$

दोनों पक्षों का p वाँ मूल लेने पर,

$$a^m = c^n$$

जिससे, $c = \sqrt[n]{a^m}$

इस प्रकार, $\sqrt[np]{a^{mp}} = \sqrt[n]{a^m}$

(ii) यदि a एक धनात्मक वास्तविक संख्या, m एक पूर्णांक तथा n एक प्राकृत संख्या है, तो

$$\sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

मान लीजिए कि $\sqrt[n]{a^m} = b$

तब, $a^m = b^n$ (1)

अब मान लीजिए कि $\sqrt[n]{a} = c$

तब, $a = c^n$

जिससे, $a^m = (c^n)^m = c^{mn} = (c^m)^n$

अतः (1) से,

$$b^n = (c^m)^n \tag{2}$$

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*प्रत्येक धनात्मक संख्या a और प्राकृत संख्या n के लिए एक अद्वितीय धनात्मक संख्या b ऐसी होती है कि $b^n = a$ हो। b, a का nवाँ मूल (nth root) कहलाता है। [इसकी उपपत्ति इस पुस्तक की सीमा के बाहर है।]

**हम एक परिमेय घातांक को सदा इस प्रकार से लिखेंगे कि इसका हर धनात्मक हो।

(2) के दोनों पक्षों का n वाँ मूल लेने पर,

$$b=c^m$$

अर्थात्, $\sqrt[n]{a^m}=\left(\sqrt[n]{a}\right)^m$

अतः विकल्पतः हम a की परिमेय घात को निम्न प्रकार परिभाषित कर सकते हैं :

यदि a एक धनात्मक वास्तविक संख्या, m एक पूर्णांक तथा n एक प्राकृत संख्या है, तो

$$a^{\frac{m}{n}}=\sqrt[n]{a^m}=\left(\sqrt[n]{a}\right)^m$$

[एक ऋणात्मक वास्तविक संख्या की परिमेय घात की परिभाषा इस पुस्तक की सीमा के बाहर है ।]

अब परिणामों (i) और (ii) तथा परिभाषा का उपयोग करते हुए हम (परिमेय) घातांकों के निम्न नियम प्राप्त करते हैं :

यदि a और b धनात्मक वास्तविक संख्याओं को, m और p पूर्णांकों को तथा n और q प्राकृत संख्याओं को व्यक्त करते हों, तो

I. $a^{\frac{m}{n}}\ a^{\frac{p}{q}}=a^{\frac{m}{n}+\frac{p}{q}}$

II. $\dfrac{a^{\frac{m}{n}}}{a^{\frac{p}{q}}}=a^{\frac{m}{n}-\frac{p}{q}}$

III. $\left(a^{\frac{m}{n}}\right)^{\frac{p}{q}}=a^{\frac{mp}{nq}}$

IV. $(ab)^{\frac{m}{n}}=a^{\frac{m}{n}}\ b^{\frac{m}{n}}$

V. $\left(\dfrac{a}{b}\right)^{\frac{m}{n}}=\dfrac{a^{\frac{m}{n}}}{b^{\frac{m}{n}}}$

(क) परिकलित कीजिए :

$(81)^{\frac{1}{4}}$; $\left(\frac{1}{49}\right)^{\frac{1}{2}}$; $(128)^{\frac{3}{7}}$; $(216)^{\frac{4}{3}}$; $9^{-\frac{1}{2}}$; $27^{-\frac{1}{3}}$; $\left(\frac{2}{3}\right)^{-2}$;

$\left(\frac{81}{16}\right)^{-\frac{3}{4}}$; $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$; $\left(3y^{-\frac{5}{3}}\right)^{-3}$

(ख) घातांकों के नियमों का प्रयोग करके सरल कीजिए। अपना उत्तर बिना ऋणात्मक घातांकों का प्रयोग किए दीजिए ।

(i) $y^0\ y^{\frac{5}{3}}\ y^{\frac{1}{6}}$ $\qquad$ (ii) $(a^{-2}y^{-3})^{-2}$

(iii) $\left(3y^{-\frac{3}{8}}\right)^{-1}$ (iv) $\left(x^{-2}y^{-\frac{2}{5}}\right)^{-1}$

(v) $\dfrac{3a^{-\frac{1}{2}}b^{\frac{4}{3}}}{2a^{\frac{3}{2}}b^{-\frac{2}{3}}}$ (vi) $\left(8y^{-3}x^{6}\right)^{-\frac{2}{3}}$

(vii) $\dfrac{(49)^{\frac{1}{2}}a^{-1}y^{-2}}{(81)^{\frac{1}{4}}a^{-2}y^{-\frac{1}{3}}}$ (viii) $\left(\dfrac{2m^{2}n^{-\frac{4}{5}}}{m^{0}n^{-\frac{3}{5}}}\right)^{-5}$

(ix) $\left(x^{\frac{1}{2}}+1\right)\left(x^{\frac{1}{2}}-1\right)$ (x) $\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^{2}$

(xi) $\dfrac{3^{-2}-2^{-2}}{3^{-2}2^{-2}}$ (xii) $\dfrac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}}$

(xiii) $\left[\left(\sqrt[3]{125}\right)-2\right]^{-2}$

5. यह सुगमतापूर्वक सिद्ध किया जा सकता है कि यदि a एक धनात्मक वास्तविक संख्या है, $(a \neq 1)$; m और p पूर्णांक हैं तथा n और q प्राकृत संख्याएँ हैं, तो

$a^{\frac{m}{n}} = a^{\frac{p}{q}}$ तभी तथा केवल तभी होगा जबकि $\frac{m}{n} = \frac{p}{q}$ हो।

निम्नलिखित में से प्रत्येक घातांकीय समीकरण को हल कीजिए :

(i) $2^{x}=4^{2x-1}$ (ii) $3^{x}=3^{-x}$ (iii) $5^{x}=125$ (iv) $3^{y-1}=9$

(v) $4^{x+2}=(16)^{x}$ (vi) $(10)^{2y-3}=1000$ (vii) $(25)^{x}=5$

(viii) $6^{y}=\dfrac{1}{216}$

अनुक्रम तथा श्रेणियाँ

इस एकक में हम अनुक्रम और श्रेणियों, विशेष रूप से, समांतर और गुणोत्तर श्रेणियों का अध्ययन करेंगे। हम सीखेंगे कि इनका योग किस प्रकार ज्ञात किया जाता है। किसी अनुक्रम की 'सीमा' (limit) की संकल्पना में स्वतः शोध की विधि का प्रयोग करते हुए अपरिमित गुणोत्तर श्रेणी के योग और उसके, एक ऐसी परिमेय संख्या ज्ञात करने में जिसका दशमलव प्रसार एक दिया हुआ असांत आवर्ती दशमलव हो, अनुप्रयोग का उल्लेख किया गया है। अंत में वर्गों और घनों के अनुक्रमों पर विचार किया गया है।

5.1 अनुक्रम क्या होते हैं ?

शीला किसी बैंक में 10% वार्षिक दर से चक्रवृद्धि ब्याज पर 15 वर्ष के लिए 1000 रुपए जमा कराती है। पहले, दूसरे, तीसरे,··· वर्ष के अन्त में उसकी राशि रुपयों में (निकटतम पैसों तक) निम्न हो जाती है :

1100, 1210, 1331, 1464.10, 1610.51, 1771.56, 1948.72, 2143.59, 2357.95, 2593.75, 2853.13, 3138.44, 3452.28, 3797.51, 4177.26

इन राशियों से बने क्रम को हम **अनुक्रम** (**sequence**) अथवा अधिक स्पष्ट रूप से **परिमित अनुक्रम** (**finite sequence**) कहते हैं।

अब 10 को 3 से भाग करने पर प्राप्त उत्तरोत्तर भागफलों पर विचार कीजिए। हमें निम्न प्राप्त होता है :

3, 3.3, 3.33, 3.333, 3.3333, इत्यादि।

ये भागफल एक **अनुक्रम** बनाते हैं। अधिक स्पष्ट शब्दों में हम इसे एक **अपरिमित अनुक्रम** (**infinite sequence**) कहते हैं। यह इस अर्थ में अपरिमित है कि इसका कहीं अंत नहीं होता।

उपरोक्त उदाहरणों में हमने ऐसे अनुक्रम प्राप्त किए हैं जिनका प्रतिरूप (pattern) स्पष्ट रूप से पहचाना जा सकता है। परन्तु ऐसा सदा नहीं होता।

उदाहरणार्थ, अनुक्रम 3,2,17,2,2, -5, $\sqrt{13}$, 4, $6\frac{1}{2}$, -5, $4\frac{108}{371}$, 2 पर विचार कीजिए।

स्पष्ट है कि इस अनुक्रम का प्रतिरूप स्पष्ट रूप से नहीं पहचाना जा सकता।

अब, एक अनुक्रम क्या होता है?

अनुक्रम * से हमारा अभिप्राय संख्याओं की, किसी नियम के अनुसार, एक निश्चित क्रम से की गई व्यवस्था (arrangement) से है। आगे चलकर हम अनुक्रम को स्पष्ट रूप से परिभाषित करेंगे।

किसी अनुक्रम में आने वाली विभिन्न संख्याएँ उसके पद (terms) कहलाती हैं। हम किसी अनुक्रम के पदों को $t_1, t_2, \ldots$ अथवा $a_1, a_2, \ldots$, इत्यादि से व्यक्त करते हैं— जहाँ अक्षर के साथ लिखी संख्या (subscript) पद की स्थिति को व्यक्त करती है। **n वाँ पद** अनुक्रम में nवीं स्थिति पर दी गई संख्या होती है तथा इसे t_n अथवा a_n से व्यक्त किया जाता है। कभी-कभी n वाँ पद अनुक्रम का **व्यापक पद (general term)** भी कहलाता है। इस प्रकार, विभिन्न राशियों के अनुक्रम में (शीला की जमा राशियाँ देखिए) प्रथम पद t_1, 1100 है तथा $t_2=1210$, $t_7=1948.72$, $t_{12}=3138.44$, $t_{15}=4177.26$ है। इसी प्रकार, उत्तरोत्तर भागफलों के उदाहरण में (10 का 3 से भाग देखिए) $a_2=3.3$, $a_6=3.33333$, इत्यादि।

अनुक्रमों के कुछ अन्य उदाहरण निम्न हैं :

विषम प्राकृत संख्याओं का अनुक्रम : 1, 3, 5, 7, 9, ...

सम घनात्मक पूर्णांकों का अनुक्रम : 2, 4, 6, 8, 10, 12, ...

अभाज्य संख्याओं का अनुक्रम : 2, 3, 5, 7, 11, 13, 17, ...

जनवरी 1978 में रविवारों
की तिथियों का अनुक्रम : 1, 8, 15, 22, 29

क्या आप बता सकते हैं कि उपरोक्त उदाहरणों में कौन से परिमित तथा कौन से अपरिमित अनुक्रम हैं?

5.2 किसी अनुक्रम का n वाँ पद अथवा व्यापक पद

प्रायः बीजगणितीय सूत्र के रूप में एक ऐसे नियम को व्यक्त करना सम्भव है जिससे किसी अनुक्रम के विभिन्न पदों को प्राप्त किया जा सकता है। उदाहरणार्थ, विषम प्राकृत संख्याओं के अनुक्रम पर विचार कीजिए। यहाँ,

$$a_1=1=2\times1-1$$
$$a_2=3=2\times2-1$$
$$a_3=5=2\times3-1$$

वस्तुतः हम देखते हैं कि इस अनुक्रम का n वाँ पद निम्न रूप से लिखा जा सकता है :

$a_n=2n-1$, जहाँ n एक प्राकृत संख्या है।

इसी प्रकार, सम घनात्मक पूर्णांकों के अनुक्रम में n वाँ पद t_n, सूत्र $t_n=2n$ से प्राप्त होता है, जहाँ n एक प्राकृत संख्या है।

परन्तु अभाज्य संख्याओं के अनुक्रम में n वीं अभाज्य संख्या प्राप्त करने के लिए कोई ज्ञात सूत्र नहीं है।

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* हम प्रस्तुत अध्ययन को वास्तविक संख्याओं के अनुक्रमों तक ही सीमित रखेंगे।

दूसरे शब्दों में, हमें यह अपेक्षा नहीं करनी चाहिए कि किसी अनुक्रम के पद आवश्यक रूप से एक सूत्र के ही रूप में दिए होंगे। परन्तु हम एक ऐसी सैद्धान्तिक योजना अथवा नियम की अवश्य अपेक्षा करते हैं जिससे n के किन्हीं मानों के लिए व्यापक पद t_n प्राप्त किया जा सके। अतः एक अनुक्रम वस्तुतः एक फलन (function) है जिसका प्रांत (domain) प्राकृत संख्याओं का समुच्चय (set) N अथवा इसका कोई उपसमुच्चय (subset) है। अब हम अनुक्रम को निम्न रूप से परिभाषित करते हैं :

कोई अनुक्रम एक फलन होता है यदि उसका प्रांत या तो N या $\{1,2,3,\ldots,k\}$ के रूप का N का कोई उपसमुच्चय हो। जब प्रांत सम्पूर्ण N होता है, तो हम इसे **अपरिमित अनुक्रम** कहते हैं। जब प्रांत $\{1,2,3,\ldots,k\}$ के रूप का N का एक उपसमुच्चय होता है, तो हम इसे **परिमित अनुक्रम** कहते हैं।

अतः हम कभी-कभी फलन के संकेतन का प्रयोग करते हैं तथा t_n के लिए $t(n)$ अथवा a_n के लिए $a(n)$ का प्रयोग करते हैं।

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : निम्न से परिभाषित अनुक्रमों के प्रथम तीन पद ज्ञात कीजिए

(i) $a_n=n(n+1)$ (ii) $t(n)=\dfrac{n}{n+1}$

(iii) $t_n=n^2-16$

हल : (i) $a_n=n(n+1)$

$n=1,2$ तथा 3 प्रतिस्थापित करने पर हम निम्न तीन पद प्राप्त करते हैं :

$a_1=1(2)=2,\ a_2=2(3)=6$ तथा $a_3=3(4)=12$

(ii) $t(n)=\dfrac{n}{n+1}$

इस प्रकार, $t(1)=\dfrac{1}{1+1}=\frac{1}{2},\quad t(2)=\frac{2}{3}$ तथा $t(3)=\frac{3}{4}$

(iii) $t_n=n^2-16$

t_n में $n=1,2$ तथा 3 प्रतिस्थापित करने पर हम निम्न तीन पद प्राप्त करते हैं :

$t_1=1^2-16=-15,$

$t_2=2^2-16=-12$ तथा

$t_3=3^2-16=-7$

उदाहरण 2 : अनुक्रम $a(n)=\dfrac{n(n+3)}{n+1}$ का 17 वाँ पद क्या है ?

हल : हम $n=17$ प्रतिस्थापित करते हैं तथा निम्न प्राप्त करते हैं :

$a\,(17)=\frac{17(20)}{18}=\frac{170}{9}$

अतः, 17 वाँ पद $\frac{170}{9}$ है।

उदाहरण 3 : निम्न अनुक्रम का 960 वाँ तथा 961वाँ पद ज्ञात कीजिए :

$$t_n = \begin{cases} \dfrac{n}{\frac{n}{96}-1}, & \text{यदि } n \text{ किसी प्राकृत संख्या का वर्ग नहीं है} . \\ 6\frac{1}{2}, & \text{यदि } n \text{ किसी प्राकृत संख्या का वर्ग है} . \end{cases}$$

हल : क्योंकि $961=31^2$, अर्थात् 961 प्राकृत संख्या 31 का वर्ग है, अतः $t_{961}=6\frac{1}{2}$

साथ ही, क्योंकि 961 एक प्राकृत संख्या का वर्ग है, अतः स्पष्ट है कि 960 किसी प्राकृत संख्या का वर्ग नहीं है। इस प्रकार,

$$t_{960}=\frac{960}{\frac{960}{96}-1}=\frac{960}{9}=106\frac{2}{3}$$

अतः, 960 वाँ पद $106\frac{2}{3}$ है तथा 961 वाँ पद $6\frac{1}{2}$ है।

उदाहरण 4 : निम्न द्वारा प्राप्त अनुक्रम के प्रथम पाँच पद ज्ञात कीजिए :

$$\begin{cases} a_1=1 \\ a_n=a_{n-1}+2, \; n\geqslant 2 \end{cases}$$

हल : यहाँ $a_1=1$

$n=2$ प्रतिस्थापित करने पर,

$a_2=a_1+2=3$

क्रमशः $n=3$, 4 तथा 5 प्रतिस्थापित करने पर हमें निम्न प्राप्त होता है :

$a_3=a_2+2=5$

$a_4=a_3+2=7$

तथा $a_5=a_4+2=9$

इस प्रकार, प्रथम पाँच पद 1,3,5,7 तथा 9 हैं।

*उदाहरण 5 : मान लीजिए $a(n)$, 9 पदों का एक परिमित अनुक्रम है जिसके पद $a(1), a(2), \ldots, a(9)$ निम्न रूप से परिभाषित हैं :

$$a(n)=\begin{cases} 1, & \text{यदि } \frac{4}{3} \text{ के दशमलव प्रसार में अंक } n \text{ असंख्य बार (infinitely many times) आता हो} \\ 2, & \text{यदि } \frac{4}{3} \text{ के दशमलव प्रसार में अंक } n \text{ विषम बार (odd number of times) आता हो} \\ 3, & \text{यदि } \frac{4}{3} \text{ के दशमलव प्रसार में अंक } n \text{ सम बार (even number of times) आता हो} \end{cases}$$

इस अनुक्रम के सभी पद ज्ञात कीजिए ।

हल : $a(1)$ के लिए हमें सर्वप्रथम यह निर्धारित करना है कि $\frac{4}{9}$ के दशमलव प्रसार में अंक 1 कितनी बार आता है । यह केवल एक बार अर्थात् विषम बार आता है । अतः $a(1)=2$

अंक 2, $\frac{4}{9}$ के दशमलव प्रसार में, 0 बार आता है (अर्थात् यह नहीं आता) । क्योंकि 0 एक सम संख्या है, अतः $a(2)=3$ । इसी प्रकार, $a(4)=a(5)=a(6)=a(7)=a(8)=a(9)=3$

अंक 3, असंख्य बार आता है । इस प्रकार, $a(3)=1$

अतः इस अनुक्रम के पद 2, 3, 1, 3, 3, 3, 3, 3, 3 हैं ।

अंत में हम एक ऐसे परिमित अनुक्रम का उदाहरण देते हैं जहाँ पदों को व्यक्त करने का नियम तो दिया है परन्तु हम स्पष्ट रूप से पदों को ज्ञात नहीं कर सकते ।

*उदाहरण 6 : मान लीजिए $a(n)$, 10 पदों का एक परिमित अनुक्रम है जिसके पद $a(1), a(2), \ldots, a(10)$ निम्न रूप से परिभाषित हैं :

$$a(n)=\begin{cases} 1, & \text{यदि } \pi \text{ के दशमलव मान में अंक } n \text{ असंख्य बार आता है} \\ 2, & \text{यदि } \pi \text{ के दशमलव मान में अंक } n \text{ विषम बार आता है} \\ 3, & \text{यदि } \pi \text{ के दशमलव मान में अंक } n \text{ सम बार आता है} \end{cases}$$

तथा $a(10)=1, 2$ अथवा 3, जो अंक 0 के उपरोक्त प्रकार से घटित होने पर निर्भर करता है ।

टिप्पणी : हम जानते हैं कि π का दशमलव मान असांत अनावर्ती है तथा अभी तक केवल 500,000 स्थानों तक ही इसका मान परिकलित किया जा सका है (देखिए अनुच्छेद 1.8) । अतः यह कहना सम्भव नहीं है कि दशमलव मान में अमुक अंक कितनी बार आता है । इस प्रकार, इस परिमित अनुक्रम के प्रथम पद तक का निर्धारण भी करना सम्भव नहीं है ।

प्रश्नावली 5.1

उन अनुक्रमों के प्रथम पाँच पद लिखिए जिनके nवें पद निम्न हैं :

1. $a_n=\dfrac{2n-3}{6}$

2. $t_n=(-1)^{n-1}5^{n+1}$

3. $t_n=2^n$

4. $a(n)=\dfrac{3n^2}{(n-\frac{1}{2})(n+1)}$

5. $a(n)=\dfrac{n(n+1)}{2}$

6. $a(n)=\dfrac{n(n+1)(2n+1)}{6}$

उन अनुक्रमों जिनके nवें पद (अर्थात् व्यापक पद) निम्न हैं, के दर्शाए गए पद ज्ञात कीजिए :

7. $t_n = 4n + 3$; 18 वाँ पद, 21 वाँ पद

8. $t_n = \frac{n^2}{3^n}$; t_4

9. $t_n = (-1)^n n^2$; t_8

10. अनुक्रम $a(n)$, $a(n) = (n-1)(n-2)(n-3)$ से परिभाषित है। दिखाइए कि इस अनुक्रम के प्रथम तीन पद शून्य हैं परन्तु शेष सभी पद धनात्मक हैं।

निम्न सूत्रों द्वारा दिए गए अनुक्रमों के प्रथम छः पद ज्ञात कीजिए:

11. $a_1 = a_2 = 2,\ a_n = a_{n-1} - 1\ (n > 2)$

12. $a_1 = -1,\ a_n = \frac{a_{n-1}}{n}\quad (n \geqslant 2)$

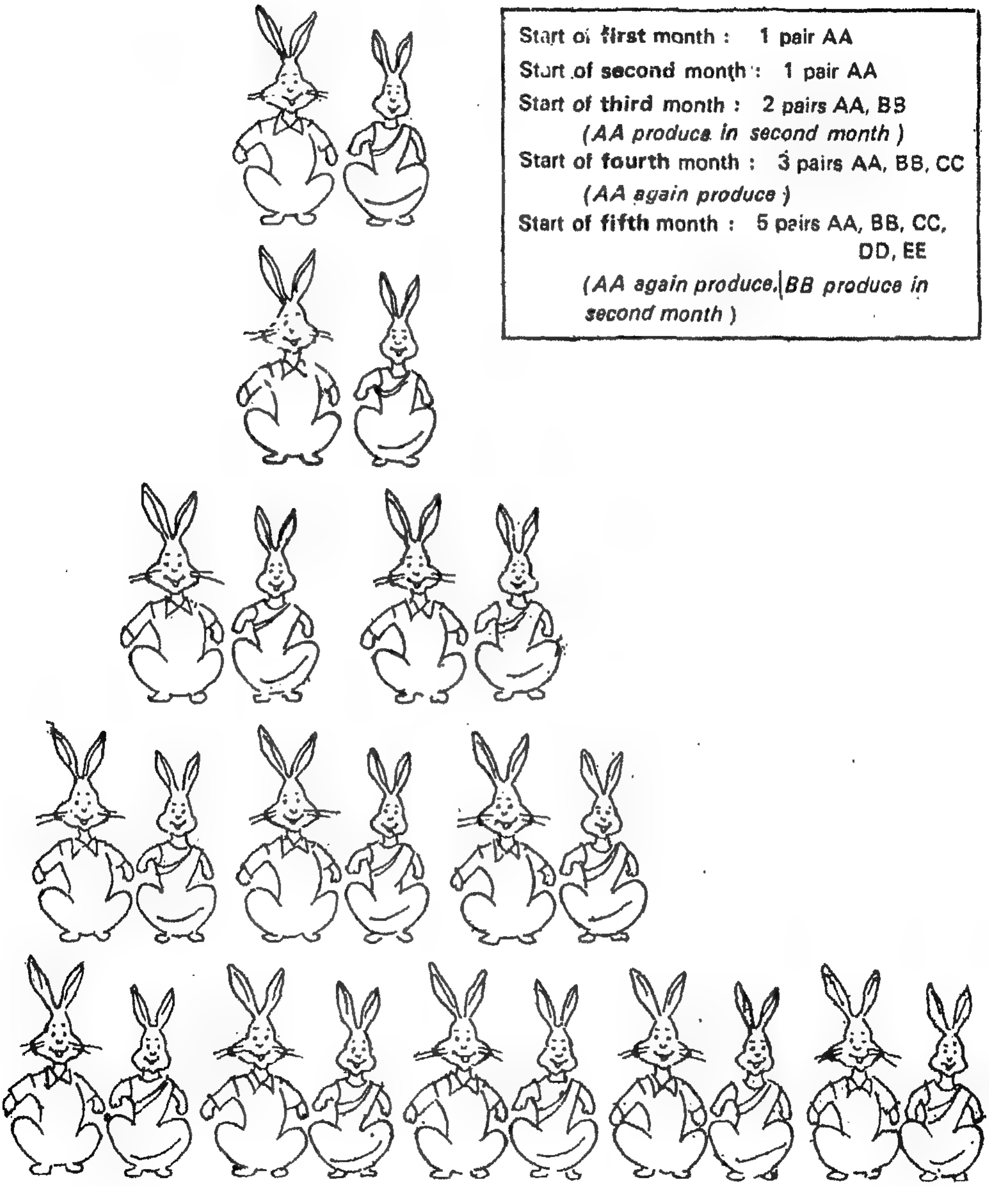
13. $a_1 = a_2 = 1,\ a_n = a_{n-1} + a_{n-2}\quad (n \geqslant 3)$

[प्रश्न 13 के अनुक्रम के पद फिबोनाशी संख्याएँ (Fibonacci numbers) कहलाती हैं, जिनका नाम इटली के महान गणितज्ञ लियोनारडो फिबोनाशी (1170—1250 ई०) के नाम पर रखा गया था। गणित के लिए उनका योगदान शीर्षस्थ है। यद्यपि हिन्दू गणित अरबों तक पहुँच चुका था तथापि मध्य युग में इसके यूरोप में प्रभाव तथा प्रसार का श्रेय फिबोनाशी को ही है।

1202 ई० में फिबोनाशी ने बीजगणित पर एक पुस्तक लिबर अबासी (Liber Abaci) लिखी तथा उसमें प्रसिद्ध खरगोश समस्या (rabbit problem) प्रस्तुत की जो निम्न है :

खरगोशों का एक जोड़ा अल्पायु होने के कारण प्रथम मास में संतानोत्पत्ति नहीं कर सकता। द्वितीय तथा प्रत्येक उत्तरोत्तर मास में वे एक नए खरगोशों के जोड़े को जन्म देते हैं। इसी प्रकार खरगोशों का प्रत्येक नया जोड़ा भी अपने द्वितीय तथा प्रत्येक उत्तरोत्तर मास में एक नए जोड़े को जन्म देता है। अगर यह मान लिया जाए कि किसी खरगोश की मृत्यु नहीं होती तो प्रत्येक मास के प्रारम्भ में खरगोशों के कितने जोड़े होंगे ?]

हम आकृति 5.1 में दर्शाई गई स्थिति प्राप्त करते हैं ।



आकृति 5.1

इस प्रकार, प्रत्येक मास के प्रारम्भ में इन जोड़ों की संख्याएँ सुप्रसिद्ध फिबोनाशी अनुक्रम बनाती हैं ।]

14. $t_n = an^2 + bn + c$ द्वारा परिभाषित अनुक्रम पर विचार कीजिए । यदि $t_2 = 3$, $t_4 = 13$ तथा $t_7 = 113$ हो, तो दिखाइए कि $3t_n = 17n^2 - 87n + 115$ ।

5.3 समांतर श्रेणियाँ (Arithmetic Progressions अर्थात् A. P.)

आइए विषम प्राकृत संख्याओं के अनुक्रम

$$1, 3, 5, 7, 9, \ldots, 2n-1, \ldots \qquad (A)$$

अथवा, सम धनात्मक पूर्णांकों के अनुक्रम

$$2, 4, 6, 8, 10, 12, \ldots, 2n, \ldots \qquad (B)$$

पर विचार करें ।

(A) में हम देखते हैं कि $t_1 = 1$, $t_2 = 3 = t_1 + 2$, $t_3 = 5 = t_2 + 2$, $t_4 = 7 = t_3 + 2$, इत्यादि । दूसरे शब्दों में, प्रथम के अतिरिक्त प्रत्येक पद एक 'निश्चित' ढंग से 'प्रगति' (progress) करता है ।

(B) में हम देखते हैं कि $t_1 = 2$, $t_2 = 4 = t_1 + 2$, $t_3 = 6 = t_2 + 2$, $t_4 = 8 = t_3 + 2$, इत्यादि । पुनः प्रथम के अतिरिक्त प्रत्येक पद एक निश्चित ढंग से 'प्रगति' करता है । हम देखते हैं कि प्रथम के अतिरिक्त कोई भी पद स्वयं से तुरन्त पिछले पद में एक निश्चित संख्या जोड़ने से प्राप्त होता है । इस प्रकार के अनुक्रम **समांतर अनुक्रम (arithmetic sequences)** अथवा **समांतर श्रेणियाँ (arithmetic progressions)** कहलाते हैं । संक्षिप्त रूप में इन्हें **A. P.** भी कहते हैं । दूसरे शब्दों में, **समांतर अनुक्रम अथवा समांतर श्रेणी वह अनुक्रम है जिसका प्रथम के अतिरिक्त प्रत्येक पद स्वयं से तुरन्त पिछले पद में एक निश्चित संख्या जोड़ने से प्राप्त होता है । निश्चित संख्या सार्व अन्तर (common difference) कहलाती है तथा इसे प्रायः 'd' से व्यक्त किया जाता है । प्रथम पद को प्रायः a से व्यक्त किया जाता है ।** इस प्रकार, एक समांतर श्रेणी मे,

$$t_n = t_{n-1} + d, \ (n > 1) \text{ होता है ।}$$

(A) में $a = 1$ तथा $d = 2$ है । (B) में $a = 2$ तथा $d = 2$ है ।

समांतर श्रेणियों के कुछ अन्य उदाहरण निम्न हैं :

$16, 11, 6, 1, -4, -9, \ldots; \ a = 16, \ d = -5$

$-1, -4, -7, -10, -13, \ldots; \ a = -1, \ d = -3$

$x - 3b, x + b, x + 5b, x + 9b, \ldots; \ a = x - 3b, \ d = 4b$

5.4 समांतर श्रेणी का nवाँ अर्थात् व्यापक पद

आइए, एक समांतर श्रेणी जिसका प्रथम पद 'a' तथा सार्व अन्तर 'd' है पर विचार करें । इस श्रेणी के प्रथम कुछ पद क्या होंगे ? हम प्रथम पद में 'd' जोड़कर द्वितीय पद प्राप्त करते हैं । इस प्रकार, $t_2 = a + d$ । इसी प्रकार, t_2 में d जोड़ने से तृतीय पद प्राप्त हो जाता है । अतः $t_3 = t_2 + d$, अर्थात् $t_3 = a + 2d$ । हम इन्हें तथा कुछ अन्य पदों को नीचे लिखते हैं :

प्रथम पद, $\quad t_1 = a \quad = \quad a + (1-1)\, d$

दूसरा पद, $t_2 = a+d = a + (2-1)\,d$

तीसरा पद, $t_3 = a+2d = a + (3-1)\,d$

चौथा पद, $t_4 = a+3d = a + (4-1)\,d$

पाँचवाँ पद, $t_5 = a+4d = a + (5-1)\,d$

क्या आप इसमें कोई प्रतिरूप देखते हैं ? हम देखते हैं कि प्रत्येक पद $a +$ (पद संख्या-1) d है। उदाहरणार्थ, 16 वाँ पद क्या होगा ?

$$t_{16} = a + (16-1)\,d = a + 15d$$

क्या अब आप बता सकते हैं कि nवाँ पद अर्थात् व्यापक पद क्या होगा ? यह सिद्ध किया जा सकता है कि

$$t_n = a + (n-1)\,d$$

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : समांतर श्रेणी 16, 11, 6, 1, -4, -9, . . . का 18वाँ, 23वाँ तथा nवाँ पद ज्ञात कीजिए।

हल : यहाँ, $a = 16$ तथा $d = -5$ है। इस प्रकार,

$$t_{18} = a + (18-1)\,d = 16 + 17\,(-5) = -69$$

$$t_{23} = a+22d = 16+22(-5) = -94$$

तथा $t_n = a+(n-1)d = 16-5(n-1) = 21-5n$

उदाहरण 2 : समांतर श्रेणी $x-3b,\ x+b,\ x+5b,\ x+9b, \ldots$ का व्यापक पद ज्ञात कीजिए।

हल : यहाँ $a = x-3b$ तथा $d = 4b$ है।

इस प्रकार, $t_n = (x-3b)+(n-1)4b$

अर्थात्, $t_n = x+4\left(n-\dfrac{7}{4}\right)b$

उदाहरण 3 : एक समांतर श्रेणी का प्रथम पद -2 तथा दसवाँ पद 16 है। इसका 15वाँ पद निर्धारित कीजिए।

हल : किसी समांतर श्रेणी के व्यापक पद के सूत्र में दो अज्ञात 'a' तथा 'd' होते हैं। इस प्रश्न में हमें दो प्रतिबन्ध अर्थात् $t_1 = -2$ तथा $t_{10} = 16$ दिए हुए हैं। इस प्रकार,

$$\left.\begin{aligned} t_1 &= a = -2 \\ t_{10} &= a+9d = 16 \end{aligned}\right\}$$

हम इनको युगपत् समीकरणों के रूप में हल करते हैं तथा निम्न प्राप्त करते हैं :

$$a = -2$$

तथा $$d = 2$$

अतः, $t_{15} = a+14d = -2+14(2) = 26$

प्रश्नावली 5.2

निम्न में से प्रत्येक समांतर श्रेणी का 'd' ज्ञात कीजिए तथा उसके अगले चार पद लिखिए :

1. $0, -3, -6, -9, \ldots$
2. $-1, \frac{1}{4}, \frac{3}{2}, \ldots$
3. $x+y, x-y, x-3y, \ldots$
4. $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \ldots$

निम्नलिखित समांतर श्रेणियों में से प्रत्येक के सामने लिखे पद ज्ञात कीजिए :

5. $a=3, d=2; t_{10}, t_n$
6. $a=\frac{1}{5}, d=\frac{2}{3}; t_{18}, t_n$
7. $-1, -2, -3, -4, \ldots ; t_{100}$
8. $n-1, n-2, n-3, \ldots ; t_m$
9. उस समांतर श्रेणी का 25वाँ पद ज्ञात कीजिए जिसका 9वाँ पद -6 तथा सार्व अंतर $\frac{5}{4}$ है।
10. एक समांतर श्रेणी के प्रत्येक पद को दुगुना कर दिया जाता है। क्या इस प्रकार प्राप्त अनुक्रम भी एक समांतर श्रेणी है ? यदि ऐसा है, तो इस श्रेणी का प्रथम पद, सार्व अंतर तथा nवाँ पद लिखिए।
11. यदि एक समांतर श्रेणी के 7वें पद का 7 गुना उसके 11वें पद के 11 गुने के बराबर है, तो दिखाइए कि उसका 18वाँ पद शून्य है।
12. यदि एक समांतर श्रेणी के mवें पद का m गुना उसके nवें पद के n गुने के बराबर हो, तो सिद्ध कीजिए कि उसका $(m+n)$वाँ पद शून्य है।
13. एक समांतर श्रेणी का दूसरा तथा r वाँ पद ज्ञात कीजिए जिसका छठा पद 12 तथा 8वाँ पद 22 है।
14. एक समांतर श्रेणी का तीसरा पद 'p' है तथा चौथा पद 'q' है। 10वाँ तथा व्यापक पद ज्ञात कीजिए।
15. समांतर श्रेणी $5, 2, -1, \ldots$ का कौन सा पद -22 है ?
16. k ज्ञात कीजिए ताकि $k+2, 4k-6$ और $3k-2$ किसी समांतर श्रेणी के तीन क्रमागत (consecutive) पद हों।

5.5 एक समांतर श्रेणी के n पदों का योग

महान जर्मन गणितज्ञ कार्ल फ्रेडरिक गौस जब प्राथमिक विद्यालय में थे तो उनके अध्यापक ने उनकी कक्षा से प्रथम 100 प्राकृत संख्याओं का योग ज्ञात करने के लिए कहा। जबकि कक्षा के शेष सभी विद्यार्थी इस समस्या के लिए संघर्ष कर रहे थे, गौस ने अति अल्प समय में इसका उत्तर प्राप्त कर लिया था। हम नीचे किसी समांतर श्रेणी के n पदों का योग ज्ञात करने के लिए, गौस द्वारा प्रयोग की गई विधि के प्रकार की, एक विधि दे रहे हैं :

मान लीजिए कि समांतर श्रेणी का प्रथम पद 'a' तथा सार्व अंतर 'd' है। आइए इस श्रेणी के प्रथम n पदों के योग को S_n से व्यक्त करें। तब,

$$S_n = a + (a+d) + (a+2d) + \dots + [a + (n-2)\,d] + [a + (n-1)\,d] \quad (1)$$

इस योग को प्राप्त करने की एक 'युक्ति' यह है कि S_n को फिर से निम्न रूप से लिखें :

$$S_n = [a + (n-1)\,d] + [a + (n-2)\,d] + [a + (n-3)\,d] + \dots + (a+d) + a \quad (2)$$

अब हम (1) तथा (2) को एक-एक पद करके जोड़ते हैं। हम देखते हैं कि (1) के किसी भी पद तथा (2) में उसके तदनुरूपी पद का योग $2a + (n-1)\,d$ है। उदाहरणार्थ,

$$\begin{aligned}
a + [a + (n-1)\,d] &= 2a + (n-1)\,d \\
(a+d) + [a + (n-2)\,d] &= 2a + (n-1)\,d \\
(a+2d) + [a + (n-3)\,d] &= 2a + (n-1)\,d \\
\cdots\cdots\cdots\cdots & \cdots\cdots\cdots \\
[a + (n-2)\,d] + (a+d) &= 2a + (n-1)\,d \\
[a + (n-1)\,d] + a &= 2a + (n-1)\,d
\end{aligned}$$

हम $2a + (n-1)\,d$ कितनी बार प्राप्त करेंगे? यह स्पष्ट है कि (1) तथा (2) में प्रत्येक S_n के n पद हैं। अतः हम निम्न प्राप्त करते हैं :

$$2\,S_n = n\,[2a + (n-1)\,d]$$

$$\text{अर्थात्,}\quad S_n = \frac{n}{2}\Big[\,2a + (n-1)d\,\Big]$$

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : प्रथम 100 प्राकृत संख्याओं का योग ज्ञात कीजिए।

हल : यहाँ $a = 1,\ d = 1$ तथा $n = 100$ है।

इस प्रकार,
$$\begin{aligned}
S_{100} &= \frac{100}{2}\Big[\,2\,(1) + (100-1)\,1\,\Big] \\
&= 50\,(101) = 5050
\end{aligned}$$

उदाहरण 2 : प्रथम n प्राकृत संख्याओं का योग ज्ञात कीजिए।

हल : यह उदाहरण 1 की तरह है।

इस प्रकार, $$S_n = \frac{n}{2}\left[2(1) + (n-1)1\right]$$
$$= \frac{n}{2}(n+1)$$

यह कि प्रथम n प्राकृत संख्याओं का योग $\frac{n(n+1)}{2}$ है, एक महत्वपूर्ण परिणाम है और इसका गणित में बहुत उपयोग होता है।

[हम देखते हैं कि उदाहरण 1, उदाहरण 2 की वह विशेष अवस्था है जबकि $n = 100$ है।]

अमरीका के समर्पित गणितीय शिक्षाविद् पॉल रोसिनब्लूम ने कहा है कि 'विचारों के माध्यम में सुन्दरता की रचना करना गणित का उद्देश्य है।' गणित की समस्याओं को एक से अधिक विधियों से हल करने में सक्षम होना हर्ष का एक स्रोत है तथा अति आनन्ददायक है। हम नीचे प्रथम प्राकृत संख्याओं का योग ज्ञात करने की एक अन्य विधि दे रहे हैं :

$$S_n = 1 + 2 + 3 + \ldots + n$$

आइए, सर्वसमिका $k^2-(k-1)^2=2k-1$ लें।

हम इसमें बारी-बारी से $k=1,2,3,\ldots,n$ प्रतिस्थापित करके निम्न प्राप्त करते हैं :

$$1^2-0^2=2(1)-1$$
$$2^2-1^2=2(2)-1$$
$$3^2-2^2=2(3)-1$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$n^2-(n-1)^2=2(n)-1$$

जब हम जोड़ते हैं तो हम पदों के कटान (cancellation) में एक प्रतिरूप देखते हैं। क्या आप यह प्रतिरूप देख सकते हैं? इनका योग क्या है? हम निम्न प्राप्त करते हैं :

$$n^2-0^2=2(1+2+3+\ldots+n)-n$$

अर्थात्, $n^2=2(S_n)-n$

इस प्रकार, $S_n = \frac{n(n+1)}{2}$

[ध्यान दीजिए हम देखते हैं कि वाम पक्ष में हम केवल दो ही पद प्राप्त करते हैं—शेष सभी पद कट जाते हैं। अतः इस विधि को दूरदर्शीय (telescopic) योग की विधि कहा जाता है।]

कभी-कभी हमें किसी समांतर श्रेणी का केवल प्रथम पद तथा nवाँ पद ही दिया होता है। आपको याद होगा कि हमें समान्तर श्रेणी के दो अज्ञातों के मान ज्ञात करने के लिए केवल दो प्रतिबन्धों की ही आवश्यकता होती है। हम इस का योग कैसे ज्ञात करते हैं? आइए, इसे अधिक सरल करने के लिए nवें पद को अंतिम पद (last term) कहें तथा इसे 'l' से व्यक्त करें।

अत:, $S_n = a + (a+d) + (a+2d) + \ldots + (l-d) + l$

पहले की तरह हम निम्न प्राप्त करते है :

$$2\ S_n = n\ (a+l)$$

अर्थात्, $$S_n = \frac{n}{2}\ (a+l)$$

दूसरे शब्दों में, प्रथम n पदों का योग, प्रथम पद तथा अंतिम पद (अर्थात् nवें पद) के मध्यमान का n गुना है।

हम कुछ और उदाहरण लेते हैं।

उदाहरण 3 : 1 तथा 100 के बीच उन सभी प्राकृत संख्याओं का योग ज्ञात कीजिए जो 3 की गुणज हैं।

हल : यहाँ $a=3$, $d=3$ तथा $l=99$ है। (क्यों ?)

सर्वप्रथम हमें n ज्ञात करना चाहिए। हम देखते है कि

$$l = a + (n-1)\ d$$

अर्थात्, $$3+3\ (n-1) = 99$$

इस प्रकार, $$n = 33$$

$$\text{तथा } S_{33} = \frac{33(3+99)}{2} = 1683$$

उदाहरण 4 : 715 योग प्राप्त करने के लिए समांतर श्रेणी 1, 4, 7, . . . के कितने पदों की आवश्यकता है ?

हल : यहाँ $a=1$, $d=3$ तथा $S_n=715$ है। हमें n निर्धारित करने की आवश्यकता है। हम S_n का सूत्र प्रयोग करके निम्न प्राप्त करते हैं :

$$715 = \frac{n}{2}\left[2\ (1) + (n-1)\ 3\right]$$

अर्थात्, $$3n^2 - n - 1430 = 0$$

यह n में द्विघात समीकरण है। इसका विविक्तकर धनात्मक है। हम द्विघात सूत्र का प्रयोग कर सकते हैं तथा निम्न प्राप्त करते हैं :

$$n = \frac{1 \pm \sqrt{1+4\ (3)\ (1430)}}{6} = \frac{1 \pm \sqrt{17161}}{6}$$

$$\text{इस प्रकार, } n = \frac{1 \pm 131}{6} \text{ अर्थात् } 22,\ \frac{-65}{3}$$

निश्चय ही पदों की संख्या $\frac{-65}{3}$ नहीं हो सकती (क्यों ?)। अतः वांछित पदों की संख्या 22 है।

उदाहरण 5 : एक समांतर श्रेणी के प्रथम तीन पदों का योग 36 तथा इनका गुणनफल 1620 है। वह श्रेणी ज्ञात कीजिए।

हल : गणित में समस्याएँ हल करने के लिए हम सदैव 'श्रेष्ठ' विधियाँ खोजते हैं। हम यह मान सकते हैं कि प्रथम तीन पद a, $a+d$ तथा $a+2d$ हैं। परन्तु इन संख्याओं का गुणनफल कुछ कठिन होगा तथा दिए गए दोनों प्रतिबन्धों को युगपत रूप से हल करने में अधिक समय लगेगा।

एक 'श्रेष्ठ' विधि यह है कि संख्याओं को $a-d$, a तथा $a+d$ मान लिया जाए। तब हमें तुरंत निम्न प्राप्त होता है :

$$36=(a-d)+a+(a+d)=3a \qquad (1)$$

$$\text{तथा} \quad a(a^2-d^2)=1620 \qquad (2)$$

(1) से, $a=12$

(2) से, $12^2-d^2=135$, अत: $d^2=9$

जिससे $d=\pm 3$

इस प्रकार हम निम्न दो समांतर श्रेणियाँ प्राप्त करते हैं :

$d=+3$: 9, 12, 15, 18, 21, 24, 27,...

$d=-3$: 15, 12, 9, 6, 3, 0, −3,...

दोनों समांतर श्रेणियों में प्रथम तीन पद दिए हुए प्रतिबन्धों को संतुष्ट करते हैं।

उदाहरण 6: एक किसान 12000 रु० में एक पुराना ट्रैक्टर खरीदता है। वह 6000 रु० का नकद भुगतान करता है तथा शेष रकम को 500 रु० की वार्षिक किस्तों में देने का वादा करता है। साथ ही भुगतान न की गई राशि पर 12% की दर से ब्याज देता है। इस प्रकार उसे ट्रैक्टर का कुल कितना मूल्य देना पड़ेगा ?

हल : किसान 6000 रुपये नकद देता है। शेष 6000 रु० की राशि वह 500 रु० की वार्षिक किस्तों में अदा करता है तथा इसके अतिरिक्त भुगतान न की गई राशि पर ब्याज भी देता है। क्या आप बता सकते हैं कि उसे ऋण चुकाने में कितने वर्ष लगेंगे ? स्पष्ट है कि 12 वर्ष लगेंगे। पहले वर्ष के अन्त में उसे कितनी राशि देनी पड़ती है ? वह निम्न राशि अदा करता है :

$$500+\frac{6000\times 12}{100} \text{ अर्थात् } 1220 \text{ रुपये।}$$

दूसरे वर्ष के अन्त में वह कितनी राशि देता है ? वह $500+\frac{5500\times 12}{100}$ अर्थात् 1160 रु० अदा करता है।

यह स्पष्ट है कि प्रत्येक वर्ष वह पिछले वर्ष से 60 रुपये कम अदा करता है जोकि उसके मूलधन के भाग 500 रु० पर ब्याज है। इस प्रकार हम एक समांतर श्रेणी प्राप्त करते हैं जिसमें $a=1220$, $d=-60$ तथा $n=12$ है।

$$\text{अत: } S_{12}=\frac{12}{2}\Big[2(1220)+11(-60)\Big]=10680$$

निस्संदेह, वह 6000 रु० नकद दे चुका है। अत: उसे ट्रैक्टर का कुल मूल्य 16680 रु० देना पड़ेगा।

प्रश्नावली 8.3

निम्न में से प्रत्येक समांतर श्रेणी के सम्मुख दिए हुए पदों का योग ज्ञात कीजिए :

1. 16, 11, 6, ...; 23 पद, n पद
2. -0.5, -1.0, -1.5, ...; 10 पद, 50 पद
3. -2, 4, 10, ...; 21 पद, 32 पद
4. -1, $\frac{1}{4}$, $\frac{3}{2}$, ...; 81 पद
5. $x+y$, $x-y$, $x-3y$, ...; 22 पद, p पद
6. 1, 3, 5, 7, ...; 100 पद, 200 पद
7. 2, 4, 6, 8, ...; 100 पद, n पद
8. 100 तथा 1000 के बीच की उन सभी प्राकृत संख्याओं का योग ज्ञात कीजिए जो 5 की गुणज हैं।
9. यदि $t_3=2$ तथा $t_7=22$ हो तो समांतर श्रेणी के प्रथम 35 पदों का योग निर्धारित कीजिए।
10. -25 योग प्राप्त करने के लिए समांतर श्रेणी $-6, -\frac{11}{2}, -5, \ldots$ के कितने पदों की आवश्यकता है ?

निम्न में से प्रत्येक श्रेणी में l, a, n, d तथा S_n में से जो नहीं हैं उनका निर्धारण कीजिए :

11. $a=-2$, $d=5$, $S_n=568$
12. $a=\frac{17}{2}$, $d=\frac{3}{2}$, $n=66$
13. $l=8$, $n=8$, $S_8=-20$
14. $d=\frac{2}{3}$, $l=10$, $n=20$
15. $a=-3030$, $l=-1530$, $n=51$
16. एक समांतर श्रेणी में $a=2$ है तथा प्रथम पाँच पदों का योग अगले पाँच पदों के योग का एक-चौथाई है। सिद्ध कीजिए कि $t_{20}=-112$ है।
[संकेत : एक समांतर श्रेणी a, $a+d$, $a+2d$,...पर विचार कीजिए।
तब $S_5=\frac{5}{2}[a+(a+4d)]$ अगले पाँच पदों में प्रथम पद $a+5d$ तथा अन्तिम पद $a+9d$ होगा, इत्यादि।]

*17. एक समांतर श्रेणी, जिसके प्रथम n पदों का योग $2n+3n^2$ है, का r वाँ पद ज्ञात कीजिए।
[संकेत : $t_r=S_r-S_{r-1}$]

*18. 3 अंकों की उन सभी संख्याओं का योग ज्ञात कीजिए जिन्हें यदि 4 से विभाजित किया जाए तो 1 शेष बचे।

19. हरि एक मकान 22000 रु० में खरीदता है। वह 4000 रु० नकद तथा शेष रकम 1000 रु० की वार्षिक किस्तों में अदा करता है तथा भुगतान न की गई राशि पर 10% वार्षिक ब्याज भी देता है। इस प्रकार उसे मकान का कुल कितना मूल्य देना पड़ेगा ?

5.6 गुणोत्तर श्रेणियाँ (Geometric Progressions अर्थात् G.P.)

आइए निम्नलिखित अनुक्रमों पर विचार करें :

$$2, 4, 8, 16, \ldots, 2^n, \ldots \qquad (A)$$

$$-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots, \left(-\frac{1}{3}\right)^n, \ldots \qquad (B)$$

$$.01, .0001, .000001, \ldots, (.01)^n, \ldots \qquad (C)$$

$$-\frac{9}{2}, \frac{9}{4}, -\frac{9}{8}, \frac{9}{16}, \ldots, 9\left(-\frac{1}{2}\right)^n, \ldots \qquad (D)$$

हम देखते हैं कि इन सभी अनुक्रमों में प्रथम पद के अतिरिक्त प्रत्येक पद एक 'निश्चित' प्रकार से 'प्रगति' करता है। ये पद कैसे 'प्रगति' करते हैं ?

(A) में हम देखते हैं कि $t_1=2,\ t_2=4=2t_1,\ t_3=8=2t_2,\ t_4=16=2t_3,$ इत्यादि।

(B) में हम देखते हैं कि $t_1=-\frac{1}{3},\ t_2=\frac{1}{9}=-\frac{1}{3}t_1,\ t_3=-\frac{1}{27}=-\frac{1}{3}t_2,$

$t_4=\frac{1}{81}=-\frac{1}{3}t_3,$ इत्यादि।

(C) में पद किस प्रकार प्रगति करते हैं ? D में किस प्रकार करते हैं ?

हम देखते हैं कि प्रथम के अतिरिक्त प्रत्येक पद अपने से तुरन्त पिछले पद को एक शून्येतर स्थिर संख्या से गुणा करने पर प्राप्त होता है। ऐसे अनुक्रम गुणोत्तर अनुक्रम (geometric sequences) अथवा गुणोत्तर श्रेणियाँ (geometric progressions) या संक्षेप में G. P. कहलाते हैं। दूसरे शब्दों में, गुणोत्तर अनुक्रम अथवा गुणोत्तर श्रेणी वह अनुक्रम होता है जिसका प्रथम के अतिरिक्त प्रत्येक पद उससे तुरन्त पिछले पद को एक शून्येतर स्थिर संख्या से गुणा करके प्राप्त होता है। स्थिर संख्या सार्व अनुपात (common ratio) कहलाती है तथा इसे प्राय: 'r' से व्यक्त किया जाता है। प्रथम पद को प्राय: 'b' से व्यक्त किया जाता है। यदि $b\neq 0$ है तो गुणोत्तर श्रेणी में कोई भी पद शून्य नहीं होगा।

इस प्रकार, एक गुणोत्तर श्रेणी में,

$$t_n=rt_{n-1}, (r\neq 0)$$

$$\text{अर्थात्}, \quad \frac{t_n}{t_{n-1}}=r \quad (\text{स्थिर})$$

(A) में हम देखते हैं कि $b=2$ तथा $r=2$ है। (B) में, $b=-\frac{1}{3}$ तथा $r=-\frac{1}{3}$ है। (C) में, $b=.01$ तथा $r=.01$ है। (D) में, $b=-\frac{9}{2}$ तथा $r=-\frac{1}{2}$ है।

5.7 एक गुणोत्तर श्रेणी का nवाँ पद अर्थात् व्यापक पद

आइए एक गुणोत्तर श्रेणी लें जिसका प्रथम पद 'b' तथा सार्व अनुपात 'r' है। इसके प्रथम कुछ पद क्या होंगे ? दूसरा पद b को r से गुणा करके प्राप्त होता है। अतः $t_2 = br$। इसी प्रकार तीसरा पद t_2 को r से गुणा करके प्राप्त होता है। अतः $t_3 = rt_2 = br^2$।

हम इन्हें तथा कुछ अन्य पदों को नीचे दे रहे हैं :

पहला पद, $t_1 = b \quad = br^{1-1}$

दूसरा पद, $t_2 = br \quad = br^{2-1}$

तीसरा पद, $t_3 = br^2 \quad = br^{3-1}$

चौथा पद, $t_4 = br^3 \quad = br^{4-1}$

पाँचवाँ पद, $t_5 = br^4 \quad = br^{5-1}$

क्या आप कोई प्रतिरूप देखते हैं? हम देखते हैं कि प्रत्येक पद $b \times r$ की [illegible] है। उदाहरणार्थ, 16वाँ पद क्या होगा ?

$$t_{16} = b \times r^{16-1} = br^{15}$$

क्या अब आप कह सकते हैं कि nवाँ पद अर्थात् व्यापक पद क्या होगा ? यह सिद्ध किया जा सकता है कि

$$\mathbf{t_n = br^{n-1}}$$

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : गुणोत्तर श्रेणी $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots$ का 10वाँ तथा n वाँ पद ज्ञात कीजिए।

हल : यहाँ $b = \frac{3}{2}$ तथा $r = \frac{1}{2}$ है।

इस प्रकार, $\quad t_{10} = br^{10-1} = br^9 = \frac{3}{2}\left(\frac{1}{2}\right)^9 = \frac{3}{1024}$

तथा $\quad t_n = br^{n-1} = \frac{3}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{3}{2^n}$

उदाहरण 2 : उन गुणोत्तर श्रेणियों के प्रथम 4 पद ज्ञात कीजिए जिनके nवें पद निम्न हैं :

(i) $t_n = 3(-2)^{n-1}$ (ii) $t_n = x^n, \ (x \neq 0)$

हल : (i) $t_n = 3(-2)^{n-1}$

इस प्रकार, $\quad t_1 = 3(-2)^0 = 3, \ t_2 = 3(-2) = -6,$

$t_3 = 3(-2)^2 = 12$ तथा $t_4 = 3(-2)^3 = -24$

अतः प्रथम 4 पद 3, −6, 12 तथा −24 हैं।

(ii) $t_n = x^n, \ (x \neq 0)$

$n = 1, 2, 3$ तथा 4 प्रतिस्थापित करके हमें क्रमशः निम्न प्रथम 4 पद प्राप्त होते हैं :

x, x^2, x^3 तथा x^4

उदाहरण 3 : गुणोत्तर श्रेणी $2, 2\sqrt{2}, 4, \ldots$ का कौन सा पद 64 है ?

हल : यहाँ $a = 2$ तथा $r = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$ है।

अब, $t_n = 2(\sqrt{2})^{n-1}$

हमें ऐसा n ज्ञात करना है कि $t_n = 64$ अर्थात् $2(\sqrt{2})^{n-1} = 64$ हो।

इस प्रकार, $(\sqrt{2})^{n-1} = 32$

अर्थात्, $2^{\frac{n-1}{2}} = 2^5$

अथवा, $\dfrac{n-1}{2} = 5$

अतः, $n = 11$

इस प्रकार 64 दी हुई श्रेणी का 11वाँ पद है।

उदाहरण 4 : एक गुणोत्तर श्रेणी का पहला पद 50 तथा चौथा पद 1350 है। उसका 5 वाँ पद ज्ञात कीजिए :

हल : यहाँ $a = 50$ तथा $t_4 = 1350$ है। हमें r को निर्धारित करने की आवश्यकता है।

अब, $t_4 = 50r^3 = 1350$

इस प्रकार, $r^3 = 27$

अर्थात्, $r = 3$

अतः, $t_5 = 50(3)^4 = 4050$

प्रश्नावली 5.4

निम्नलिखित गुणोत्तर श्रेणियों में से प्रत्येक का 'r' ज्ञात कीजिए तथा प्रत्येक के अगले चार पद लिखिए :

1. $-3, 1, -\frac{1}{3}, \ldots$
2. $-\frac{2}{3}, -6, -54, \ldots$
3. $5, 0.5, 0.05, \ldots$

4. $\sqrt{3}, \sqrt{6}, 2\sqrt{3}, 2\sqrt{6}, \ldots$

5. $\frac{x}{y}, \frac{1}{x}, \frac{y}{x^3}, \ldots$

निम्नलिखित गुणोत्तर श्रेणियों में प्रत्येक के सम्मुख दिए पद ज्ञात कीजिए :

6. $b=1,\ r=1.2\ ;\ t_4,\ t_n$

7. $128,\ -96,\ 72,\ \ldots\ ;\ t_{11},\ t_n$

8. $100,\ -110,\ 121,\ \ldots\ ;\ t_n$

9. उस गुणोत्तर श्रेणी का 12वाँ पद निर्धारित कीजिए जिसका 8वाँ पद 192 तथा सार्व अनुपात 2 है।

10. एक गुणोत्तर श्रेणी के 5वें, 8वें तथा 11वें पद क्रमशः p, q तथा s हैं। दिखाइए कि $q^2=ps$ ।

11. एक गुणोत्तर श्रेणी का चौथा पद उसके दूसरे पद का वर्ग है तथा उसका प्रथम पद -3 है। उसका 7वाँ पद निर्धारित कीजिए।

12. एक गुणोत्तर श्रेणी, जिसका प्रथम पद 'b' तथा सार्व अनुपात 'r' है, के सभी पदों का वर्ग किया जाता है। क्या इस प्रकार से प्राप्त अनुक्रम भी एक गुणोत्तर श्रेणी है ? यदि ऐसा है, तो इसका प्रथम पद, सार्व अनुपात तथा nवाँ पद ज्ञात कीजिए।

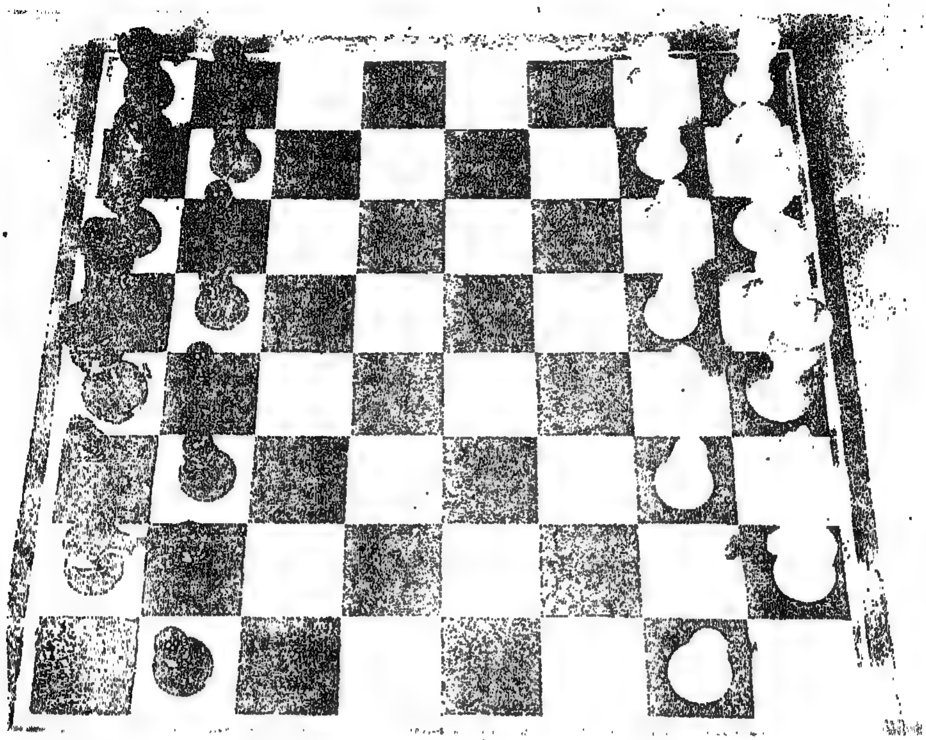
13. x का (के) मान ज्ञात कीजिए ताकि $-\frac{2}{7},\ x,\ -\frac{7}{2}$ एक गुणोत्तर श्रेणी के तीन क्रमागत पद हों।

14. दो गुणोत्तर श्रेणियाँ दी हुई हैं जिनमें से एक श्रेणी का प्रथम पद 'b' तथा सार्व अनुपात 'r' है तथा दूसरी श्रेणी का प्रथम पद 'c' तथा सार्व अनुपात 's' है। दिखाइए कि दोनों श्रेणियों के तदनुरूपी पदों के गुणनफल से प्राप्त अनुक्रम भी एक गुणोत्तर श्रेणी है। इस प्रकार प्राप्त श्रेणी का प्रथम पद तथा सार्व अनुपात ज्ञात कीजिए। यह भी दिखाइए कि, जब $c \neq 0$, तो दोनों श्रेणियों के तदनुरूपी पदों के भागफल से प्राप्त अनुक्रम भी एक गुणोत्तर श्रेणी है। उसका प्रथम पद तथा सार्व अनुपात ज्ञात कीजिए।

15. किसी संवर्धन (culture) में जीवाणुओं की संख्या प्रत्येक घंटे में पहले से दुगुनी हो जाती है। यदि संवर्धन में प्रारम्भ में जीवाणुओं की संख्या 30 हो, तो दूसरे घंटे के अन्त में जीवाणुओं की संख्या क्या होगी ? चौथे तथा n वें घंटे के अन्त में जीवाणुओं की संख्याएँ क्या-क्या होंगी ?

5.8 एक गुणोत्तर श्रेणी के n पदों का योग

शतरंज (chess) के खेल के आविष्कारक के विषय में एक अति रुचिपूर्ण कहानी है। क्या आपने शतरंज का बोर्ड देखा है ?



आकृति 5.2 : शतरंज का बोर्ड

आविष्कारक से फारस (Persia) का राजा इतना प्रसन्न हुआ कि उसने आविष्कारक को मुँह माँगा इनाम देना चाहा। आविष्कारक ने इच्छा व्यक्त की कि शतरंज के बोर्ड के पहले वर्ग में गेहूँ का एक दाना रख दिया जाए, दूसरे वर्ग में दो दाने तथा तीसरे में चार दानें, इत्यादि। आविष्कारक ने कुल मिलाकर गेहूँ के कितने दाने माँगे थे ? स्पष्ट है कि 64 वें वर्ग में 2^{63} दानों की आवश्यकता होगी। जैसा कि हम देखेंगे कि यद्यपि आविष्कारक की माँग अति अल्प प्रतीत होती है परन्तु वस्तुतः ऐसा है नहीं।

आइए अब यह सीखें कि एक गुणोत्तर श्रेणी, जिसका प्रथम पद 'b' तथा सार्व अनुपात 'r' है, के प्रथम n पदों का योग किस प्रकार ज्ञात करते हैं। यदि हम n पदों के योग को S_n से व्यक्त करें, तो

$$S_n = b + br + br^2 + \ldots + br^{n-2} + br^{n-1} \tag{1}$$

आइए (1) के दोनों पक्षों को r से गुणा करें। हमें निम्न प्राप्त होता है :

$$r\,S_n = br + br^2 + \ldots + br^{n-2} + br^{n-1} + br^n \tag{2}$$

ध्यान दीजिए कि हमने (2) में गुणनफल के दक्षिण पक्ष को इस प्रकार लिखा है कि इसका प्रथम पद (1) के दक्षिण पक्ष के दूसरे पद के ठीक नीचे आए; इसका दूसरा पद (1) के दक्षिण पक्ष के तीसरे पद के ठीक

नीचे आए, इत्यादि। हम ऐसा क्यों करते हैं? यह स्पष्ट है कि जब हम $r\,S_n$ को S_n में से घटाते हैं तो समान पद आपस में कट जाएँगे। इस प्रकार,

$$S_n - r\,S_n = b - br^n$$

अर्थात्, $S_n\,(1-r) = b - br^n$

अब, यदि $r \neq 1$, तो $S_n = \dfrac{b(1-r^n)}{1-r}$

इस प्रकार, किसी गुणोत्तर श्रेणी के प्रथम n पदों का योग

$$S_n = \frac{b\,(1-r^n)}{1-r}\,,\ r \neq 1 \text{ है।}$$

स्पष्ट है कि जब $r = 1$, तो $S_n = nb$ होता है।

आइए अब देखें कि आविष्कारक ने गेहूँ के कितने दाने माँगे थे? स्पष्ट है कि $b = 1$, $r = 2$ तथा $n = 64$। इस प्रकार,

$$S_{64} = \frac{1(2^{64}-1)}{2-1} = 2^{64}-1$$

क्या आपको मालूम है कि यह संख्या कितनी बड़ी है? यह कहा जाता है कि आविष्कारक ने जितना गेहूँ माँगा था उससे पूरे फारस देश को 7 सेंटी मीटर से भी अधिक गहरी गेहूँ की परत से ढका जा सकता था। आधुनिक भारत के लिए यह परत लगभग 4 से० मी० गहरी होती। यह कहने की आवश्यकता नहीं है कि जब राजा को यह अनुभव हुआ कि आविष्कारक की माँग पूरी करना असम्भव है तो वह बहुत खिन्न हुआ। हम इस पर विचार करना नहीं चाहेंगे कि राजा ने आविष्कारक को इनाम दिया या नहीं।

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : गुणोत्तर श्रेणी 128, — 96, 72, ... के प्रथम 11 पदों तथा प्रथम n पदों का योग ज्ञात कीजिए।

हल : यहाँ, $b = 128$, $r = \dfrac{-96}{128} = -\dfrac{3}{4}$ है।

क्योंकि $r \neq 1$ है, अतः हम योग के सूत्र का प्रयोग कर सकते हैं और निम्न प्राप्त करते हैं :

$$S_{11} = \frac{128\left[1-\left(\frac{-3}{4}\right)^{11}\right]}{1-\left(\frac{-3}{4}\right)} = 128\left[1+\frac{3^{11}}{4^{11}}\right]\times\frac{4}{7}$$

अर्थात्, $$S_{11} = \frac{2(4^{11}+3^{11})}{7\times 4^7} = \frac{4^{11}+3^{11}}{7\times 2^{15}}$$

तथा, $$S_n = \frac{128\left[1-\left(\frac{-3}{4}\right)^n\right]}{1-\left(\frac{-3}{4}\right)} = \frac{2[\ 4^n - (-1)^n 3^n]}{7(4^{n-4})}$$

अर्थात्, $$S_n = \frac{4^n - (-1)^n\, 3^n}{7 \times 2^{3n-6}}$$

उदाहरण 2 : गुणोत्तर श्रेणी $1, a, a^2, a^3, \ldots;\ a \neq 1$ के प्रथम n पदों का योग ज्ञात कीजिए।

हल : यहाँ, $b=1$ तथा $r=a\neq 1$ है।

इस प्रकार, $$S_n = \frac{1(1-a^n)}{1-a}$$

अर्थात्, $$S_n = \frac{1-a^n}{1-a}$$

उदाहरण 3 : एक गुणोत्तर श्रेणी में $b=1$ तथा $r=\sqrt{2}$ है। S_{20} ज्ञात कीजिए।

हल : $$S_{20} = \frac{1[1-(\sqrt{2})^{20}]}{1-\sqrt{2}} = \frac{2^{10}-1}{\sqrt{2}-1}$$

आइए, हर का परिमेयकरण (rationalization) करें। हम अंश तथा हर को $\sqrt{2}+1$ से गुणा करते हैं। हमें निम्न प्राप्त होता है :

$$S_{20} = \frac{(2^{10}-1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

अर्थात्, $$S_{20} = 1023\,(\sqrt{2}+1)$$

उदाहरण 4 : गुणोत्तर श्रेणी $3, 3^2, 3^3, \ldots$ के कितने पदों की आवश्यकता होगी ताकि योग 120 प्राप्त हो ?

हल : यहाँ, $b=3$, $r=3$ तथा $S_n=120$ है। हमें n निर्धारित करना है।

अब, $$\frac{3(1-3^n)}{1-3} = 120$$

इस प्रकार, $$1-3^n = -80$$

अर्थात्, $$3^n = 81 = 3^4$$

अतः, $$n=4$$

इस प्रकार, पदों की वांछित संख्या 4 है।

उदाहरण 5 : एक गुणोत्तर श्रेणी के प्रथम तीन पदों का योग $\frac{39}{10}$ है तथा उनका गुणनफल 1 है। प्रथम पद, सार्व अनुपात तथा तीनों पद ज्ञात कीजिए।

हल : एक 'सुन्दर' विधि यह है कि प्रथम तीन पदों को $\frac{b}{r}$, b तथा br मान लिया जाए (क्यों ?)।

तब, $$\frac{b}{r} + b + br = \frac{39}{10} \qquad (1)$$

तथा, $$\left(\frac{b}{r}\right)(b)(br) = 1 \qquad (2)$$

(2) से हमें तुरंत $b^3=1$ अर्थात्* $b=1$ प्राप्त होता है।

---

*हम $b^3-1=0$ का केवल वास्तविक मूल ही ले रहे हैं।

(1) में प्रतिस्थापित करने पर,

$$\frac{1}{r}+1+r=\frac{39}{10}$$

अर्थात्, $10r^2-29r+10=0$

यह r में एक द्विघात समीकरण है। इस प्रकार,

$$r=\frac{29\pm\sqrt{841-4(10)(10)}}{20}=\frac{29\pm 21}{20}$$

अर्थात्, $r=\frac{5}{2}$ तथा $\frac{2}{5}$

अत: हमें निम्न दो गुणोत्तर श्रेणियाँ प्राप्त होती हैं :

$$r=\frac{5}{2}:\quad \frac{2}{5}, 1, \frac{5}{2}, \frac{25}{4}, \ldots$$

$$r=\frac{2}{5}:\quad \frac{5}{2}, 1, \frac{2}{5}, \frac{4}{25}, \ldots$$

दोनों गुणोत्तर श्रेणियों के प्रथम तीन पद दिए हुए प्रतिबन्धों को संतुष्ट करते हैं।

उदाहरण 6 : निम्नलिखित अनुक्रम का n पदों तक योग ज्ञात कीजिए :

$$9, 99, 999, 9999, \ldots, \underbrace{99\ldots9}_{(n \text{ बार } 9)}, \ldots$$

हल : यह पूर्णतया एक गुणोत्तर श्रेणी नहीं है। परन्तु हम इसे निम्न प्रकार से लिखकर एक गुणोत्तर श्रेणी से सम्बद्ध कर सकते हैं :

$$10-1,\ 10^2-1,\ 10^3-1,\ 10^4-1,\ \ldots,\ 10^n-1,\ \ldots$$

यदि S_n, n पदों के योग को व्यक्त करता है, तो

$$S_n=10+10^2+\ldots+10^n-n$$

$$=\frac{10(1-10^n)}{1-10}-n$$

अर्थात्, $S_n=\frac{10}{9}(10^n-1)-n$

प्रश्नावली 5.5

निम्नलिखित गुणोत्तर श्रेणियों में से प्रत्येक के सम्मुख दिए पदों का योग ज्ञात कीजिए ;

1. $1, \frac{2}{3}, \frac{4}{9}, \ldots$; 10 पद, n पद

2. .15, .015, .0015, ...; 8 पद, 20 पद
3. $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots$; n पद
4. $1, -a, a^2, -a^3, \ldots$; n पद $(a \neq 1)$
5. $x^3, x^5, x^7 \ldots$; n पद $(x \neq \pm 1)$
6. $2, -\frac{1}{2}, \frac{1}{8}, \ldots$; 12 पद, n पद
7. $\sum\limits_{k=1}^{11} (2+3^k)$ का मान ज्ञात कीजिए।

निम्नलिखित में से प्रत्येक का मान ज्ञात कीजिए:

8. $\sum\limits_{j=1}^{6} 4\left(\frac{1}{3}\right)^{j-1}$
9. $\sum\limits_{i=5}^{11} 4^i$
10. $\sum\limits_{l=1}^{18} (2^l + 3^{l-1})$
11. एक गुणोत्तर श्रेणी के प्रथम तीन पदों का योग $\frac{13}{12}$ तथा उनका गुणनफल -1 है। गुणोत्तर श्रेणी ज्ञात कीजिए।
12. गुणोत्तर श्रेणी $3, \frac{3}{2}, \frac{3}{4}, \ldots$ के कितने पदों की आवश्यकता होगी ताकि योग $\frac{3069}{512}$ प्राप्त हो?
13. एक गुणोत्तर श्रेणी के प्रथम तीन पदों का योग 16 है तथा अगले तीन पदों का योग 128 है। गुणोत्तर श्रेणी का प्रथम पद, सार्व अनुपात तथा प्रथम n पदों का योग निर्धारित कीजिए।
14. एक व्यक्ति अपने चार मित्रों को एक पत्र लिखता है। वह प्रत्येक को यह कहता है कि वह इस पत्र की प्रतिलिपि अपने चार मित्रों को इस निर्देश के साथ भेजें कि वे भी इस श्रृंखला को इसी प्रकार दोहराएँ। यह मानते हुए कि यह श्रृंखला नहीं टूटती तथा एक पत्र को भेजने का डाक खर्च 25 पैसे है, पत्रों के आठवें समुच्चय को भेजे जाने तक कुल डाक व्यय निर्धारित कीजिए।
15. एक गुणोत्तर श्रेणी दी हुई है जिसमें $b=729$ तथा सातवाँ पद 64 है। S_7 ज्ञात कीजिए।

*16. निम्न अनुक्रम के प्रथम n पदों का योग ज्ञात कीजिए :

$$7, 77, 777, \ldots, \underbrace{77\ldots7}_{(n \text{ बार } 7)}, \ldots$$

17. अनुक्रम $1, -1, 1, -1, \ldots, (-1)^{n+1}, \ldots$ के प्रथम n पदों का योग ज्ञात कीजिए।

5.9 अपरिमित गुणोत्तर श्रेणियाँ (Infinite Geometric Progressions)

आइए गुणोत्तर श्रेणी 1, $\frac{2}{3}$, $\frac{4}{9}$, ... पर विचार करें।

हम पहले ही प्रश्नावली 5.5 के प्रश्न 1 में इस श्रेणी का n पदों तक योग ज्ञात कर चुके हैं। आपको याद होगा कि

$$S_n=3\left[1-\left(\tfrac{2}{3}\right)^n\right]=3-3\left(\tfrac{2}{3}\right)^n$$

यदि पदों की संख्या बहुत बड़ी हो जाए तो क्या होगा ? आइए $\left(\frac{2}{3}\right)^n$ के आचरण का अध्ययन करें जबकि n का मान बड़ा होता जाता है।

n	1	5	10	20	40
$\left(\frac{2}{3}\right)^n$	.6667	.1316872428	.01734152992	.00030072866	.0000009043772695

हम देखते हैं कि जैसे-जैसे n बड़ा होता जाता है, $\left(\frac{2}{3}\right)^n$ शून्य के निकट पहुँचता जाता है। गणित की भाषा में हम यह कहते हैं कि जैसे-जैसे n बड़ा होता जाता है, $\left(\frac{2}{3}\right)^n$ शून्य की ओर अग्रसर (approaches) होता रहता है। [पाठक ध्यान दें कि $\left(\frac{2}{3}\right)^n$ शून्य की ओर अग्रसर होता है, परन्तु यह कभी भी शून्य के बराबर नहीं होता।] हम पुनः यह कहते हैं कि S_n, 3 की ओर अग्रसर होता है। इसे हम निम्न प्रकार लिखते हैं :

जैसे-जैसे n बड़ा होता जाता है, $\left(\frac{2}{3}\right)^n\to 0$ तथा $S_n\to 3$। कभी-कभी हम यहाँ तक भी कह देते हैं कि इस गुणोत्तर श्रेणी के असंख्य पदों का (या श्रेणी का अनंत तक) योग 3 है।

आइए, एक अन्य उदाहरण लें। हम गुणोत्तर श्रेणी $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, ... पर विचार करते हैं। इस श्रेणी के प्रथम n पदों का योग $S_n=1-\left(\frac{1}{2}\right)^n$ है।

पुनः, आइए $\left(\frac{1}{2}\right)^n$ के आचरण का अध्ययन करें जबकि n बड़ा होता जाता है।

n	1	5	10	20
$\left(\frac{1}{2}\right)^n$	.5	.03125	.0009765625	.00000095367431640625

हम देखते हैं कि जैसे-जैसे n बड़ा होता जाता है, $\left(\frac{1}{2}\right)^n$ शून्य के निकट आता जाता है। वास्तव में यह $\left(\frac{2}{3}\right)^n$ की अपेक्षा तीव्र गति से शून्य की ओर अग्रसर होता है। इस प्रकार, जैसे-जैसे n बड़ा होता जाता है, $\left(\frac{1}{2}\right)^n\to 0$ तथा $S_n\to 1$।

इन दोनों उदाहरणों में हमने देखा कि $r<1$ है। वास्तव में यह सिद्ध* किया जा सकता है कि यदि $|r|<1$, तो जैसे-जैसे n बड़ा होता जाता है, $r^n \to 0$।

आइए अब देखें कि ऐसी स्थितियों में S_n का क्या होता है। एक गुणोत्तर श्रेणी के लिए, जिसका प्रथम पद 'b' तथा सार्व अनुपात 'r' है, हमें निम्न ज्ञात है :

$$S_n = \frac{b(1-r^n)}{1-r} = \frac{b}{1-r} - \frac{br^n}{1-r} \qquad (1)$$

क्योंकि $|r|<1$, $r^n \to 0$। इस प्रकार, $\frac{br^n}{1-r} \to 0$।

अतः $S_n \to \frac{b}{1-r}$

दूसरे शब्दों में, एक गुणोत्तर श्रेणी, जिसका प्रथम पद 'b' तथा सार्व अनुपात 'r' ($|r|<1$) है, के असंख्य पदों का योग $\frac{b}{1-r}$ होता है।

हम इसे इस प्रकार लिखते हैं :

S_∞ **अथवा** $S = \frac{b}{1-r}$

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : अपरिमित गुणोत्तर श्रेणी $5, \frac{20}{7}, \frac{80}{49}, \ldots$ का योग ज्ञात कीजिए।

हल : यहाँ $b=5$ तथा $r=\frac{4}{7}$ है। साथ ही, $|r|<1$ है।

इस प्रकार $S = \frac{5}{1-\frac{4}{7}} = \frac{35}{3}$

उदाहरण 2 : गुणोत्तर श्रेणी $\frac{-3}{4}, \frac{3}{16}, \frac{-3}{64}, \ldots$ के लिए S ज्ञात कीजिए।

हल : यहाँ $b=\frac{-3}{4}$ तथा $r=-\frac{1}{4}$ है। साथ ही, $|r|<1$ है।

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* यदि $r=0$ हो, तो सभी n के लिए $r^n=0$।

यदि $0<|r|<1$, तो $\frac{1}{|r|}>1$ तथा $\frac{1}{|r|}$ को $1+s$ के रूप में लिखा जा सकता है जबकि $s>0$।

अब आपको याद होगा कि $(1+s)^n \geqslant 1+ns$

इस प्रकार, $|r|^n < \frac{1}{1+ns} \leqslant \frac{1}{n}\left(\frac{1}{s}\right)$

जिससे, जब $n \to \infty$, $r^n \to 0$

इस प्रकार, $S=\dfrac{-2}{1-(-\frac{1}{4})}=-\frac{8}{5}$

अब हम एक गुणोत्तर श्रेणी, जिसमें $|r|<1$ है, के असंख्य पदों के योग का एक सुन्दर अनुप्रयोग (application) देते हैं। क्या आपको याद है कि एकक I में हमने सीखा था कि एक परिमेय संख्या को, जिसका प्रसार एक असांत आवर्ती दशमलव हो, किस प्रकार ज्ञात किया जाता है? ऐसी परिमेय संख्या ज्ञात करने के लिए हम एक अपरिमित गुणोत्तर श्रेणी के योग का प्रयोग करेंगे। उदाहरणार्थ, संख्या 0.333... पर बिचार कीजिए। [अनुच्छेद 1.5 का उदाहरण 1 भी देखिए।] इस दशमलव को हम निम्न प्रकार से लिख सकते हैं:

$$0.333\ldots=0.3+0.03+0.003+\ldots \qquad (3)$$

निस्संदेह (3) में दक्षिण पक्ष एक अपरिमित गुणोत्तर श्रेणी के योग को प्रदर्शित करता है जिसमें $b=0.3$ तथा $r=0.1$ $(|r|<1)$ है। इसका योग क्या होगा? यह योग $\dfrac{0.3}{1-0.1}$ अर्थात् $\frac{1}{3}$ है।

इस प्रकार, $\frac{1}{3}$ वह परिमेय संख्या है जिसे यदि दशमलव के रूप में व्यक्त किया जाए तो प्रसार $0.\bar{3}$ प्राप्त होगा।

आइए कुछ और उदाहरण लें।

उदाहरण 3: वह परिमेय संख्या ज्ञात कीजिए जिसे यदि दशमलव के रूप में व्यक्त किया जाए तो प्रसार $0.23\bar{4}$ प्राप्त हो। [अनुच्छेद 1.5 का उदाहरण 2 भी देखिए।]

हल: हम इस प्रकार लिखते हैं:

$$0.23\bar{4}=0.23+0.004+0.0004+\ldots$$
$$=0.23+\frac{0.004}{1-0.1}=0.23+\frac{4}{900}$$
$$=\frac{211}{900}$$

अतः, $\dfrac{211}{900}$ अभीष्ट परिमेय संख्या है।

उदाहरण 4: वह परिमेय संख्या ज्ञात कीजिए जिसे यदि दशमलव में व्यक्त किया जाए तो प्रसार $1.\overline{56}$ प्राप्त हो।

हल: हम इसे इस प्रकार लिखते हैं:

$$1.\overline{56}=1+0.56+0.0056+\ldots$$
$$=1+\frac{0.56}{1-0.01}=1+\frac{56}{99}$$
$$=\frac{155}{99}$$

इस प्रकार, $\dfrac{155}{99}$ वांछित परिमेय संख्या है।

यदि $|r|>1$ हो, तो S_n क्या होगा? स्पष्ट है कि यदि $|r|>1$, तो जैसे-जैसे n बड़ा होता जाता है, वैसे-वैसे $|r^n|$ बड़ा होता जाता है। **इस स्थिति में हम कहते हैं कि गुणोत्तर श्रेणी के असंख्य पदों के योग का अस्तित्व नहीं है।**

प्रश्नावली 5.6

निम्नलिखित गुणोत्तर श्रेणियों में से प्रत्येक में r का निर्धारण कीजिए। जाँच कीजिए कि $|r| < 1$ है। S_∞ अथवा S भी ज्ञात कीजिए।

1. $1, \frac{1}{3}, \frac{1}{9}, \ldots$
2. $7, -1, \frac{1}{7}, -\frac{1}{49}, \ldots$
3. $6, 1.2, .24, \ldots$
4. $50, 42.5, 36.125, \ldots$
5. $0.3, 0.18, 0.108, \ldots$
6. $10, -9, 8.1, \ldots$

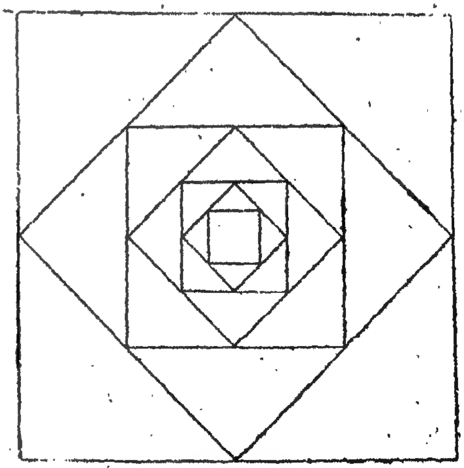
निम्नलिखित दशमलवों में से प्रत्येक के लिए एक परिमेय संख्या ज्ञात कीजिए जिसका प्रसार वह दशमलव हो :

7. $0.6\overline{8}$
8. $1.\overline{15}$
9. $0.\overline{712}$

*10. $22.378\overline{2378}$

11. $0.\overline{10}$
12. एक गुणोत्तर श्रेणी का प्रथम पद 2 है तथा असंख्य पदों तक इसका योग 6 है। सार्व अनुपात ज्ञात कीजिए।
13. एक गुणोत्तर श्रेणी का सार्व अनुपात $-\frac{4}{5}$ है तथा इसका असंख्य पदों तक योग $\frac{80}{9}$ है। प्रथम पद ज्ञात कीजिए।

*14. एक दिए हुए वर्ग की भुजाओं के मध्य-बिन्दुओं को मिलाकर एक अन्य वर्ग खींचा गया है। इसी प्रकार एक तीसरा वर्ग दूसरे वर्ग के अन्तर्गत खींचा गया है तथा यही प्रक्रिया असंख्य बार दोहराई गई है। (देखिए आकृति 5.3) यदि पहले वर्ग की एक भुजा 16 सें० मी० हो, तो सभी वर्गों के क्षेत्रफलों का योग निर्धारित कीजिए।



आकृति 5.3

5.10 वर्गों तथा घनों के अनुक्रम

अब हम दो अन्य अनुक्रमों—प्राकृत संख्याओं के वर्गों तथा घनों के अनुक्रमों पर विचार करेंगे। इनका योग प्राप्त करना गणित की सुन्दरता का एक सुखद अनुभव है। आइए पहले प्राकृत संख्याओं के **वर्गों के अनुक्रम** अर्थात् $1^2, 2^2, 3^2, \ldots, n^2, \ldots$ पर विचार करें। हम इस अनुक्रम के n पदों का योग अर्थात् S_n कैसे ज्ञात करेंगे? आइए दूरदर्शीय योगों की विधि का प्रयोग करें जैसा हमने प्रथम n प्राकृत संख्याओं का योग प्राप्त करने की स्थिति में किया था। हम किस सर्वसमिका से आरम्भ करें? क्या आप अनुमान लगा सकते हैं?

हम सर्वसमिका $k^3-(k-1)^3=3k^2-3k+1$ पर विचार करते हैं।

बारी-बारी से $k=1, 2, 3, \ldots, n$ प्रतिस्थापित करके हम निम्न प्राप्त करते हैं:

$$1^3-0^3=3(1)^2-3(1)+1$$
$$2^3-1^3=3(2)^2-3(2)+1$$
$$3^3-2^3=3(3)^2-3(3)+1$$
$$\ldots \quad \ldots \quad \ldots \quad \ldots$$
$$n^3-(n-1)^3=3(n)^2-3(n)+1$$

जोड़ने पर, $n^3-0^3=3S_n-3(1+2+\ldots+n)+n$ (1)

परन्तु $1+2+3+\cdots+n=\dfrac{n(n+1)}{2}$

(1) में प्रतिस्थापित करने पर हमें निम्न प्राप्त होता है:

$$3S_n=n^3+\frac{3n(n+1)}{2}-n=\frac{2n^3+3n^2+n}{2}$$

अर्थात्, $3\,S_n=\dfrac{n(2n^2+3n+1)}{2}$

इस प्रकार, $\mathbf{S}_n=\dfrac{\mathbf{n(n+1)\ (2n+1)}}{\mathbf{6}}$

इस सूत्र का उपयोग करके, उदाहरणार्थ, हम प्रथम 10 प्राकृत संख्याओं के वर्गों का योग ज्ञात करते हैं। हमें निम्न प्राप्त होता है:

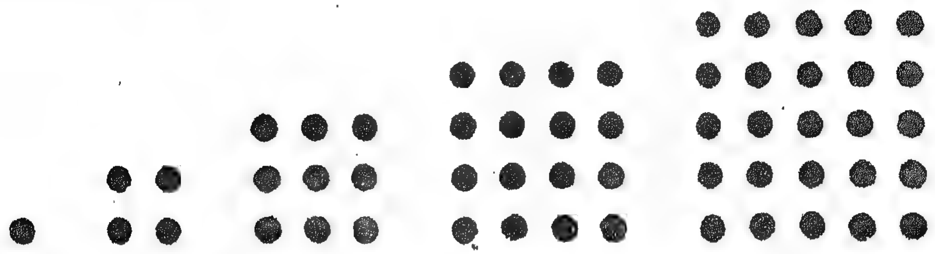
$$S_{10}=\frac{10\times11\times21}{6}=385$$

[पाठक को चाहिए कि वह इसी प्रकार सिद्ध करे कि प्रथम n प्राकृत संख्याओं के घनों का योग $\left(\dfrac{n(n+1)}{2}\right)^2$ होता है। आप किस सर्वसमिका से आरम्भ करेंगे?

संकेत: सर्वसमिका $k^4-(k-1)^4=4k^3-6k^2+4k-1$ लीजिए।]

ऐसे प्रमाण मिलते हैं कि लगभग 4000 वर्ष पूर्व बेबीलोन के निवासी समांतर तथा गुणोत्तर अनुक्रमों के विषय में जानते थे। विभिन्न मृत्तिका शिलाओं (clay tablets) में से एक शिला में, जिसके विषय में विश्वास किया जाता है कि वह बेबीलोन के निवासियों के काल की है, संख्याओं 1 से 60 तक के वर्ग तथा संख्याओं 1 से 32 तक के घन अंकित हैं।

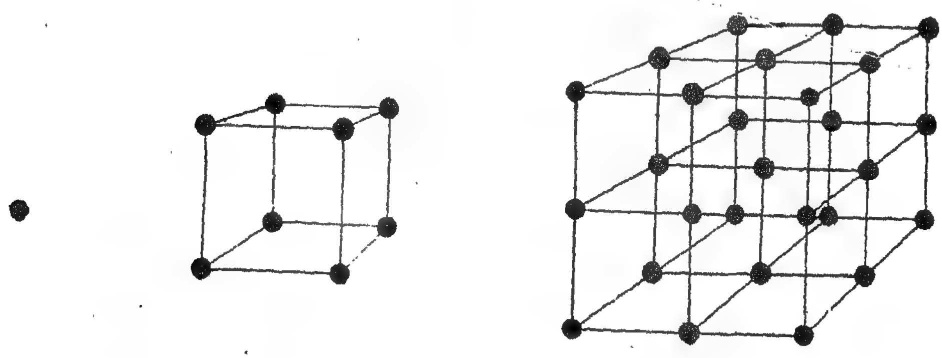
यूनान के निवासी संख्याओं का निरूपण ज्यामितीय आकारों में बिन्दुओं (dots) की सहायता से किया करते थे । उदाहरणार्थ, 4^2 को चार बिन्दुओं की भुजा के वर्ग के रूप में निरूपित किया जाता था जैसा कि नीचे आकृति 5.4 में दिखाया गया है :



$1^2=1$ $2^2=4$ $3^2=9$ $4^2=16$ $5^2=25$

आकृति 5.4 : यूनानियों द्वारा वर्ग संख्याओं का ज्यामितीय निरूपण

इसी प्रकार वे घन संख्याओं, उदाहरणार्थ, 27 को तीन बिन्दुओं की भुजा के घन के रूप में निरूपित करते थे जैसा कि नीचे आकृति 5.5 में दिखाया गया है :



$1^3=1$ $2^3=8$ $3^3=27$

आकृति 5.5 : यूनानियों द्वारा घन संख्याओं का ज्यामितीय निरूपण

यद्यपि समांतर अनुक्रम का योग सम्भवतया पहले ही ज्ञात था परन्तु **आर्किमीडीज** (287-212 ई० पू०) जिन्हें **'पूर्वकालीन महान गणितज्ञ'** कहा जाता है, के समय से पहले साहित्य में इसके सूत्र का कहीं स्पष्ट रूप से विवरण नहीं मिलता । आर्किमीडीज न केवल एक महान गणितज्ञ थे बल्कि अपने आविष्कारों के कारण वे एक बहुचर्चित व्यक्ति भी थे । उनके विषय में एक सुप्रसिद्ध कहानी है जो विशेषकर भौतिकी के विद्यार्थियों के लिए अत्यन्त रुचिपूर्ण है । राजा का विश्वास था कि जो ताज उसने बनवाया है, उसमें अशुद्ध सोना है । उसने आर्किमीडीज से कहा कि वे ताज की बनावट को क्षति पहुँचाए बिना उसे जाँच करने की विधि ज्ञात करे । आर्किमीडीज बहुत दिनों तक सोचते रहे परन्तु कोई विधि ज्ञात नहीं कर सके । एक दिन जब वे पानी में तैर रहे थे तो उन्होंने अनुभव किया कि पानी ने उनका शरीर आंशिक रूप से 'ऊपर उठा दिया' है । वे प्रसन्नता से उछल

पड़े। उन्होंने आज के एक प्रसिद्ध सिद्धांत, अर्थात् "जब कोई वस्तु पानी में डुबोई जाती है तो वह अपने भार के समान पानी का स्थानान्तरण करती है", के आधार का आविष्कार कर लिया था। उन्होंने ताज में मिश्रित वस्तुओं की जाँच में इस ही सिद्धांत का प्रयोग किया। प्रसंग की बात है, प्राकृत संख्याओं के वर्गों का सूत्र देने का श्रेय भी आर्किमिडीज को ही है।

हिन्दू गणितज्ञ समांतर अनुक्रमों के विषय में पहले से ही जानते थे परन्तु वर्गों तथा घनों के योग के सूत्र सर्वप्रथम आर्यभट्ट (जन्म 476 ई०) ने अपनी सुप्रसिद्ध कृति **आर्यभटीयम्**, जो लगभग 499 ई० में लिखी गई थी, में दिए थे। उन्होंने p वें पद से आरम्भ करके (प्रथम पद से नहीं) समांतर श्रेणी के n पदों के योग का सूत्र भी दिया।

एक अपरिमित गुणोत्तर श्रेणी के योग के लिए एक व्यापक व्यंजक सर्वप्रथम एक फ्रांसीसी फ्रैंकोइस वीटा (1540-1603 ई०) ने दिया था—जो व्यवसाय से वकील थे तथा शौक के रूप में उन्होंने गणित का अध्ययन किया।

अन्त में हम कुछ रोचक उदाहरणों पर विचार करते हैं जिनमें समांतर तथा गुणोत्तर दोनों ही श्रेणियाँ सम्बद्ध हैं।

उदाहरण 1 : यदि a, b तथा c एक समांतर श्रेणी के तीन क्रमागत पद हों तो k^a, k^b तथा k^c एक गुणोत्तर श्रेणी के तीन क्रमागत पद होंगे। k को शून्येतर वास्तविक संख्या मानिए।

हल : क्योंकि a, b तथा c समांतर श्रेणी में हैं,

अतः, $b-a=c-b$

इसलिए, $k^{b-a}=k^{c-b}$

इस प्रकार, $\frac{k^b}{k^a}=\frac{k^c}{k^b}$, जिससे निष्कर्ष निकलता है कि k^a, k^b तथा k^c एक गुणोत्तर श्रेणी के क्रमागत पद हैं।

उदाहरण 2 : यदि किसी गुणोत्तर श्रेणी के mवें, nवें तथा pवें पद एक अन्य गुणोत्तर अनुक्रम के तीन क्रमागत पद हों, तो सिद्ध कीजिए कि m, n तथा p एक समांतर अनुक्रम के तीन क्रमागत पद होंगे।

हल : $t_m=br^{m-1}$,

$t_n=br^{n-1}$,

$t_p=br^{p-1}$, जहाँ 'b' गुणोत्तर श्रेणी का प्रथम पद तथा 'r' सार्व अनुपात है।

हमें दिया है कि

$$\frac{t_n}{t_m}=\frac{t_p}{t_n} \qquad \text{(क्यों ?)}$$

अर्थात्, $$\frac{br^{n-1}}{br^{m-1}}=\frac{br^{p-1}}{br^{n-1}}$$

इस प्रकार, $r^{2n-2}=r^{p+m-2}$, जिससे निष्कर्ष निकलता है कि

$$2n=p+m$$

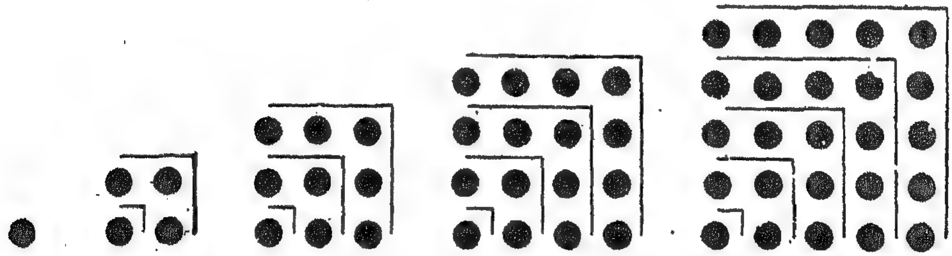
अर्थात्, $n-m=p-n$

अतः हम इस निष्कर्ष पर पहुँचते हैं कि m, n तथा p एक समांतर श्रेणी के तीन क्रमागत पद हैं।

प्रश्नावली 5.7

1. सिद्ध कीजिए कि समांतर श्रेणी 1, 3, 5, ... का n पदों तक योग n^2 है।

[हम देखते हैं कि प्रथम n विषम प्राकृत संख्याओं का योग nवीं वर्ग संख्या है, जिसका यूनानियों को भी आभास हुआ था। वास्तव में उन्होंने इस सम्बन्ध को ज्यामितीय रूप से दर्शाया था जैसा कि आकृति 5.6 में दिखाया गया है।]



$1=1^2$ $1+3=2^2$ $1+3+5=3^2$ $1+3+5+7=4^2$ $1+3+5+7+9=5^2$

आकृति 5.6 : $1+3+5+\ldots+(2n-1)=n^2$

2. सिद्ध कीजिए कि प्रथम n प्राकृत संख्याओं के घनों का योग $\left[\frac{n(n+1)}{2}\right]^2$ होता है।

3. यदि $\frac{1}{x+y}$, $\frac{1}{2y}$ तथा $\frac{1}{y+z}$ एक समांतर श्रेणी के तीन क्रमागत पद हों, तो सिद्ध कीजिए कि x, y तथा z एक गुणोत्तर श्रेणी के तीन क्रमागत पद हैं।

1 मुख्य संकल्पनाएँ

अनुक्रम, अनुक्रम के पद	दूरदर्शीय योग की विधि
परिमित तथा अपरिमित अनुक्रम	गुणोत्तर श्रेणी—गुणोत्तर श्रेणी का प्रथम पद, सार्व अनुपात
अनुक्रम का nवाँ पद अथवा व्यापक पद	गुणोत्तर श्रेणी का nवाँ पद
फिबोनाशी अनुक्रम	गुणोत्तर श्रेणी के n पदों तक का योग
समांतर श्रेणी—समांतर श्रेणी का प्रथम पद, सार्व अंतर	अपरिमित गुणोत्तर श्रेणियाँ
समांतर श्रेणी का nवाँ पद	वर्गों के अनुक्रम—वर्ग संख्याएँ
समांतर श्रेणी के n पदों तक का योग	घनों के अनुक्रम—घन संख्याएँ

2 अग्रिम अध्ययन हेतु सुझाव

रुचिपूर्ण ऐतिहासिक विवरणों सहित अनुक्रमों के विषय में एक उत्तम पुस्तक निम्न है :

H. R. Jacobs: **Mathematics, A Human Endeavour,** W. H. Freeman and Company, San Francisco (U.S.A.), 1970.

स्कूल गणित श्रृंखला की लायब्रेरी में एक 'छोटी' रुचिपूर्ण पुस्तिका निम्न है:

S. N. Gelfand, etc : **Sequences, Combinations, Limits,** Translated and adapted from the Russian by L. Cohn and Joan Teller; The M.I.T. Press, Massachusetts (U.S.A.), 1969.

हिन्दू गणितज्ञों के योगदान के लिए पाठक अनुच्छेद 1.12 में दी गई पुस्तकें [4] तथा [5] देखें तथा अन्य ऐतिहासिक विवरण के लिए अनुच्छेद 1.12 की पुस्तक [7] देखें। इनके अतिरिक्त एक अन्य बहुमूल्य पुस्तक निम्न है:

M. Kline: **Mathematical Thought from Ancient to Modern Times.** Oxford University Press, New York (U.S.A.), 1972.

एकक VI

गणितीय आगमन का सिद्धांत

गणित के किसी भी गहन अध्ययन के लिए गणितीय आगमन का सिद्धांत मौलिक है। इस 'सिद्धांत' के एक रूप की चर्चा की गई है तथा कुछ मौलिक परिणामों को सिद्ध करने में इसके अनुप्रयोगों को दर्शाया गया है।

6.1 भूमिका

विज्ञान सम्बन्धी निष्कर्षों को निकालने में आमतौर पर तर्कण (reasoning) के दो मौलिक प्रक्रमों (processes) का प्रयोग किया जाता है। इनमें से एक है **निगमन (deduction)** का प्रक्रम अर्थात् **व्यापक (general) से विशिष्ट (particular) के तर्कण का प्रक्रम**। उदाहरणार्थ, हम कहते हैं कि वे सभी पूर्णांक जिनके अंकों का योग 3 का गुणज है, 3 से विभाज्य होते हैं। क्योंकि 345 एक ऐसा पूर्णांक है जिसके अंकों का योग 3 का गुणज है अत: 345, 3 से विभाज्य है। एक व्यापक कथन से हम एक विशिष्ट कथन के विषय में निष्कर्ष निकाल रहे हैं।

दूसरा प्रक्रम है **आगमन (induction)** का प्रक्रम अर्थात् **विशिष्ट से व्यापक के तर्कण का प्रक्रम**। उदाहरणार्थ, हम कहते हैं कि 30 एक ऐसा पूर्णांक है जो 5 से विभाज्य है। अत: वे सभी पूर्णांक जिनका अन्त शून्य पर होता है, 5 से विभाज्य होते हैं। एक विशिष्ट कथन से हम एक व्यापक निष्कर्ष निकाल रहे हैं। विज्ञान सम्बन्धी तर्कण में ऐसे निष्कर्ष को एक **विवेकपूर्ण अनुमान (conjecture अथवा educated guess)** कहा जाता है।

क्या हम कोई अन्य निष्कर्ष भी निकाल सकते थे ? हाँ, उदाहरणार्थ हम यह तर्क भी दे सकते थे कि 30, 5 से विभाज्य है तथा 30 दो अंकों का एक पूर्णांक है। अत: दो अंकों वाले सभी पूर्णांक 5 से विभाज्य होते हैं। निश्चय ही यह निष्कर्ष असत्य है। (क्यों ?)

विज्ञान सम्बन्धी ज्ञान के विकास में दोनों प्रक्रम ही एक अधिकृत स्थान रखते हैं। इस एकक में हम अपना सम्बन्ध यह बताने में रखेंगे कि गणित में आगमनिक तर्कण (inductive reasoning) कैसे प्रयुक्त होता है, इसे सही ढंग से किस प्रकार प्रयोग किया जाए तथा इसका गणितीय आगमन के सिद्धांत (principle of mathematical induction) से, यदि कोई सम्बन्ध है, तो वह क्या है।

6.2 गणित में आगमन अथवा आगमनिक तर्कण का स्थान

विकास की प्रक्रिया में गणित आगमन का विज्ञान है। आगमन प्रेक्षण से प्रारम्भ होता है। हम प्रेक्षण करते हैं तथा एक अनुमानित निष्कर्ष पर पहुँचने के लिए सहज ज्ञान का प्रयोग करते हैं। अनुमानित इसलिए क्योंकि यह केवल एक संभावित तथ्य अथवा विवेकपूर्ण अनुमान ही है। यह अनुमान सत्य भी हो सकता है परन्तु इसे व्यवस्थित निगमनिक तर्कण (deductive reasoning) द्वारा सिद्ध किया जाना चाहिए। अथवा यह असत्य भी हो सकता है, परन्तु एक ऐसा विरोधउदाहरण (**counterexample**) खोजकर जहाँ यह अनुमान गलत हो जाए, इसे दर्शाया जाना चाहिए।

एक महत्वपूर्ण प्रश्न यह है कि अशुद्ध निष्कर्ष किस प्रकार प्राप्त न किए जाएँ? दूसरे शब्दों में, एक अनुमान प्राप्त कर लेने के बाद इसकी सत्यता को कैसे स्थापित किया जाए अथवा इसकी संगतता को कैसे असत्य प्रमाणित किया जाए? इससे पहले कि हम इस अति महत्वपूर्ण प्रश्न का उत्तर प्राप्त करने का प्रयत्न करें, हम आगमन अथवा आगमनिक तर्कण के प्रयोग के कुछ और उदाहरणों का अध्ययन करते हैं।

उदाहरण 1 : हम देखते हैं कि प्राकृत संख्याओं 5, 15, 25, 35, 125, 625 में से प्रत्येक 5 से विभाज्य है। अतः हम निष्कर्ष निकालते हैं कि ये सभी प्राकृत संख्याएँ जिनके इकाई के स्थान पर अंक 5 है, 5 से विभाज्य होते हैं। पुनः यह एक अनुमान है जिसे व्यवस्थित निगमनिक तर्कण द्वारा सिद्ध किया जाना चाहिए अथवा एक विरोधउदाहरण खोज कर इसे असत्य प्रमाणित किया जाना चाहिए। [क्या आप यह सिद्ध कर सकते हैं कि यह अनुमान वास्तव में सत्य ही है?]

उदाहरण 2 : $2^{2^1}+1$, अर्थात् 5, एक अभाज्य संख्या है। पुनः $2^{2^2}+1$, अर्थात् 17, एक अभाज्य संख्या है। साथ ही $2^{2^3}+1$ अर्थात् 257, भी एक अभाज्य संख्या है। अतः हम निष्कर्ष निकालते हैं कि प्रत्येक प्राकृत संख्या n के लिए $2^{2^n}+1$ एक अभाज्य संख्या होती है। वास्तव में, महान फ्रांसीसी गणितज्ञ पियरी दि फर्मे (1601—1665) का यह अनुमान था कि प्रत्येक प्राकृत संख्या n के लिए $2^{2^n}+1$ एक अभाज्य संख्या है। उन्होंने अपने समय के गणितज्ञों को चुनौती दी थी कि वे इसे असत्य प्रमाणित करें। परन्तु अनेकों वर्ष बाद स्विटजरलैंड के गणितज्ञ लियोनार्ड ऑयलर (1707-1783) ने यह दिखाया था कि $2^{2^5}+1$ अर्थात् 4294967297, एक अभाज्य संख्या नहीं है क्योंकि यह 641 से विभाज्य है।

हमने प्रेक्षण किया और एक अनुमान पर पहुँचे। फिर हमने एक विरोधउदाहरण खोज लिया जिससे हमारा अनुमान असत्य प्रमाणित हो गया।

उदाहरण 3 : हम देखते हैं कि निम्नलिखित गुणनखण्डों में से प्रत्येक में विभिन्न पदों के गुणांकों के निरपेक्ष मान या तो 1 से कम अथवा 1 के बराबर हैं :

$$x^1-1=(x-1)$$
$$x^2-1=(x-1)\ (x+1)$$
$$x^3-1=(x-1)\ (x^2+x+1)$$

$$x^4-1=(x-1)\ (x+1)\ (x^2+1)$$
$$x^5-1=(x-1)\cdot(x^4+x^3+x^2+x+1)$$

अतः हमारा यह अनुमान है कि जब x^n-1 को, जहाँ n एक प्राकृत संख्या है, ऐसे गुणनखण्डों में, जिनके गुणांक पूर्णांक हों, में व्यक्त किया जाए तो किसी भी गुणांक का निरपेक्ष मान 1 से अधिक नहीं होगा। इस व्यापक कथन को सिद्ध करने के सभी प्रयास विफल हुए। परन्तु 1941 ई० में रूसी गणितज्ञ वी इवानोव ने एक विरोध उदाहरण प्रस्तुत कर दिया। उन्होंने यह ज्ञात कर लिया कि $x^{105}-1$ का एक गुणनखंड इस अनुमान का खण्डन करता है। यह गुणनखंड, जो घात 48 का एक बहुपद है, निम्न है :

$$x^{48}+x^{47}+x^{46}-x^{43}-x^{42}-2x^{41}-x^{40}-x^{39}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}$$
$$+x^{31}-x^{28}-x^{26}-x^{24}-x^{22}-x^{20}+x^{17}+x^{16}+x^{15}+x^{14}+x^{12}-x^{9}-x^{8}$$
$$-2x^{7}-x^{6}-x^{5}+x^{2}+x+1$$

पुनः हमने प्रेक्षण किया तथा एक अनुमान पर पहुँच गए। फिर हम एक विरोधउदाहरण प्राप्त करने में समर्थ हो गए जिससे हमारा अनुमान असत्य प्रमाणित हो गया।

वैज्ञानिक का कोई अनुमान, अनुमान ही रहता है चाहे हम इसके पक्ष में कितने ही उदाहरण प्राप्त कर लें। गणित में ऐसे अनेकों अनुमान हैं जो प्रमाण की अनुपस्थिति में अनुमान ही हैं, यद्यपि इनके पक्ष में हमारे पास वस्तुतः हजारों उदाहरण उपलब्ध हैं। हम ऐसे एक अनुमान का उदाहरण देते हैं।

1742 ई० में सी० गोल्डबाक (1690-1764) ने स्विटजरलैंड के प्रसिद्ध गणितज्ञ लियोनार्ड ऑयलर को पत्र लिखकर एक समस्या प्रस्तुत की। गोल्डबाक ने प्रेक्षण किया कि 2 के अतिरिक्त प्रत्येक सम प्राकृत संख्या, जिस पर उन्होंने विचार किया था, दो अभाज्य संख्याओं के योग के रूप में व्यक्त की जा सकती है। उदाहरणार्थ, $4=2+2$, $6=3+3$, $8=3+5$, इत्यादि। उन्होंने ऑयलर से पूछा कि क्या यह प्रत्येक सम प्राकृत संख्या के लिए सत्य है। ऑयलर, गोल्डबाक की समस्या का हल नहीं दे सके अर्थात् वे न तो इसका कोई प्रमाण दे सके और न ही कोई विरोध उदाहरण दे सके। वास्तव में कोई अन्य भी आज तक इसका हल नहीं दे सका है। यह समस्या आज तक अनुमान ही बनी हुई है।

संक्षेप में, विज्ञान सम्बन्धी ज्ञान के विकास में आगमन अथवा आगमनिक तर्कण का महत्वपूर्ण योगदान है। साथ ही इसका सही ढंग से प्रयोग करना भी उतना ही महत्वपूर्ण है। आगमन का प्रयोग करके एक अनुमान पर पहुँचने के पश्चात उस अनुमान को सिद्ध करने का कठिन कार्य आरम्भ होता है।

यदि अनुमान एक कथन, उदाहरणार्थ $P(n)$, के रूप में है, जो प्राकृत संख्याओं से सम्बन्धित है तो गणितीय आगमन का सिद्धांत इसकी उपपत्ति की एक विधि प्रदान करता है। इस सिद्धांत के एक रूप का अगले अनुच्छेद में वर्णन किया गया है।

6.3 गणितीय आगमन का सिद्धांत—एक रूप

मान लीजिए $P(n)$ प्राकृत संख्या n से सम्बन्धित एक कथन है तब यदि

(क) $P(1)$ सत्य है, तथा

(ख) जब-जब $P(k)$ सत्य है तो $P(k+1)$ भी सत्य है,

तो हम निष्कर्ष निकालते हैं कि $P(n)$ सभी प्राकृत संख्याओं n के लिए सत्य है।

दूसरे शब्दों में, यह सिद्ध करने के लिए कि कोई कथन $P(n)$ सभी प्राकृत संख्याओं n के लिए सत्य है हमें दो चरण अवश्य पूरे करने चाहिए; एक हमें सिद्ध करना चाहिए कि $P(1)$ सत्य है। दूसरे, हमें यह सिद्ध करना चाहिए कि जब भी $P(k)$ सत्य है तो $P(k+1)$ भी सत्य है।

अब हम उपपत्ति की इस विधि के प्रयोग को दर्शाने के लिए कुछ उदाहरण लेते हैं।

उदाहरण 1 : दिखाइए कि प्रथम n विषम प्राकृत संख्याओं का योग n^2 होता है।

हल : निश्चय ही यह प्राकृत संख्या n से सम्बन्धित एक कथन है। हम इसे $P(n)$ से व्यक्त करते हैं तथा निम्न प्रकार से लिखते हैं :

$$P(n) : 1+3+5+\ldots+(2n-1)=n^2$$

प्रथम, हमें यह सिद्ध करना चाहिए कि $P(1)$ सत्य है। अब $P(1)$ क्या है ? यह एक कथन है कि प्रथम 1 विषम संख्या का योग 1^2 है।

दूसरे शब्दों में, $P(1) : 1=1^2$

क्या $P(1)$ सत्य है ? निश्चय ही, यह सत्य है।

दूसरे, हमें यह सिद्ध करना चाहिए कि जब भी $P(k)$ सत्य है $P(k+1)$ भी सत्य है।

अतः आइए हम $P(k)$ को सत्य मान लें। अर्थात्

$$P(k) : \quad 1+3+5+\ldots+(2k-1)=k^2 \tag{1}$$

अब हम यह सिद्ध करना चाहते हैं कि जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है।

$P(k+1)$ क्या है ?

$$P(k+1) : 1+3+5+\ldots+[2(k+1)-1]=(k+1)^2 \tag{2}$$

क्या $P(k+1)$ सत्य है ? आइए देखें। आइए (2) के वाम पक्ष की जाँच करें। अब,

$$\begin{aligned} \text{वाम पक्ष} &= 1+3+5+\ldots+(2k-1)+[2(k+1)-1] \\ &= k^2+(2k+1), \text{ क्योंकि } P(k) \text{ सत्य है} \\ &= (k+1)^2 \\ &= (2) \text{ का दक्षिण पक्ष} \end{aligned}$$

इस प्रकार, जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है। अतः हमने यह दिखा दिया है कि सभी प्राकृत संख्याओं n के लिए $P(n)$ सत्य है, अर्थात् प्रथम n विषम प्राकृत संख्याओं का योग n^2 होता है।

उपपत्ति की यह विधि वास्तव में उपपत्ति कि एक शक्तिशाली विधि है। क्या आप देख सकते हैं कि हमने कथनों के एक अपरिमित अनुक्रम को सिद्ध कर लिया है ? दूसरे शब्दों में, यह सिद्ध करके कि $P(n)$ सभी प्राकृत संख्याओं n के लिए सत्य है, हमने यह सिद्ध कर दिया है कि निम्नलिखित में से प्रत्येक कथन सत्य है :

$$P(1) : \quad 1=1^2=1$$
$$P(2) : \quad 1+3=2^2=4$$
$$P(3) : \quad 1+3+5=3^2=9$$

$P(4): \quad 1+3+5+7=4^2=16$

....

$P(k): \quad 1+3+5+\ldots+(2k-1)=k^2$

$P(k+1): 1+3+5+\ldots+(2k+1)=(k+1)^2$

....

उदाहरण 2 : सिद्ध कीजिए कि $n^3+(n+1)^3+(n+2)^3$, प्रत्येक प्राकृत संख्या n के लिए, 9 से विभाज्य है।

हल : पुनः यह प्राकृत संख्या n से सम्बंधित एक कथन है। हम इसे $P(n)$ से व्यक्त करते हैं तथा निम्न प्राप्त करते हैं :

$P(n): n^3+(n+1)^3+(n+2)^3$, 9 से विभाज्य है।

प्रथम, हमें सिद्ध करना चाहिए कि $P(1)$ सत्य है। $P(1)$ क्या है? यह एक कथन है कि $1^3+2^3+3^3$, 9 से विभाज्य है। क्या $1^3+2^3+3^3$, 9 से विभाज्य है? निश्चय ही यह सत्य है क्योंकि, $1^3+2^3+3^3=1+8+27=36$ तथा 36, 9 से विभाज्य है।

अब, आइए मान लें कि $P(k)$ सत्य है, अर्थात्

$k^3+(k+1)^3+(k+2)^3$, 9 से विभाज्य है। (1)

हम यह भी सिद्ध करना चाहते हैं कि जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है। अब $P(k+1)$ क्या है?

$P(k+1): (k+1)^3+(k+2)^3+(k+3)^3$, 9 से विभाज्य है। (2)

क्या $P(k+1)$ सत्य है? आइए देखें। हम देखते हैं कि

$$(k+1)^3+(k+2)^3+(k+3)^3=(k+1)^3+(k+2)^3+k^3+9k^2+27k+27$$

$$=(k+1)^3+(k+2)^3+k^3+9(k^2+3k+3) \quad (3)$$

अब क्या (3) में दिया गया व्यन्जक 9 से विभाज्य है? निश्चय ही, $(k+1)^3+(k+2)^3+k^3$, 9 से विभाज्य है क्योंकि $P(k)$ सत्य है। पुनः $9(k^2+3k+3)$ का एक गुणनखंड 9 है, अतः यह 9 से विभाज्य है। निस्संदेह दोनों का योग भी 9 से विभाज्य है।

इस प्रकार जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है।

अतः हमने यह दिखा दिया है कि प्रत्येक प्राकृत संख्या n के लिए $P(n)$ सत्य है अर्थात् प्रत्येक प्राकृत संख्या n के लिए $n^3+(n+1)^3+(n+2)^3$, 9 से विभाज्य होता है।

पुनः हमने कथनों के एक अपरिमित अनुक्रम को सिद्ध कर लिया है।

उदाहरण 3 : सिद्ध कीजिए कि प्रथम n प्राकृत संख्याओं के वर्गों का योग $\frac{n(n+1)(2n+1)}{6}$ होता है।

हल : दूरदर्शीय योगों की विधि से हम यह पहले ही दिखा चुके हैं कि वास्तव में यह ऐसा ही है। (देखिए अनुच्छेद 5.10) अब हम गणितीय आगमन के सिद्धांत का प्रयोग करेंगे तथा यह सिद्ध करेंगे कि सभी प्राकृत संख्याओं n के लिए $P(n)$ सत्य है।

अब, $P(n): 1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$

क्या $P(1)$ सत्य है ? $P(1): 1^2 = \frac{1(1+1)(2+1)}{6}$

निश्चय ही वाम पक्ष = दक्षिण पक्ष

इस प्रकार, $P(1)$ वास्तव में सत्य है।

अब मान लीजिए कि $P(k)$ सत्य है।

$$P(k): 1^2+2^2+3^2+\ldots+k^2 = \frac{k(k+1)(2k+1)}{6} \qquad (1)$$

हम सिद्ध करना चाहते हैं कि जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है। $P(k+1)$ क्या है?

$$P(k+1): 1^2+2^2+3^2+\ldots+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

क्या $P(k+1)$ सत्य है ? आइए देखें। आइए इसके वाम पक्ष की जाँच करें।

वाम पक्ष $= 1^2+2^2+3^2+\ldots+k^2+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6}+(k+1)^2$, क्योंकि $P(k)$ सत्य है।

$$= \frac{k+1}{6}\left[k(2k+1)+6(k+1)\right]$$

$$= \frac{k+1}{6}(2k^2+7k+6)$$

$= \frac{(k+1)(k+2)(2k+3)}{6}$ = दक्षिण पक्ष ,

इस प्रकार जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है।

अतः हमने यह सिद्ध कर दिया है कि सभी प्राकृत संख्याओं n के लिए $P(n)$ सत्य है। पुनः हमने कथनों के एक अपरिमित अनुक्रम को सिद्ध कर दिया है।

उदाहरण 4 : सिद्ध कीजिए कि सभी प्राकृत संख्याओं n के लिए $(1+x)^n \geqslant 1+nx$, जबकि $x > -1$ हो।

हल : इस कथन को, जिसे सिद्ध किया जाना है, हम $P(n)$ से व्यक्त करते हैं, अर्थात्

$$P(n): (1+x)^n \geqslant 1+nx,\ x > -1$$

क्या $P(1)$ सत्य है ? $P(1): (1+x) \geqslant (1+x)$, जबकि $x > -1$ है।

निश्चय ही $P(1)$ सत्य है।

अब, मान लीजिए कि $P(k)$ सत्य है, अर्थात्

$P(k): (1+x)^k \geqslant 1+kx$, जबकि $x > -1$ है। (1)

हम यह सिद्ध करना चाहते हैं कि जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है।

अब, $P(k+1)$ क्या है ?

$P(k+1): (1+x)^{k+1} \geqslant 1+(k+1)x$, जबकि $x > -1$ है।

$P(k+1)$ में, वाम पक्ष $= (1+x)^{k+1}$

$= (1+x)^k(1+x)$

परन्तु क्योंकि $x > -1$ (अर्थात्, $1+x > 0$) है तथा $(1+x)^k \geqslant 1+kx$ है, अतः यह निष्कर्ष निकलता है कि

$$(1+x)^{k+1} \geqslant (1+kx)\ (1+x)$$

अर्थात्, $(1+x)^{k+1} \geqslant 1+x+kx+kx^2$ (2)

पुनः क्योंकि k एक प्राकृत संख्या है तथा x^2 सदैव ऋणेतर है, अतः kx^2 सदैव ऋणेत्तर है। अतः यदि हम (2) के दक्षिण पक्ष से kx^2 को छोड़ दें तो (2) में दी गई असमिका तब भी सत्य होगी। इस प्रकार हम निम्न प्राप्त करते हैं :

$$(1+x)^{k+1} \geqslant 1+x+kx$$

अर्थात्, $(1+x)^{k+1} \geqslant 1+(k+1)x$

इस प्रकार, जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है।

अतः हमने यह सिद्ध कर लिया है कि सभी प्राकृत संख्याओं n के लिए $P(n)$ सत्य है। पुनः हमने कथनों के एक अपरिमित अनुक्रम को सिद्ध कर लिया है।

किसी कथन को गणितीय आगमन के सिद्धांत से सिद्ध करने के लिए ये दोनों चरण आवश्यक हैं, अन्यथा यह हो सकता है कि हम कोई असंगत परिणाम सिद्ध कर लें। आइए निम्न उदाहरण पर विचार करें:

उदाहरण 5 : कोई प्राकृत संख्या अपने से तुरन्त पश्चात् आने वाली प्राकृत संख्या के समान होती है अर्थात्

$$P(n) :\ n = n+1$$

हल : आइए मान लें कि $P(k)$ सत्य है, अर्थात्

$$P(k) : k = k+1 \qquad (1)$$

हम यह सिद्ध करना चाहते हैं कि $P(k+1)$ सत्य है।

अब, $P(k+1) :\ k+1 = k+2$

$P(k+1)$ में,

$$\text{वाम पक्ष} = k+1 \qquad (2)$$

क्योंकि $P(k)$ को सत्य मान लिया गया है, अतः $k = k+1$। इस प्रकार (2) में

$$\text{वाम पक्ष} = (k+1)+1 = k+2$$

जो कि, निस्संदेह, $P(k+1)$ का दक्षिण पक्ष है। अतः हमने यह सिद्ध कर दिया है कि जब भी $P(k)$ सत्य है तो $P(k+1)$ भी सत्य है। परन्तु क्या हम यह कह सकते हैं कि $P(n)$ सभी प्राकृत संख्याओं n के लिए सत्य है ? नहीं, निश्चय ही नहीं। हमने यह जाँच नहीं की कि $P(1)$ सत्य है अथवा नहीं। यदि हमने यह जाँच की होती तो हमें ज्ञात हो जाता कि $P(1)$ असत्य है। दूसरे शब्दों में, हमने तुरन्त ही एक विरोध-उदाहरण प्राप्त कर लिया होता जिससे यह स्थापित हो जाता कि दिया हुआ कथन असत्य है।

उदाहरण 6 : सभी प्राकृत संख्याओं n के लिए n^2-n+41 एक अभाज्य संख्या है।

हल : आइए, हम इस कथन को कि n^2-n+41 एक अभाज्य संख्या है $P(n)$ से व्यक्त करें।

क्या P(1) सत्य है ? निश्चय ही यह सत्य है क्योंकि $P(1) = 1^2-1+41 = 41$। अब मान लीजिए कि हम यह जाँच करने का प्रयत्न ही नहीं करते कि जब भी $P(k)$ सत्य है, $P(k+1)$ भी सत्य है

तथा यह निष्कर्ष निकाल लेते हैं कि सभी प्राकृत संख्याओं n के लिए $P(n)$ सत्य है। इस प्रकार हम एक गम्भीर गलती कर जाएंगे।

न केवल $n=1$, 2 अथवा 3 के लिए ही $P(n)$ सत्य है अपितु यह 40 या इससे छोटी प्रत्येक प्राकृत संख्या के लिए भी सत्य है। परन्तु $n=41$ के लिए हम देखते हैं कि n^2-n+41 एक अभाज्य संख्या नहीं है।

अतः हम पुनः यह कहेंगे कि गणितीय आगमन के सिद्धांत का प्रयोग करते समय दोनों चरणों की जाँच अवश्य की जानी चाहिए अन्यथा यह सम्भव है कि हम कुछ हानिकारक निष्कर्षों पर पहुँच जाए।

प्रश्नावली 6.1

निम्नलिखित में से प्रत्येक कथन को सिद्ध करने के लिए गणितीय आगमन के सिद्धांत का प्रयोग कीजिए :

1. किसी समांतर श्रेणी जिसका प्रथम पद a तथा सार्व अंतर d हो, का nवाँ पद $a+(n-1)d$ होता है।
2. प्रथम n प्राकृत संख्याओं का योग $\frac{n(n+1)}{2}$ होता है।
   (अनुच्छेद 5.5 का उदाहरण 2 भी देखिए।)
3. सभी प्राकृत संख्याओं n के लिए $n(n+1)(n+2)$, 6 से विभाज्य होता है।
4. $1+4+7+\ldots+(3n-2)=\frac{n(3n-1)}{2}$
5. $4+8+12+\ldots+4n=2n(n+1)$
6. $1^3+2^3+3^3+\ldots+n^3=\left[\frac{n(n+1)}{2}\right]^2$
7. $1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

*8. $a+(a+d)+(a+2d)+\ldots+[a+(n-1)d]=\frac{n}{2}\left[2a+(n-1)d\right]$
   [आपको याद होगा कि यह एक समांतर श्रेणी के प्रथम n पदों का योग है। (अनुच्छेद 5.5 भी देखिए।)]

*9. $b+br+br^2+\ldots+br^{n-1}=\frac{b(1-r^n)}{1-r}, r\neq 1$
   [आपको याद होगा कि यह एक गुणोत्तर श्रेणी के प्रथम n पदों का योग है। (अनुच्छे 5.8 भी देखिए।)]

10. $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$

11. $1.2+2.3+3.4+\dots+n(n+1)=\frac{n(n+1)(n+2)}{3}$

12. $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

13. प्रत्येक प्राकृत संख्या n के लिए $2^n>n$ होता है।

14. प्रत्येक प्राकृत संख्या n के लिए $3^{2n}-1$, 8 से विभाज्य होता है।
[संकेत : $3^{2(k+1)}-1=9(3^{2k}-1)+8$ लिखिए।]

15. प्रत्येक प्राकृत संख्या n के लिए $10^{2n-1}+1$, 11 से विभाज्य होता है।
[संकेत : $10^{2k-1}+1=10^2(10^{2k-1}+1)-99$ लिखिए।]

16. प्रत्येक प्राकृत संख्या n के लिए, $2^{3n}-1$, 7 से विभाज्य होता है।
[संकेत : $2^{3k+3}-1=8(2^{3k}-1)+7$ लिखिए।]

17. $1+2+3+\dots+n<\frac{1}{8}(2n+1)^2$

18. n अवयवों के किसी समुच्चय के सभी उपसमुच्चयों की संख्या 2^n होती है।

19. दिखाइए कि यदि कथन $P(n)$: $2+4+6+\dots+2n=n(n+1)+2$, $n=k$ के लिए सत्य हो तो यह $n=k+1$ के लिए भी सत्य है। क्या हम निष्कर्ष निकाल सकते हैं कि $P(n)$ प्रत्येक प्राकृत संख्या n के लिए सत्य है?

20. सिद्ध कीजिए कि x^n-y^n प्रत्येक प्राकृत संख्या n के लिए $x-y$ से विभाज्य होता है, जहाँ $x-y\neq 0$ है।
[संकेत : $x^{k+1}-y^{k+1}=x^{k+1}-x^ky+x^ky-y^{k+1}$]

6.4 'आगमन' तथा इसका 'सिद्धांत'

'गणितीय आगमन के सिद्धांत' में 'आगमन' शब्द का प्रयोग दुर्भाग्यपूर्ण है। यह एक मिथ्या नाम है। इस 'सिद्धांत' में आगमनिक कुछ भी नहीं है। वास्तव में यह सिद्धांत आगमनिक तर्कण के प्रक्रम से बहुत भिन्न है। यह सत्य है कि किसी निष्कर्ष अथवा अनुमान पर पहुँचने के लिए आगमन अथवा आगमनिक तर्कण का प्रयोग किया जाता है। परन्तु जब हम किसी निष्कर्ष अथवा अनुमान पर पहुँच जाते हैं तो यह 'सिद्धांत' इसकी संगतता (सत्यता) प्रमाणित करने में हमें एक गूढ़ तथा सुन्दर विधि प्रदान करता है। जब तक हम ऐसा नहीं करते अथवा जब तक हम कोई विरोधउदाहरण प्राप्त नहीं कर लेते यह एक संभावित तथ्य अथवा विवेकपूर्ण अनुमान ही रह जाता है चाहे हम इसके पक्ष में कितने ही उदाहरण प्राप्त कर लें। दूसरे शब्दों में, आगमनिक तर्कण का प्रयोग $P(n)$ के रूप की अभिधारणा (postulate) करने अथवा अनुमान लगाने में किया जाता है। परन्तु जैसे ही $P(n)$ के रूप की अभिधारणा कर ली जाती है आगमन का कार्य समाप्त हो जाता है तथा 'गणितीय आगमन का सिद्धांत' इसका स्थान ले लेता है।

6.5 गणितीय आगमन की उत्पत्ति (Origin of Mathematical Induction)

यद्यपि गणितीय आगमन के सिद्धांत की उत्पत्ति का श्रेय फ्रांसीसी गणितज्ञ ब्लेज़ पास्कल (1623-1662) को दिया जाता है, इटली के गणितज्ञ फ्रांसिस्को मौरोलिकस (1494-1575) ने इससे पूर्व अपने लेखों में इस

सिद्धांत का प्रयोग किया था। हमें हिन्दुओं (भास्कर, 1153 ई०) तथा यूनानियों के लेखों से भी गणितीय आगमन के सिद्धांत के विषय में कुछ संकेत प्राप्त होते हैं।

'Induction' (आगमन) शब्द का प्रयोग सम्भवतः सर्वप्रथम ब्रिटिश गणितज्ञ जॉन वालिस (1616-1703) ने किया था। बाद में स्विटजरलैंड के गणितज्ञ जेम्स बर्नूली (1655-1705) ने इस नाम को प्रयोग किए बिना द्विपद-प्रमेय (Binomial Theorem) की उपपत्ति प्रदान करने में इस 'सिद्धांत' का प्रयोग किया। द्विपद-प्रमेय का अध्ययन हम एकक VIII में करेंगे।

"गणितीय आगमन" का उन अर्थों में प्रयोग जिसका वर्णन हमने ऊपर किया है, सर्वप्रथम ब्रिटिश गणितज्ञ ऑगस्टस डीमोरगान (1806-1871) ने Penny Cyclopedia, लंदन, 1838 में छपे अपने लेख "Induction (Mathematics)" में किया था। यह नाम उस समय के गणितज्ञों द्वारा तुरंत स्वीकृत कर लिया गया और लगभग चालीस वर्षों के अंतराल में सभी स्थानों के गणितज्ञों ने इसका प्रयोग करना प्रारंभ कर दिया।

6.6 मुख्य संकल्पनाएँ

निगमन	गणितीय आगमन का सिद्धांत
आगमन अथवा आगमनिक तर्कण	

6.7 अग्रिम अध्ययन हेतु सुझाव

आगमन पर दो श्रेष्ठ पुस्तकें निम्न हैं :

[1] G. Polya : **Induction and Analogy in Mathematics**, Princeton University Press, New Jersey (U. S. A.), 1954.

[2] G. Polya : **Patterns of Plausible Influences**, Princeton University Press, New Jersey (U. S. A.), 1954.

इस विषय पर एक अन्य उत्तम पुस्तक निम्न है :

[3] I.S. Sominiskii : **The Method of Mathematical Induction**, Blaisdell Publishing Company, New York (U. S. A.), 1961.

गणितीय आगमन की उत्पत्ति के विषय में पाठक निम्न दो लेख देखें :

[4] W. H. Bussey : **The Origin of Mathematical Induction**, American Mathematical Monthly (Journal), Vol. 24, 1917, pp. 199—206.

[5] F. Cajori : **Origin of the Name 'Mathematical Induction'**, American Mathematical Monthly (Journal), Vol. 25, 1918, pp. 197—201.

एक परिमित समुच्चय के सभी उपसमुच्चयों की संख्या के विषय में पाठक निम्न लेख देखें :

[6] M. S. Arora and J. D. Allen : **A Non-Combinatorial Proof of the Number of All Subsets of a Finite Set**. The Mathematics Teacher (Journal), Vol. **LXIII**, April 1970, pp. 312.

इस विषय पर एक अन्य रुचिपूर्ण लेख निम्न है :

[7] J. V. Uspensky : **A Curious Case of the Use of Mathematical Induction in Geometry**, American Mathematical Monthly (Journal), Vol 34, 1927, pp. 247—250.

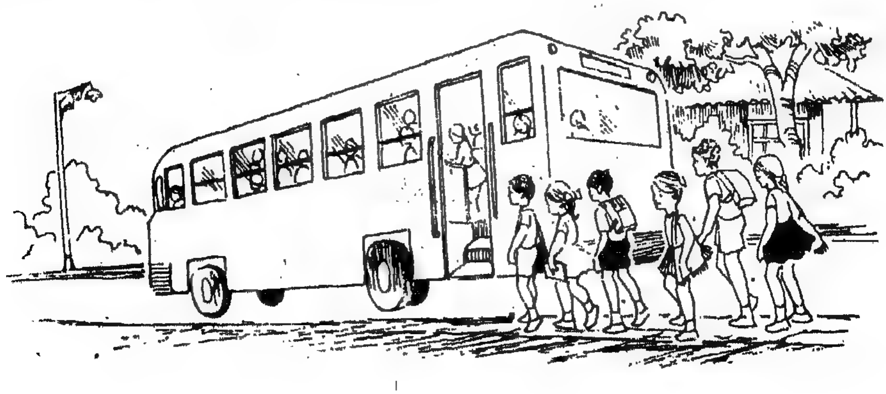
एकक VII

बिना गिनती किए कैसे गिनें?

इस एकक में हमने क्रमचयों (permutations) तथा संचयों (combinations) की संकल्पनाओं का परिचय दिया है जो संचय विन्यास के विज्ञान (combinatorics), जिसे चयन का गणित (mathematics of choice) भी कहते हैं, के अध्ययन में अति मौलिक हैं। ये संकल्पनाएँ हमें वास्तविक गिनती किए बिना गिनती की 'उत्तम' विधियाँ प्रदान करती हैं।

7.1 भूमिका

एक समय की बात है कि एक संयुक्त परिवार में विद्यालय जाने वाले 6 बच्चे थे। उनकी एक दूसरे से खूब बनती थी परन्तु स्कूल बस में चढ़ते समय उनमें धक्का-मुक्की व लड़ाई हो जाती थी तथा उनमें गम्भीर वाद विवाद होता था। राज कहता 'मैं यहाँ सबसे पहले आया था अतः मैं ही पहले बस में चढ़ूँगा'। मोहन कहता, 'नहीं! मैं सबसे छोटा हूँ, मुझे पहले चढ़ना चाहिए' इत्यादि। इस प्रकार बस में सवार होने के लिए उनमें विवाद होता था तथा एक दूसरे को धकेलते हुए वे लड़ पड़ते थे। जब वे आपस में इस समस्या को न सुलझा सके तो उन्होंने अपने दादाजी से कहा कि वह बस में सवार होने के लिए उनके लिए किसी व्यवस्थित क्रम की खोज करें। उनके दादाजी बहुत निष्पक्ष व्यक्ति थे। वे उनमें से किसी का भी पक्ष नहीं लेना चाहते थे। अतः उन्होंने कहा कि वे बस के लिए पंक्ति बनाएँ तथा यह देखे बिना कि कौन पहले आया या कौन सबसे छोटा है, वे प्रतिदिन पंक्ति में अपना स्थान परिवर्तन कर लें। सब से बड़े लड़के अरुण ने, जिसे गणित का



ज्ञान था कुछ परिकलना करके कहा 'क्या आप जानते हैं कि हमें कितने विन्यास (arrangements) प्राप्त होंगे तथा सभी सम्भव विन्यासों को समाप्त करने में हमें कितना समय लगेगा ?' उसने अपने प्रश्न का उत्तर स्वयं ही दे दिया। उसने कहा, 'कुल 720 विन्यास होंगे। यह मानकर कि वर्ष में विद्यालय जाने के 240 दिन होते हैं, हमें सभी सम्भव विन्यासों को समाप्त करने में 3 वर्ष लगेंगे। यदि हम प्रातः तथा सायंकाल दोनों समय स्थान परिवर्तन करें फिर भी हमें $1\frac{1}{2}$ वर्ष का समय लगेगा ।

अरुण को इस संख्या का ज्ञान कैसे हुआ ? निश्चय ही उसने सभी विन्यासों को लिखकर उनकी गिनती नहीं की। उसने गणन की एक विधि का प्रयोग किया जिसका अध्ययन एक व्यापक शीर्षक संचय विन्यास का गणित (Combinatorial Mathematics or Combinatorics) अथवा चयन का गणित (Mathematics of choice) के अन्तर्गत आता है।

संचय विन्यास के गणित के अध्ययन में एक मूलभूत प्रश्न है, 'कितने ?' अरुण ने यह प्रश्न पूछा था कि, 'छः बच्चों को एक पंक्ति में खड़ा करने की कितनी विधियाँ हैं ?'

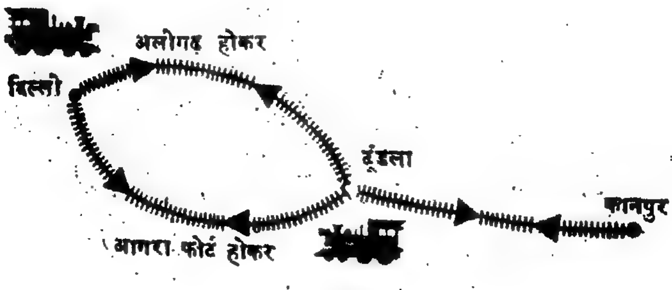
एक परीक्षा में 10 सत्य-असत्य प्रश्न हैं। इनके उत्तरों के कितने अनुक्रम सम्भव हैं ? एक कक्षा में 24 विद्यार्थी हैं। कितनी विधियों से इनमें से 1 मुख्य तथा 3 उपमुख्य बच्चों का चयन सम्भव है, जबकि प्रत्येक विद्यार्थी इन दोनों पदों में से कोई भी पद प्राप्त कर सकता हो ? एक सिक्के को 3 बार उछाला जाता है। कुल कितने परिणाम सम्भव हैं ? इन तथा इसी प्रकार के अन्य प्रश्नों, जिनमें 'कितने ?' पूछा जाता है, के उत्तर संचय विन्यास के गणित द्वारा प्राप्त होते हैं।

संचय विन्यास के गणित का क्षेत्र विस्तृत है। हम केवल कुछ मूल गणन विधियों का ही अध्ययन करना चाहेंगे ताकि हम उपरोक्त प्रकार के प्रश्नों के उत्तर प्राप्त कर सकें। परन्तु इससे पहले कि हम ऐसा करें हमें एक मूलभूत गणन सिद्धांत सीख लेना चाहिए।

7.2 गणन का मूलभूत सिद्धांत (The Fundamental Principle of Counting)

आइए पहले निम्नलिखित उदाहरणों का अध्ययन करें :

उदाहरण 1: दिल्ली और टूंडला के बीच दो रेल मार्ग हैं। एक अलीगढ़ होकर और दूसरा आगरा फोर्ट होकर तथा टूंडला और कानपुर के बीच एक ही रेल मार्ग है। साथ ही, दिल्ली और कानपुर के बीच टूंडला होकर जाने के अतिरिक्त कोई अन्य रेल मार्ग नहीं है।



आकृति 7.1

कोई व्यक्ति दिल्ली से कानपुर की यात्रा कितनी विधियों से कर सकता है? स्पष्ट है कि ऐसी 2 विधियाँ हैं। वह या तो दिल्ली-टूंडला (अलीगढ़ होकर)-कानपुर मार्ग से या दिल्ली-टूंडला (आगरा फोर्ट होकर)-कानपुर मार्ग से यात्रा कर सकता है।

उदाहरण 2 : एक सिक्का दो बार उछाला जाता है तथा प्राप्त परिणामों को लिख लिया जाता है। कुल कितने परिणाम सम्भव हैं? यदि हम जिस सिक्के पर 'Head' (चित) आए उसे 'H' से तथा जिस पर 'Tail' (पट) आए उसे 'T' से व्यक्त करें तो हमें कुल सम्भव परिणाम HH, HT, TH तथा TT प्राप्त होंगे। [उदाहरणार्थ, HT वह परिणाम है जब पहली बार उछालने पर सिक्के पर 'Head' तथा दूसरी बार उछालने पर 'Tail' प्राप्त होता है।] इस प्रकार हम देखते हैं कि कुल 4 सम्भव परिणाम हैं।

उदाहरण 3 : रवि फिल्म देखने जाता है। सिनेमा हॉल के दो प्रवेश द्वार तथा तीन निकास द्वार हैं। रवि कितनी विधियों से हॉल में प्रवेश करके बाहर निकल सकता है? यदि हम प्रवेश द्वारों को I और II से तथा निकास द्वारों को A, B और C से व्यक्त करें तो रवि के लिए कुल संभव चयन IA, IB, IC, II A, II B, II C हैं। [उदाहरणार्थ IC वह चयन है जब रवि प्रवेश द्वार I से प्रवेश करता है तथा निकास द्वार C से बाहर निकलता है।] इस प्रकार हम देखते हैं कि रवि के लिए कुल 6 चयन सम्भव है।

उदाहरण 4 : अंकों 1, 2, 3, 4 तथा 5 से दो अंकों की कितनी संख्याएँ बनाई जा सकती हैं? आइए पहले दो अंकों की उन सभी संख्याओं को लिख लें जो दिए हुए अंकों से बनाई जा सकती हैं। इस प्रकार हम 12, 13, 14, 15, 23, 24, 25, 34, 35, 45, 51, 21, 31, 41, 32, 42, 52, 43, 53, 54 प्राप्त करते हैं। निस्संदेह हमने यह मान लिया है कि किसी अंक की पुनरावृत्ति नहीं होती। हम देखते हैं कि हम 20 संख्याएँ प्राप्त करते हैं।

उपरोक्त चार उदाहरण एक व्यापक सिद्धान्त, जिसे गणन का मूलभूत सिद्धान्त कहा* जाता है, के प्रयोग को दर्शाते हैं। इस सिद्धान्त का कथन निम्न है :

यदि कोई घटना m भिन्न विधियों से घटित हो सकती है तथा इसके घटित होने के पश्चात एक अन्य घटना n भिन्न विधियों से घटित हो सकती है तो दिए हुए क्रम में दोनों घटनाओं के घटित होने की कुल भिन्न विधियाँ $m \times n$ होंगी।** दूसरे शब्दों में, यदि कोई घटना m भिन्न विधियों में से किसी एक से घटित हो सकती है तथा इस घटना के घटित होने के पश्चात् एक अन्य घटना n भिन्न विधियों में से किसी एक से घटित हो सकती है तो दिए हुए क्रम में दोनों घटनाओं के घटित होने की भिन्न विधियाँ $m \times n$ होंगी।

उदाहरण 1 में दिल्ली से टूंडला के दो मार्ग हैं तथा टूंडला से कानपुर का 1 मार्ग है। अतः दिए हुए क्रम में कुल विधियों की संख्या $2 \times 1 = 2$ है।

उदाहरण 2 में पहली बार उछाले जाने पर सिक्के पर या तो 'Head' या 'Tail' आता है। दूसरे शब्दों में, पहली बार उछाले जाने पर सम्भव परिणामों की संख्या 2 है। पहली बार के प्रत्येक परिणाम के तदनुसार दूसरी बार उछालने पर दो सम्भव परिणाम हैं। अतः भिन्न परिणामों की कुल संख्या $2 \times 2 = 4$ है।

उदाहरण 3 में रवि या तो प्रवेश द्वार *I* या *II* से प्रवेश कर सकता है। प्रवेश के प्रत्येक चयन के

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*कुछ लेखक इस सिद्धान्त को गुणन सिद्धांत (**multiplication principle**) कहते हैं। परन्तु हम इसके लिए गणन का मूलभूत सिद्धांत के नाम का ही प्रयोग करेंगे।

**दो से अधिक घटनाओं के लिए भी यही सिद्धांत सुगमतापूर्वक लागू किया जा सकता है।

तदनुसार रवि के पास बाहर निकलने के तीन चयन हैं। इसके परिणामस्वरूप कुल विधियों की संख्या, जिनसे रवि प्रवेश करके बाहर निकल सकता है, $2 \times 3 = 6$ है।

उदाहरण 4 में क्योंकि हमने कल्पना कर ली है कि अंकों की पुनरावृत्ति नहीं हो सकती, अतः इकाई के स्थान के लिए 5 चयन हैं। 5 अंकों में से किसी भी एक को इकाई के स्थान पर रखा जा सकता है। पुनः इकाई के स्थान के लिए प्रत्येक चयन के तदनुसार हमारे पास दहाई के स्थान के लिए 4 चयन हैं। शेष 4 अंकों में से किसी भी एक को दहाई के स्थान पर रखा जा सकता है। अतः गणन के मूलभूत सिद्धांत का प्रयोग करके दोनों स्थानों को इस विशिष्ट क्रम में $5 \times 4 = 20$ विधियों से भरा जा सकता है।

हम गणन के मूलभूत सिद्धांत के प्रयोग के कुछ अन्य उदाहरण लेते हैं।

उदाहरण 5 : अंकों 1, 2, 3, 4 तथा 5 से दो अंकों की कितनी सम संख्याएँ बनाई जा सकती है ?

हल : आइए मान लें कि अंकों की पुनरावृत्ति हो सकती है। क्योंकि वांछित संख्याएँ सम होनी चाहिए, अत: इस स्थिति में इकाई का अंक 2 अथवा 4 ही होना चाहिए।

अत: हमारे पास इकाई के स्थान के लिए 2 चयन हैं। इकाई के स्थान के लिए प्रत्येक चयन के तदनुसार हम दहाई के स्थान के लिए दिए हुए 5 अंकों में से किसी भी एक का चयन कर सकते हैं। अतः दो अंकों की सम संख्याओं की कुल संख्या $2 \times 5 = 10$ है।

क्या आप इन 10 संख्याओं को लिख सकते हैं ? यदि अंकों की पुनरावृत्ति की अनुमति न हो तो 2 अंकों की सम संख्याओं की संख्या क्या होगी ?

उदाहरण 6 : अंकों 1, 2, 3, 4 तथा 5 से तीन अंकों की कितनी संख्याएँ बनाई जा सकती हैं, यदि यह मान लिया जाए कि अंकों की पुनरावृत्ति नहीं हो सकती ?

हल : इकाई के स्थान के लिए हमारे पास पांच अंकों में से किसी भी एक का चयन है। जब हम इसका चयन कर लेते हैं तो हमारे पास केवल चार अंक शेष रहते हैं जिनमें से कोई भी एक दहाई का स्थान ले सकता है। इस प्रकार स्पष्ट है कि सैकड़े के स्थान के लिए हमारे पास केवल 3 चयन हैं। हम इसे निम्न रूप से प्रदर्शित करते हैं :

3	4	5
सै	द	इ

अत: गुणन के मूलभूत सिद्धांत से इस क्रम में तीनों स्थान $5 \times 4 \times 3 = 60$ विधियों से भरे जा सकते हैं।

इस प्रकार दिए हुए 5 अंकों से 3 अंकों की संख्याओं की संख्या 60 है।

[पाठक को चाहिए कि वह उस स्थिति में 3 अंकों की संख्याओं की संख्या निर्धारित करे जबकि अंकों की पुनरावृत्ति की अनुमति हो।]

अब हम देखते हैं कि अरुण ने अपना उत्तर कैसे प्राप्त किया था। पंक्ति में 6 स्थान भरे जाने हैं। 6 बच्चों में से कोई भी एक पहला स्थान ग्रहण कर सकता है। इसके पश्चात् स्थान लेने वाले केवल 5 बच्चे शेष बचते हैं। शेष 5 बच्चों में से कोई भी एक बच्चा पंक्ति में दूसरा स्थान ले सकता है, इत्यादि।

अत: गणन के मूलभूत सिद्धांत द्वारा इस क्रम में 6 स्थान $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ विधियों से भरे जा सकते हैं।

प्रश्नावली 7.1

1. एक सिक्का तीन बार उछाला जाता है तथा परिणामों को लिख लिया जाता है। सम्भव परिणामों की संख्या निर्धारित करने के लिए गणन के मूलभूत सिद्धांत का प्रयोग कीजिए तथा फिर परिणामों की सूची बनाइए।
*2. यदि एक सिक्का चार बार उछाला जाए तो कितने परिणाम सम्भव हैं? पाँच बार उछालने से कितने परिणाम सम्भव हैं ? क्या आप अपने उत्तरों में कोई प्रतिरूप (pattern) देखते हैं ? क्या आप बता सकते हैं कि यदि एक सिक्का n बार उछाला जाए तो कितने परिणाम सम्भव हैं ?
3. किसी परीक्षा में 10 सत्य-असत्य प्रश्न है। उत्तरों के कितने अनुक्रम सम्भव हैं ?
4. एक अलमारी में 4 पुस्तकों, जिनमें भौतिकी, रसायन, जैविकी तथा गणित में से प्रत्येक की एक-एक पुस्तक है, को एक शेल्फ पर कितनी विधियों से व्यवस्थित किया जा सकता है ?
5. अंकों 1, 2, 3, 4 तथा 5 से 2—अंकों की कितनी सम संख्याएँ बनाई जा सकती हैं, यदि अंकों की पुनरावृत्ति की अनुमति न हो ?
6. अंकों 1, 2, 3, 4 तथा 5 से 3—अंकों की कितनी संख्याएँ बनाई जा सकती हैं यदि अंकों की पुनरावृत्ति की अनुमति हो ?
7. 5 व्यक्ति 5 नलों से कितनी विधियों से पानी ले सकते हैं यदि किसी भी नल को प्रयोग के बिना न छोड़ा जाए?
8. शब्द 'KNIFE' के अक्षरों से 5 अक्षरों के कितने भिन्न शब्द बनाए जा सकते है यदि अक्षरों की पुनरावृत्ति की अनुमति न हो ?
[शब्दों का शब्द-कोषीय अर्थ हो या न हो।]
9. 12 बच्चे एक दौड़ में भाग लेते हैं। कितनी विधियों से पहले तीन स्थान लिए जा सकते हैं ?
10. भिन्न रंगों के 7 झंडे हैं। इनसे कितने भिन्न सिगनल (संकेत) बनाए जा सकते हैं यदि एक सिगनल में दो झंडों (एक के नीचे एक) के प्रयोग की आवश्यकता पड़ती है ?
11. 1000 तथा 10000 के बीच ऐसे कितने पूर्णांक हैं जिनमें 4, 5 अथवा 6 के अतिरिक्त अन्य कोई अंक न हो ?
12. एक परीक्षा में शुद्ध उत्तर-चयन (multiple choice) के 6 प्रश्न हैं। यदि पहले तीन प्रश्नों में से प्रत्येक के 4 विकल्प तथा अगले तीन प्रश्नों में से प्रत्येक के 5 विकल्प हों तो उत्तरों के कुल कितने अनुक्रम सम्भव हैं ?
13. यदि अंकों की पुनरावृत्ति की अनुमति हैं तो 1, 2, 3, 4, 5 तथा 9 से 3-अंकों की ऐसी कितनी संख्याएँ बनाई जा सकती हैं जिनमें से प्रत्येक 600 से कम हो ?

7.3 एक उपयोगी संकेतन

इससे पहले कि हम क्रमचयों और संचयों की संकल्पनाओं का परिचय दें हम एक उपयोगी संकेतन को सीखेंगे जो प्रथम n प्राकृत संख्याओं के गुणनफल को प्रदर्शित करने के लिए अति सुविधाजनक है। गुणनफल के इस संकेतन* में विस्मय बोधक चिन्ह का प्रयोग होता है, जो गुणनफल में आने वाली सब से बड़ी प्राकृत संख्या के साथ बाईं ओर लगा होता है।

इस प्रकार, $3! = 3\times2\times1 = 6$

$5! = 5\times4\times3\times2\times1 = 120$

$6! = 6\times5\times4\times3\times2\times1 = 720$

इनमें से प्रत्येक गुणनफल क्रमगुणित अर्थात् फैक्टोरियल (**factorial**) कहलाता है। हम $3!$ को 'तीन फैक्टोरियल' पढ़ते हैं, $5!$ को "पांच फैक्टोरियल" पढ़ते हैं, इत्यादि। $n!$ को निम्न रूप से परिभाषित करते हैं :

$n! = n(n-1)(n-2)\ldots(3)(2)(1)$ तथा $n!$ को 'n फैक्टोरियल' पढ़ते हैं, जहाँ n एक प्राकृत संख्या है।

हम देखते हैं कि $1! = 1$

हम यह भी देखते हैं कि

$$n! = n(n-1)!$$

साथ ही, $$n! = n(n-1)\ (n-2)!$$

तथा, $$n! = n(n-1)\ (n-2)\ (n-3)!$$

हम n पर लगे प्रतिबन्धों का कथन देने की चिंता नहीं करेंगे। हम मान लेते हैं कि जहाँ भी उपरोक्त सम्बन्धों में आने वाले फैक्टोरियल परिभाषित हैं, ये सम्बन्ध लागू रहते हैं।

हम देखेंगे कि $0!$ को आगे आने वाले परिणामों में प्रयोग करने के लिए $0! = 1$ से परिभाषित करना सुविधाजनक है।

प्रश्नावली 7.2

1. निम्नलिखित में से प्रत्येक को गुणनफलों के रूप में लिखिए तथा इनका मान ज्ञात कीजिए।
   (i) $4!$ (ii) $2!$ (iii) $7!$ (iv) $8!$
2. $2! + 3!$ को परिकलित कीजिए। क्या $2! + 3! = 5!$ है?
3. $(4!)\ (2!)$ को परिकलित कीजिए। क्या $(4!)\ (2!) = 8!$ है?

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*कुछ लेखक $n!$ के स्थान पर $\underline{n|}$ का प्रयोग करते हैं। परन्तु हम $n!$ के प्रयोग को ही प्राथमिकता देंगे।

4. $\frac{8!}{4!}$ को परिकलित कीजिए। क्या $\frac{8!}{4!} = 2!$ है?

[संकेत : गुणधर्म $n! = n(n-1)!$, इत्यादि का प्रयोग कीजिए।]

5. $(n-r)!$ का मान निकालिए जबकि

(i) $n=6$, $r=2$ हो।

(ii) $n=9$, $r=5$ हो।

6. $\frac{n!}{(n-r)!}$ का मान ज्ञात कीजिए जबकि

(i) $n=10$, $r=4$ हो।

(ii) $n=12$, $r=3$ हो।

(iii) $r=1$ हो।

(iv) $r=2$ हो।

(v) $r=3$ हो।

7. $\frac{n!}{r!\,(n-r)!}$ का मान ज्ञात कीजिए जबकि

(i) $n=6$, $r=2$ हो।

(ii) $n=7$, $r=4$ हो।

(iii) $n=15$, $r=12$ हो।

*8. सिद्ध कीजिए कि सभी प्राकृत संख्याओं n के लिए

$(2n)! = 2^n\ (n!)\ [1.3.5 \ldots (2n-1)]$ होता है।

[संकेत : $(2n)! = 2n(2n-1)(2n-2)\ldots(3)(2)(1)$]

7.4 क्रमचय

7.4.1 आइए अनुच्छेद 7.2 के उदाहरण 2 पर विचार करें। एक सिक्के को दो बार उछालने पर प्राप्त परिणाम TH, परिणाम HT से भिन्न है। (क्यों ?) दूसरे शब्दों में, H तथा T के घटित होने के क्रम को हमें ध्यान में रखना है। इसी प्रकार, उदाहरणार्थ, उदाहरण 4 में 12, 2-अंकों की एक सम संख्या है। परन्तु यदि हम 1 तथा 2 के घटित होने के क्रम को परस्पर बदल लें तो इस प्रकार प्राप्त संख्या अब 2-अंकों की सम संख्या नहीं होगी।

उन स्थितियों से जिनमें घटनाओं के घटित होने का क्रम महत्वपूर्ण है, क्रमचयों (**permutations**) की प्राप्ति होती है।

कुछ दी हुई वस्तुओं (**objects**) में से एक बार में कुछ को या सभी को लेकर एक निश्चित क्रम में की गई व्यवस्था या विन्यास को क्रमचय (**permutation**) कहा जाता है। अतः उदाहरण 2 में HH, HT, TH तथा TT में से प्रत्येक परिणाम एक क्रमचय है। उदाहरण 3 में प्रत्येक प्रवेश द्वार तथा इसके बाद प्रत्येक निकास द्वार का चयन एक क्रमचय है। उदाहरण 4 में 2-अंकों की प्रत्येक सम संख्या एक क्रमचय है। हम आगे क्रमचयों के कुछ और उदाहरण दे रहे हैं।

उदाहरण 1 : तीन अक्षरों A, B तथा C के क्रमचयों की संख्या निर्धारित कीजिए। सभी क्रमचयों को लिखिए।

हल : क्योंकि तीनों अक्षरों को व्यवस्थित किया जाना है तथा किसी विशेष व्यवस्था में प्रत्येक अक्षर के घटित होने के क्रम को ध्यान में रखना है, अतः हम प्रत्येक अक्षर के घटित होने के स्थान को पहले, दूसरे तथा तीसरे स्थान से व्यक्त कर सकते हैं।

3	2	1
पहला	दूसरा	तीसरा

अब पहले स्थान में हम या तो A, या B या C को लिख सकते हैं। दूसरे शब्दों में, पहले स्थान में एक अक्षर लिखने के लिए हमारे पास 3 चयन हैं।

दूसरे स्थान के लिए हमारे पास शेष बचे दो अक्षरों में से किसी एक का चयन है। (क्यों ?)

तथा तीसरे स्थान के लिए हमारे पास अंत में बचे एक अक्षर का ही चयन है।

अतः गणन के मूलभूत सिद्धान्त द्वारा हम इस क्रम में तीनों स्थानों को $3 \times 2 \times 1 =$ 6 विधियों से भर सकते हैं।

आइए इन छः क्रमचयों को लिखें। ये निम्न हैं :

ABC, ACB, BCA, BAC, CAB तथा CBA

[इस प्रकार शीर्षों A, B तथा C वाले त्रिभुज का नाम उपरोक्त छः विधियों में से किसी एक विधि से लिखा जा सकता है।]

अब हम भिन्न वस्तुओं के एक समुच्चय के क्रमचयों की संख्या निर्धारित करने के लिए दो महत्वपूर्ण प्रमेयों को सिद्ध करेंगे।

प्रमेय 1 : n भिन्न वस्तुओं को एक बार में सभी को लेकर उनके क्रमचयों की संख्या* $P(n, n)$

$$P(n, n) = n(n-1)(n-2)\ldots(3)(2)(1) = n! \text{ होती है।}$$

उपपत्ति : हम n वस्तुओं को एक पंक्ति में n स्थानों में व्यवस्थित करना चाहते हैं। आइए उनके घटित होने के स्थानों को पहले, दूसरे, तीसरे, . . . , nवें स्थान से व्यक्त करें।

n	$(n-1)$	$(n-2)$	. . .	$[n-(n-1)]$
पहला	दूसरा	तीसरा		nवाँ

अब पहले स्थान के लिए हम n वस्तुओं में से किसी एक का चयन कर सकते हैं। दूसरे शब्दों में हमारे पास पहले स्थान के लिए वस्तुओं के n चयन है।

दूसरे स्थान के लिए केवल $(n-1)$ वस्तुएँ शेष बचती हैं। अतः हमारे पास $(n-1)$ चयन हैं।

इसी प्रकार तीसरे स्थान के लिए $(n-2)$ चयन हैं, इत्यादि।

[... जिस समय हम nवें स्थान पर पहुँचेंगे हम $n-1$ वस्तुओं का प्रयोग कर चुकेंगे और इसलिए हमारे पास $[n-(n-1)]$ वस्तुएँ शेष बचेंगी। इस प्रकार, nवें स्थान के लिए हमारे पास $[n-(n-1)]$ अर्थात् 1 चयन है।

*कुछ अन्य संकेतन जिनका प्रयोग किया जाता है, वे हैं "P_n, ^nP_n", $P_{n,n}$ इत्यादि।

अतः गणन के मूलभूत सिद्धांत द्वारा n भिन्न वस्तुओं को एक बार में सभी को लेकर प्राप्त क्रमचयों की संख्या $P(n, n)$ निम्न गुणनफल है :

$$n(n-1)(n-2)\ldots(3)(2)(1)$$

अर्थात् $$P(n, n) = n!$$

प्रमेय 2 : n भिन्न वस्तुओं में से एक बार में r $(r \leq n)$ वस्तुएँ लेकर प्राप्त क्रमचयों की संख्या P (n, r)

$$\mathbf{P(n, r) = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}}$$ **होती हैं।**

प्रमेय 2 की उपपत्ति अपेक्षाकृत सरल है। पाठक को चाहिए कि वह प्रमेय 1 की उपपत्ति का अनुसरण करके इसकी उपपत्ति दे। यह दिखाने के लिए कि

$$n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$ है,

हमें केवल गुणनफल के अंश तथा हर को $(n-r)!$ से गुणा करना है। इस प्रकार,

$$n(n-1)(n-2)\ldots(n-r+1) = \frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)!}{(n-r)!}$$

परन्तु दक्षिण पक्ष का अंश केवल गुणनफल

$n(n-1)(n-2)\ldots(n-r+1)(n-r)(n-r-1)\ldots(3)(2)(1)$ ही है जो कि निस्सन्देह $n!$ है।

इस प्रकार,

$$n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 2 : राकेश, हरि, करुणा तथा सलीम को, जिन सब की शैक्षणिक योग्यताएँ समान हैं, एक कारखाने में चार पदों पर नियुक्त किया जाना है। उन विधियों की संख्या निर्धारित कीजिए, जिनमें

(i) पहले तथा दूसरे पद भरे जा सकते हैं।
(ii) पहले तीन पद भरे जा सकते हैं।
(iii) सभी चारों पद भरे जा सकते हैं।

हल : (i) पहला पद चारों व्यक्तियों में से किसी एक द्वारा भरा जा सकता है। चारों में से किसी एक का चयन कर लेने के बाद शेष तीन व्यक्ति बचते हैं। इनमें से कोई भी एक दूसरे पद पर नियुक्त किया जा सकता है।
अतः वांछित विधियों की संख्या 4×3 अर्थात् 12 है।

(ii) प्रमेय 2 द्वारा, वांछित विधियों की संख्या
$P(4, 3) = 4 \times 3 \times 2 = 24$ है।

(iii) प्रमेय 1 द्वारा वांछित पदों की संख्या
$P(4, 4) = 4 \times 3 \times 2 \times 1 = 24$ है।

उदाहरण 3 : एक म्यूनिसिपल चुनाव में किसी पद के लिये 6 प्रत्याशी हैं। बैलट (ballot) पर उनके नामों की सूची कितनी विधियों से बनाई जा सकती है।

हल : हम 6 नामों को 6 स्थानों में व्यवस्थित करने की विधियों की संख्या निर्धारित करना चाहते हैं। स्पष्ट रूप से वांछित संख्या

$P(6, 6) = 6! = 720$ है।

इस प्रकार बैलट पर 6 प्रत्याशियों के नाम 720 विधियों से लिखे जा सकते हैं।

***उदाहरण 4 :** 1 से 1000 तक की प्राकृत संख्याओं में कितनी ऐसी हैं जिनके किसी भी अंक की पुनरावृत्ति नहीं होती ?

हल : हमें 1 से 1000 तक की 1-अंक की संख्याओं, 2-अंकों की संख्याओं तथा 3-अंकों की संख्याओं की जाँच करनी है तथा देखना है कि इनमें से कितनी ऐसी हैं जिनके सभी अंक भिन्न हैं।

1-अंक की संख्याओं से कोई समस्या उत्पन्न नहीं होती। स्पष्ट है कि ऐसी 9 संख्याएँ हैं और प्रत्येक का अंक भिन्न है।

आइए अब 2-अंकों की उन संख्याओं की संख्या निर्धारित करें जिनके किसी अंक की पुनरावृत्ति नहीं होती। दहाई के स्थान में हम 1, 2, ..., 9 में से कोई भी एक अंक ले सकते हैं। (स्पष्ट है कि हम 0 को दहाई के स्थान पर नहीं ले सकते।) इस प्रकार दहाई के स्थान के लिए कुल $P(9, 1)$ अर्थात् 9 चयन है। इकाई के स्थान के लिए हम या तो 1 से 9 तक के शेष 8 अंकों में से कोई भी एक अंक ले सकते हैं (क्योंकि हम अंकों की पुनरावृत्ति करना नहीं चाहते) या अंक 0 ले सकते हैं। इस प्रकार इकाई के स्थान के लिए पुनः $P(9, 1)$ अर्थात 9 चयन हैं। अतः भिन्न अंकों वाली 2-अंकों की संख्याओं की संख्या $9 \times 9 = 81$ है।

इसी प्रकार, हम 3-अंकों की संख्याओं की संख्या निर्धारित कर सकते हैं जिनमें किसी भी अंक की पुनरावृत्ति नहीं होती। यह संख्या

$9 \times 9 \times 8 = 648$ है।

इस प्रकार, 1 से 1000 तक की प्राकृत संख्याओं में कुल $9 + 81 + 648 = 738$ प्राकृत संख्याएँ ऐसी हैं जिनके किसी भी अंक की पुनरावृत्ति नहीं होती।

उदाहरण 5 : n का (के) मान ज्ञात कीजिए ताकि

(i) $P(n, 5) = 42\, P(n, 3)$; $n > 4$

(ii) $30\, P(n, 6) = P(n+2, 7)$; $n \geq 6$

हल : (i) हमें दिया है कि

$$P(n, 5) = 42\, P(n, 3)$$

अर्थात्, $$\frac{n!}{(n-5)!} = 42\left[\frac{n!}{(n-3)!}\right]$$

इस प्रकार, $$\frac{(n-3)!}{(n-5)!} = 42$$

अर्थात्, $$(n-3)(n-4) = 42$$

अथवा, $$n^2 - 7n - 30 = 0$$

जिससे, $$n = 10, -3$$

स्पष्ट है कि n ऋणात्मक नहीं हो सकता। इस प्रकार n का वांछित मान 10 है।

(ii) हमें दिया है कि

$$30\,P(n, 6) = P(n+2, 7)$$

अर्थात्, $$30\left[\frac{n!}{(n-6)!}\right] = \frac{(n+2)!}{(n-5)!}$$

इस प्रकार, $$\frac{30\,(n-5)!}{(n-6)!} = \frac{(n+2)!}{n!}$$

अर्थात्, $$30(n-5) = (n+2)(n+1)$$

अथवा, $$n^2 - 27n + 152 = 0$$

जिससे, $$n = 8 \text{ या } 19$$

इस प्रकार, n के वांछित मान 8 तथा 19 हैं।

7.4.2 क्रमचय जब कि कुछ वस्तुएँ एक सी हों

17वीं शताब्दी से पहले किसी पुस्तक के छपने में प्रायः अनेक वर्ष लगते थे। साथ ही, उस समय इतनी पत्रिकाएँ भी नहीं होती थीं जिनमें शोधकर्ता अपने परिणाम प्रकाशित करा सकते। अतः अधिकतर शोध परिणामों को निजी पत्रों द्वारा अपने साथियों तक पहुँचाया जाता था। परन्तु इससे शोधकर्ता को क्षति पहुँचने की सम्भावना बनी रही तथा 17वीं शताब्दी के अंत में हमने शोध की प्राथमिकता के विषय में उठने वाले अनेकों विवादों के विषय में पढ़ा।

इस प्रकार की क्षति की रोकथाम के लिए शोधकर्ताओं ने सूचना संचार की एक ऐसी पद्धति का 'आविष्कार' किया जिसमें वे अक्षरों का स्थान परिवर्तन करके वाक्यों का प्रयोग करते थे। इस प्रकार बने नये वाक्यों को **वर्ण विपर्यास (anagrams)** कहा जाता है। उदाहरणार्थ, शब्द **luccalus**, **calculus** शब्द का एक वर्ण विपर्यास है।

जब **क्रिश्चियन हाइगन्ज** (1629-1695) ने शनि-वलय (Saturn's ring) की खोज की तो उसने निम्नलिखित वर्ण-वपर्यास की रचना की :

aaaaaaa, ccccc, d, eeeee, g, h, iiiiiii, lll, mm, nnnnnnnnn, oooo, pp, q, rr, s, ttttt, uuuuu

जिसका यदि उचित क्रम में व्यवस्थित करके अंग्रेजी में अनुवाद किया जाए तो इसका अर्थ निम्न है :

"Surrounded by a thin flat ring not suspended anywhere, inclined to the ecliptic."

उपर्युक्त का हिन्दी अनुवाद निम्न है :

"एक पतले चपटे वलय (ring) से घिरा हुआ, किसी ओर लटका हुआ नहीं तथा रविमार्ग (ecliptic) की ओर झुका हुआ"।

वर्ण विपर्यासों के अवकूटन (decoding) करने में उन्हें विभिन्न क्रमों में पुनः व्यवस्थित करना सम्बद्ध होता है।

इससे हमें n वस्तुओं के क्रमचयों की संख्या के विषय में सीखने की प्रेरणा मिलती है, जबकि उनमें से कुछ वस्तुएँ एक सी हों।

आइए, उदाहरणार्थ A, A तथा C तीनों अक्षरों को एक बार में एक साथ लेकर प्राप्त क्रमचयों पर विचार करें। आइए सभी सम्भव क्रमचयों को लिखें। हम निम्न प्राप्त करते हैं :

AAC, ACA, CAA

[आपको याद होगा कि यदि तीनों वस्तुएँ भिन्न-भिन्न हों तो क्रमचयों की संख्या $P(3, 3) = 3!$ होगी।]

आइए अब A, A, C तथा D अक्षरों के क्रमचयों पर विचार करें। हम निम्न प्राप्त करते हैं :

$AACD$, $AADC$, $ACDA$, $ACAD$, $ADAC$, $ADCA$, $CAAD$, $CADA$, $CDAA$, $DAAC$, $DACA$, $DCAA$

[आपको याद होगा कि यदि चारों वस्तुएँ भिन्न-भिन्न हों तो क्रमचयों की संख्या $P(4, 4) = 4!$ होगी।]

अब हम n वतुस्ओं, जिनमें से कुछ एक सी हैं, के क्रमचयों की संख्या निर्धारित करने के लिए एक महत्वपूर्ण प्रमेय सिद्ध करेंगे।

प्रमेय 3 : n वस्तुओं, जिनमें m वस्तुएँ, एक प्रकार की तथा $(n-m)$ दूसरी प्रकार की हैं, को एक बार में एक साथ लेकर प्राप्त क्रमचयों की संख्या $\dfrac{n!}{m!(n-m)!}$ **होती है।**

उपपत्ति : आइए क्रमचयों की वाँछित संख्या को, उदाहरणार्थ, x से व्यक्त करें।

आइए इन x क्रमचयों में से एक विशेष क्रमचय उदाहरणार्थ, वह क्रमचय लें जिसमें एक प्रकार की m वस्तुएँ एक साथ ली गई हैं तथा इनके पश्चात् दूसरी प्रकार की $(n-m)$ वस्तुएँ एक साथ ली गई हैं।

$\underline{1}$ $\underline{2}$ . . . $\underline{m}$ $\underline{m+1}$ $\underline{m+2}$ $\underline{m+3}$ . . . $\underline{n}$

एक प्रकार की दूसरी प्रकार की

$\longleftarrow m$ वस्तुएँ $\longrightarrow$ $\longleftarrow (n-m)$ वस्तुएँ $\longrightarrow$

आइए अब मान लें कि इन m वस्तुओं में से सभी परस्पर भिन्न हैं तथा शेष $(n-m)$ वस्तुओं से भी भिन्न हैं। इन m वस्तुओं को एक बार में एक साथ लेकर प्राप्त क्रमचयों की संख्या $P(m, m) = m!$ है।

इसी प्रकार, यदि $(n-m)$ वस्तुओं में से प्रत्येक को दूसरी से भिन्न मान लिया जाए तथा इनको m वस्तुओं से भी भिन्न मान लिया जाए तो इन $(n-m)$ वस्तुओं को एक बार में एक साथ लेकर प्राप्त क्रमचयों की संख्या $P(n-m, n-m) = (n-m)!$ होगी।

अतः मूलभूत सिद्धान्त द्वारा, x क्रमचयों में से इस एक विशेष क्रमचय से $m!\,(n-m)!$ क्रमचय प्राप्त होते हैं, जबकि सभी वस्तुएँ भिन्न हैं।

यह स्पष्ट है कि यदि एक प्रकार की m वस्तुएँ तथा दूसरी प्रकार की $n-m$ वस्तुएँ एक साथ व्यवस्थित न भी हों जैसा कि ऊपर दर्शाया गया है तो भी ज्यों ही हम इन्हें भिन्न मान लेते हैं तो एक विशेष प्रकार की वस्तुओं को उन्हीं द्वारा पहले ही घेरे गए स्थानों में पुनः व्यवस्थित करने पर हम $m!\,(n-m)!$ क्रमचय प्राप्त करते हैं।

इस प्रकार, हम x क्रमचयों से कुल $x\ (m\ !)\ (n-m)\ !$ क्रमचयों को प्राप्त होते हैं, यदि सभी n वस्तुएँ भिन्न हों।

परन्तु यदि सभी n वस्तुएँ भिन्न हों, तो क्रमचयों की संख्या $P\ (n, n) = n\ !$ होती है।

इस प्रकार, $x\ (m\ !)\ (n-m)\ ! = n\ !$

जिससे, $x = \dfrac{n\ !}{m\ !\ (n-m)\ !}$

आइए, अक्षरों A, A तथा C के क्रमचयों के उदाहरण पर वापिस आ जाएँ। यहाँ, $n=3$, $m=2$ है। इस प्रकार, $(n-m) = 1$

अतः, $x = \dfrac{3\ !}{2\ !\ \ 1\ !} = 3$

क्या आप बता सकते हैं कि n वस्तुओं, जिनमें p वस्तुएँ एक प्रकार की, q वस्तुएँ दूसरी प्रकार की तथा शेष (n—p—q) वस्तुएँ एक अन्य प्रकार की हैं, के क्रमचयों की संख्या क्या होगी ? वाँछित संख्या

$\dfrac{n\ !}{p\ !\ q\ !\ (n-p-q)\ !}$ होगी।

[पाठक को चाहिए कि वह प्रमेय 3 के आधार पर उपरोक्त की उपपत्ति दे। पाठक इस का आगे व्यापकीकरण भी करे।]

आइए, अक्षरों A, A, C तथा D के क्रमचयों के उदाहरण पर पुनः विचार करें। यहाँ $n = 4$, $p = 2$, $q = 1$ हैं। इस प्रकार, $(n-p-q) = 1$

अतः क्रमचयों की वाँछित संख्या

$\dfrac{4\ !}{2\ !\ \ 1\ !\ \ 1\ !} = 12$ है।

आइए हाइगन्ज के वर्ण विपर्यास के क्रमचयों की संख्या परिकलित करें। इसमें 61 अक्षर हैं जिनमें $7a$, $5c$, $1d$, $5e$, $1g$, $1h$, $7i$, $3l$, $2m$, $9n$, $4o$, $2p$, $1q$, $2r$, $1s$, $5t$ तथा $5u$ हैं। इस प्रकार क्रमचयों की संख्या

$$\frac{61\ !}{7!\ \ 5!\ \ 1!\ \ 5!\ \ 1!\ \ 1!\ \ 7!\ \ 3!\ \ 2!\ \ 9!\ \ 4!\ \ 2!\ \ 1!\ \ 2!\ \ 1!\ \ 5!\ \ 5!}$$

है जोकि एक बहुत बड़ी संख्या है जिसे परिकलित करने का प्रयत्न करने के लिए हम पाठक को नहीं कहेंगे। यह जानना सुखकर है कि इस वर्ण विपर्यास का कूटवाचन (decode) करने के लिए शोधकर्ता अपनी सहज बुद्धि का प्रयोग करता है तथा केवल वही क्रमचय लेता है जो कि 'व्यावहारिक' हैं।

हम कुछ और उदाहरण लेते हैं।

उदाहरण 6 : शब्द 'EATERANTEATER' के अक्षर कितनी भिन्न विधियों से व्यवस्थित किए जा सकते हैं ?

हल : इस शब्द में 13 अक्षर हैं जिनमें 4E, 3A, 3T, 2R तथा 1N है। इस प्रकार सभी सम्भव भिन्न व्यवस्थाओं की संख्या

$$\frac{13\ !}{4!\ \ 3!\ \ 3!\ \ 2!\ \ 1!} = 3603600 \text{ है।}$$

उदाहरण 7 : एक बैग में 5 लाल, 4 सफेद तथा 3 नीले संगमरमर के टुकड़े हैं। उन्हें एक-एक करके निकाला जाता है तथा एक पंक्ति में लगाया जाता है। यह मानकर कि सभी 12 टुकड़े निकाल लिए जाते हैं, भिन्न व्यवस्थाओं की संख्या निर्धारित कीजिए।

हल : यहाँ $n=12$, $p=5$, $q=4$ तथा $(n-p-q)=3$ है।

अतः वाँछित संख्या $\dfrac{12!}{5!\ 4!\ 3!}=27720$ है।

उदाहरण 8 : बीसवीं शताब्दी की जैविकी की एक महत्वपूर्ण उपलब्धि यह है कि जीव वैज्ञानिक 'अनुवंशिक कूट' (genetic code) को तोड़ सकने में समर्थ हो गए हैं। क्योंकि अब वे यह निर्धारित कर सकते हैं कि किसी जीव के जीन अगले वंश तक किस प्रकार पहुँच जाते हैं।

इन जीनों में डेसॉक्सीरिबोनुक्लेइक एसिड (DNA) के बहुत भारी अणु होते हैं जो स्वयं 4 प्रकार के छोटे अणुओं की रैखिक शृंखलाओं में होते हैं। ये 4 प्रकार निम्न हैं ऐडेनीन (A), साइटोसिन (C)-ग्वानीन (G) तथा थायमीन (T)।

एक शृंखला में 12 अणुओं की भिन्न व्यवस्थाओं की संख्या निर्धारित कीजिए, यदि शृंखला में A, C, G तथा T में से प्रत्येक के तीन-तीन अणु हों।

हल : यहाँ $n=12$ है। प्रत्येक छोटे अणु A, C, G तथा T के तीन एक जैसे अणु हैं। इस प्रकार, शृंखला में व्यवस्थाओं की वाँछित संख्या

$$\frac{12\,!}{3\,!\ 3\,!\ 3\,!\ 3\,!}=369600 \text{ है।}$$

उदाहरण 9 : छः एक जैसे सिक्कों को एक पंक्ति में व्यवस्थित किया जाता है। 4 heads तथा 2 tails प्राप्त करने की विधियों की संख्या निर्धारित कीजिए।

हल : हम 6 वस्तुओं को, जिनमें 4 एक प्रकार की (heads) तथा 2 अन्य प्रकार की (tails) हैं, व्यवस्थित करना चाहते हैं। अतः वाँछित विधियों की संख्या

$$\frac{6\,!}{4\,!\ 2\,!}=15 \text{ है।}$$

प्रश्नावली 7.3

1. निम्नलिखित में से प्रत्येक का मान ज्ञात कीजिए :

(i) $P(10, 3)$ (ii) $P(9, 5)$

(iii) $P(20, 4)$ (iv) $P(75, 2)$

2. सिद्ध कीजिए कि सभी प्राकृत संख्याओं n के लिए
$P(n, n) = P(n, n-1)$

3. रेनु अर्थशास्त्र की 3, इतिहास की 2 तथा भाषा की 4 पुस्तकों को अलमारी में व्यवस्थित करना चाहती है। निम्नलिखित को निर्धारित कीजिए :
(क) सभी सम्भव विन्यासों की संख्या।
(ख) सभी सम्भव विन्यासों की संख्या, यदि एक विषय की सभी पुस्तकें एक साथ रहें।

4. शब्द 'EQUATION' के अक्षरों से बने 5-अक्षरों के भिन्न शब्दों की संख्या निर्धारित कीजिए।

5. दिखाइए कि 1000 से 10000 के बीच कुल 4536 ऐसी संख्याएँ हैं जिनके किसी भी अंक की पुनरावृत्ति नहीं होती। इन में से कितनी संख्याएं विषम संख्याएं हैं ?

6. शब्द 'EQUATION' के अक्षरों से बने 8-अक्षरों के भिन्न शब्दों की संख्या ज्ञात कीजिए, यदि प्रत्येक शब्द एक स्वर (vowel) से आरम्भ होता हो।

7. 8 विद्यार्थियों को एक परीक्षा में बैठना है जिनमें से 3 विद्यार्थियों को गणित में तथा शेष 5 विद्यार्थियों को अन्य विषयों में परीक्षा देनी है। उन्हें एक पंक्ति में कितनी विधियों से बैठाया जा सकता है यदि गणित के परीक्षार्थी एक दूसरे के साथ नहीं बैठ सकते हैं ?

8. 4 लड़कों तथा 3 लड़कियों को एक पंक्ति में 7 कुर्सियों पर कितनी विधियों से बैठाया जा सकता है यदि दो लड़के या दो लड़कियां एक साथ न बैठ सकें ?

*9. दिल्ली में पंजीकृत गाड़ियों की लाइसेन्स प्लेटों पर (अंग्रेजी वर्णमाला) के तीन अक्षर, तथा उनके बाद 1, 2, 3, अथवा 4 अंक लिखे होते हैं। बाईं ओर का पहला अक्षर 'D' ही हो सकता है। 1-अंक की संख्या में 0 की अनुमति नहीं है। निस्संदेह, अंकों की पुनरावृत्ति हो सकती है। लाइसेंस प्लेटों की सम्भव संख्या निर्धारित कीजिए।

10. एक जलपानगृह में खाने की वस्तुओं की सूची (menu) में 5 सब्जियां, 3 मीट (meat), 2 सलाद तथा 4 ब्रेड लिखी हुई हैं। एक ग्राहक कितनी विधियों से अपने खाने का चयन कर सकता है यदि वह 1 मीट, 1 सब्जी, 1 सलाद तथा 1 ब्रेड चुने ?

11. एक सिनेमा हॉल की टिकट खिड़की पर 10 व्यक्ति कितनी विधियों से पंक्ति बना सकते हैं ?

12. अंकों 1, 2, 0, 2, 4, 2 तथा 4 से 7-अंकों वाली कितनी संख्याएँ बनाई जा सकती हैं ?

13. 6 लड़कों तथा 5 लड़कियों को एक ग्रुप फोटो के लिए कितनी विधियों से व्यवस्थित किया जा सकता है यदि लड़कियों को एक पंक्ति में कुर्सियों पर बैठना है तथा लड़कों को एक पंक्ति में उनके पीछे खड़े होना है ?

14. अंकों 4, 2, 5, 0, 6 तथा 7 से 6-अंकों की कितनी भिन्न संख्याएँ बनाई जा सकती हैं ? (मान लीजिए कि अंकों की पुनरावृत्ति नहीं हो सकती।) इनमें से कितनी संख्याओं के दहाई के स्थान में अंक 0 होगा ?

15. अंकों 1, 2, 3, 4 तथा 5 से कितनी भिन्न संख्याएँ बनाई जा सकती हैं, यदि यह मान

लिया जाए कि संख्याओं में अंकों की पुनरावृत्ति नहीं हो सकती ? इनमें से कितनी संख्याएँ सम होंगी ?

16. अंकों 0, 1, 2, 3, 4, 5 तथा 6 से 100 से 1000 के बीच कितनी भिन्न संख्याएँ बनाई जा सकती हैं, यदि यह मान लिया जाए कि किसी संख्या में अंकों की पुनरावृत्ति नहीं हो सकती ? इनमें से कितनी संख्याएँ 5 से विभाज्य होंगीं ?

17. n ज्ञात कीजिए यदि

(i) $P(n, 6)=3P(n, 5)$ (ii) $P(n, 4)=20P(n, 2)$

(iii) $2P(n, 3)=P(n+1, 3)$

18. r ज्ञात कीजिए यदि

(i) $P(10, r)=2P(9, r)$ (ii) $5P(4, r)=6P(5, r-1), r\geqslant 1$

(iii) $4P(6, r)=P(6, r+1)$

19. सिद्ध कीजिए कि सभी प्राकृत संख्याओं n तथा r के लिए, जिन के लिए संकेत परिभाषित हैं,

$P(n, r)=P(n-1, r)+r\,P(n-1, r-1)$ होता है।

20. शब्द 'KURUKSHETRA' के अक्षर कितनी भिन्न विधियों से व्यवस्थित किए जा सकते हैं ?

21. तार के संदेशों* में मौर्स कोड (Morse Code) का प्रयोग किया जाता है। इस कोड में अंग्रेजी वर्णमाला के अक्षरों, संख्याओं तथा मात्रा चिन्हों (punctuation marks) को बिन्दुओं (.) तथा डैशों (—) से दर्शाया जाता है। उदाहरणार्थ, अक्षर E को एक बिन्दु द्वारा तथा अक्षर T को एक डैश द्वारा दर्शाया जाता है। अन्य अक्षरों को 2 या अधिक संकेतों से दर्शाया जाता है।

(क) दो चिन्हों, उदाहरणार्थ, . ., . — , इत्यादि के प्रयोग से कितने अक्षरों को दर्शाया जा सकता है ?

(ख) तीन चिन्हों, उदाहरणार्थ, . . ., . — . इत्यादि के प्रयोग से कितने अक्षरों को दर्शाया जा सकता है ?

(ग) दिखाइए कि अंग्रेजी वर्णमाला के प्रत्येक अक्षर को 4 या इससे कम संकेतों के प्रयोग से दर्शाना सम्भव है।

टिप्पणी : 26 अक्षरों के अतिरिक्त हमें 0 से 9 तक के 10 अंकों तथा मात्रा चिन्हों को भी संदेश रूप में भेजने की आवश्यकता होती है। अतः तार के संदेशों के लिए 5 या इससे कम संकेतों के अनुक्रमों की आवश्यकता होती है।

22. तीन एक जैसे सिक्के एक पंक्ति में व्यवस्थित किए जाते हैं। उन विधियों की संख्या निर्धारित कीजिए जिसमें 0 heads तथा 3 tails प्राप्त हों। साथ ही 1 head तथा 2

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*मौर्स कोड (जिसे मौर्स वर्णमाला भी कहते हैं) की खोज एक अमरीकी सेम्युअल एफ० बी० मौर्स (1791-1872) ने की थी।

tails प्राप्त करने की विधियों की संख्या निर्धारित कीजिए, इत्यादि ।

इस प्रकार निम्न सारणी को पूर्ण कीजिए :

3 सिक्कों को व्यवस्थित करने से प्राप्त heads की संख्या	0	1	2	3
विधियों की संख्या				

टिप्पणी : सारणी में दूसरी पंक्ति की संख्याओं का जोड़ $8=2^3$ होना चाहिए ।

23. चार एक जैसे सिक्के एक पंक्ति में व्यवस्थित किए गए हैं । उन विधियों की संख्या निर्धारित कीजिए जिनमें 0 heads प्राप्त होते हैं । साथ ही, 1 head प्राप्त करने की विधियों की संख्या निर्धारित कीजिए, इत्यादि ।
इस प्रकार निम्न सारणी को पूर्ण कीजिए :

4 सिक्कों को व्यवस्थित करने से प्राप्त heads की संख्या	0	1	2	3	4
विधियों की संख्या					

टिप्पणी : सारणी में दूसरी पंक्ति की संख्याओं का जोड़ $16=2^4$ होना चाहिए ।

24. एक खम्भे (pole) पर 3 लाल, 2 पीले तथा 2 हरे झंडों को व्यवस्थित करके कितने भिन्न संकेत भेजे जा सकते हैं ?
(मान लीजिए कि एक संकेत को भेजने में सभी सातों झंडों का प्रयोग होता है ।)

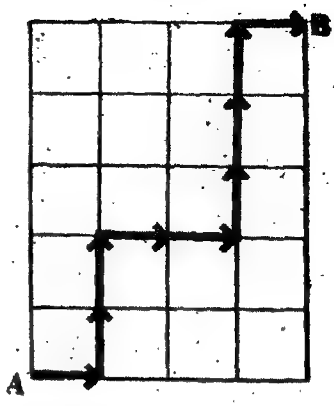
25. कितनी विधियों से दो 10-पैसे के, दो 20-पैसे के, तीन 25-पैसे के तथा एक 50-पैसा का सिक्का 8 बच्चों में इस प्रकार विभाजित किए जा सकते हैं, कि प्रत्येक बच्चे को केवल एक ही सिक्का प्राप्त हो ?

26. अंकों 1, 2, 5, 5 तथा 4 का प्रयोग करके 5-अंकों की कितनी सम संख्याएँ बनाई जा सकती हैं ?

27. **ग्रिड-समस्या :** एक व्यक्ति को A से चलकर B तक जाना है । परन्तु उस पर यह प्रतिबन्ध है, कि वह A के केवल दाईं ओर या A के ऊपर की ओर ही चल सकता है, परन्तु यह आवश्यक नहीं कि वह इसी क्रम से चले । उसका ऐसा एक पथ आकृति 7.2

में दर्शाया गया है। A से B तक के पथों की संख्या निर्धारित कीजिए।

[संकेत : वह व्यक्ति चाहे कोई भी पथ चुने, उसे 4 बार दाईं ओर तथा 5 बार ऊपर की ओर चलना होगा।]



आकृति 7.2: 4×5 ग्रिड

7.5 संचय

आपको याद होगा कि n वस्तुओं को व्यवस्थित करने में किसी क्रमचय में वस्तुओं के घटित होने का क्रम महत्वपूर्ण होता है। अब हम गिनने की उन समस्याओं पर विचार करते हैं जहाँ घटित होने का क्रम महत्वपूर्ण नहीं होता। आइए उदाहरणार्थ, चार में से दो व्यक्तियों को 'चुनने' की विधियों की संख्या निर्धारित करें। आइए चारों व्यक्तियों को A, B, C तथा D से व्यक्त करें। हम पहले से ही जानते हैं कि 4 वस्तुओं में से एक बार में 2 को लेने से प्राप्त क्रमचयों की संख्या $P(4, 2) = 12$ होती है। ये क्रमचय निम्न हैं :

AB, AC, AD, BC, BD, CD, BA, CA, DA, CB, DB तथा DC

परन्तु यदि घटित होने के क्रम का कोई महत्व नहीं हो तो AB तथा BA समान हैं। इसी प्रकार AC तथा CA समान हैं, इत्यादि। इस प्रकार हम 4 में से 2 व्यक्तियों को निम्नलिखित विधियों से चुन सकते हैं :

AB, AC, AD, BC, BD तथा CD

हम देखते हैं कि 4 में से 2 व्यक्तियों को चुनने की 6 विधियाँ हैं।

उन स्थितियों से, जिनमें वस्तुओं के एक दिए हुए समुच्चय में से एक उपसमुच्चय को चुना जाना होता है, संचयों (**Combinations**) की प्राप्ति होती है।

दी हुई भिन्न वस्तुओं में से सभी या कुछ वस्तुओं के चुनने को संचय कहा जाता है। एक संचय में वस्तुओं के चुनने का क्रम महत्वपूर्ण नहीं होता।

हम नीचे n भिन्न वस्तुओं में से एक बार में r वस्तुएँ लेकर संचयों की संख्या निर्धारित करने के लिए एक सूत्र देते रहे हैं।

प्रमेय 4 : n भिन्न वस्तुओं में से एक बार में r वस्तुएँ लेकर संचयों की संख्या* $C(n, r)$,

$$C(n, r) = \frac{n!}{r!\,(n-r)!}$$ होती है, जहाँ $r \leqslant n$।

उपपत्ति : आपको याद होगा कि n भिन्न वस्तुओं में से एक बार में r, $(1 \leqslant r \leqslant n)$ वस्तुएँ लेकर प्राप्त

क्रमचयों की संख्या $P(n, r) = \dfrac{n!}{(n-r)!}$ होती है।

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* $C(n, r)$ के लिए अन्य संकेतन ${}^nC_r$; C^n_r; $C_{n,r}$; $\binom{n}{r}$ हैं।

क्योंकि वस्तुओं के व्यवस्थितिकरण के क्रम का महत्व नहीं है, अतः यह स्पष्ट है कि r वस्तुओं के प्रत्येक संचय के लिए $P(r,r)=r\,!$ विन्यास हैं। दूसरे शब्दों में,

$$P(n,r)=r\,!\;C(n,r)$$

इस प्रकार, $C(n,r)=\dfrac{P(n,r)}{r\,!}=\dfrac{n\,!}{r\,!\,(n-r)!}$

अत : n भिन्न वस्तुओं में से एक बार में r वस्तुएँ लेकर प्राप्त संचयों की संख्या

$C(n,r)=\dfrac{n\,!}{n\,!\,(n-r)\,!}$ होती है जहाँ $r \leqslant n$ है।

टिप्पणी 1 : स्पष्ट है कि n वस्तुओं को एक बार में सभी को लेकर संचयों की संख्या 1 है। इसकी जाँच प्रमेय 4 द्वारा भी, जब $r=n$ हो, की जा सकती है।

जब $r=n$ है तो

$$C(n,\ n)=\frac{n\,!}{n\,!\,(n-n)\,!} \qquad (1)$$

हम पहले ही परिभाषित कर चुके हैं कि $0!=1$।

इस प्रकार, सरल करने पर (1) से हम $\mathbf{C\ (n,\ n)=1}$ प्राप्त करते हैं।

टिप्पणी 2 : दी हुई n भिन्न वस्तुओं में से r वस्तुओं के चुनने के बाद प्रत्येक बार हमारे पास शेष $(n-r)$ वस्तुएँ बच रहती हैं। अतः यह स्पष्ट है कि n भिन्न वस्तुओं में से एक बार में r वस्तुओं को लेकर प्राप्त संचयों की संख्या n भिन्न वस्तुओं में से एक बार में $(n-r)$ वस्तुएँ लेकर प्राप्त संचयों की संख्या के समान होती है। दूसरे शब्दों में,

$$\mathbf{C(n,\ r)=C(n,\ n-r)}$$

[पाठक को चाहिए कि वह $C(n,\ r)$ के लिए सूत्र का प्रयोग करके उपरोक्त गुणधर्म को सिद्ध करे।]

टिप्पणी 3 : n वस्तुओं में से एक बार में किसी को भी न लेकर संचयों की संख्या भी 1 है। स्पष्ट है कि जब हम किसी वस्तु का चयन नहीं करते तो ऐसा करना सभी n वस्तुओं को शेष छोड़ने के समान है तथा हम यह जानते हैं कि ऐसा करने की केवल एक विधि है। इस प्रकार,

$$\mathbf{C(n,\ 0)=1}.$$

अतः प्रमेय 4 तब भी लागू रहती है जब कि $r=0$ हो।

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : मान ज्ञात कीजिए :

(i) $C(10,\ 4)$ (ii) $C(12,7)$
(iii) $C(6,\ 6)$ (iv) $C(9,\ 0)$

हल :

(i) $C(10,\ 4)=\dfrac{10\,!}{4\,!\,(10-4)\,!}=\dfrac{10\,!}{4\,!\,6\,!}=\dfrac{10\times9\times8\times7\times6\,!}{4\times3\times2\times1\times6\,!}=210$

(ii) $C(12, 7) = \frac{12\,!}{7\,!\,5\,!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7\,!}{5 \times 4 \times 3 \times 2 \times 1 \times 7\,!} = 792$

(iii) $C(6, 6) = \frac{6\,!}{6\,!\,0\,!} = 1$

(iv) $C(9, 0) = \frac{9\,!}{0\,!\,9\,!} = 1$

उदाहरण 2 : एक विद्यालय में बैडमिंटन के 5 'अच्छे' खिलाड़ी हैं। अन्तर-स्कूल टूर्नामेन्ट के लिए 4 खिलाड़ियों की एक टीम को भेजा जाना है। टीम को कितनी विधियों से चुना जा सकता है ?

हल : हम 5 में से 4 खिलाड़ियों को चुनने की विधियों की संख्या का निर्धारण करना चाहते हैं। प्रमेय 4 का प्रयोग करके हम निम्न प्राप्त करते हैं :

$$C(5, 4) = \frac{5\,!}{4\,!\,1\,!} = 5$$

[आइए बैडमिंटन के 5 'अच्छे' खिलाड़ियों को A, B, C, D तथा E से व्यक्त करें। इन खिलाड़ियों में से 4 खिलाड़ियों की निम्न भिन्न टीमें बनाई जा सकती हैं :

$ABCD$, $ABCE$, $ABDE$, $ACDE$ तथा $BCDE$]

उदाहरण 3 : 5 लड़कों तथा 4 लड़कियों में से 3 लड़कों तथा 3 लड़कियों की एक टीम को कितनी विधियों से चुना जा सकता है ?

हल : 5 में से 3 लड़कों को चुनने की विधियों की संख्या $C(5, 3) = 10$ है। (क्यों ?) 4 में से 3 लड़कियों को चुनने की विधियों की संख्या $C(4, 3) = 4$ है।

अतः मूलभूत सिद्धांत द्वारा वांछित विधियों की संख्या $10 \times 4 = 40$ है।

उदाहरण 4 : एक वृत्त के 6 बिन्दुओं से होकर कितनी रेखाएँ खींची जा सकती हैं ?

हल : क्योंकि दो बिन्दुओं से केवल एक रेखा खींची जा सकती है, अतः वृत्त के 6 बिन्दुओं से खींची जा सकने वाली रेखाओं की संख्या :

$$C(6, 2) = \frac{6\,!}{2\,!\,4\,!} = 15 \text{ है।}$$

[पाठक को चाहिए कि वृत्त के n बिन्दुओं के लिए इस समस्या का व्यापकीकरण करें।]

उदाहरण 5 : ताश के 52 पत्तों की गड्डी में से 5-पत्तों के संचयों की संख्या निर्धारित कीजिए।

हल : 5-पत्तों के संचयों की संख्या निम्न है :

$$C(52, 5) = \frac{52\,!}{5\,!\,47\,!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1}$$
$$= 2598960$$

उदाहरण 6 : जाँच कीजिए कि

$$C(8, 4) + C(8, 3) = C(9, 4)$$

हल : $C(8, 4)=\frac{8!}{4!\ 4!}=70$

$$C(8, 3)=\frac{8!}{3!\ 5!}=56$$

इस प्रकार, $C(8, 4)+C(8, 3)=70+56=126$

अब, $C(9, 4)=\frac{9!}{4!\ 5!}=\frac{9\times8\times7\times6}{4\times3\times2\times1}$

$$=126$$

अत: हम देखते है कि

$$C(8,4)+C(8, 3)=C(9, 4)$$

अब हम $C(n, r)$ के एक महत्वपूर्ण गुणधर्म को सिद्ध करते हैं, जो संचयों के गणित के अध्ययन में बहुत उपयोगी है।

प्रमेय 5 : यदि n तथा r इस प्रकार की प्राकृत संख्याएँ हैं कि $1 \leqslant r \leqslant n$, तो

$$C(n, r)+C(n, r-1)=C(n+1, r)$$

उपपत्ति : इसकी उपपत्ति अति सरल है। हम वाम पक्ष लेते हैं तथा $C(n, r)$ की परिभाषा का प्रयोग करते हैं। इस प्रकार हम निम्न प्राप्त करते हैं।

$$C(n, r)+C(n, r-1)=\frac{n!}{r!\ (n-r)!}+\frac{n!}{(r-1)!\ (n-r+1)!} \quad (1)$$

आपको याद होगा कि $r!=r(r-1)!$

तथा $(n-r+1)!=(n-r+1)(n-r)!$

इस प्रकार, (1) से

$$C(n, r)+C(n, r-1)=n!\left[\frac{1}{r(r-1)!\ (n-r)!}+\frac{1}{(r-1)!\ (n-r+1)\ (n-r)!}\right]$$

$$=n!\left[\frac{n-r+1+r}{r(r-1)!\ (n-r+1)\ (n-r)!}\right]$$

$$=n!\left[\frac{n+1}{r!\ (n-r+1)!}\right]$$

$$=\frac{(n+1)!}{r!\ (n-r+1)!}=C(n+1, r)$$

अत:, $C(n, r)+C(n, r-1)=C(n+\ \ , r)$

अब हम इसके लिए एक वैकल्पिक उपपत्ति देते हैं जो संचयों से सम्बन्धित तर्कों पर आधारित है। वास्तव में कुछ लेखक इसे **संचयों के गणित की उपपत्ति (combinatorial proof)** तक कह देते हैं।

आपको याद होगा कि $(n+1)$ वस्तुओं में से एक बार में r वस्तुओं को लेकर संचयों की संख्या $C(n+1, r)$ होती है।

आइए, दी हुई $(n+1)$ भिन्न वस्तुओं में से किसी एक वस्तु पर ध्यान केन्द्रित करें। मान लीजिए हम इसे, उदाहरणार्थ, S से व्यक्त करते हैं। स्पष्ट रूप से दो सम्भावनाएँ (possibilities) हैं।

(P1) यह विशिष्ट वस्तु S चयन में सम्मिलित है।

(P2) S चयन में सम्मिलित नहीं है।

सम्भावना $P1$ के लिए, जबकि S चयन में सम्मिलित है, शेष $(r-1)$ वस्तुएँ शेष बची $(n+1)-1 = n$ वस्तुओं में से चुनी जानी चाहिए। ऐसा $C(n, r-1)$ विधियों से किया जा सकता है।

सम्भावना $P2$ के लिए, जबकि S चयन में सम्मिलित नहीं है, स्पष्ट है कि r वस्तुएँ शेष बची $(n+1)-1 = n$ वस्तुओं में से चुनी जानी चाहिए। निस्संदेह ऐसा $C(n, r)$ विधियों से किया जा सकता है।

इस प्रकार $(n+1)$ भिन्न वस्तुओं में से r वस्तुओं को चुनने की कुल विधियों की संख्या

$C(n, r-1) + C(n, r)$ है।

अर्थात्, $$C(n+1, r) = C(n, r) + C(n, r-1)$$

हम कुछ और उदाहरण लेते हैं।

उदाहरण 7 : निम्नलिखित में से प्रत्येक का मान ज्ञात कीजिए :

(*i*) $C(10, 4) + C(10, 5)$

(*ii*) $C(61, 57) - C(60, 56)$

हल : (*i*) हम प्रमेय 5 का प्रयोग करके निम्न प्राप्त करते हैं :

$$C(10, 4) + C(10, 5) = C(11, 5) = \frac{11!}{5!\ 6!} = 462$$

(*ii*) पुनः हम प्रमेय 5 का प्रयोग करते हैं। हमें निम्न प्राप्त होता है :

$$C(60, 56) + C(60, 57) = C(61, 57)$$

इस प्रकार, $$C(61, 57) - C(60, 56) = C(60, 57) = \frac{60!}{57!\ 3!} = 34220$$

अन्त में हम $C(n, r)$ तथा $C(n, r-1)$ के बीच एक आवर्ती सम्बन्ध **(recurrence relation)** सिद्ध करते हैं।

प्रमेय 6 : यदि n तथा r इस प्रकार की प्राकृत संख्याएँ हैं कि $1 \leqslant r \leqslant n$, तो

$$\frac{C(n, r)}{C(n, r-1)} = \frac{n-r+1}{r}$$

उपपत्ति : आपको याद होगा कि :

$$C(n, r) = \frac{n!}{r!\ (n-r)!}$$

तथा $$C(n, r-1) = \frac{n!}{(r-1)!\ (n-r+1)!}$$

इस प्रकार, $$\frac{C(n, r)}{C(n, r-1)} = \frac{n!\ (r-1)!\ (n-r+1)!}{r!\ (n-r)!\ n!}$$

अर्थात्, $$\frac{C(n, r)}{C(n, r-1)} = \frac{n-r+1}{r}$$

जिससे प्रमेय सिद्ध हो जाता है।

हम कब क्रमचयों के सूत्र का तथा कब संचयों के सूत्र का प्रयोग करें ? पाठक ध्यान दें कि **'विन्यास'**, **'क्रम-सम्बन्ध'**, **'क्रमचय'**, **'व्यवस्थित करना'**, **'पंक्ति बद्ध करना'** इत्यादि शब्द प्रायः उन समस्याओं में मुख्य शब्द हैं जिनमें क्रमचयों के सूत्रों के प्रयोग की आवश्यकता होती है। दूसरी ओर शब्द **'चुनाव'** **'उपसम्मुचय'**, **'कमेटी'**, **'चुनना'** इत्यादि शब्द प्रायः उन समस्याओं में मुख्य शब्द हैं जिनमें संचयों के सूत्रों के प्रयोग की आवश्यकता होती है।

प्रश्नावली 7.4

1. मान ज्ञात कीजिए :
   (*i*) $C(8, 5)$
   (*ii*) $C(15, 4)$

2. $C(n, r)$ के लिए सूत्र का प्रयोग करके सिद्ध कीजिए कि
   $C(n, r) = C(n, n-r)$

3. एक वृत्त के n बिन्दुओं से कितनी रेखाएँ खींची जा सकती हैं ? इन n बिन्दुओं से कितने त्रिभुज खींचे जा सकते हैं ?

4. मान ज्ञात कीजिए :
   (*i*) $C(50, 47)$
   (*ii*) $C(16, 10)$
   (*iii*) $C(19, 17) + C(19, 18)$
   (*iv*) $C(25, 22) - C(24, 21)$
   (*v*) $C(31, 26) - C(30, 26)$

5. यदि $C(n, 10) = C(n, 12)$ हो तो n का निर्धारण कीजिए तथा इससे $C(n, 5)$ का मान ज्ञात कीजिए।

6. यदि $C(2n, 3) : C(n, 2) = 12 : 1$ हो, तो n ज्ञात कीजिए।

7. सिद्ध कीजिए कि

$C(n, r) = \frac{n}{r} C(n-1, r-1)$, जहाँ n तथा r प्राकृत संख्याएँ हैं तथा $r < n$ है।

8. प्रत्येक शब्द के लिए 3 व्यंजनों (consonants) तथा 2 स्वरों (vowels) का प्रयोग करके 5 व्यंजनों तथा 4 स्वरों से कितने शब्द बनाए जा सकते हैं ?

9. प्रत्येक शब्द के लिए 3 स्वरों तथा 2 व्यंजनों का प्रयोग करके शब्द 'INVOLUTE' के अक्षरों से कितने शब्द बनाए जा सकते हैं ?

10. एक ठेकेदार को 2 बढ़ईयों की आवश्यकता है। समान योग्यता वाले 5 व्यक्ति इस पद के लिए आवेदन देते हैं। ठेकेदार कितनी विधियों से 2 व्यक्तियों का चुनाव कर सकता है ?

11. एक परीक्षा में एक विद्यार्थी को 5 में से 4 प्रश्नों के उत्तर देने हैं। परन्तु प्रश्न 1 तथा 2 अनिवार्य है। उन विधियों को निर्धारित कीजिए जिनसे विद्यार्थी प्रश्नों का चयन कर सकता है।

12. एक थैली में से, जिसमें 4 काली तथा 5 लाल गेंदे हैं, 6 गेंदे निकाली जाती हैं। उन विधियों की संख्या निर्धारित कीजिए जिनसे 3 काली तथा 3 लाल गेंदें निकाली जा सकती हैं।

13. 11 भिन्न वस्तुओं को 5 तथा 6 वस्तुओं के दो समूहों में कितनी विधियों से विभाजित किया जा सकता है ?

14. एक परीक्षा पत्र को, जिसमें 12 प्रश्न हैं, दो भागों A तथा B में विभाजित किया गया है। भाग A में 7 तथा भाग B में 5 प्रश्न हैं। एक परीक्षार्थी को प्रत्येक भाग से कम से कम 3 प्रश्न चुनकर कुल 8 प्रश्न हल करने हैं। परीक्षार्थी कितनी विधियों से प्रश्नों का चुनाव कर सकता है ?

15. एक विद्यालय में पुनर्मिलन सभा में कुल 16 विद्यार्थी उपस्थित होते हैं। प्रत्येक विद्यार्थी प्रत्येक अन्य से ठीक एक बार हाथ मिलाता है। हाथ मिलाने की कुल संख्या निर्धारित कीजिए।

16. 52 पत्तों की ताश की एक गड्डी से 5-पत्तों के संचयों की संख्या निर्धारित कीजिए यदि 5-पत्तों के प्रत्येक संचय में एक इक्का अवश्य हो।

17. एक सिक्का 5 बार उछाला जाता है। उन विधियों की संख्या निर्धारित कीजिए जिनसे 0 heads (अर्थात् 0 heads तथा 5 tails) प्राप्त हो सकते हैं। साथ ही, 1 head

(अर्थात् 1 head तथा 4 tails) प्राप्त करने की विधियों की संख्या निर्धारित कीजिए, इत्यादि। तदुपरान्त, निम्न सारणी को पूर्ण कीजिए :

heads की संख्या	0	1	2	3	4	5
विधियों की संख्या						

[प्रश्नावली 7.3 के प्रश्न 22 तथा 23 भी देखिए।]

18. एक सिक्के को 6 बार उछालने से 4 heads तथा 2 tails प्राप्त करने की विधियों की संख्या निर्धारित कीजिए।

[अनुच्छेद 7.4 का उदाहरण 9 भी देखिए।]

19. 6 लड़कों तथा 5 लड़कियों में से 5 सदस्यों की एक कमेटी चुनी जानी है। कमेटी चुनने की विधियों की संख्या निर्धारित कीजिए यदि कमेटी में कम से कम 1 लड़का तथा 1 लड़की अवश्य हों।

20. सिद्ध कीजिए कि k क्रमागत (consecutive) प्राकृत संख्याओं का गुणनफल $k!$ से विभाज्य होता है।

[संकेत : $C(n,k)$ की परिभाषा का प्रयोग कीजिए।]

21. एक थैले में 4 लाल, 3 सफेद तथा 2 नीले मार्बल हैं। 3 मार्बलों को यादृच्छिक रूप से (randomly) निकाला जाता है। उन विधियों की संख्या निर्धारित कीजिए जिनसे इस चयन में कम से कम 1 सफेद मार्बल को अवश्य चुना जा सके।

7.6 ऐतिहासिक दृष्टि से

क्रमचयों तथा संचयों की संकल्पनाओं का इतिहास भारत में जैन धर्म के उदय के समय से या सम्भवतया इससे भी पहले से माना जा सकता है। परन्तु इनका श्रेय जैनियों को जाता है जिन्होंने इसको विषय-वस्तु का **विकल्प** शीर्षक के अन्तर्गत एक स्वतः पूर्ण विषय के रूप में विवेचन किया।

भारत में जैन धर्म के आगमन से पूर्व वैदिक काल में हमें ऐसी परिकल्पनाएँ मिलती हैं जिनमें कि श्लोकों और छंदों को परिवर्तित करने की विधियों की संख्याएँ ज्ञात की गई हैं।

ईसा से पूर्व छठी शताब्दी में **सुश्रुत** ने अपनी औषधीय कृति **सुश्रुत संहिता** में तर्क दिया है कि 6 भिन्न रसों में से एक बार में एक, एक बार में दो, इत्यादि लेकर 63 संचय बनाए जा सकते हैं। ईसा से पूर्व लगभग तीसरी शताब्दी में एक संस्कृत विद्वान **पिंगला** ने अपनी कृति **छंद-सूत्र** में कुछ दिए हुए अक्षरों में से एक बार में एक, एक बार में दो, इत्यादि लेकर मंत्रों की संख्या निर्धारित करने की विधि प्रदान की है।

जैनियों में, निस्सन्देह, **महावीर** ने गणित की विभिन्न शाखाओं में महत्वपूर्ण योगदान किया है। वे संसार के प्रथम गणितज्ञ हैं जिन्हें क्रमचयों तथा संचयों के व्यापक सूत्र प्रदान करने का श्रेय प्राप्त है।

भास्कर (जन्म 1114 ई०) ने क्रमचयों तथा संचयों की विषय-वस्तु का उल्लेख अपनी सुप्रसिद्ध कृति **लीलावती** में **अंक-पाशा** शीर्षक के अन्तर्गत किया है।

महावीर द्वारा $C(n, r)$ तथा $P(n, r)$ के लिए पहले से प्रदान किए गए व्यापक सूत्रों के अतिरिक्त भास्कर ने इस विषय पर अनेकों महत्वपूर्ण प्रमेयों तथा परिणामों का भी उल्लेख किया है।

भारत से बाहर क्रमचयों तथा संचयों की विषय-वस्तु का अल्प आरम्भ चीन में प्रसिद्ध पुस्तक I-किंग (परिवर्तनों की पुस्तक) में हुआ। इस कृति का अनुमानित समय देना कठिन है क्योंकि 213 ई० पू० वहाँ के राजा ने देश में सभी पुस्तकों अथवा हस्तलिपियों को जला डालने की आज्ञा दे दी थी। परन्तु सौभाग्यवश उन आदेशों का पूर्ण रूप से पालन नहीं किया गया।

यूनान तथा इसके पश्चात् लेटिन वासियों ने भी क्रमचयों तथा संचयों के सिद्धांत पर कुछ छुटपुट कार्य किया है। उनका बहुत सा कार्य भारत में किए गए कार्य का पुनः आविष्कार ही था। यूनानियों ने मुख्य रूप से अपना ध्यान क्रमचयों के सिद्धांतों पर केन्द्रित किया। उदाहरणार्थ, यूनानी दार्शनिक जेनोक्रेटस (Xenocrates) ने कुल सम्भव संधियों (syllables) की संख्या 1002000000000 परिकलित की थी। यह लगभग 350 ई० पू० में हुआ था। एक अन्य यूनानी दार्शनिक क्रिसीपस (Chrysippus) (280-207 ई० पू०) ने 10 स्वयं तथ्यों (axioms) के कुल सम्भव विन्यासों की संख्या परिकलित की। परन्तु उन्होंने केवल इतना कहा कि यह संख्या 1000000 से अधिक है। लैटिन लेखकों में से ए० एम० एस० बोथिस (A.M.S. Boethius) (480-524 ई०) ने लगभग 510 ई० में $C(n,2)$ निर्धारित करने का एक नियम प्रदान किया।

मध्य काल में अरब तथा हिब्रू लेखक अपने खगोल-विज्ञान के अध्ययन के लिए क्रमचयों तथा संचयों की संकल्पनाओं में जुटे रहे। उदाहरणार्थ, रब्बी बेनेज़रा (Rabbi ben Ezra) ने ज्ञात ग्रहों में एक बार में दो, एक बार में तीन, इत्यादि को लेकर संचयों की संख्या निर्धारित की। ऐसा लगभग 1140 ई० में किया गया। ऐसा प्रतीत होता है कि Rabbi ben Ezra $C(n,r)$ के सूत्र को नहीं जानते थे। परन्तु उन्हें यह ज्ञात था कि n तथा r के विशिष्ट मानों के लिए $C(n,r)=C(n,n-r)$ होता है। 1321 ई० में एक अन्य हिब्रू लेखक लेवी बेन गर्सन (Levi ben Gerson) ने $P(n,r)$ तथा $P(n,n)$ के लिए सूत्र दिए तथा $C(n,r)$ के लिए निम्न व्यापक सूत्र प्रस्तुत किया :

$$C(n,r)=\frac{n(n-1)\ (n-2)\ldots(n-r+1)}{r!}$$

पहली पुस्तक जिसमें क्रमचयों तथा संचयों की विषय-वस्तु का पूर्ण विवरण दिया गया है वह है Ars Conjectandi जिसे स्विटज़रलैंड के जेकब बर्नूली (Jakob Bernoulli) (1654-1705 ई०) ने लिखा था। यह पुस्तक उनकी मृत्यु के पश्चात 1713 ई० में प्रकाशित हुई थी। निश्चय ही इस पुस्तक में क्रमचयों तथा संचयों के सिद्धांत का वैसा ही वर्णन मिलता है जैसा कि यह आज जाना जाता है।

7.7 मुख्य संकल्पनाएँ

गणन का मूलभूत सिद्धांत	संचय —$C(n,r)$
फैक्टोरियल—$n!$	वस्तुओं के क्रमचय जबकि
क्रमचय—$P(n,n)$, $P(n, r)$	उनमें से कुछ एक सी हों

7.8 अग्रिम अध्ययन हेतु सुझाव

इस विषय पर एक श्रेष्ठ पुस्तक निम्न है :

[1] Ivan Niven : **Mathematics of Choice or How to Count without Counting**, The L.W. Singer Company, New York (U.S.A.), 1965.

रुचिपूर्ण अध्ययन के लिए एक रूसी लेखक की छोटी सी निम्न पुस्तक है :

[2] N. Vilenkin : **Combinatorial Mathematics for Recreation**, Mir Publishers, Moscow (U.S.S.R), 1972.

संकल्पनाओं का प्रारम्भिक वर्णन निम्न पुस्तक में मिलता है :

[3] H.R. Jacobs : **Mathematics, A Human Endeavour**, W. H. Freeman and Company, California (U S.A.), 1970.

क्रमचयों तथा संचयों के विषय के ऐतिहासिक संदर्श (perspective) के लिए पाठक अनुच्छेद 1.12 में दी गई पुस्तकें [4], [5] तथा [7] देखें।

एकक VIII

द्विपद प्रमेय

गणितीय आगमन के सिद्धांत का प्रयोग करके एक धनात्मक पूर्णांकीय घातांक के लिए द्विपद प्रमेय को सिद्ध किया गया है। द्विपद गुणांकों के सरल गुणधर्मों का अध्ययन किया गया है। तदुपरान्त द्विपद प्रमेय को ऋणात्मक और परिमेय घातांकों के लिए भी लागू किया गया है। संख्याओं के मूल और घातों के ज्ञात करने में इसके अनुप्रयोगों पर विचार किया गया है।

8.1 भूमिका

पाठक पहले से ही 'द्विपद' की संकल्पना से परिचित हैं। आइए देखें कि यदि हम द्विपद $(x+y)$ की घात एक ऋणेतर पूर्णांकीय घातांक* लें** तो हमें क्या प्राप्त होता है।

$$(x+y)^0 = 1$$
$$(x+y)^1 = x+y$$
$$(x+y)^2 = x^2+2xy+y^2$$
$$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$$
$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$
$$(x+y)^5 = x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$$

उपरोक्त प्रत्येक अवस्था में हम कहते हैं कि हमने द्विपद को प्रसारित (expand) किया है। इन द्विपद प्रसारों (binomial expansions) में हम क्या देखते है? हम देखते हैं कि

1. प्रसार में पदों की संख्या घातांक से सदैव एक अधिक है।

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*वास्तव में अगर देखें तो यह $x+y = 0$ के लिए मान्य नहीं है जब तक कि हम राशि 0^0 का मान निर्दिष्ट न कर लें।

**पाठक को चाहिए कि वह वास्तविक गुणन द्वारा इनमें से प्रत्येक की जाँच करे।

2. प्रत्येक प्रसार में
   (क) (x में) प्रथम पद का घातांक द्विपद के घातांक के समान है। (इससे निष्कर्ष निकलता है कि प्रथम पद में y का घातांक शून्य है।)
   (ख) इसके पश्चात आने वाले प्रत्येक पद में x का घातांक 1 घट जाता है तथा साथ ही साथ y का घातांक 1 बढ़ जाता है।
   (ग) यह क्रम तब तक चलता रहता है जब तक कि हम अंतिम पद पर न पहुँच जाएँ। इस अवस्था में x का घातांक शून्य तथा y का घातांक द्विपद के घातांक के समान हो जाता है।
3. प्रत्येक प्रसार के प्रत्येक पद में x तथा y के घातांकों का योग द्विपद के घातांक के समान होता है।
4. किसी प्रसार में विभिन्न पदों के गुणांक (coefficients) एक प्रतिरूप (pattern) बनाते हैं। आइए इस प्रतिरूप की जाँच करें :

द्विपद का घातांक	विभिन्न पदों के गुणांक
0	1
1	1—1
2	1—2—1
3	1—3—3—1
4	1—4—6—4—1
5	1 5 10 10 5 1

(क) गुणांकों की प्रत्येक पंक्ति दोनों ओर 1 से परिबद्ध है।

(ख) किसी भी पंक्ति में कोई अन्य गुणांक इससे तुरन्त पहले की पंक्ति के दो गुणांकों, अर्थात् एक तुरन्त बांया गुणांक तथा दूसरा तुरन्त दांया गुणांक, के योग के समान होता है। इसे त्रिभुज बनाने वाले बिन्दुकित (dotted) चिन्हों से दर्शाया गया है। उदाहरणार्थ, घातांक 4 की तदनरूपी पंक्ति में हम देखते हैं कि $4 = 1+3$, $6 = 3+3$, $4 = 3+1$। यह पंक्ति दोनों ओर 1 से परिबद्ध है।

गुणांकों की उपरोक्त व्यवस्था को आजकल सामान्यतया पास्कल त्रिभुज (Pascal's Triangle) कहा जाता है। फ्रांस में जन्मे ब्लेज् पास्कल (1623 ई०—1662 ई०) ने 1653 में एक त्रिभुज की रचना की जो उपरोक्त त्रिभुज के समान ही था परन्तु इसे कुछ परिवर्तित रूप में लिखा गया था। उन्होंने द्विपद प्रसार के

गुणांकों को प्राप्त करने के लिए इस त्रिभुज का प्रयोग किया था। इसे 1665 ई० में प्रकाशित किया गया तथा यह अंकगणितीय त्रिभुज (arithmetical triangle) के नाम से जाना जाता था।

त्रिभुज के जिस रूप का प्रयोग हमने किया है वह इटली के Nicola of Brescia, जिन्हें आमतौर पर टारटगलिया (1499 ई०—1557 ई०) के नाम से जाना जाता था, तथा जर्मनी के माइकेल स्टाइफेल (1486 ई०—1567 ई०) और साईमन स्टीविन (1548 ई०—1620 ई०) को ज्ञात था।

पूर्वकालीन हिन्दू गणितज्ञ एक ऋणेतर पूर्णांक $n \leq 7$ के लिए $(x+y)^n$ के प्रसार में आने वाले गुणांकों को जानते थे। इन गुणांकों की एक आरेख के रूप में व्यवस्था (arrangement) जिसे मेरु-प्रस्तर कहा जाता है, सर्व प्रथम ईसा से पूर्व तीसरी शताब्दी में पिंगला द्वारा दी गई थी। मेरु-प्रस्तर की त्रिभुजीय व्यवस्था का वर्णन चीनी गणितज्ञ चु शी-काई द्वारा 1303 ई० में किए गए कार्य में भी मिलता है। इस प्रकार, पास्कल इस त्रिभुज का जन्मदाता नहीं था। परन्तु उस ने इस त्रिभुज के बहुत से गुणधर्मों के महत्व को दर्शाया तथा अपनी कृतियों में इसका इतना विस्तृत प्रयोग किया कि इस त्रिभुज के साथ पास्कल का नाम दृढ़ता से जुड़ गया।

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : प्रसारित कीजिए

(i) $(a+4)^5$ (ii) $(2-3x^2)^4$

हल : (i) $(a+4)^5 = (a^5)(4)^0 + 5(a)^4(4)^1 + 10(a)^3(4)^2 + 10(a)^2(4)^3 + 5(a)^1(4)^4 + (a)^0(4)^5$

सरल करने पर हम निम्न प्राप्त करते हैं :

$$(a+4)^5 = a^5 + 20a^4 + 160a^3 + 640a^2 + 1280a + 1024$$

$$(ii)\ (2-3x^2)^4 = (2)^4 + 4(2)^3(-3x^2)^1 + 6(2)^2(-3x^2)^2 + 4(2)^1(-3x^2)^3 + (-3x^2)^4$$

$$= 16 - 96x^2 + 216x^4 - 216x^6 + 81x^8$$

[पाठक को चाहिए वह ध्यान दे कि हमने किस प्रकार पास्कल त्रिभुज से विभिन्न गुणांकों को प्रयुक्त किया है।]

उदाहरण 2 : $\left(\frac{4}{7}x - q^2\right)^5$ के प्रसार में चौथा पद ज्ञात कीजिए।

हल : पास्कल त्रिभुज से चौथे पद का गुणांक 10 होगा।

अतः चौथा पद $= 10\left(\frac{4}{7}x\right)^2\left(-q^2\right)^3$

$$= \frac{-160}{49}x^2q^6$$

[पाठक को याद होगा कि किसी भी पद में y का घातांक पद संख्या से 1 कम होता है। फिर x का घातांक सुगमतापूर्वक प्राप्त किया जा सकता है क्योंकि किसी पद में x तथा y के घातांकों का योग द्विपद के घातांक के समान होता है।]

पास्कल त्रिभुज का प्रयोग $(x+y)^6$, $(x+y)^7$, इत्यादि के प्रसारों में गुणांक ज्ञात करने के लिए किया जा सकता है। परन्तु पास्कल त्रिभुज की किसी विशेष पंक्ति के गुणांकों को ज्ञात करने के लिए हमें उससे तुरन्त पहले की पंक्ति के गुणांकों का प्रयोग करना पड़ता है और ऐसा करने के लिए फिर उससे तुरन्त पहले की पंक्ति के गुणांकों का प्रयोग होता है, इत्यादि। अतः यदि हमें किसी वांछित पंक्ति के गुणांकों का प्रयोग करने के लिए पास्कल त्रिभुज का निर्माण करना पड़े तो यह अति असुविधाजनक होगा तथा इसमें समय भी अत्यधिक लगेगा। क्या हम किसी अन्य अधिक सुविधाजनक विधि से गुणांकों को प्राप्त नहीं कर सकते ? आइए देखें।

आइए, उदाहरणार्थ, $(x+y)^3$ के प्रसार की जाँच करें। हमारे पास एक पद x^3 में, एक पद x^2y में, एक पद xy^2 में तथा एक पद y^3 में है। आइए इसे निम्न रूप में लिखें तथा गुणनखंडों को प्रथम, द्वितीय तथा तृतीय से प्रदर्शित करें :

$$(x+y)^3 = \underset{\text{प्रथम}}{(x+y)}\ \underset{\text{द्वितीय}}{(x+y)}\ \underset{\text{तृतीय}}{(x+y)}$$

अब हम, कोई पद, उदाहरणार्थ, x^2y किस प्रकार प्राप्त करते हैं ? यह दो x तथा एक y का गुणनफल है। इस प्रकार हम प्रथम पद के x, द्वितीय पद के x तथा तृतीय पद के y का गुणा कर सकते हैं अथवा हम प्रथम पद के x, द्वितीय पद के y तथा तृतीय पद के x का गुणा कर सकते हैं, इत्यादि। अतः हमें दो ऐसे गुणनखंड चुनने की आवश्यकता है जिनमें से प्रत्येक से x ले सकें तथा एक ऐसा गुणनखंड चुनने की आवश्यकता है जिसमें से y ले सकें और फिर इन्हें लेकर हम परस्पर गुणा करते हैं। हम तीन गुणनखंडों में से एक गुणनखंड को कितनी विधियों से चुन सकते हैं ? [इस गुणनखंड से हम y ले लेते हैं तथा शेष बचे दो गुणनखंडों में से प्रत्येक से x लेते हैं।] स्पष्ट है कि हम ऐसा $C(3,1)=\frac{3!}{1!\,2!}=3$ विधियों से कर सकते हैं।

अतः पद x^2y का गुणांक 3 है।

इसी प्रकार के तर्क द्वारा हम देखते हैं कि पद, उदाहरणार्थ, x^3 तीनों गुणनखंडों में से प्रत्येक से x लेकर और इन्हें परस्पर गुणा करने से प्राप्त किया जा सकता है। अर्थात् y लेने के लिए हमें किसी गुणनखंड को नहीं चुनना है। ऐसा $C(3,0)=1$ विधि से किया जा सकता है। **अतः पद x^3 का गुणांक 1 है।**

[पाठक को चाहिए कि वह इसी प्रकार के तर्क द्वारा पदों xy^2 तथा y^3 के गुणांकों का निर्धारण करे।]

अतः $(x+y)^3$ के प्रसार में गुणांकों को निम्न प्रकार से लिखा जा सकता है :

$C(3,0)$; $C(3,1)$; $C(3,2)$; $C(3,3)$

[पाठक यह जाँच करें कि $(x+y)^4$ के प्रसार में गुणांकों को इस प्रकार से लिखा जा सकता है :

$C(4,0)$; $C(4,1)$; $C(4,2)$; $C(4,3)$; $C(4,4)$ तथा $(x+y)^5$ के प्रसार में इन्हें

$C(5,0)$; $C(5,1)$; $C(5,2)$; $C(5,3)$, $C(5,4)$; $C(5,5)$ के रूप में लिखा जा सकता है, इत्यादि।]

क्योंकि ये गुणांक गुणनखंडों के संचयों (combinations) की संख्या से प्राप्त होते हैं अतः प्रायः इन्हें **संचय विन्यास गुणांक (combinatorial coefficients)** कहा जाता है।

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 3 : $(x^2+2y)^6$ को प्रसारित कीजिए।

हल : हमें निम्न प्रसार प्राप्त होता है :

$$(x^2+2y)^6=C(6,0)\ (x^2)^6+C(6,1)\,(x^2)^5\ (2y)+C(6,2)\ (x^2)^4\ \ (2y)^2$$
$$+C(6,3)\,(x^2)^3(2y)^3+\ C(6,4)\,(x^2)^2\ (2y)^4$$
$$+C(6,5)\,x^2\ (2y)^5+C(6,6)\ (2y)^6$$
$$=\frac{6!}{0!\ 6!}x^{12}+\frac{6!}{1!\ 5!}x^{10}\ (2y)+\frac{6!}{2!\,4!}x^8\ \ (4y^2)$$
$$+\frac{6!}{3!\ 3!}x^6\ (8y^3)+\frac{6!}{4!\ 2!}\ x^4\ (16y^4)$$
$$+\frac{6!}{5!\ 1!}\ x^2\ (32y^5)+\frac{6!}{6!\ 0!}\ (64y^6)$$
$$=1\ x^{12}+\ 6\ x^{10}\,(2y)+15\ x^8\ (4y^2)+20\ x^6(8y^3)$$
$$+15\ x^4(16y^4)+\ 6\ x^2\ (32y^5)+\ 1\ (64y^6)$$

अर्थात्, $$(x^2+2y)^6=x^{12}+12x^{10}y+60x^8y^2+160x^6y^3+240x^4y^4$$
$$+192\ x^2\ y^5+64\ y^6$$

उदाहरण 4 : $(x+\frac{2}{5}y)^4$ के प्रसार में तीसरा पद लिखिए ।

हल : तीसरा पद $=C(4,2)\,(x)^{4-2}(\frac{2}{5}y)^2$

$$=\frac{4\ !}{2!\ 2!}\ x^2\left(\frac{2}{5}y\right)^2$$
$$=6x^2\left(\frac{4}{25}\ y^2\right)$$

अर्थात्, तीसरा पद $=\dfrac{24}{25}\ x^2y^2$

क्या अब आप किसी द्विपद $(x+y)$ की nवीं घात के लिए सूत्र का अनुमान लगा सकते हैं ? यह निम्न है :

$$(x+y)^n=C(n,0)x^n+C(n,1)x^{n-1}y+C(n,2)x^{n-2}y^2+\ \dots\ +C(n,r)x^{n-r}y^r+\dots+C(n.n)y^n$$

अर्थात्, $(x+y)^n=\sum\limits_{r=0}^{n}\ C(n,r)\ x^{n-r}\ y^r$

इसके लिए कि यह वास्तव में सत्य है उपपत्ति की आवश्यकता है। अगले अनुच्छेद में हम उपरोक्त को सिद्ध करने के लिए गणितीय आगमन के सिद्धांत का प्रयोग करेंगे ।

प्रश्नावली 8.1

निम्नलिखित को प्रसारित कीजिए :

1. $(x^2+y^2)^3$
2. $\left(x+\dfrac{1}{x}\right)^2$

3. $(2x-3y)^5$ 4. $\left(-\frac{1}{2}x^2+\frac{1}{4}y\right)^4$

5. $\left(-3x-\frac{1}{3x}\right)^3$

6. $\left(x+\frac{1}{x}\right)^4$ के प्रसार में मध्य पद लिखिए।

7. $\left(\frac{x}{8}+y^2\right)^5$ के प्रसार में x^3 का गुणांक ज्ञात कीजिए।

8. $\left(3x-\frac{y^3}{6}\right)^4$ के प्रसार में तीसरे पद का गुणांक ज्ञात कीजिए।

9. निम्नलिखित के गुणांक निर्धारित करने के लिए उस प्रकार का तर्क दीजिए जैसा कि अनुच्छेद 8.1 में दिया गया है :
   (i) $(x+y)^3$ में xy^2 का
   (ii) $(x+y)^3$ में y^3 का
   (iii) $(x+y)^4$ में x^2y^2 का
   (iv) $(x+y)^5$ में x^2y^3 का

8.2 ऋणेतर पूर्णांकीय घातांकों के लिए द्विपद प्रमेय (The Binomial Theorem For Non-negative Integral Exponents)

द्विपद $(x+y)$ की nवीं घात के लिए सूत्र को द्विपद प्रमेय (Binomial Theorem) अथवा **द्विपद प्रसार (Binomial expansion)** कहा जाता है। हम द्विपद प्रमेय को उस स्थिति के लिए सिद्ध करते हैं जबकि घातांक n एक ऋणेतर पूर्णांक है।

स्पष्ट है कि जब $n=0$ है तो $(x+y)^0=1$

अब हम द्विपद प्रमेय को उस स्थिति में सिद्ध करने के लिए, जब घातांक n एक प्राकृत संख्या है, गणितीय आगमन के सिद्धांत का प्रयोग करेंगे।

प्रमेय 1 : यदि n एक प्राकृत संख्या है, तो

$$(x+y)^n=C(n,0)x^n+C(n,1)x^{n-1}y+C(n,2)x^{n-2}y^2+\ldots+C(n,r)x^{n-r}y^r+\ldots+C(n,n)y^n$$

अर्थात्, $$(x+y)^n=\sum_{r=0}^{n} C(n,r)x^{n-r}y^r$$

उपपत्ति: आइए, द्विपद प्रसार सम्बन्धी कथन को, जबकि n एक प्राकृत संख्या है, $P(n)$ से व्यक्त करें।

अर्थात्, $$P(n) : (x+y)^n=C(n,0)\, x^n+C(n,1)x^{n-1}\, y+C(n,2)x^{n-2}y^2+\ldots +C(n,r)x^{n-r}y^r+\ldots+C(n,n)y^n \quad (1)$$

क्या P(1) सत्य है ? $P(1)$ क्या है ? यह निम्न कथन है :

$$(x+y)^1=C(1,0)\, x+C(1,1)y \quad (2)$$

परन्तु $C(1,0)=1$ तथा $C(1,1)=1$

इस प्रकार (2) से,

$$(x+y)^1=x+y$$

जो कि वस्तुतः सत्य है।

अब मान लीजिए कि $P(k)$ सत्य है, अर्थात्

$$P(k) : (x+y)^k=C(k,0)x^k+C(k,1)x^{k-1}y+C(k,2)x^{k-2}y^2 + \ldots + C(k,r)x^{k-r}y^r+\ldots+C(k,k)\ y^k \quad (3)$$

हम यह दिखाना चाहते हैं कि $P(k+1)$ सत्य है, अर्थात्

$$P(k+1) : (x+y)^{k+1}=C(k+1,0)x^{k+1}+C(k+1,1)x^ky+\ldots + C(k+1,r)x^{k+1-r}y^r+\ldots+C(k+1,k+1)y^{k+1} \quad (4)$$

आइए (4) के वाम पक्ष की जाँच करें। हम निम्न प्राप्त करते हैं :

$$(x+y)^{k+1}=(x+y)^k(x+y) \quad (5)$$

$(x+y)^k$ के प्रसार में $(k+1)$ पद हैं। (5) के दक्षिण पक्ष में हमें $(x+y)^k$ तथा $(x+y)$ को गुणा करने की आवश्यकता है। अब जब हम $(k+1)$ पदों को $(x+y)$ के दो पदों से गुणा करेंगे तो हम कितने पद प्राप्त करेंगे ? हम $(k+1)+(k+1)=(2k+2)$ पद प्राप्त करेंगे। हम x की समान घातों के गुणांकों को इकट्ठा करते हैं तथा निम्न प्राप्त करते हैं :

$$(x+y)^k(x+y)=C(k,0)x^{k+1}+[C(k,1)+C(k,0)]x^ky$$
$$[C(k,2)+C(k,1)]\ x^{k-1}\ y^2+\ldots+$$
$$[C(k,r)+C(k,r-1)]\ x^{k-r+1}\ y^r+\ldots+C(k,k)\ y^{k+1} \quad (6)$$

अब, $C(k,0)=C(k+1,0)$

तथा, $C(k,k)=C(k+1,\ k+1)$

पुनः एकक VII के प्रमेय 5 द्वारा, $1 \leqslant r \leqslant k$ के लिए हमें निम्न प्राप्त होता है :

$$C(k,r)+C(k,r-1)=C(k+1,r)$$

अर्थात्,

$$C(k,1)+C(k,0)=C(k+1,1)$$
$$C(k,2)+C(k,1)=C(k+1,2)$$
$$C(k,3)+C(k,2)=C(k+1,3)$$

इत्यादि।

अब (6) को निम्न प्रकार से लिखा जा सकता है :

$$(x+y)^k(x+y)=C(k+1,0)x^{k+1}+C(k+1,1)x^ky$$
$$+C(k+1,\ 2)x^{k-1}\ y^2+\ldots+C(k+1,r)x^{k-r+1}y^r$$
$$+\ldots+C(k+1,k+1)y^{k+1}$$

जो कि (4) से, निस्संदेह, $(x+y)^{k+1}$ का प्रसार है।

इस प्रकार हमने सिद्ध कर दिया है कि

(A) $P(1)$ सत्य है, तथा

(B) जब भी $P(k)$ सत्य है, $P(k+1)$ सत्य है ।

इससे $P(n)$ सभी प्राकृत संख्याओं n के लिए सिद्ध हो जाता है । हम नीचे कुछ महत्वपूर्ण टिप्पणियाँ दे रहे हैं :

टिप्पणी 1 : हम देखते हैं कि इस प्रसार में $(r+1)$ वाँ पद, T_{r+1} निम्न है :

$$T_{r+1}=C(n,r)x^{n-r}y^r$$

कुछ लेखक $(r+1)$ वें पद को व्यापक पद भी कहते हैं ।

टिप्पणी 2 : प्रमेय 1 का कथन निम्नलिखित रूप से भी व्यक्त किया जाता है :
हम $x=1$ तथा $y=a$ मान लेते हैं । हम कहते हैं कि यदि n एक प्राकृत संख्या है तो,

$$(1+a)^n=1+C(n,1)a+C(n,2)a^2+\ldots+C(n,r)a^r+\ldots+C(n,n)a^n$$

इस प्रसार के लिए $(r+1)$ वाँ पद $T_{r+1}=C(n,r)a^r$ है ।
अब हम कुछ उदाहरण लेते हैं ।

उदाहरण 1 : निम्नलिखित में से प्रत्येक का प्रसार प्राप्त करने के लिए द्विपद प्रमेय का प्रयोग कीजिए

(i) $(x^2-y)^7$ (ii) $\left(a+\frac{1}{b}\right)^{10}$

हल : (i) हम निम्न प्राप्त करते हैं :

$$(x^2-y)^7=C(7,0)\,(x^2)^7+C(7,1)\,(x^2)^6(-y)+C(7,2)\,(x^2)^5(-y)^2$$
$$+C(7,3)\,(x^2)^4\,(-y)^3+C(7,4)\,(x^2)^3(-y)^4+C(7,5)\,(x^2)^2\,(-y)^5$$
$$+C(7,6)\,(x^2)(-y)^6+C(7,7)\,(-y)^7$$
$$=x^{14}-7x^{12}y+\frac{7!}{2!\,5!}x^{10}y^2-\frac{7!}{3!\,4!}x^8y^3$$
$$+\frac{7!}{4!\,3!}x^6y^4-\frac{7!}{5!\,2!}x^4y^5+7x^2y^6-y^7$$

अर्थात्, $(x^2-y)^7=x^{14}-7x^{12}y+21x^{10}y^2-35x^8y^3+35x^6y^4-21x^4y^5+7x^2y^6-y^7$

(ii) हम निम्न प्राप्त करते हैं :

$$\left(a+\frac{1}{b}\right)^{10}=\sum_{r=0}^{10}C(10,r)\,(a)^{10-r}\left(\frac{1}{b}\right)^r$$

हम विभिन्न संचय विन्यास गुणांकों को परिकलित करते हैं तथा निम्न प्राप्त करते हैं :

$C(10,0)=C(10,10)=1$; $C(10,1)=C(10,9)=10$;

$C(10,2)=C(10,8)=45$; $C(10,3)=C(10,7)=120$;

$C(10,4)=C(10,6)=2$[illegible]

इस प्रकार,

$$\left(a+\frac{1}{b}\right)^{10} = a^{10} + 10\,\frac{a^9}{b} + 45\,\frac{a^8}{b^2} + 120\,\frac{a^7}{b^3} + 210\,\frac{a^6}{b^4} + 252\,\frac{a^5}{b^5}$$

$$+ 210\,\frac{a^4}{b^6} + 120\,\frac{a^3}{b^7} + 45\,\frac{a^2}{b^8} + 10\,\frac{a}{b^9} + \frac{1}{b^{10}}$$

उदाहरण 2 : $(x+3)^9$ के प्रसार में छठा पद ज्ञात कीजिए।

हल : आपको याद होगा कि $(x+y)^n$ के प्रसार में $(r+1)$ वाँ पद

$$T_{r+1} = C(n, r)x^{n-r}y^r \text{ होता है।}$$

अब, $n = 9, \quad r = 5$ है। इस प्रकार,

$$T_6 = C(9, 5)\, x^{9-5}\, (3)^5$$

$$= \frac{9\,!}{5\,!\ 4\,!}\, x^4\, (243) = (126)\ (243)\ x^4$$

अर्थात्, $T_6 = 30618\ x^4$

अतः $(x+3)^9$ के प्रसार में छठा पद $30618\ x^4$ है।

उदाहरण 3 : $\left(z-\frac{1}{z}\right)^{12}$ के प्रसार में z से स्वतंत्र पद को लिखिए तथा उसे सरल कीजिए।

हल : z से स्वतंत्र पद को लिखने के लिए हमें वह पद परिकलित करना चाहिए जिसमें z तथा $\left(-\frac{1}{z}\right)$ के घातांक समान हों। स्पष्ट है कि यह घातांक 6 होना चाहिए। अतः वांछित पद 7वाँ पद अर्थात् T_7 है। इस प्रकार,

$$T_7 = C\ (12, 6)\ z^{12-6}\left(-\frac{1}{z}\right)^6$$

$$= \frac{12\,!}{6\,!\ 6\,!}\ z^6\ \left(\frac{1}{z^6}\right)$$

$$= 924$$

अतः $\left(z-\frac{1}{z}\right)^{12}$ के प्रसार में z से स्वतंत्र पद 924 है।

उदाहरण 4 : सिद्ध कीजिए कि n अवयवों के एक परिमित समुच्चय के सभी उपसमुच्चयों की संख्या 2^n होती है।

हल : आपको याद होगा कि रिक्त समुच्चय प्रत्येक समुच्चय का उपसमुच्चय होता है। केवल $1 = C\,(n,0)$ उपसमुच्चय अर्थात् रिक्त समुच्चय ऐसा है जिसमें कोई भी अवयव नहीं है।

अब 1 अवयव के कितने उपसमुच्चय हैं ? आपको याद होगा कि n भिन्न वस्तुओं में से एक बार में 1 वस्तु लेकर प्राप्त संचयों की संख्या $C(n, 1)$ होती है। इस प्रकार 1-अवयव वाले उपसमुच्चयों की संख्या, $C(n, 1)$ है।

इसी प्रकार, 2-अवयवों के उपसमुच्चयों की संख्या $C(n, 2)$ है, 3-अवयवों के उपसमुच्चयों की संख्या $C(n, 3)$ है, इत्यादि। अन्त में, n अवयवों के उपसमुच्चयों की संख्या $C(n, n)$ है। यह निस्संदेह स्वयं वह समुच्चय ही है।

अतः कुल उपसमुच्चयों की संख्या निम्न है :

$$C(n,0)+C(n,1)+C(n,2)+\ldots+C(n,r)+\ldots+C(n,n) \qquad (1)$$

आपको $(1+a)^n$ के प्रसार के बारे में याद होगा। इसके संचय विन्यास गुणांक वैसे ही हैं जैसे कि हम चाहते हैं केवल इस अंतर के साथ कि इस प्रसार में a भी आते हैं। अत: यदि हम $(1+a)^n$ के प्रसार में $a=1$ ले लें तो हमें निम्न प्राप्त होता है :

$$(1+1)^n = 1+C(n,1)+C(n,2)+\ldots+C(n,r)+\ldots+C(n,n)$$

अर्थात्, $$2^n = C(n,0)+C(n,1)+C(n,2)+\ldots+C(n,r)+\ldots+C(n,n)$$

अतः n अवयवों के एक परिमित समुच्चय के सभी उपसमुच्चयों की संख्या 2^n है।

[प्रश्नावली 6.1 का प्रश्न 18 भी देखिए।]

उदाहरण 5 : $\left(2x^2-\dfrac{3}{x}\right)^{11}$ के प्रसार में उस पद का गुणांक ज्ञात कीजिए जिसमें x^{10} आता हो।

हल : आपको याद होगा कि $(r+1)$ वाँ पद, T_{r+1} निम्न है :

$$T_{r+1}=C(11,r)\,(2x^2)^{11-r}\left(-\frac{3}{x}\right)^r$$

अर्थात्, $$T_{r+1}=C(11,r)\,2^{11-r}\,(-3)^r\,x^{22-3r} \qquad (1)$$

हम उस पद का गुणांक ज्ञात करना चाहते हैं जिसमें x^{10} आता है।

इस प्रकार, $22-3r=10$

जिससे, $r=4$

अब (1) से हम निम्न प्राप्त करते हैं :

$$T_5=C(11,4)2^{11-4}(-3)^4\,x^{22-12}$$

$$=\frac{11\,!}{4\,!\,7\,!}\,2^7\,(81)x^{10}$$

अर्थात्, $T_5=330\,(128)\,(81)\,x^{10}$

अतः, $\left(2x^2-\dfrac{3}{x}\right)^{11}$ के प्रसार में x^{10} का गुणांक $330\,(128)\,(81)=3421440$ है।

उदाहरण 6 : यदि $(1+a)^n$ के द्विपद प्रसार में, a^{r-1}, a^r तथा a^{r+1} के गुणांक समांतर श्रेणी में हों, तो सिद्ध कीजिए कि

$$n^2-n(4r+1)+4r^2-2=0$$

हल : आपको याद होगा कि इस प्रसार में $C(n, r-1)$, a^{r-1} का गुणांक है। इसी प्रकार $C(n,r)$ तथा $C(n, r+1)$ क्रमशः a^r तथा a^{r+1} के गुणांक हैं।

हमें दिया हुआ है कि $C(n, r-1)$, $C(n,r)$ तथा $C(n, r+1)$ समांतर श्रेणी में हैं।

अर्थात्, $$2\,C(n,r) = C(n,r-1) + C(n,r+1) \tag{1}$$

हम (1) के दोनों पक्षों को $C(n,r)$ से भाग देते हैं तथा निम्न प्राप्त करते हैं :

$$\frac{C(n,r-1)}{C(n,r)} + \frac{C(n,r+1)}{C(n,r)} = 2 \tag{2}$$

अब, $$\frac{C(n,r-1)}{C(n,r)} = \frac{n!\; r!\; (n-r)!}{(r-1)!\;(n-r+1)!\; n!} = \frac{r}{n-r+1}$$

तथा, $$\frac{C(n,r+1)}{C(n,r)} = \frac{n!\; r!\; (n-r)!}{(r+1)!\;(n-r-1)!\; n!} = \frac{n-r}{r+1}$$

इस प्रकार (2) से हमें निम्न प्राप्त होता है :

$$\frac{r}{n-r+1} + \frac{n-r}{r+1} = 2$$

जिससे, $$r(r+1) + (n-r)(n-r+1) = 2(n-r+1)(r+1) \tag{3}$$

सरल करने पर (3) से निम्न प्राप्त होता है :

$$n^2 - n(4r+1) + 4r^2 - 2 = 0$$

उदाहरण 7 : $(1+a)^n$ के प्रसार में हम संचय विन्यास गुणांक $C(n,r)$ को C_r से व्यक्त करेंगे। हम पहले ही अनुच्छेद 8.2 के उदाहरण 4 में इन संचय विन्यासों के गुणांकों के एक महत्वपूर्ण गुणधर्म को सिद्ध कर चुके हैं जो कि निम्नलिखित है :

$$C_0 + C_1 + C_2 + \ldots + C_n = 2^n$$

सिद्ध कीजिए कि

(i) $C_0 + C_2 + C_4 + \ldots = 2^{n-1}$

(ii) $C_1 + C_3 + C_5 + \ldots = 2^{n-1}$

हल : आइए $(1+a)^n$ के प्रसार में $a = -1$ प्रतिस्थापित करें।

हमें निम्न प्राप्त होता है :

$$(1-1)^n = C_0 - C_1 + C_2 - C_3 + C_4 - \ldots$$

अर्थात्, $$0 = C_0 - C_1 + C_2 - C_3 + C_4 - \ldots$$

इस प्रकार, $$C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots$$

परन्तु, $$C_0 + C_1 + C_2 + C_3 + \ldots + C_n = 2^n$$

अतः, $$C_0 + C_2 + C_4 + \ldots = 2^{n-1}$$

इसी प्रकार, $$C_1 + C_3 + C_5 + \ldots = 2^{n-1}$$

उदाहरण 8 : सिद्ध कीजिए कि

$$(C_0)^2 + (C_1)^2 + (C_2)^2 + \ldots + (C_n)^2 = \frac{(2n)!}{(n!)^2}$$

जहाँ $C_0, C_1, C_2,$ इत्यादि के सामान्य अर्थ हैं।

हल : आपको याद होगा कि C_r संचय विन्यास गुणांक $C(n,r)$ है जो कि $(1+a)^n$ के प्रसार में a^r का गुणांक है। हमें इन गुणांकों के वर्गों के योग की आवश्यकता है। अत: हम यहाँ सहज ज्ञान का प्रयोग करेंगे।

आइए $(1+a)^n$ तथा $(a+1)^n$ का गुणा करें। हमें निम्न प्राप्त होता है :

$$(1+a)^n (a+1)^n = (C_0 + C_1 a + C_2 a^2 + \ldots + C_r a^r + \ldots + C_n a^n) \times (C_0 a^n + C_1 a^{n-1} + \ldots + C_r a^{n-r} + \ldots + C_n) \quad (1)$$

अब, (1) के दक्षिण पक्ष में a^n का गुणांक क्या है ? स्पष्ट है कि यह

$$(C_0)^2 + (C_1)^2 + \ldots + (C_r)^2 + \ldots + (C_n)^2 \text{ है।} \quad (2)$$

परन्तु, $$(1+a)^n \ (a+1)^n = (1+a)^{2n} \quad (3)$$

अब, (3) के दक्षिण पक्ष में a^n का गुणांक क्या है ?

स्पष्ट है कि यह निम्न है :

$$C(2n,n) = \frac{(2n)!}{(n!)\ (n!)} = \frac{(2n)\ !}{(n!)^2} \quad (4)$$

(2) तथा (4) से a^n के गुणांकों को बराबर करने पर हमें वाँछित परिणाम अर्थात्

$(C_0)^2 + (C_1)^2 + \ldots + (C_r)^2 + \ldots + (C_n)^2 = \dfrac{(2n)!}{(n!)^2}$ प्राप्त हो जाता है।

प्रश्नावली 8.2

1. निम्नलिखित में से प्रत्येक के प्रसार में कुल कितने पद हैं :

   (i) $\left(\frac{2}{p}+\frac{p}{2}\right)^8$?

   (ii) $(x^2-y)^{16}$?

2. $(5-2y)^{11}$ के प्रसार में y^9 के गुणांक को लिखिए तथा उसे सरल कीजिए।

3. निम्नलिखित में से प्रत्येक का प्रसार कीजिए :

   (i) $(a-2b)^5$

   (ii) $\left(\frac{2}{3}t-\frac{3}{2t}\right)^6$

   (iii) $(4x-5y)^5$

   (iv) $(y^2+3x)^8$

4. $(-3a-b)^5$ के प्रसार में चौथा पद निर्धारित कीजिए।

5. $(\frac{1}{2}a+\frac{1}{3}b)^8$ के प्रसार में मध्य पद निर्धारित कीजिए।

6. यदि T_r, $(1+a)^n$ के प्रसार में, जो कि a की घातों के आरोही क्रम में है, rवाँ पद है तो सिद्ध कीजिए कि

$$r(r+1)\ T_{r+2}=(n-r+1)(n-r)a^2\ T_r$$

7. सिद्ध कीजिए कि $(1+a)^{m+n}$ के प्रसार में a^m तथा a^n के गुणांक समान हैं।

8. $\left(x^3-\dfrac{1}{x}\right)^{12}$ के प्रसार में x से स्वतंत्र पद के गुणांक को लिखिए तथा उसे सरल कीजिए।

9. यदि C_r, $(1+a)^n$ के प्रसार में संचय विन्यास गुणांक $C(n,r)$ को व्यक्त करता है, तो सिद्ध कीजिए कि

(i) $$\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+\ldots+n\frac{C_n}{C_{n-1}}=\frac{n(n+1)}{2}$$

[संकेत : $\dfrac{C_r}{C_{r-1}}$ क्या है ?]

(ii) $$(C_0+C_1)(C_1+C_2)\ldots(C_{n-1}+C_n)=\frac{C_0C_1\ldots C_{n-1}(n+1)^n}{n!}$$

[संकेत : वाम पक्ष को $C_0C_1\ldots C_{n-1}$ से भाग दीजिए। हमें

$\left(1+\dfrac{C_1}{C_0}\right)\left(1+\dfrac{C_2}{C_1}\right)\ldots\left(1+\dfrac{C_n}{C_{n-1}}\right)$, इत्यादि प्राप्त होता है।]

*10. यदि C_r, $(1+a)^n$ के प्रसार में संचय विन्यास गुणांक $C(n,r)$ को व्यक्त करता है, तो सिद्ध कीजिए कि

$$C_0+2C_1+3C_2+\ldots+(n+1)\ C_n=2^n+n(2^{n-1})$$

[संकेत : वाम पक्ष को $C_0+C_1+C_2+\ldots+C_n+(C_1+2C_2+3C_3+\ldots+nC_n)$ के रूप में लिखिए। कोष्ठकों में दिए व्यंजक को rC_r, इत्यादि के मान लिखकर सरल कीजिए।]

11. $(x^2+a^2)^5$ के प्रसार में दो मध्य पद निर्धारित कीजिए।

12. $\left(x-\dfrac{1}{x}\right)^{11}$ के प्रसार में उस पद को लिखिए और सरल कीजिए जिसमें x^5 आता हो।

13. सिद्ध कीजिए कि $(1+x)^{2n}$ के प्रसार में x^n का गुणांक $(1+x)^{2n-1}$ के प्रसार में x^n के गुणांक का दुगुना है।

14. सिद्ध कीजिए कि

$$C_0C_1+C_1C_2+\ldots+C_{n-1}C_n=\frac{2^n(n)[(1)(3)(5)\ldots(2n-1)]}{(n+1)!}$$, जहाँ

C_0, C_1, C_2, इत्यादि के सामान्य अर्थ हैं।

[संकेत : $(1+a)^n(a+1)^n$ तथा $(1+a)^{2n}$ में a^{n+1} के गुणांकों की तुलना कीजिए।]

15. दिखाइए कि यदि द्विपद का घातांक 3 है तो पास्कल त्रिभुज की तदनुरूपी पंक्ति में गुणांकों का योग 2^3 है। यह भी दिखाइए कि यदि द्विपद का घातांक 5 है तो पास्कल त्रिभुज की तदनुरूपी पंक्ति में गुणांकों का योग 2^5 है।

16. $(1+x)^n$ के प्रसार में तीन विशिष्ट क्रमागत (consecutive) गुणांक 1 :3: 5 के अनुपात में हैं। n निर्धारित कीजिए।

17. $(x^2-y)^{10}$ के प्रसार में उस पद को लिखिए और सरल कीजिए जिसमें x^8 आता हो।

8.3 परिमेय घातांकों के लिए द्विपद प्रमेय

अब तक हमने एक द्विपद की n वीं घात के लिए सूत्र प्राप्त किया है, जबकि n एक ऋणेतर पूर्णांक है। हम नीचे परिमेय घातांकों के लिए द्विपद प्रमेय का कथन दे रहे हैं :

प्रमेय 2 : यदि m एक ऋणात्मक पूर्णांक है अथवा एक परिमेय संख्या है, तो

$$(b+x)^m = b^m + mb^{m-1}x + \frac{m(m-1)}{2!}b^{m-2}x^2 + \dots + \frac{m(m-1)(m-2)\dots(m-r+1)}{r!}b^{m-r}x^r + \dots$$

निस्सन्देह, यदि $|x| < b$ हो।

इस प्रमेय की उपपत्ति इस पुस्तक की सीमा के बाहर है। परन्तु हम इस प्रमेय के कुछ अनुप्रयोगों को सीखेंगे।

हम पहले कुछ टिप्पणियाँ दे रहे हैं।

टिप्पणी 1 : ध्यान दीजिए कि गुणांक $m, \frac{m(m-1)}{2!}$, इत्यादि संचय विन्यास गुणांकों जैसे दिखते हैं। परन्तु आपको याद होगा कि $C(n, r)$ प्राकृत संख्याओं n तथा पूर्ण संख्याओं r के लिए ही परिभाषित है। अतः किसी अन्य स्थिति में इसका कोई अर्थ नहीं है।

टिप्पणी 2 : जब m एक ऋणात्मक पूर्णांक है अथवा एक परिमेय संख्या है तो $(b+x)^m$ के प्रसार में पदों की संख्या अपरिमित होती है। इस परिणाम को हम उच्चतर कक्षाओं में सिद्ध करेंगे।

टिप्पणी 3 : निम्नलिखित परिणाम अत्यन्त उपयोगी हैं तथा प्रमेय 2 का प्रयोग करके इन्हें सुगमतापूर्वक प्राप्त किया जा सकता है :

$$\frac{1}{1+x} = (1+x)^{-1} = 1 - x + x^2 - x^3 + \dots \quad ; |x| < 1$$

$$\frac{1}{(1+x)^2} = (1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots \quad ; |x| < 1$$

$$\frac{1}{1-x} = (1-x)^{-1} = 1 + x + x^2 + x^3 + \dots \quad ; |x| < 1$$

$$\frac{1}{(1-x)^2} = (1-x)^{-2} = 1+2x+3x^2+4x^3+\ldots \qquad ; |x| < 1$$

अब हम कुछ उदाहरण लेते हैं।

उदाहरण 1 : निम्नलिखित में से प्रत्येक का, x की घातों के आरोही क्रम में, पहले तीन पदों तक प्रसार ज्ञात कीजिए :

(*i*) $\dfrac{1}{(2+x)^4}$ (*ii*) $\dfrac{1}{\sqrt[3]{6-3x}}$

यह भी बताइए कि ये प्रसार कब मान्य (valid) हैं।

हल : (*i*) $\dfrac{1}{(2+x)^4} = (2+x)^{-4}$

यह प्रसार तभी मान्य है जबकि $|x| < 2$ हो। हम प्रमेय 2 का प्रयोग करते हैं। हम देखते हैं कि $b = 2$ तथा $m = -4$ है।

इस प्रकार, $(2+x)^{-4} = 2^{-4} + (-4)\, 2^{-5}(x) + \dfrac{(-4)(-5)}{2!}\, 2^{-6}(x^2) + \ldots$

$$= \frac{1}{16} - \frac{1}{8}x + \frac{20}{128}x^2 + \ldots$$

अत: $\dfrac{1}{(2+x)^4}$ का प्रथम तीन पदों तक प्रसार $\dfrac{1}{16} - \dfrac{1}{8}x + \dfrac{5}{32}x^2$ है।

(ii) $\dfrac{1}{\sqrt[3]{6-3x}} = (6-3x)^{-\frac{1}{3}}$

यह प्रसार तभी मान्य है जबकि $|-3x| < 6$ अर्थात् $|x| < 2$ हो। हम पुन: प्रमेय 2 का प्रयोग करते हैं। यहाँ $m = -\frac{1}{3}$ है।

इस प्रकार,

$$(6-3x)^{-\frac{1}{3}} = 6^{-\frac{1}{3}} + \left(-\frac{1}{3}\right)6^{-\frac{1}{3}-1}(-3x) + \frac{(-\frac{1}{3})(-\frac{1}{3}-1)}{2!}\,6^{-\frac{1}{3}-2}(-3x)^2 + \ldots$$

$$= \frac{1}{\sqrt[3]{6}} + \frac{1}{6(\sqrt[3]{6})}x + \frac{1}{18(\sqrt[3]{6})}x^2 + \ldots$$

अत: $\dfrac{1}{\sqrt[3]{6-3x}}$ का प्रथम तीन पदों तक प्रसार

$$\frac{1}{\sqrt[3]{6}} + \frac{1}{6(\sqrt[3]{6})}x + \frac{1}{18(\sqrt[3]{6})}x^2 \text{ है।}$$

उदाहरण 2 : $\sqrt[3]{7.60}$ को 4 दशमलव स्थानों तक परिकलित कीजिए।

हल : $\sqrt[3]{7.60}$ को ज्ञात करने के लिए हमें द्विपद प्रमेय का प्रयोग करने की आवश्यकता है। परन्तु इस प्रकार की सन्निकटन समस्याओं (**approximation problems**) के विषय में पहले हम एक टिप्पणी करेंगे।

[illegible] तुलना में x छोटा है तो $(b+x)^m$ का प्रथम सन्निकटन (**first approximation**) $b^m + m\,b^{m-1}x$, अर्थात् द्विपद प्रसार के प्रथम दो पदों, द्वारा प्राप्त होता है। $(b+x)^m$ का दूसरा सन्निकटन प्रथम तीन पदों अर्थात् $b^m + m\,b^{m-1}x + \frac{m(m-1)}{2!}\,b^{m-2}x^2$, द्वारा प्राप्त होता है और इसी प्रकार आगे के सन्नि[illegible] होते हैं।

किसी विशिष्ट समस्या में चुना जाने वाला सन्निकटन वांछित शुद्धता के अंश पर निर्भर करता है। परन्तु हम पुनः यह कहते है कि x, b की तुलना में छोटा होना चाहिए। अतः सन्निकटनों को $(1+y)^n$ के प्रसार पर, जबकि y छोटा है, आधारित किया जाना चाहिए।

अब हम अपनी समस्या की ओर ध्यान देते हैं। हम लिखते हैं कि

$$(7.60)^{\frac{1}{3}} = (8-0.40)^{\frac{1}{3}} = (8)^{\frac{1}{3}}(1-0.05)^{\frac{1}{3}} = 2(1-0.05)^{\frac{1}{3}}$$

$$= 2\left[1+\tfrac{1}{3}(-0.05)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(-0.05)^2+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(-0.05)^3+\ldots\right]$$

$$= 2[1-0.01666-0.000277-0.000007+\ldots]$$

क्योंकि हम $\sqrt[3]{7.60}$ को चार दशमलव स्थानों तक परिकलित करना चाहते हैं अतः हम उन पदों की उपेक्षा* कर सकते हैं जिनमें $(-0.05)^3$ तथा (-0.05) की अन्य उच्चतर घातें आती हैं क्योंकि चौथे दशमलव स्थान में इनका कोई योगदान नहीं है।

इस प्रकार, $\sqrt[3]{7.60} = 2(.98307) = 1.96614$

अतः 4 दशमलव स्थानों तक,

$$\sqrt[3]{7.60} = 1.9661$$

उदाहरण 3 : $(1.02)^6$ को पाँच दशमलव स्थानों तक परिकलित कीजिए।

हल : हम लिखते हैं कि

$$(1.02)^6 = (1+.02)^6$$

$$= 1 + 6(.02) + \frac{6.5}{2!}(.02)^2 + \frac{6.5.4}{3!}(.02)^3$$

$$+ \frac{6.5.4.3}{4!}(.02)^4 + \frac{6.5.4.3.2}{5!}(.02)^5 + (.02)^6$$

$$= 1 + 0.12 + 0.006 + 0.00016 + 0.0000024 +$$

$$= 1.1261624$$

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* इसके लिए स्पष्ट कथन यह होगा कि ऐसे पदों, जिनमें $(-0.05)^3$ तथा (-0.05) की अन्य उच्चतर घातें आती हैं, के योग के दुगुने का चौथे दशमलव स्थान में कोई योगदान नहीं है। परन्तु इस कथन की उपपत्ति इस पुस्तक की सीमा के बाहर है।

अतः, पांच दशमलव स्थानों तक

$$(1.02)^6 = 1.12616$$

उदाहरण 4 : यह मानकर कि x इतना छोटा है कि x^2 तथा x की अन्य उच्चतर घातों को छोड़ा जा सकता है,

$$\frac{(1+\frac{3}{4}x)^{-4}\,(16-3x)^{\frac{1}{2}}}{(8+x)^{\frac{2}{3}}}$$ का मान ज्ञात कीजिए।

हल : हम देखते हैं कि

$$\frac{(1+\frac{3}{4}x)^{-4}(16-3x)^{\frac{1}{2}}}{(8+x)^{\frac{2}{3}}} = \frac{(1+\frac{3}{4}x)^{-4}(16)^{\frac{1}{2}}(1-\frac{3}{16}x)^{\frac{1}{2}}}{(8)^{\frac{2}{3}}\left(1+\frac{x}{8}\right)^{\frac{2}{3}}}$$

$$=\left(1+\frac{3}{4}x\right)^{-4}\left(1-\frac{3x}{16}\right)^{\frac{1}{2}}\left(1+\frac{x}{8}\right)^{-\frac{2}{3}}$$

$$=\left[1-4\left(\frac{3}{4}\right)x+\ldots\right]\left[1-\frac{1}{2}\left(\frac{3x}{16}\right)+\ldots\right]\left[1-\frac{2}{3}\left(\frac{x}{8}\right)+\ldots\right]$$

$$=\left(1-3x+\ldots\right)\left(1-\frac{3}{32}x+\ldots\right)\left(1-\frac{1}{12}x+\ldots\right)$$

$$=\left(1-\frac{3}{32}x-3x+\ldots\right)\left(1-\frac{1}{12}x+\ldots\right)$$

$$=\left(1-\frac{1}{12}x-\frac{3}{32}x-3x+\ldots\right)$$

$$=1-\frac{305}{96}x \quad \left[x^2 \text{ तथा } x \text{ की अन्य उच्चतर घातों को छोड़ देने पर}\right]$$

प्रश्नावली 8.3

1. निम्नलिखित में से प्रत्येक का प्रथम तीन पदों तक x की घातों के आरोही क्रम में प्रसार ज्ञात कीजिए :

   (i) $\sqrt{2-3x}$

   (ii) $(27-6x)^{-\frac{2}{3}}$

   (iii) $\dfrac{1}{\left(x^2+\dfrac{1}{x}\right)^{\frac{4}{5}}}$

यह भी बताइए कि प्रत्येक प्रसार कब मान्य है।

2. $(1+x)^{-2}$ के प्रसार में प्रथम चार पद प्राप्त कीजिए यदि यह दिया है कि $|x| > 1$ ।
[**संकेत** : $(1+x)^{-2} = x^{-2}\left(1+\frac{1}{x}\right)^{-2}$ लिखिए। अब $\left|\frac{1}{x}\right| < 1$ है तथा प्रसार संभव है।]

3. निम्नलिखित में से प्रत्येक को 4 दशमलव स्थानों तक परिकलित कीजिए :

(i) $\sqrt[4]{16.08}$ (ii) $(999)^{-\frac{1}{3}}$

(iii) $(10001)^{\frac{3}{4}}$ (iv) $(2.002)^8$

4. यह मानकर कि निम्नलिखित में से प्रत्येक का प्रसार सम्भव है, प्रत्येक में x^3 का गुणांक ज्ञात कीजिए :

(i) $\dfrac{3-2x}{(1+3x)^2}$ (ii) $\dfrac{(1-4x)^2\,(1-2x^2)^{\frac{1}{2}}}{(4-x)^{\frac{3}{2}}}$

5. यह मानकर कि x इतना छोटा है कि x^2 तथा x की अन्य उच्चतर घातों की उपेक्षा की जा सकती है,

$$\frac{(1-2x)^{\frac{2}{3}}\,(4+5x)^{\frac{3}{2}}}{\sqrt{1-x}}$$ का मान ज्ञात कीजिए।

6. दिखाइए कि $\dfrac{(1+y)^2}{(1-y)^2}$ के प्रसार में y^n का गुणांक $4n$ है।

7. सिद्ध कीजिए कि

$$(1+x+x^2+x^3+\ldots)(1-x+x^2-x^3+\ldots) = (1+x^2+x^4+x^6+\ldots)$$

*8. यह मानकर कि x इतना छोटा है कि x^3 तथा x की अन्य उच्चतर घातों की उपेक्षा की जा सकती है,

$(1-\frac{3}{2}x)^5\,(2+3x)^6$ का मान ज्ञात कीजिए।

9. $\sqrt{101}$ को 6 दशमलव स्थानों तक ज्ञात करने के लिए $(100+x)^{\frac{1}{2}}$ के द्विपद प्रसार का प्रयोग कीजिए।

10. $(0.998)^8$ का 6 दशमलव स्थानों तक मान ज्ञात कीजिए।
[**संकेत** : $(1+x)^8$ के द्विपद प्रसार का प्रयोग कीजिए।]

8.4 ऐतिहासिक दृष्टिकोण

$n=2$ की स्थिति के लिए द्विपद प्रसार यूनानी गणितज्ञ **यूक्लिड** (लगभग 300 ई० पू०) को ज्ञात था। जहाँ तक प्राकृत संख्याओं की उच्चतर घातों के लिए द्विपद प्रसार का सम्बन्ध है, इसका श्रेय अरब गणितज्ञ उमर खय्याम (1048- 1122 ई०) को जाता है। उन्होंने संख्याओं के मूल ज्ञात करने में इस प्रसार का प्रयोग किया तथा दावा किया कि उन्होंने एक द्विपद की nवीं घात (n एक प्राकृत संख्या है) के लिए एक सूत्र प्राप्त किया है जिसके विषय में उन्होंने बताया कि उन्होंने उसका कथन अपनी एक अन्य कृति में दिया है।

परन्तु उनकी वह कृति आज तक नहीं मिल सकी है। पद 'द्विपद गुणांक' (binomial coefficient) सर्वप्रथम जर्मनी के गणितज्ञ माइकेल स्टिफेल (1486-1567 ई०) ने लगभग 1544 ई० में दिया था।

ऋणात्मक पूर्णांकीय तथा परिमेय संख्याओं के घातांकों के लिए द्विपद प्रमेय का व्यापकीकरण (generalization) सर आईज़ेक न्यूटन (1642—1727 ई०) ने 1665 ई० में किया था। उन्होंने इस व्यापकीकरण की उपपत्ति प्रदान नहीं की थी परन्तु यह सुझाव अवश्य दिया था कि यह संख्याओं के मूल ज्ञात करने में अत्यन्त उपयोगी है।

द्विपद प्रमेय की उपपत्तियों का श्रेय लेखकों की एक शृंखला को जाता है। ब्रिटिश गणितज्ञ कोलिन मैकलॉरिन (1698—1746 ई०) ने n के परिमेय मानों के लिए एक उपपत्ति दी थी। जियोवैनी फ्रांसेस्को एम० एम० साल्वैमिनी (1708-1791 ई०) तथा जर्मन गणितज्ञ अब्राहम जी कास्टनर (1719—1800 ई०) ने स्वतन्त्र रूप से n के पूर्णांकीय मानों के लिए द्विपद प्रमेय को सिद्ध किया था। स्विटज़रलैंड के सुप्रसिद्ध गणितज्ञ लियोनार्ड ऑयलर (1707—1783 ई०) ने भी 1744 ई० में भिन्नों वाले घातांकों के लिए द्विपद प्रमेय की एक उपपत्ति दी थी। अंत में, नार्वे के गणितज्ञ नील्स हैनरिक ऐबल (1802—1829 ई०) ने, जो एक निर्धन पादरी के पुत्र थे, सम्मिश्र संख्याओं वाले घातांकों के लिए द्विपद प्रमेय को सिद्ध किया था।

8.5 मुख्य संकल्पनाएँ

पास्कल त्रिभुज	संचय विन्यास गुणांक
ऋणेतर पूर्णांकीय घातांकों के लिए द्विपद प्रमेय	परिमेय घातांकों के लिए द्विपद प्रमेय
व्यापक पद	द्विपद प्रमेय के प्रयोग से सन्निकटन

8.6 अग्रिम अध्ययन हेतु सुझाव

पाठक बीजगणित की किसी मानक पाठ्यपुस्तक, उदाहरणार्थ, अनुच्छेद 3.12 की पुस्तक [1] को देखें। कुछ अन्य मानक पाठ्यपुस्तकें निम्न हैं :

[1] C. Smith : **A Treatise on Algebra, 5th Edition.**
Macmillan and Company Ltd., London (U.K.), 1957.

एक पुस्तक, जिसका अध्ययन गणित के प्रत्येक गम्भीर विद्यार्थी के लिए आवश्यक है, निम्न है :

[2] G. H. Hardy : **A Course of Pure Mathematics, 10th Edition.**
The Cambridge University Press, London (U.K.), 1963.

इस विषय के उच्चस्तरीय विवरण के लिए अनुच्छेद 1.12 में दी गई पुस्तक [2] देखें।

ऐतिहासिक संदर्भ के लिए पाठक अनुच्छेद 1.12 में दी गई पुस्तक [7], अनुच्छेद 2.7 में दी गई पुस्तक [3], इत्यादि को देखें।

विविध प्रश्नावली II

(एकक V, VI, VII तथा VIII पर)

1. निम्नलिखित में से प्रत्येक अनुक्रम के प्रथम चार पद लिखिए जिनके nवें पद निम्न हैं :

   (i) $1+(-1)^n$

   (ii) $\dfrac{3^n}{2^n+1}$

   (iii) $\dfrac{2+(-2)^n}{n+1}$

   (iv) $\log\left(1+\dfrac{1}{n}\right)$

   (v) $\dfrac{n+4}{n+1}$

2. निम्नलिखित में से प्रत्येक अनुक्रम के, जिनके nवें पद दिए हैं, सम्मुख लिखे पद ज्ञात कीजिए :

   (i) $2^n-\frac{5}{2}$; 8वाँ पद ,12वाँ पद

   (ii) $\dfrac{n^2+2n}{2n}$; 16वाँ पद, $(n-1)$ वाँ पद

   (iii) $2n-11$; 11वाँ पद, 20वाँ पद

   (iv) $\frac{2}{3}(n+1)(2n+1)$; 12वाँ पद

3. निम्नलिखित में से प्रत्येक अनुक्रम के प्रथम 6 पद लिखिए :

   (i) $a_1=4, a_{n+1}=2na_n$

   (ii) $a_1=\frac{1}{2}$, $a_2=-1$, $a_{n+2}=a_n\, a_{n+1}$

4. निम्नलिखित द्वारा परिभाषित अनुक्रम का 21वाँ तथा 42वाँ पद ज्ञात कीजिए :

$$t_n=\begin{cases} 0, \text{ यदि } n \text{ विषम है} \\ 1, \text{ यदि } n \text{ सम है} \end{cases}$$

5. ऐसे अनुक्रम के प्रथम 6 पद ज्ञात कीजिए जिसका प्रथम पद 1 है तथा $(n+1)$वाँ पद, nवें पद में n जोड़ने से प्राप्त होता है ।

6. निम्नलिखित में से प्रत्येक समांतर श्रेणी का सार्व अंतर ज्ञात कीजिए तथा उसके अगले 6 पद लिखिए :

   (i) $-9, -8\frac{3}{5}, -8\frac{1}{5}, \ldots$

(ii) $\sqrt{3}, 1, 2-\sqrt{3}, \ldots$

(iii) $2x-3y, -2x+3y, -6x+9y, \ldots$

7. निम्नलिखित में से प्रत्येक समांतर श्रेणी के सम्मुख लिखा पद ज्ञात कीजिए :

(i) $6, 5\frac{1}{3}, 4\frac{2}{3}, \ldots ; t_7$

(ii) $\frac{3n-1}{n}, \frac{3n-3}{n}, \frac{3n-5}{n}, \ldots ; t_r$

(iii) $a=\frac{1}{3}, d=-2; t_{20}$

8. k का मान निर्धारित कीजिए ताकि $\frac{2}{3}$, k तथा $\frac{5}{8}k$ एक समांतर श्रेणी के तीन क्रमागत पद हों ।

9. क्या 310 समांतर श्रेणी 3, 8, 13, 18, ... का एक पद है ?

10. समांतर श्रेणी 17, $14\frac{1}{2}$, 12, ..., -38 में पदों की संख्या निर्धारित कीजिए ।

11. समांतर श्रेणी 5, 13, 21, ... का कौन सा पद 181 है ?

12. एक समांतर श्रेणी का तीसरा पद 1 है तथा छठा पद -11 है । इस श्रेणी का 15 वाँ तथा r वाँ पद निर्धारित कीजिए ।

13. एक समांतर श्रेणी का पहला पद -6 तथा 12 वाँ पद 13 है । सार्व अंतर ज्ञात कीजिए ।

14. एक समांतर श्रेणी का 7 वाँ पद 34 तथा 15 वाँ पद 74 है । इस श्रेणी का प्रथम पद तथा 40 वाँ पद निर्धारित कीजिए ।

15. हमें एक समांतर श्रेणी दी गई है जिसका प्रथम पद a तथा सार्व अंतर d है ।

(क) यदि इसके प्रत्येक पद में एक ही संख्या k जोड़ दी जाए तो क्या इस प्रकार प्राप्त श्रेणी भी एक समांतर श्रेणी होगी ? यदि ऐसा है, तो उसका सार्व अन्तर ज्ञात कीजिए ।

(ख) यदि इसके प्रत्येक पद को एक ही संख्या c से गुणा किया जाए तो क्या इस प्रकार प्राप्त श्रेणी भी एक समांतर श्रेणी होगी ? यदि ऐसा है, तो उसका सार्व अन्तर ज्ञात कीजिए ।

16. यदि x, y तथा z एक समांतर श्रेणी में हों तो दिखाइए कि $(xy)^{-1}$, $(zx)^{-1}$ तथा $(yz)^{-1}$ भी समांतर श्रेणी में हैं ।

17. यदि एक समांतर श्रेणी का p वाँ पद q तथा q वाँ पद p है, तो सिद्ध कीजिए कि इसका r वाँ पद $p+q-r$ है ।

18. निम्नलिखित में से प्रत्येक समांतर श्रेणी का उसके सम्मुख दिए पदों तक योग ज्ञात कीजिए :

(i) $0, -7, -14, \ldots$; 45 पदों तक

(ii) $1\frac{1}{7}, 1, \frac{6}{7}, \ldots$; 12 पदों तक

(iii) $(x+y)^2, x^2+y^2, (x-y)^2, \ldots$; 10 पदों तक

19. 50 से 500 के बीच उन सभी पूर्णांकों का योग ज्ञात कीजिए जो 7 से विभाज्य हैं।
20. समांतर श्रेणी $13, 6, -1, \ldots$ के पदों की संख्या n ज्ञात कीजिए ताकि $S_n = -92$ हो।
21. ऐसी समांतर श्रेणी, जिसका तीसरा पद 1 तथा छठा पद -11 है, के प्रथम 32 पदों का योग ज्ञात कीजिए।
22. एक ऐसी समांतर श्रेणी निर्धारित कीजिए, जिसके प्रथम n पदों का योग an^2+bn हैं।
23. एक समांतर श्रेणी के प्रथम तीन पदों का योग 9 है तथा उनके वर्गों का योग 35 है। S_n ज्ञात कीजिए।
24. यदि S_n एक समांतर श्रेणी के प्रथम n पदों के योग को व्यक्त करता है तो सिद्ध कीजिए कि $S_{30} = 3(S_{20} - S_{10})$
25. निम्नलिखित में से प्रत्येक गुणोत्तर श्रेणी के अगले चार पद लिखिए :

    (i) $7, -1, \frac{1}{7}, \ldots$

    (ii) $\sqrt{2}, \sqrt{10}, 5\sqrt{2}, \ldots$

    (iii) $\frac{1}{x^2}, x, x^4, \ldots$

26. निम्नलिखित में से प्रत्येक गुणोत्तर श्रेणी के सम्मुख दिए पद ज्ञात कीजिए :

    (i) $12, 8, \frac{16}{3}, \ldots$; t_{10}, t_6

    (ii) $\frac{1}{\sqrt{3}}, 2\sqrt{3}, 12\sqrt{3}, \ldots$; t_{19}

    (iii) $1, -x^2, x^4, \ldots$; t_{28}

    (iv) $b = 3, r = 1.1$; t_5

27. यदि a, b, c तथा d एक गुणोत्तर श्रेणी में हों तो सिद्ध कीजिए कि $a+b, b+c$ तथा $c+d$ भी गुणोत्तर श्रेणी में हैं।
28. एक गुणोत्तर श्रेणी का 18 वाँ पद निर्धारित कीजिए जिसका पाँचवाँ पद 1 तथा सार्व अनुपात $\frac{2}{3}$ है।
29. हमें एक गुणोत्तर श्रेणी दी गई है जिसका प्रथम पद b तथा सार्व अनुपात r है।

    (i) यदि इसके प्रत्येक पद को $k\ (\neq 0)$ से गुणा किया जाये तो क्या इस प्रकार प्राप्त श्रेणी भी एक गुणोत्तर श्रेणी होगी? यदि ऐसा है, तो इसका सार्व अनुपात ज्ञात कीजिए।

    (ii) यदि इसके प्रत्येक पद का वर्ग किया जाये तो क्या इस प्रकार प्राप्त श्रेणी भी एक गुणोत्तर श्रेणी होगी ? यदि ऐसा है, तो इसका सार्व अनुपात ज्ञात कीजिए।

30. एक गुणोत्तर श्रेणी का तीसरा पद उसके पहले पद का वर्ग है। यदि दूसरा पद 8 है, तो इसका छठा पद ज्ञात कीजिए।

31. निम्नलिखित में से प्रत्येक गुणोत्तर श्रेणी का उसके सम्मुख दिए पदों तक योग ज्ञात कीजिए :

(*i*) 0.6, 0.06, 0.006, ... ; 10 पदों तक

(*ii*) $4\frac{1}{11}, -3\frac{3}{11}, 2\frac{34}{55}, \ldots$; 8 पदों तक

(*iii*) $a+b, ab^2+b^3, ab^4+b^5, \ldots$; r पदों तक

32. $\sum_{j=1}^{10}\left[\left(\frac{1}{2}\right)^{j-1}+\left(\frac{1}{5}\right)^{j+1}\right]$ का मान ज्ञात कीजिए।

33. गुणोत्तर श्रेणी 3, 6, 12,... के पदों की संख्या n ज्ञात कीजिए ताकि $S_n=381$ प्राप्त हो।

34. एक गुणोत्तर श्रेणी के प्रथम तीन पदों का योग 21 है जबकि अगले तीन पदों का योग 168 है। इस श्रेणी का प्रथम पद, सार्व अनुपात तथा उसके प्रथम 6 पद ज्ञात कीजिए।

35. ऐसी गुणोत्तर श्रेणी का तीसरा पद ज्ञात कीजिए जिसका सार्व अनुपात 3 है तथा जिसके प्रथम 7 पदों तक का योग 2186 है।

36. एक गुणोत्तर श्रेणी के प्रथम तीन पदों का योग 13 है तथा उनके वर्गों का योग 91 है। वह गुणोत्तर श्रेणी ज्ञात कीजिए।

37. यदि S_n एक गुणोत्तर श्रेणी के प्रथम n पदों तक के योग को व्यक्त करता है, तो सिद्ध कीजिए कि

$$(S_{10}-S_{20})^2=S_{10}(S_{30}-S_{20})$$

38. निम्नलिखित में से प्रत्येक अपरिमित गुणोत्तर श्रेणी के लिए S_∞ ज्ञात कीजिए :

(*i*) $24, 18, 13\frac{1}{2}, \ldots$

(*ii*) $\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \ldots$

39. निम्नलिखित प्रत्येक दशमलव के लिए एक परिमेय संख्या ज्ञात कीजिए जिसका प्रसार वह दशमलव हो :

(*i*) $1.\overline{36}$ (*ii*) $3.01\overline{09}$

(*iii*) $0.\overline{437}$

40. ऐसी गुणोत्तर श्रेणी के लिए S_∞ [illegible] प्रथम पद 28 तथा चौथा पद $\frac{4}{49}$ है।

41. यदि गुणोत्तर श्रेणी $p, 1, \frac{1}{p}, \ldots$ के लिए $S_\infty, \frac{25}{4}$ हो तो p का (के) मान ज्ञात कीजिए।

42. एक गुणोत्तर श्रेणी के अपरिमित पदों का योग 23 है तथा उनके वर्गों का योग 69 है। प्रथम पद तथा सार्व अनुपात ज्ञात कीजिए।

43. एक समबाहु त्रिभुज की एक भुजा 24 सें० मी० है। इसकी भुजाओं के मध्य-बिन्दुओं को मिला कर एक अन्य त्रिभुज बनाया जाता है। इस प्रकार प्राप्त त्रिभुज की भुजाओं के मध्य-बिन्दुओं को मिलाकर फिर एक अन्य त्रिभुज बनाया जाता है। इस क्रम को अपरिमित बार दोहराया जाता है। सभी त्रिभुजों की परिमापों का योग ज्ञात कीजिए।

44. एक गुणोत्तर श्रेणी के प्रथम तीन पदों का गुणनफल 1000 है। यदि हम इसके द्वितीय पद में 6 तथा तृतीय पद में 7 जोड़ दें तो तीनों पद एक समांतर श्रेणी बनाते हैं। गुणोत्तर श्रेणी के पदों को ज्ञात कीजिए।

45. यदि p, q तथा r समांतर श्रेणी में और x, y तथा z गुणोत्तर श्रेणी में हों, तो सिद्ध कीजिए कि

$$x^{q-r} y^{r-p} z^{p-q} = 1$$

46. गणितीय आगमन के सिद्धान्त के प्रयोग से सिद्ध कीजिए कि सभी प्राकृत संख्याओं n के लिए $n(n+1)(n+2)(n+3)$, 24 से विभाज्य है।

47. निम्नलिखित में से प्रत्येक कथन को सिद्ध करने के लिए गणितीय आगमन के सिद्धांत का प्रयोग कीजिए :

(i) $5+15+45+\ldots+5(3)^{n-1}=\frac{5}{2}(3^n-1)$

(ii) $\frac{1}{a(a+1)}+\frac{1}{(a+1)(a+2)}+\ldots+\frac{1}{(a+n-1)(a+n)}=\frac{n}{a(a+n)}$

48. सिद्ध कीजिए कि सभी प्राकृत संख्याओं n के लिए

$11^{n+2}+12^{2n+1}$, 133 से विभाज्य है।

49. यह सिद्ध करने के लिए कि प्रत्येक प्राकृत संख्या n के लिए

$1.3+3.5+5.7+\ldots+(2n-1)(2n+1)=\frac{n(4n^2+6n-1)}{3}$ है, गणितीय आगमन के सिद्धांत का प्रयोग कीजिए।

50. सिद्ध कीजिए कि प्रत्येक प्राकृत संख्या n के लिए

$1.1! + 2.2! + 3.3! + \ldots + n.n! = (n+1)! - 1$ है।

51. दिखाइए कि यदि कथन $P(n)$: $2^n < n!$, $n=k$ के लिए सत्य है तो यह $n=k+1$ के लिए भी सत्य है। क्या हम यह निष्कर्ष निकाल सकते हैं कि $P(n)$ प्रत्येक प्राकृत संख्या n के लिए सत्य है ?

52. एक पासा (*die*) एक बार उछाला जाता है तथा परिणाम को लिख लिया जाता है।

कुल सम्भव परिणाम कितने होंगे ? यदि पासा दो बार उछाला जाए तो कुल सम्भव परिणाम कितने होंगे ? तीन बार उछालने से कितने परिणाम प्राप्त होंगे ? क्या आप अपने उत्तरों में कोई प्रतिरूप देखते हैं ?

[पाठक को चाहिए कि वह एक पासे को, उदाहरणार्थ, n बार उछालने से प्राप्त परिणामों के लिए एक सूत्र की खोज करे ।]

53. अंकों 8, 1, 5, 3 तथा 4 से दो अंकों की कितनी संख्याएं बनाई जा सकती हैं यदि यह मान लिया जाए कि

(क) अंकों की पुनरावृत्ति हो सकती है ?

(ख) अंकों की पुनरावृत्ति नहीं हो सकती ?

54. अंकों 2, 8, 5, 6 तथा 9 से 1000 से छोटी कितनी संख्याएँ बनाई जा सकती हैं यदि

(i) किसी अंक की पुनरावृत्ति नहीं होती हो ?

(ii) अंकों की पुनरावृत्ति हो सकती हो ?

55. 'DELHI' शब्द के अक्षरों से 5 अक्षरों के कितने भिन्न शब्द बनाए जा सकते हैं ? इनमें से कितने शब्द D से आरम्भ होंगे तथा I पर समाप्त होंगे ?

56. अंकों 1, 2, 3, 5 तथा 7 से कितनी भिन्न प्राकृत संख्याएँ बनाई जा सकती है यदि यह मान लिया जाए कि अंकों की पुनरावृति नहीं हो सकती ? इनमें से कितनी संख्याएँ सम होंगी ?

57. एक बस की तीन खाली सीटों पर दो भिन्न कितनी विधियों से बैठ सकते है ?

58. इन्दिरा और पिंकी चार कालेजों में कितनी विधियों से प्रवेश ले सकती हैं यदि वे दोनों एक ही कालेज में प्रवेश न लेती हों ?

59. सभी अक्षरों को एक साथ लेकर, शब्द 'HEXAGON' के अक्षरों के कुल क्रमचय ज्ञात कीजिए ।

60. चार पर्यटक ठहरने के लिए किसी नगर के 5 होटलों में से चयन कर सकते हैं । उनमें से प्रत्येक पर्यटक एक भिन्न होटल में ठहरना चाहता है । वे कितनी विधियों से होटलों का चयन कर सकते हैं ?

61. एक विविध मनोरंजन कार्यक्रम में पाँच गानों को प्रस्तुत किया जाना है । इन्हें कितने विभिन्न क्रमों से प्रस्तुत किया जा सकता है ?

62. ताश के 52 पत्तों की गड्डी में से दो पत्तों को एक एक करके इस प्रकार निकाला जाता है कि एक बार में निकाला गया पत्ता गड्डी में वापिस नहीं रखा जाता । कितनी विधियों से गड्डी में से दो पत्तों को निकाला जा सकता है ?

63. किसी प्रतियोगिता में कुल 10 प्रवेश-पत्र हैं । कितनी विधियों से पहले तीन इनाम दिए जा सकते हैं ?

64. दिए हुए 8 झंडों से 3 झंडों के कितने भिन्न संकेत (झंडों को ऊर्ध्वाधर, एक के नीचे एक रखते हुए) बनाए जा सकते हैं ?

65. एक गणित क्लब में 18 सदस्य हैं। कितनी विधियों से सदस्य एक अध्यक्ष, एक उपाध्यक्ष, एक सचिव तथा एक कोषाध्यक्ष का चुनाव कर सकते हैं यदि कोई भी सदस्य एक बार में उपरोक्त में से केवल एक ही पद ग्रहण कर सकता हो ?

66. निम्नलिखित में से प्रत्येक का मान ज्ञात कीजिए :

(i) $P(8,5)$ (ii) $P(13,3)$

(iii) $P(6,6)$ (iv) $P(27,2)$

67. 'ALLAHABAD' शब्द के अक्षरों को कितनी भिन्न विधियों से व्यवस्थित किया जा सकता है ?

68. 9 बच्चों में 4 सेब, 3 केले तथा 2 संतरे कितनी विधियों से बाँटे जा सकते हैं ताकि प्रत्येक बच्चे को केवल एक ही फल प्राप्त हो ?

69. r ज्ञात कीजिए यदि

$$P(10,r+1) : P(11,r)=30 : 11$$

70. n निर्धारित कीजिए यदि

$$P(n,4) : P(n+1,5)=1 : 9$$

*71. एक पोल पर 4 सफेद, 1 नीले तथा 3 लाल झंडों को व्यवस्थित करके संदेश भेजे जाते हैं। यदि संदेश उसी क्रम से प्रसारित किए जाएँ जिसमें रंगों को व्यवस्थित किया गया है तो दिखाइए कि यदि सभी आठों झंडों का प्रयोग किया जाए तो कुल 280 भिन्न संदेश भेजे जा सकते हैं। यदि केवल 6 झंडों का प्रयोग किया जाए तो कुल कितने संदेश प्रसारित किए जा सकते हैं ?

72. यदि $C(n,r)$, n भिन्न वस्तुओं में से एक बार में r वस्तुओं को एक साथ लेकर प्राप्त संचयों की संख्या को व्यक्त करता है, तो सिद्ध कीजिए कि

$$C(n,r) + 2C(n,r-1) + C(n,r-2) = C(n+2,r)$$

73. यदि $P(n,r)$, n भिन्न वस्तुओं में से एक बार में r वस्तुओं को एक साथ लेकर प्राप्त क्रमचयों को व्यक्त करता है, तो सिद्ध कीजिए कि

$$P(n,r) = (n-r+1)\,P(n,r-1)$$

74. अंकों 2, 3, 4, 5 तथा 9 से पाँच अंकों की कितनी संख्याएँ बनाई जा सकती हैं यदि यह मान लिया जाए कि किसी अंक की पुनरावृत्ति नहीं होती ? इनमें से कितनी संख्याएँ 5 से विभाज्य हैं ?

75. अंकों 3, 4, 5, 8 तथा 9 से कितनी सम संख्याएँ बनाई जा सकती हैं ?

76. जानवरों का एक प्रशिक्षक कितनी विधियों से 6 सिंह तथा 5 बाघों को एक पंक्ति में व्यवस्थित कर सकता है यदि दो बाघ एक साथ न रखे जाएं ?

77. दिखाइए कि n भिन्न पुस्तकों को एक अलमारी में व्यवस्थित करने की कुल $(n-1)!\ (n-2)$ विधियाँ हैं जबकि दो विशिष्ट पुस्तकों को एक साथ न रखा जाए।

78. यदि $P(n, 5) = 12P(n, 3)$ हो, तो n निर्धारित कीजिए।

79. यदि $P(2n, 3) = 100\,P(n, 2)$ है, तो दिखाइए कि n का मान 13 है।

80. **Public Administration Review** के नवम्बर-दिसम्बर अंक में **Defence Digest** से निम्नलिखित **'buzz—phrases'** (वे वाक्यांश या शब्द जिनका प्रयोग अन्य व्यक्तियों को प्रभावित करने के लिए किया जाता है) प्रकाशित हुए :

BUZZ WORDS

Column 1		Column 2		Column 3	
0	Integrated	0	Management	0	Options
1	Total	1	Organizational	1	Flexibility
2	Systematized	2	Monitored	2	Capability
3	Parallel	3	Reciprocal	3	Mobility
4	Functional	4	Digital	4	Programming
5	Responsive	5	Logistical	5	Concept
6	Optimal	6	Transitional	6	Time-phase
7	Synchronized	7	Incremental	7	Projection
8	Compatible	8	Third-generation	8	Hardware
9	Balanced	9	Policy	9	Contingency

यह देखा गया कि कुछ व्यक्तियों को 3-शब्दों के वाक्यांशों का प्रयोग करने की आदत है जिनका चयन वे स्तम्भ 1, 2 तथा 3 में से प्रत्येक से इसी क्रम से एक एक शब्द लेकर करते हैं। उदाहरणार्थ, 'Synchronized Transitional Programming'। इसमें सन्देह है कि शायद इन वाक्यांशों का प्रयोग करने वाले वास्तव में इनका अर्थ जानते हैं या नहीं। यह अनुभव किया गया है कि इन वाक्यांशों का प्रयोग अन्य व्यक्तियों को प्रभावित करने के लिए किया जाता है। इस प्रकार के 3-शब्दों के कुल कितने वाक्यांश संभव हैं?

81. यदि $\dfrac{C(n, r)}{C(n, r+1)} = \dfrac{1}{2}$ तथा $\dfrac{C(n, r+1)}{C(n, r+2)} = \dfrac{2}{3}$ हो, तो n तथा r के मान निर्धारित कीजिए।

82. अंकों 1, 2, 3, 6 तथा 8 से 23000 से बड़ी 5 अंकों की कितनी संख्याएँ बनाई जा सकती हैं?

83. यदि $\alpha = C(n, 2)$ हो, तो सिद्ध कीजिए कि $C(\alpha, 2) = 3C(n+1, 4)$ है।

84. (क) कितने क्रमित युग्म (x, y) समीकरण $x+y=100$ के हल हैं ?
[कल्पना कीजिए कि x और y धनात्मक पूर्णांक हैं ।]

*(ख) दिखाइए कि कुल 4851 क्रमित त्रियुग्म (ordered triples) ऐसे हैं जो कि समीकरण $x+y+z=100$ के हल है, जहाँ x, y और z धनात्मक पूर्णांक हैं ।

[संकेत : मान लीजिए $x=1$ है। धनात्मक पूर्णांकों y और z में समीकरण $y+z=99$ के कितने हल हैं ? फिर मान लीजिए $x=2$ है । धनात्मक पूर्णांकों y और z में समीकरण $y+z=98$ के कितने हल हैं ? इत्यादि ।]

85. राम के 5 मित्र हैं । वह कितनी विधियों से एक या एक से अधिक मित्रों को पार्टी में आमन्त्रित कर सकता है ?

86. 6 लड़कों तथा 5 लड़कियों में से 4 सदस्यों की एक कमेटी का चुनाव किया जाना है । कमेटी में कम से कम एक लड़के तथा कम से कम एक लड़की को सम्मिलित करना आवश्यक है । कितनी विधियों से हम कमेटी का चुनाव कर सकते हैं ?

87. द्विपद प्रमेय का प्रयोग करके निम्नलिखित में से प्रत्येक का प्रसार कीजिए :

(i) $(2x-3x^2)^5$

(ii) $(x^2+2y^3)^6$

(iii) $(1-x^2)^4$

(iv) $(\frac{1}{3}a-\frac{1}{2}b)^7$

88. $(x-y)^{48}$ के प्रसार में 40 वें पद को लिखिए तथा उसे सरल कीजिए ।

89. $\left(y^2-\frac{2}{y^3}\right)^5$ के प्रसार में 'y' से स्वतंत्र पद को लिखिए तथा उसे सरल कीजिए ।

90. $\left(a y^{\frac{1}{2}}-bx^3\right)^{17}$ के प्रसार में x^{18} वाले पद को लिखिए तथा उसे सरल कीजिए ।

91. $(a-2b)^4(a+b)^3$ के प्रसार में उस पद को लिखकर सरल कीजिए जिसमें a^2b^5 आता हो ।

92. $(1+y)^n$ के प्रसार में पाँचवें, छठे तथा सातवें पद के गुणांक समांतर श्रेणी में हैं । n निर्धारित कीजिए ।

93. यदि K_r, $(1+2y)^{10}$ के प्रसार में y की घातों के आरोही क्रम में y^{r-1} का गुणांक है, तो r निर्धारित कीजिए जबकि $\frac{K_{r+2}}{K_r}=4$ है ।

*94. यदि C_r, $(1+x)^n$ के प्रसार में x^r का गुणांक है तो सिद्ध कीजिए कि

$$C_0+3C_1+5C_2+\ldots+(2n+1)\,C_n=(n+1)\,2^n$$

95. निम्नलिखित में से प्रत्येक प्रसार में, x अथवा y की घातों के आरोही क्रम में, प्रथम चार पदों को लिखिए तथा उन्हें सरल कीजिए :

(i) $(2a+x)^{-3}$

(ii) $(1-2x^2)^{\frac{3}{2}}$

(iii) $\left(x+\frac{1}{x}\right)^{-\frac{5}{2}}$

यह भी लिखिए कि ये प्रसार कब मान्य हैं।

96. $\left(1-2y^{\frac{1}{3}}\right)^{-4}$ के प्रसार में उस पद को लिखिए तथा सरल कीजिए जिसमें y^3 आता हो।

97. निम्नलिखित में से प्रत्येक को 4 दशमलव स्थानों तक परिकलित करने के लिए द्विपद प्रमेय का प्रयोग कीजिए :

(i) $(1.025)^8$

(ii) $(1.04)^{-\frac{5}{2}}$

98. यदि x इतना छोटा है कि x^3 तथा x की उच्चतर घातों को छोड़ा जा सकता है, तो $(1-\frac{3}{2}x)^5\,(2+3x)^6$ का मान ज्ञात कीजिए।

99. $\left(y+\frac{1}{y}\right)^6\left(y-\frac{1}{y}\right)^{12}$ के प्रसार में y से स्वतंत्र पद को लिखिए तथा उसे सरल कीजिए।

100. यदि x इतना छोटा है कि x^4 तथा x की उच्चतर घातों को छोड़ा जा सकता है, तो

$(2-\frac{3}{2}x)^4\,(2+\frac{3}{8}x^2)^2-(2+x)^6$ का मान ज्ञात कीजिए।

उत्तरमाला

एकक I

प्रश्नावली 1.1

3. (i) 0.96875

(ii) 0.688

(iii) $0.\overline{538461}$

(iv) $0.\overline{9411764705882352}$

4. $\frac{2353}{990}$

5. $\frac{223760}{9999}$

प्रश्नावली 1.2

3. हाँ, हाँ

4. हाँ

5. 6.557

एकक II

प्रश्नावली 2.1

1. 7, 3

2. $\sqrt{3}, \frac{\sqrt{2}}{7}$

3. 3, −5

4. 0, 7

5. $\sqrt{5}$, 0

6. $2, \sqrt{2}$

7. 2, 7

8. $\frac{5}{4}, \frac{2}{7}$

9. 0, 1

10. $\frac{2+2\sqrt{2}}{3}$, 0.

11. 0, 6

प्रश्नावली 2.2

11. $0+2i$

12. $1-2i$

13. $2+3i$

14. $3+0i$

15. $4-i$

16. $-\frac{1}{2}-\frac{1}{3}i$

17. नहीं

प्रश्नावली 2.3

1. $-9+i$

2. $-\frac{7}{2}+i$

3. $11+4i$

4. $-4+3i$

5. $2(\sqrt{3}+1)+5i$

6. $2+5i$

7. $8-7i$

प्रश्नावली 2.4

1. $1+0i$

2. $6+4i$

3. $-209+144i$

4. $44-6i$

5. $-19+59i$

6. $-1-2\sqrt{6}i$

7. $-10+0i$

8. $9+19i$

9. $\frac{73}{4}+0i$

10. $21+20i$

11. $-2-\frac{3}{2}i$

12. $-286-259i$

13. $-\frac{99}{4}-\frac{107}{8}i$

14. $54\sqrt{5}-378i$

18. $\pm\frac{6}{11}i$

19. $\pm 12i$

20. $\pm\sqrt{31}\,i$

21. $\pm\frac{2}{5}\sqrt{5}i$

22. -24

23. $-\frac{5}{3}$

24. $-21+14i$

25. $26+2i$

प्रश्नावली 2.5

1. $3+7i$

2. $-\sqrt{5}-3i$

3. $-3-7i$

4. $11-60i$

5. $-3-6i$

6. $\frac{3}{25}+\frac{4}{25}i$

7. $\frac{\sqrt{3}}{52}-\frac{7}{52}i$

8. $\frac{6}{39}+\frac{\sqrt{3}}{39}i$

9. i

10. $\frac{4}{25}+\frac{3}{25}i$

11. $\frac{22}{41}-\frac{7}{41}i$

12. $-\frac{22}{41}-\frac{7}{41}i$

13. $1+i$

14. $\frac{6}{25}+\frac{17}{25}i$

15. $-\frac{6}{13}-\frac{9}{13}i$

16. $\frac{22}{17}+\frac{3}{17}i$

17. $\frac{1}{2}i$

18. $12i$

19. $\frac{7}{5}-\frac{11}{5}i$

एकक III

प्रश्नावली 3.1

1. $-\frac{7}{3}, -1$

2. $2, \frac{5}{2}$

3. $-4\sqrt{3}, \frac{2\sqrt{3}}{3}$

4. $\frac{5}{2}, -2.$

5. $1\pm\frac{\sqrt{3}}{2}$

6. $-0.5, 0.35$

7. $\frac{1}{4}, \frac{1}{4}$

8. $-\frac{3}{2}, \frac{3}{5a} (a\neq 0)$

9. $-2, 1$

10. $-\frac{1}{2}, -\frac{1}{2}$

11. 0

12. $\pm 3, \pm 2$

13. $\frac{1}{3}, -\frac{1}{4}$

14. $-2, -\frac{3}{4}$

15. 3

16. कोई वास्तविक मूल नहीं

17. $\frac{9}{13}, \frac{4}{13}$

18. $1\pm\frac{1}{2}\sqrt{43}$

19. कोई मूल नहीं

20. कोई वास्तविक मूल नहीं

21. $-\frac{1}{2}$

22. लम्बाई 18 मी०, चौड़ाई 8 मी०

23. $\frac{5}{4}, \frac{4}{5}$

24. 17, 37

25. 16, 48

26. 36.

27. $1, -\frac{37}{4}$

28. $-\frac{3\sqrt{3}}{4}, -\frac{3}{4}$

29. $\frac{16}{7}, \frac{4}{7}$

30. $\frac{10}{3}, -\frac{13}{3}$

31. $-\frac{3}{4}, 0$

32. $\frac{11\sqrt{3}}{6}, \frac{3}{4}$

33. $x^2+7x+12=0$

34. $2x^2+3x-2=0$

35. $x^2-6x+6=0$

36. $x^2-49=0$

37. $4x^2+5x-6=0$

38. $x^2-80=0$

39. $21x^2+23x+6=0$

40. $x^2-4x-14=0$

प्रश्नावली 3.2

1. $3\pm 2i$
2. $-8\pm 3i$
3. $\frac{2}{3}\pm\frac{1}{3}i$
4. $\sqrt{5}\pm 3i$
5. $-3\sqrt{3}, -\frac{2}{3}\sqrt{3}$
6. $\frac{-4\sqrt{3}\pm 2\sqrt{6}}{3}$
7. $\frac{3\sqrt{2}\pm 2\sqrt{5}i}{19}$
8. $\frac{7\sqrt{3}\pm\sqrt{285}\ i}{12}$
9. $1\pm\sqrt{3}i$
10. $\pm\frac{\sqrt{33}}{11}, \pm\frac{\sqrt{3}}{3}i$
11. अवास्तविक सम्मिश्र तथा संयुग्मी
12. अवास्तविक सम्मिश्र तथा संयुग्मी
13. अवास्तविक सम्मिश्र तथा संयुग्मी
14. वास्तविक तथा समान
15. वास्तविक तथा असमान
16. अवास्तविक सम्मिश्र तथा संयुग्मी
17. 4
18. ± 6
19. 25
20. 16
21. $0, -1$
22. $\frac{7\sqrt{3}}{12}, \frac{1}{4}$
23. $-\frac{2}{5}, -\frac{27}{2}$
24. $\pm\frac{b}{a^2}\sqrt{b^2-4ac}$ यदि $b^2-4ac\geqslant 0$;

    $\pm\frac{b}{a^2}\sqrt{-(b^2-4ac)}$ यदि $b^2-4ac<0$
25. $-\frac{b}{c}$
26. $\frac{b^2-2ac}{ac}$
27. $\frac{b^4+2a^2c^2-4ab^2c}{a^4}$
28. $\frac{b^6+9a^2b^2c^2-6ab^4c-2a^3c^3}{a^6}$
29. $\pm 3\sqrt{5}$
30. $\pm\frac{1}{2}$

प्रश्नावली 3.3

1. $15x^2+11x-12=0$
2. $x^2-14x+58=0$

3. $x^2-2x-2=0$

4. $x^2+14x+113=0$

5. $x^2-(4+\sqrt{2})x+3(1+\sqrt{2})=0$

6. $4x^2-17x+4=0$

7. $(m^2-n^2)x^2+4mnx-m^2+n^2=0$

8. $x^2+(a-b)^2=0$

9. $28x^2-20x+1=0$

10. (i) $acx^2-(b^2-2ac)x+ac=0$

(ii) $cx^2+bx+a=0$

प्रश्नावली 3.4

1. $\pm\frac{\sqrt{2}}{2}(1-3i)$

2. $\pm(3-2i)$

3. $\pm(1-i)$

4. $\pm(4+3i)$

5. $\pm(2-5i)$

6. $\pm(1+\sqrt{3}i)$

7. $\pm(5-4i)$

8. $\pm\frac{\sqrt{2}}{2}(1+i)$

9. $\pm\frac{\sqrt{2}}{2}(1-i)$

प्रश्नावली 3.5

1. $3i; -2i$

2. $-2i, -2i$

3. $4-3i, 3+2i$

4. $3i, \frac{3}{2}+\frac{1}{2}i$

5. $2i, 3\sqrt{2}$

6. $i, 1-i$

7. $-i, -2i$

8. $i, -\frac{1}{3}i$

9. $\pm(1-i)$

10. $\pm(4+3i)$

11. $\pm(2-5i)$

प्रश्नावली 3.6

1. (i) $x\geqslant\frac{2}{5}$, $x\leqslant-\frac{2}{3}$

(ii) $x>6, x<1$

(iii) $x>2, x<-2$

(iv) $-3\leqslant x\leqslant 3$

(v) सभी वास्तविक x

2. (i) कोई हल नहीं

(ii) $x\geqslant 7, x\leqslant -3$

(iii) $-2\leqslant x\leqslant\frac{1}{2}$

(iv) $-4\leqslant x\leqslant 3$

विविध प्रश्नावली I

(एकक I, II और III पर)

1. (i), (iv), (vii), (viii), (ix)

2. हाँ, हाँ, हाँ

3. (i) $3.\overline{09}$ (ii) $0.\overline{2}$

(iii) $0.32\overline{5}$ (iv) $0.\overline{567}$

(v) $0.\overline{142857}$ (vi) $0.\overline{12}$

4. (i) $\frac{17}{3}$

(ii) $\frac{25}{11}$

(iii) $\frac{90}{11}$

(iv) $\frac{7847}{33300}$

(v) $\frac{1187}{5000}$

(vi) $\frac{2374}{9999}$

10. $\sqrt{11}-\sqrt{3}$, $\sqrt{8}$, $\sqrt{2}+\sqrt{6}$, $\sqrt{3}+\sqrt{5}$.

12. हाँ

13. (i) 2, 3

(ii) 4, 1

(iii) 3, -3

15. (i) $0+0i$

(ii) $0-i$

(iii) $-1+0i$

(iv) $-\frac{1}{3}+\frac{1}{5}i$

(v) $\sqrt{2}+\frac{3}{4}i$

(vi) $\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{3}i$

16. (i) $8+7\sqrt{2}i$

(ii) $-5-9i$

(iii) $1-i$

(iv) $14-8i$

(v) $13+4i$

(vi) $16-37i$

(vii) $-6-28i$

(viii) $0+26i$

(ix) $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

(x) $1+0i$

(xi) $\frac{(13+2\sqrt{2})}{3}-\frac{(13\sqrt{2}-2)}{3}i$

(xii) $-\frac{7\sqrt{2}+\sqrt{3}}{50}-\frac{7\sqrt{3}-\sqrt{2}}{50}i$

(xiii) $19-59i$

(xiv) $-5-5i$

(xv) $\frac{525}{169}-\frac{92}{169}i$

(xvi) $0+i$

(xvii) $\frac{1}{10}+\frac{1}{10}i$

(xviii) $-10+2i$

(xix) $-\frac{1}{2}-\frac{1}{4}i$

19. (i) $\frac{1}{8}-\frac{\sqrt{7}}{56}i$

(ii) $-\frac{5}{26}-\frac{1}{26}i$

21. (i) $-40+42i$

(ii) $-34-5\sqrt{7}\,i$

(iii) $\frac{71}{4}-\frac{19}{4}i$

22. (i) $\pm\frac{21}{25}i$

(ii) $\pm 4\sqrt{2}\,i$

(iii) $\pm\frac{2\sqrt{2}}{27}i$

23. (i) $8i$
(ii) $-132+171i$
(iii) $9-\frac{4\sqrt{3}-1}{2}i$

24. (i) $2, -\frac{7}{4}$
(ii) $\frac{3\sqrt{2}}{4}\pm\frac{\sqrt{30}}{4}i$
(iii) $\frac{1}{12}\pm\frac{\sqrt{95}}{12}i$
(iv) $-\frac{7}{2}\pm\frac{\sqrt{7}}{2}i$
(v) $-\frac{7}{10}\pm\frac{\sqrt{11}}{10}i$
(vi) $1, -6, -\frac{5}{2}\pm\frac{\sqrt{39}}{2}i$

25. (i) अवास्तविक सम्मिश्र तथा संयुग्मी
(ii) अवास्तविक सम्मिश्र तथा संयुग्मी
(iii) अवास्तविक सम्मिश्र तथा संयुग्मी
(iv) वास्तविक तथा असमान
(v) वास्तविक तथा समान

26. (i) 0, 12
(ii) 21
(iii) 5, -3

27. (i) 0, 3
(ii) $-\frac{5}{3}, -\frac{4}{3}$
(iii) $2, -\sqrt{5}$

31. $2, -25$

34. $x^2-6x+11=0$

35. $2x^2-25x+82=0$

36. (i) $x^2-2\sqrt{2}\,x-7=0$
(ii) $x^2+x+1=0$
(iii) $x^2-2x+2=0$
(iv) $12x^2-25x+12=0$
(v) $x^2-x-3+\sqrt{3}=0$

37. (i) $\pm(1+i)$
(ii) $\pm(1+4i)$
(iii) $\pm\left[\sqrt{\frac{\sqrt{34}-3}{2}}+\sqrt{\frac{\sqrt{34}+3}{2}}\,i\right]$
(iv) $\pm(3+i)$
(v) $\pm(3-i)$
(vi) $\pm(1+3i)$
(vii) $\pm(1-3i)$

38. (i) $3-i, -1+2i$
(ii) $1-i, \frac{4}{5}-\frac{2}{5}i$
(iii) $2+i, 1-3i$

40. $-\frac{3}{4}+\frac{11}{2}i$

41. (i) $-1<x<3$
(ii) $\left[1-\frac{\sqrt{2}}{2}\right]\leqslant x\leqslant\left[1+\frac{\sqrt{2}}{2}\right]$
(iii) कोई हल नहीं
(iv) $x\geqslant 6,\ x\leqslant-\frac{3}{2}$

42. (i) सभी x
(ii) $-4\leqslant x\leqslant 5$
(iii) कोई हल नहीं

एकक IV

पुनरावलोकन हेतु प्रश्नावली

1. 81; -8; $\frac{1}{1024}$; $128\sqrt{2}$; $\frac{4}{25}$

2. (i) 1; -64; 64

(ii) 1024; 256; $\frac{1}{729}$

(iii) $\frac{125}{8}$; $\frac{1}{y^6}$; $\frac{1}{x^2}$; x^5; $\frac{1}{x^2}$

(iv) $\frac{1}{64}$; a^6; $1 (c \neq 0)$; $8b^9$

(v) $a^{12}b^8$; $-\frac{27}{8}x^6y^3$; $-5x^4y^5$

(vi) $\frac{x^4}{y^6}$; $\frac{16a^{12}}{b^{16}}$; $-\frac{243x^{10}y^{15}}{32a^5b^{10}}$

(vii) $-7a^8b^5$; $\frac{75b^{2m}}{a^{3m}}$

3. (क) $\frac{1}{125}$; $\frac{1}{9}$; $\frac{1}{49}$; $\frac{25}{9}$

(ख) (i) 64; 32; $\frac{1}{216}$

(ii) 121; $\frac{1}{a^3}$

(iii) $\frac{4}{3}$; $\frac{1}{4096}$; 512; 4

(iv) $\frac{1}{5832}$; $\frac{625}{65536}$; $\frac{3}{32}$

(v) $\frac{9}{25}$; 28; 419904

(vi) $\frac{a}{m}$; $\frac{3}{2m^2n^3}$

4. (क) 3; $\frac{1}{7}$; 8; 1296; $\frac{1}{3}$; $\frac{1}{3}$; $\frac{9}{4}$;

$\frac{8}{27}$; $\frac{1}{4}$; $\frac{1}{27}y^5$

(ख) (ii) $y^{\frac{11}{6}}$ (ii) a^4y^6

(iii) $\frac{1}{81}y^{\frac{3}{2}}$ (iv) $x^8y^{\frac{8}{5}}$

(v) $\frac{3b^2}{2a^2}$ (vi) $\frac{y^2}{4x^4}$

(vii) $\frac{7a}{3y^{\frac{5}{8}}}$ (viii) $\frac{n}{32m^{10}}$

(ix) $x-1$

(x) $x+y+2x^{\frac{1}{2}}y^{\frac{1}{2}}$ (xi) -5

(xii) $\frac{xy}{x+y}$ (xiii) 625

5. (i) $\frac{2}{3}$ (ii) 0

(iii) 3 (iv) 3

(v) 2 (vi) 2

(vii) $\frac{1}{2}$ (viii) —

एकक V

प्रश्नावली 5.1

1. $-\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$

2. $25, -125, 625, -3125, 15625$

3. $2, 4, 8, 16, 32$

4. $3, \frac{8}{3}, \frac{27}{10}, \frac{96}{35}, \frac{5}{9}$

5. 1, 3, 6, 10, 15
6. 1, 5, 14, 30, 55
7. 75, 87
8. $\frac{16}{81}$
9. 512
11. 2, 2, 1, 0, −1, −2
12. $-1, -\frac{1}{2}, -\frac{1}{6}, -\frac{1}{24}, -\frac{1}{120}, -\frac{1}{720}$
13. 1, 1, 2, 3, 5, 8

प्रश्नावली 5.2

1. −3; −12, −15, −18, −21
2. $\frac{5}{4}$; $\frac{11}{4}$, 4, $\frac{21}{4}$, $\frac{13}{2}$
3. $-2y$; $x-5y$, $x-7y$, $x-9y$, $x-11y$
4. $\frac{1}{6}$; $\frac{2}{3}$, $\frac{5}{6}$, 1, $\frac{7}{6}$
5. 21; $2x+1$
6. $\frac{173}{15}$; $\frac{2}{3}n-\frac{7}{15}$
7. −100
8. $n-m$
9. 14
10. हाँ
13. −8, $5r-8$
14. $7q-6p$; $4p-3q+(q-p)n$
15. 10 वाँ
16. 3

प्रश्नावली 5.3

1. −897; $\frac{n}{2}(37-5n)$
2. −27.5; −637.5
3. 1218; 2912
4. 3969
5. $22(x-20y)$; $p[x-(p-2)y]$
6. 10000; 40000
7. 10100; $n(n+1)$
8. 98450
9. 2310
10. 5, 20
11. $n=16$, $l=73$
12. $l=103$, $S_{64}=3568$
13. $a=-13$, $d=3$
14. $a=-\frac{8}{3}$, $S_{20}=73\frac{1}{3}$
15. $d=30$, $S_{51}=-116280$
17. $6r-1$
18. 123525
19. 39100 रु०

प्रश्नावली 5.4

1. $-\frac{1}{3}$; $\frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, -\frac{1}{243}$
2. 9; −486, −4374, −39366, −354294
3. 0.1; 0.005, 0.0005, 0.00005, 0.000005
4. $\sqrt{2}$; $4\sqrt{3}$, $4\sqrt{6}$, $8\sqrt{3}$, $8\sqrt{6}$

5. $\frac{y}{x^3}; \frac{y^2}{x^5}, \frac{y^3}{x^7}, \frac{y^4}{x^9}, \frac{y^5}{x^{11}}$

6. 1.728; $(1.2)^{n-1}$

7. $128\left[-\frac{3^{10}}{4^{10}}\right]$; $(-1)^{n-1}\ 128\left[\frac{3}{4}\right]^{n-1}$

8. $(-1)^{n-1}\ 100(1.1)^{n-1}$

9. 3072

11. —2187

12. हाँ ; b^2; r^2; b^2r^{2n-2}

13. ± 1

14. $bc, rs; \frac{b}{c}, \frac{r}{s}$

15. 120, 480, $30(2^n)$

प्रश्नावली 5.5

1. $3\left[1-\frac{2^{10}}{3^{10}}\right]$; $3\left[1-\frac{2^n}{3^n}\right]$

2. $\frac{1}{6}[1-(0.1)^8]$; $\frac{1}{6}[1-(0.1)^{20}]$

3. $\frac{\sqrt{7}}{2}(\sqrt{3}+1)\left(3^{\frac{n}{2}}-1\right)$

4. $\frac{1-(-a)^n}{1+a}$

5. $\frac{x^3(1-x^{2n})}{1-x^2}$

6. $\frac{8}{5}\left[1-\frac{1}{4^{12}}\right]$; $\frac{8}{5}\left[1-\frac{(-1)^n}{4^n}\right]$

7. $22+\frac{3}{2}(3^{11}-1)$

8. $6\left(1-\frac{1}{3^6}\right)$

9. $\frac{4^5}{3}(4^7-1)$

10. $2(2^{18}-1)+\frac{1}{2}(3^{18}-1)$

11. $\frac{3}{4}, -1, \frac{4}{3}, -\frac{16}{9}, \ldots; \frac{4}{3}, -1, \frac{3}{4}, -\frac{9}{16},$

12. 10

13. $\frac{16}{7}$; 2 ; $\frac{16}{7}(2^n-1)$

14. 21845 रु०

15. 2059

16. $\frac{7}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

17. $\begin{cases} 0, \text{ यदि } n \text{ सम है} \\ 1, \text{ यदि } n \text{ विषम है} \end{cases}$

प्रश्नावली 5.6

1. $\frac{1}{3}; \frac{3}{2}$

2. $-\frac{1}{7}; \frac{49}{8}$

3. 0.2; 7.5

4. 0.85; $333\frac{1}{3}$

5. 0.6; 0.75

6. —0.9; $5\frac{5}{19}$

7. $\frac{31}{45}$

8. $\frac{144}{99}$

9. $\frac{712}{999}$

10. $\frac{223760}{9999}$

11. $\frac{10}{99}$

12. $\frac{2}{3}$

13. 16

14. 512 वर्ग सें० मी०

एकक VII

प्रश्नावली 7.1

1. 8
2. 16; 32; हाँ; 2^n
3. 1024
4. 24
5. 8
6. 125
7. 120
8. 120
9. 1320
10. 42
11. 81
12. 8000
13. 180

प्रश्नावली 7.2

1. (i) $4\times3\times2\times1$; 24
   (ii) 2×1; 2
   (iii) $7\times6\times5\times4\times3\times2\times1$; 5040
   (iv) $8\times7\times6\times5\times4\times3\times2\times1$; 40320
2. 8; नहीं
3. 48; नहीं
4. 1680; नहीं
5. (i) 24
   (ii) 24
6. (i) 5040
   (ii) 1320
   (iii) n
   (iv) $n(n-1)$
   (v) $n(n-1)(n-2)$
7. (i) 15
   (ii) 35
   (iii 455

प्रश्नावली 7.3

1. (i) 720
   (ii) 15120
   (iii) 116280
   (iv) 5550
3. (क) 362880
   (ख) 1728
4. 6720
5. 2240

6. 25200
7. 14400
8. 144
9. 6759324
10. 120
11. 3628800
12. 360
13. 86400
14. 600; 120
15. 325; 130
16. 180; 55
17. (i) 8
    (ii) 7
    (iii) 5
18. (i) 5
    (ii) 3
    (iii) 2
20. 4989600
21. (क) 4 (ख) 8
22. 1; 3; 3; 1
23. 1; 4; 6; 4; 1
24. 210
25. 1680
26. 24
27. 126

प्रश्नावली 7.4

1. (i) 56
   (ii) 1365
3. $\frac{n(n-1)}{2}$; $\frac{n(n-1)(n-2)}{6}$
4. (i) 19600
   (ii) 8008
   (iii) 190
   (iv) 276
   (v) 142506
5. 22; 26334
6. 5
8. 7200
9. 2880
10. 10
11. 3
12. 40
13. 462
14. 420
15. 120
16. 886656
17. 1; 5; 10; 10; 5; 1
18. 15
19. 455
21. 64

एकक VIII

प्रश्नावली 8.1

1. $x^6+3x^4y^2+3x^2y^4+y^6$

2. $x^2+2+\frac{1}{x^2}$

3. $32x^5-240x^4y+720x^3y^2-1080x^2y^3+810xy^4-243y^5$

4. $\frac{1}{16}x^8-\frac{1}{8}x^6y+\frac{3}{32}x^4y^2-\frac{1}{32}x^2y^3+\frac{1}{256}y^4$

5. $-27x^3-9x-\frac{1}{x}-\frac{1}{27x^3}$

6. 6

7. $\frac{5}{256}y^4$

8. $\frac{3}{2}x^2y^6$

प्रश्नावली 8.2

1. (i) 9
   (ii) 17

2. -704000

3. (i) $a^5-10a^4b+40a^3b^2-80a^2b^3+80ab^4-32b^5$

   (ii) $\frac{64}{729}t^6-\frac{32}{27}t^4+\frac{20}{3}t^2-20+\frac{135}{4t^2}-\frac{243}{8t^4}+\frac{729}{64t^6}$

   (iii) $1024x^5-6400x^4y+16000x^3y^2-20000x^2y^3+12500xy^4-3125y^5$

   (iv) $y^{16}+24y^{14}x+252y^{12}x^2+1512y^{10}x^3+5670y^8x^4+13608y^6x^5+20412y^4x^6+17496y^2x^7+6561x^8$

4. $-90a^2b^3$

5. $\frac{35}{648}a^4b^4$

8. -220

11. $10x^6a^4,\ 10x^4a^6$

12. $-165x^6$

16. 7

17. $210x^6y^4$

प्रश्नावली 8.3

1. (ii) $\sqrt{2}-\frac{3\sqrt{2}}{4}x-\frac{9\sqrt{2}}{32}x^2,\ |x|<\frac{2}{3}$

   (ii) $\frac{1}{9}+\frac{4}{243}x+\frac{20}{6561}x^2,\ |x|<\frac{9}{2}$

   (iii) $x^{\frac{4}{3}}-\frac{4}{3}x^{\frac{13}{3}}+\frac{14}{9}x^{\frac{22}{3}},\ |x|<1$

2. $\frac{1}{x^2}-\frac{2}{x^3}+\frac{3}{x^4}-\frac{4}{x^5}$

3. (i) 2.0025
   (ii) 0.1
   (iii) 1000.075
   (iv) 258.0552

4. (i) -918

   (ii) $\frac{13027}{8192}$

5. $8+\frac{25}{3}x$

8. $64+96x-720x^2$

9. 10.049875

10. 0.984112

विविध प्रश्नावली II

(एकक V, VI, VII तथा VIII पर)

1. (i) 0, 2, 0, 2

(ii) $1, \frac{9}{5}, 3, \frac{81}{17}$

(iii) $0, 2, -\frac{3}{2}, \frac{18}{5}$

(iv) लघु 2, लघु $\frac{3}{2}$, लघु $\frac{4}{3}$, लघु $\frac{5}{4}$

(v) $\frac{5}{2}, 2, \frac{7}{4}, \frac{8}{5}$

2. (i) $253\frac{1}{2}, 4093\frac{1}{2}$

(ii) $9, \frac{n+1}{2}$

(iii) 11, 29

(iv) $216\frac{2}{3}$

3. (i) 4, 8, 32, 192, 1536, 15360

(ii) $\frac{1}{2}, -1, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}$

4. 0, 1

5. 1, 2, 4, 7, 11, 16

6. (i) $d=\frac{2}{5}; -7\frac{4}{5}, -7\frac{2}{5}, -7, -6\frac{3}{5}, -6\frac{1}{5}, -5\frac{4}{5}$

(ii) $d=1-\sqrt{3}; 3-2\sqrt{3}, 4-3\sqrt{3}, 5-4\sqrt{3}, 6-5\sqrt{3}, 7-6\sqrt{3}, 8-7\sqrt{3}$

(iii) $d=-4x+6y; -10x+15y, -14x+21y, -18x+27y, -22x+33y, -26x+39y, -30x+45y$

7. (i) 2

(ii) $\frac{3n-(2r-1)}{n}$

(iii) $-37\frac{2}{3}$

8. $\frac{16}{33}$

9. नहीं

10. 23

11. 23वाँ पद

12. $-47, 13-4r$

13. $\frac{19}{11}$

14. 4, 199

15. (क) हाँ, d

(ख) हाँ, cd

18. (i) -6930

(ii) $4\frac{2}{7}$

(iii) $10(x^2+y^2-7xy)$

19. 17696

20. 8

21. -1696

22. $b+a, b+3a, b+5a, \ldots$

23. n^2 अथवा $(6n-n^2)$

25. (i) $-\frac{1}{7^2}, \frac{1}{7^3}, -\frac{1}{7^4}, \frac{1}{7^5}$

(ii) $5\sqrt{10}, 25\sqrt{2}, 25\sqrt{10}, 125\sqrt{2}$

(iii) $x^7, x^{10}, x^{13}, x^{16}$

26. (i) $\frac{2048}{6561}, \frac{128}{81}$

(ii) $\frac{1}{\sqrt{3}} 6^{18}$

(iii) $-x^{54}$

(iv) 4.3923

28. $\frac{2^{13}}{3^{18}}$

29. (i) हाँ, r

(ii) हाँ, r^2

30. 128

31. (i) $\frac{2}{3}[1-(0.1)^{10})]$

(ii) $\frac{25}{11}\left[1-\frac{4^9}{5^9}\right]$

(iii) $\frac{(a+b)(1-b)^{2r}}{(1-b^2)}$

32. $2\left(1-\frac{1}{2^{10}}\right)+\frac{1}{20}\left(1-\frac{1}{5^{10}}\right)$

33. 7

34. 3; 2; 3, 6, 12, 24, 48, 96

35. 18

36. 1, 3, 9, ... या 9, 3, 1, ...

38. (i) 96 (ii) $2\sqrt{2}$

39. (i) $\frac{15}{11}$

(ii) $\frac{828}{275}$

(iii) $\frac{437}{999}$

40. $32\frac{2}{3}$

41. 5, $\frac{5}{4}$

42. $\frac{69}{13}, \frac{10}{13}$

43. 144 सें० मी०

44. 5, 10, 20 या 20, 10, 5

52. 6; 6^2; 6^3; हाँ

53. (क) 25 (ख) 20

54. (i) 85 (ii) 155

55. 120; 6

56. 325; 65

57. 6

58. 12

59. 5040

60. 120

61. 120

62. 2652

63. 720

64. 336

65. 73440

66. (i) 6720 (ii) 1716
(iii) 720 (iv) 702

67. 7560

68. 1260

69. 5

70. 8

71. 185

74. 120; 24

75. 130

76. 1814400

78. 7

80. 1000

81. 14; 4

82. 90

84. 360

85. 31

86. 310

87. (i) $x^5[32-240x+720x^2-1080x^3+810x^4-243x^5]$

(ii) $x^{12}+12x^{10}y^3+60x^8y^6+160x^6y^9+240x^4y^{12}+192x^2y^{15}+64y^{18}$

(iii) $1-4x^2+6x^4-4x^6+x^8$

(iv) $\frac{a^7}{3^7}-\frac{7}{2.3^6}a^6b+\frac{21a^5b^2}{3^5.2^2}-\frac{35}{3^4.2^3}a^4b^3+\frac{35}{3^3.2^4}a^3b^4-\frac{21a^2b^5}{3^2.2^5}+\frac{7ab^6}{3.2^6}-\frac{b^7}{2^7}$

88. $-123410x^4y^{39}$

89. 40

90. $12376a^{11}b^6y^{\frac{11}{3}}x^{18}$

91. $-24a^2b^5$

92. 7, 14

93. 5

95. (i) $\frac{1}{8a^3}-\frac{x}{16a^4}+\frac{3x^2}{16a^5}-\frac{5x^3}{32a^6}$; $(x<2a)$

(ii) $1-3x^2+\frac{3}{2}x^4+\frac{x^6}{2}$; $(2x^2<1)$

(iii) $\frac{1}{x^{\frac{5}{2}}}-\frac{5}{2x^{\frac{9}{2}}}+\frac{35}{8x^{\frac{13}{2}}}-\frac{105}{16x^{\frac{17}{2}}}$; $(x^2>1)$

96. $64y^2$

97. (i) 1.2184 (ii) .9066

98. $64+96x-720x^2$

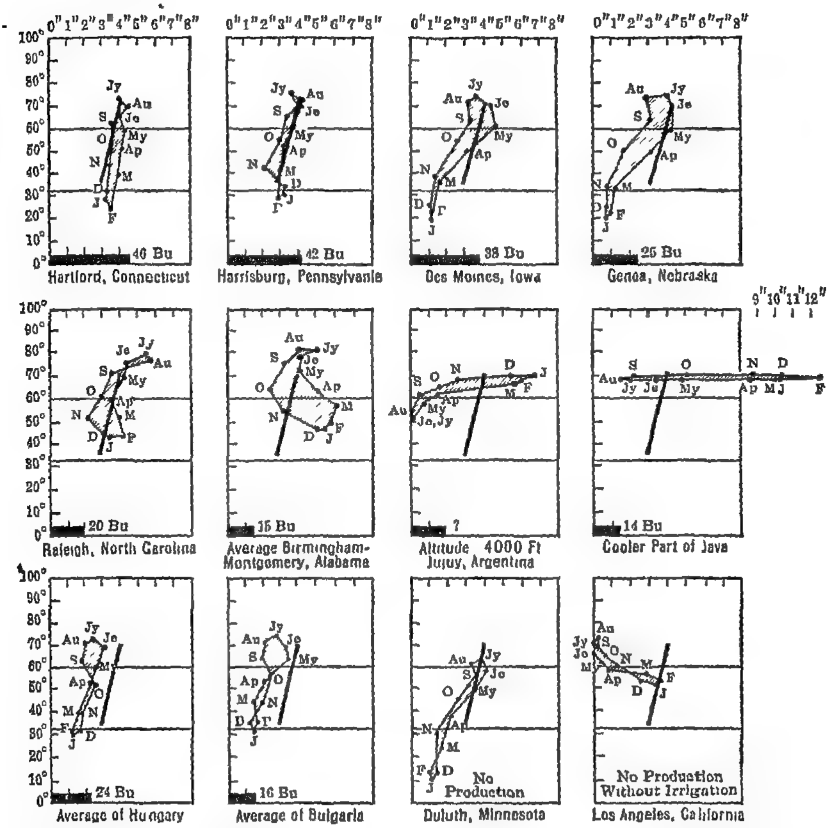
99. 198

100. $-340x^3-394x$

ing where corn shall be raised from the Rocky Mountains westward The peculiar irregularities there are due to the fact that high altitude renders large sections too cool for corn, but this, of course, is an effect of climate In the West, just as in the East, the yield is highest where the July temperature averages from 72° to 76°. The difficulty of detecting the effect of relief on yield per acre in B313 is also illustrated in a comparison of West Virginia and Florida West Virginia is quite rugged, Florida almost flat, but West Virginia averages 32 bushels of corn per acre and Florida only 14 Part of this difference is directly climatic, part is due to poor methods of cultivation in Florida, and a good deal to poor soil there Indirectly, however, both the cultivation and the soil owe much of their quality to climate In B313 the influence of the soil is clearly evident in the way in which the isopleths tend to bulge either northward or southward in the belt of rich black soil from South Dakota to Texas The good soil causes the yield per acre to be higher than one would expect on the basis of climate alone In an earlier chapter, however, we saw that this good soil owes its character largely to the fact that a certain type of climate has prevailed there for a long time Thus we conclude that the main features of B313 are directly climatic, or else arise from conditions such as soil and methods of cultivation which owe much of their quality to the climate of the past In other words, *climate, either directly or indirectly, is the main factor in determining the yield of corn*

318 Productivity and Climographs of Corn Having made this discovery, we can use the Connecticut climate as a standard by which to determine how far and in what ways the climate elsewhere departs from the optimum for corn This is important because it illustrates a method which can be used to advantage with almost any product In A318 the shaded areas represent the climographs of the places named below The slanting straight line is inserted in the same place in each diagram. It represents a standard type of climate, as will be explained later Here it simply serves as a basis for comparing the climographs with one another The heavy bars indicate the average yield of corn per acre for 20 years A careful study of A318 will tell us much about corn and optima, it will also make us familiar with the use of climographs and with types of climate which we shall frequently use later It will further illustrate more clearly the principle stated in Paragraph 317, which may be restated thus: *when all the farms in any given area of considerable size are taken together, the yield per acre of practically all crops depends upon climate far more than is usually realized* Cultivation is certainly of great im-

portance, but it varies in harmony with the climate, as we shall see later. Thus the prosperity of the two thirds of the world's people who depend on farms for a living is influenced to a remarkable degree by atmospheric conditions



A318—Local Climographs and Their Relation to Yield of Corn. Black bars indicate 20-year average yields per acre

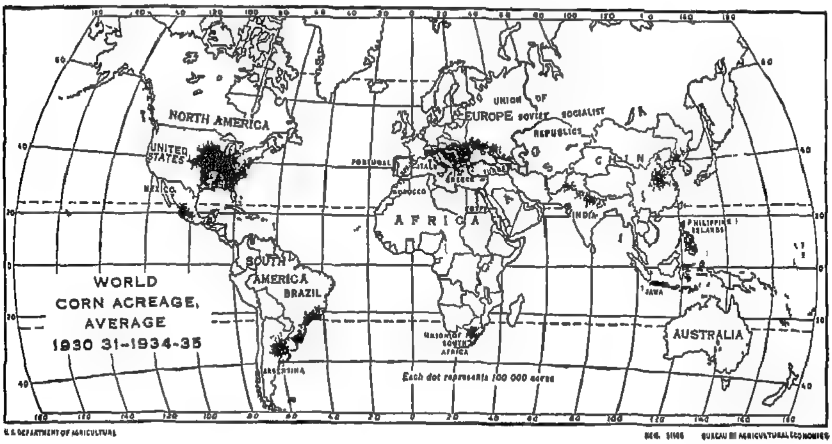
319 The four upper climographs of A318 show what happens to both climate and corn as one goes inland from the east coast in the part of the world where corn thrives best Hartford, only 40 miles from the sea, and Harrisburg, in eastern Pennsylvania, 140 miles from the open ocean, represent the eastern part of the strip of high yield per acre shown in B313 Des Moines in central Iowa represents the conditions near the western end of the strip, where the yield has declined from 46 bushels to about 40, while Genoa in the eastern, corn-raising part of Nebraska, represents an area where the climate has become

decidedly less favorable Since all four places lie in approximately the same latitude and differ only a little in height above sealevel, a comparison of the two eastern climographs with the two from the interior of the continent illustrates the effect of an east-coast climate compared with that of a continental interior

320 The two eastern climographs differ from the two in the interior in three prominent respects, which illustrate the influence of continents upon climate Each of the differences has a distinct effect upon the yield of corn (1) The interior is hotter than the coastal region in summer The average difference during June, July, and August is only about 2°, but that is enough to allow the interior to have many more of the dry, scorching days which are one of the chief reasons why the yield of corn per acre is less than in the eastern states (2) In winter there is a greater difference, about 7°, but at this season the interior is colder than the coast instead of warmer Because of this, Iowa and Nebraska get frosts earlier in the fall than the regions nearer the coast, thus preventing the corn from filling out as fully as possible at the end of the growing season (3) The interior has a marked summer maximum and winter minimum of rainfall, as appears from the way in which the slope of the climographs from the right above to the left below becomes more pronounced as one goes from Hartford to Genoa Hartford has 12 7 inches of rain in June, July, and August, and the rain and melted snow of the 3 winter months amount to 10 3 inches Des Moines, on the other hand, has 12 9 inches of precipitation in May, June, and July, but only 3 6 inches in December, January, and February. (4) Although A318 does not show it, variability is a fourth important handicap of continental climates The great contrasts between summer and winter which have just been mentioned are one evidence of this. Another is the fact that both the rainfall and the temperature of any given month vary more from year to year in the interior than near the sea This increases the tendency toward both drought and frost, as well as toward hot, dry spells in summer.

321 CORN IN REGIONS THAT DEPART FROM THE OPTIMUM The second row of climographs in A318 shows how both corn and climate change as one goes to lower latitudes and to regions warmer than those of greatest productivity In North Carolina the yield of corn averages 20 bushels This low yield occurs in spite of the fact that the fields are well fertilized In that state, according to the United States Department of Agriculture, the fertilizer and manure given to the average acre of corn are worth 6 times as much as in Nebraska where the yield per acre averages 25 bushels, they are worth 2½ times

as much as in Iowa (38 bushels), and two thirds as much as in Pennsylvania (42 bushels) A318 shows that in North Carolina the rainfall is about right, as in most of the eastern half of the United States, but temperatures higher than those farther north are a disadvantage. In Alabama the yield drops to only 15 bushels, even though the value of the fertilizer and manure per acre is greater than in Iowa. Part of the trouble lies in the fact that there is less selection of seed and less care in cultivation than farther north Other disadvantages are that here, as in most warm regions, the soil is comparatively poor, especially in nitrates, and insect pests and parasites are active All these conditions, as we have seen, are to a considerable degree indirect effects of climate, but the climate also has a direct effect on the corn The heavy



A322—World Map of Corn Acreage

rainfall and high temperature cause a strong departure from the optimum

322 The next two climographs in A318 take us to low latitudes of the southern hemisphere (A322) In order to find the most favorable temperatures, it is necessary to go up into the mountains or plateaus At Jujuy, in northwestern Argentina, we are at an altitude of about 4,000 feet and are just south of the Tropic of Capricorn The planting season is October, but that month is much too warm for the best germination of the seed Therefore, the corn is at a disadvantage, even if part of it is helped by irrigation The next two months are ideal in both temperature and rainfall, as is evident from their agreement with Hartford, but January and February are too rainy Hence, the corn does not fill out its ears properly and the kernels do not harden well Moreover, insects and bacteria have a much better

chance to do damage than in regions where cool weather soon checks their activity. The corn crop, like every other, is helped both by the optimum weather for its own growth and by weather that is unfavorable for its enemies

323 The middle line of climographs in A318 ends in Java, only a few degrees from the equator Note how the climograph is compressed into a long, narrow horizontal strip because the rainfall varies greatly, while the temperature varies scarcely at all At certain elevations in Java one can find temperatures that are ideal for the sprouting of corn, for its growth, and for its ripening, but no one place has all these conditions Nevertheless, at elevations of 3,000 to 6,000 feet where the best Javanese corn is grown, the climate for several months is favorable after the corn is once started It is also delightful for human beings, except that the uniform temperature is monotonous Most of the Javanese corn, however, is grown at low altitudes on the neighboring island of Madura The soil there is not so good as the dark volcanic soil of Java, and there is less water for irrigation, so that rice cannot be so fully relied on Hence the people turn to corn as a main food crop The climatic conditions depart so far from the optimum that the average yield per acre falls to scarcely more than 12 bushels, the average for all Java being 14 Poor cultivation and poor soil have much to do with this, but in such a climate even the best cultivation would not give yields like those of Iowa or Connecticut

324 All of Canada except southern Ontario and the St Lawrence Valley as far as Montreal lies beyond the northern limit of corn So does most of Europe, except the south and southeast from Portugal through southern France and Hungary to southern Russia and Bulgaria Most of Asia is either too cold, as in the north, too dry, as in the west and center, especially in summer, or else too warm and moist As a result the whole mainland raises scarcely 5 per cent as much as the United States Even where the summer rainfall is abundant, as in China, it is often difficult to raise corn without irrigation because the spring is too dry and the summer too wet In warm, tropical regions the combined effects of heat and moisture usually limit corn to restricted areas in the highlands This handicap and the great desert combine to cause Africa to raise less than half as much as South America, although much more than Asia The African corn grows mainly in Egypt, with the help of irrigation, and in the parts of South Africa where summer rains prevail In South America, Argentina is the only country with a really good yield per acre Nevertheless, a yield of about 30 bushels per acre in Argentina, where corn

is very important, is not so good as that of the North Atlantic and New England States where the total production of corn is small, or of the Corn Belt where it is large Corn forms the most important food in Brazil and many Andean regions Nevertheless, the small yield per acre there shows that either the climate, the soil, the cultivation, or some other condition departs from the optimum much more than in Argentina

325 *Why Europe Is Not a Great Corn Continent* Although the corn of foreign countries as a whole has a yield per acre far less than in the wedge from southern New England to Iowa, the southern tip of Ontario and the Swiss and Italian valleys at the southern base of the Alps almost rival New England Certain irrigated areas, such as Egypt, stand at the Corn Belt level, or higher, as does a small area in New Zealand Austria and northern Italy average close to 30 bushels per acre Otherwise even the best regions, such as Hungary and Japan, raise only about 25 bushels per acre One of the significant facts about all this is that the regions where corn is the staple food, such as Brazil and Rumania, are not the ones where the yield per acre is highest We have already seen that this is true also in the United States

326 In Europe corn gets more space than wheat only in the southeastern countries of Yugoslavia and Rumania. It gets more than half as much space as wheat in another southeastern country, Bulgaria, in a central country, Hungary, and in a far southwestern country, Portugal. These corn regions show a strong climatic resemblance to those of the United States, but with interesting differences, as appears in the lower line of A318 Hungary, although nearly ideal in temperature, is a little too dry at all seasons Hence, a yield of 24 bushels per acre is about what one would expect This suggests the value of cyclonic storms Both the United States and Europe have such storms, but they are more numerous and bring more rain in the interior of the United States than in that of Europe Hungary is nearer the sea than Iowa, but because it has fewer and weaker cyclonic storms it gets less rain, especially in summer. Moreover, it is shut off from the Mediterranean Sea by mountains so that there is no free inward sweep like that which brings moisture from the Gulf of Mexico to Iowa These facts in themselves, even if all other conditions were similar, would make its crop smaller and less regular and its farmers poorer than those of Iowa Bulgaria is not quite so favored as Hungary Even its best portion, represented by Philippopolis in A318, is a little drier than Hungary, and the danger from drought is greater Most of the corn-raising part of the country is both warmer and drier than Philippopolis, so that Bulgaria's yield of 16 bushels per acre as

the average for all sorts of farms is about as much as could be expected. Here, as in every other case, it is surprising to see how closely the yield per acre harmonizes with the climate.

327. Europe is not a corn continent. If the United States had no areas where the climate approaches the optimum for corn any more closely than does the climate in the main corn areas of Europe, our total yield would probably be less than 750 million bushels instead of 2,750 million. In other words, our climate is so different from that of Europe that we raise more than 2,000 million bushels of corn in climates which have no exact counterpart in Europe. Most of this is raised in the Corn Belt (A251), an area of half a million square miles, which is popularly supposed to be much like the interior of Europe in climate. Of course there is similarity, but there is enough difference so that, even with their intensive methods of cultivation, the Europeans do not rival the yields obtained in the better parts of the United States. The fact that intensive cultivation in Europe does not produce yields as great as much less intensive cultivation in states such as Ohio indicates how greatly corn depends on climate. The same is true of almost every other crop, as we shall soon see in the case of wheat.

Chapter XIV

WHEAT, AN EXAMPLE OF LAND UTILIZATION

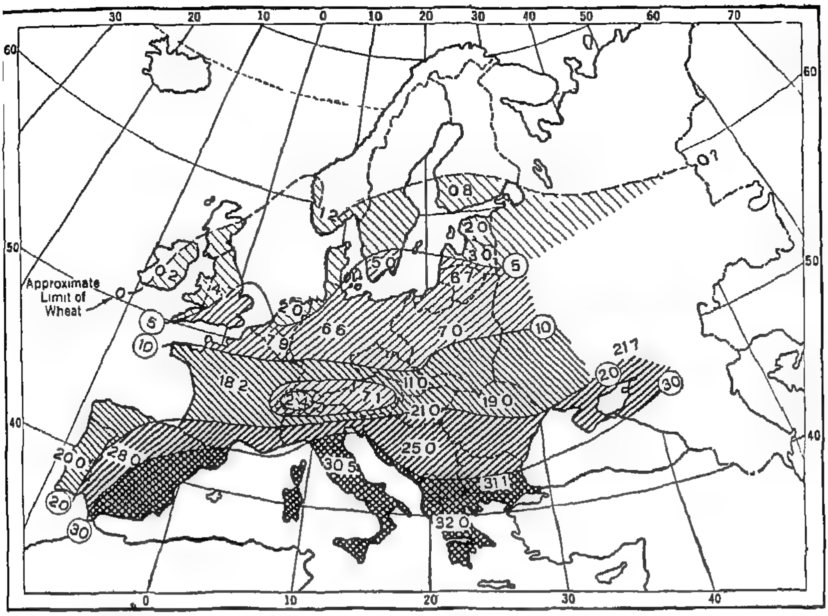
328 Worldwide Distribution of Wheat The law of optima is so important that we will examine the optimum for a crop more widely distributed than corn We have seen that climate, soil, and relief limit wheat to about one tenth of the lands of the earth, and that it is cultivated on only one tenth of this tenth Nevertheless, no other crop is so widespread, nor does any other provide so large a share of human food Wheat supplies about 30 per cent of the calories or heat units in the diet of the United States, and still more in that of western and southern Europe This wide use is due to several causes (1) Long experience has proved that cereals are the best kind of staple food crops by reason of their nutritive value, ease of cultivation, yield per acre, keeping qualities, small bulk, and adaptability to diverse climates and soils (2) Among the cereals wheat is especially well balanced in proteins, carbohydrates, and vitamins Only oats excel it in this respect, but they are twice as bulky and do not keep so well (3) Wheat is also the most appetizing of the cereals as a steady diet Hence, wherever it is possible, the majority of mankind choose wheat rather than rice, corn, rye, barley, or millet (4) Wheat will grow in a greater variety of climates than any other food crop except barley The fact that it can be sown either in the fall or spring helps in this respect So, too, does the fact that many new varieties, adapted to extremes of climate and soil, have been developed (5) Wheat is so hard, firm, and free from oil that it keeps better than any other cereal, and can stand long transportation

329. Diverse Qualities of Wheat Regions With wheat, just as with corn or any other crop, the optimum is not necessarily found where the crop is raised most abundantly In other words, the optimum for the *growth of the crop itself* is one thing, and the optimum for the *business of raising the crop* is another For the present we are interested in the optimum for the crop The optimum for the business will be considered later The wide contrast between the two kinds of optima is evident when B and D330, showing the yield of wheat per acre in Europe and the United States, are compared with

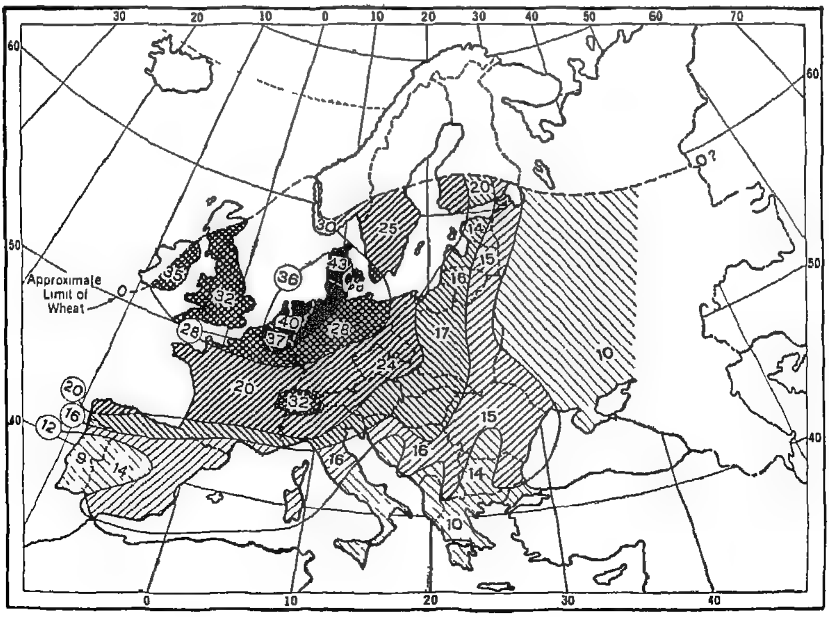
A and C330, showing the percentage of harvested land devoted to the crop in the same regions. With wheat, as with other crops, the five main factors in determining the yield per acre, and hence the optimum for productivity, are the physical factors of soil, relief, and climate, the biological factor of pests, and the human factor of cultivation, taking this last word in the broadest sense to include selection of seed and improvement of varieties, as well as fertilization, plowing, and so forth

330 I *Wheat and the Soil* Wheat is raised in a great variety of soils from the rather poor sandy loams of Flanders and the pale brown stony soils of Maine to the richest black earth in southern Russia, Siberia, Australia, and the plains from Saskatchewan to Texas In any one locality the yield per acre is much larger upon rich black soil than upon poor pale soils. Nevertheless, when the average yield per acre for large areas is considered, high yields and good soil do not go together The soil around the North Sea is far poorer than in southern Russia, but B330 shows a yield per acre about three times as great in the poor soil as in the good British Columbia and Maine have not nearly such good soil as Alberta, the Dakotas, and Kansas, but they produce twice as much wheat per acre (D330) The *total production per square mile* may be either greater or smaller on the good soils than on the poor That depends not only on yield per acre, but on relief, economic demand, and other conditions which determine what percentage of the land is used for all crops and for wheat in particular. The point to be emphasized here is that the general distribution of both acreage and yield of wheat, as indicated in A and B330, show even less relation to the general distribution of types of soil than does that of corn. This by no means indicates that soil is unimportant, but merely that its influence is concealed by other conditions.

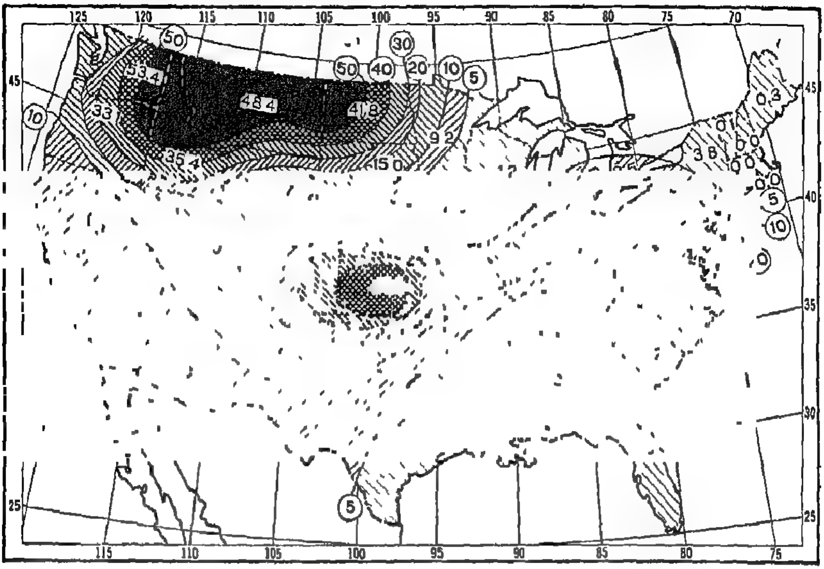
331 II *Wheat in Relation to Relief and Pests* Although the relief of the earth's surface is a factor in the general optimum for wheat, it does not play any appreciable part in determining the appearance of B and D330 Mountainous Switzerland and level Netherlands both get high yields per acre, and so do rugged Vermont and the level parts of the plateaus of eastern Washington In similar fashion level Russia and mountainous Greece get low yields, as do level North Dakota and rugged Kentucky Locally, to be sure, the effect of relief is great, but it fades into insignificance when regions with widely different climates are considered Pests likewise may be dismissed briefly As local factors in causing a high or low yield per acre they certainly play an important part, but their effect varies almost in harmony with that of climate. If the weather is good for wheat, pests have relatively



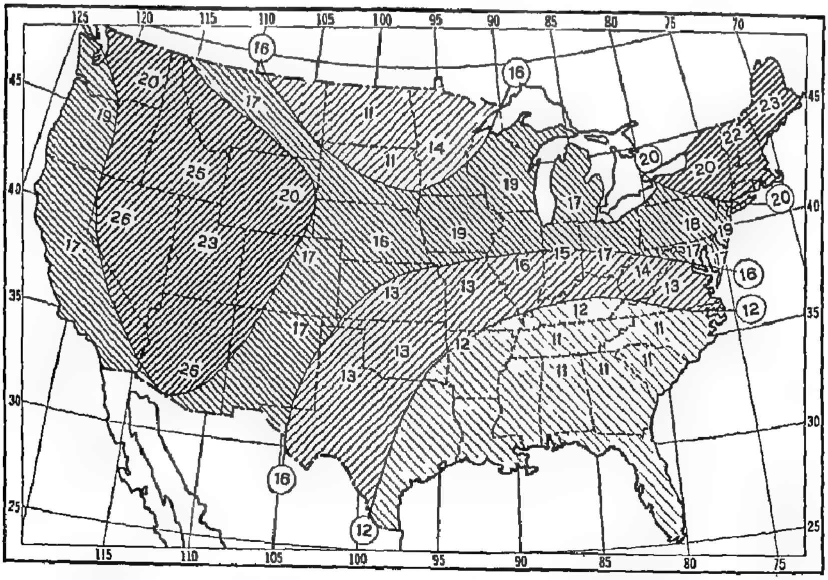
A330—Percentage of Harvested Land Devoted to Wheat in Europe



B330—Average Annual European Yield of Wheat per Acre, in Bushels, 1910-1929.



C330—Percentage of Harvested Land Devoted to Wheat in the United States



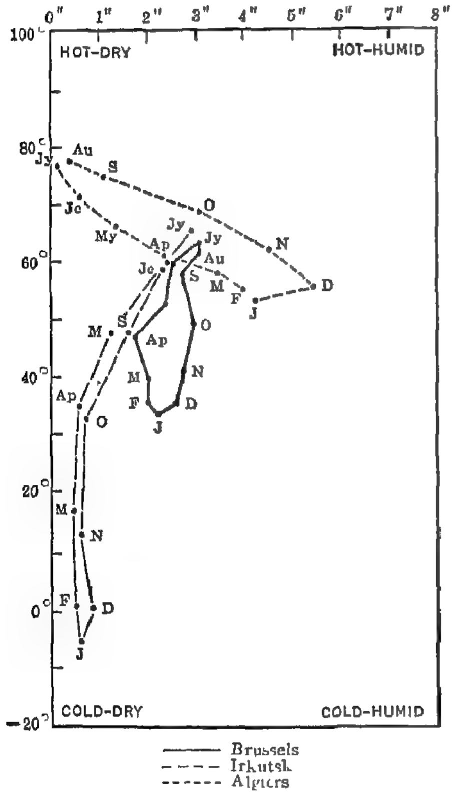
D330—Average Annual American Yield of Wheat per Acre, in Bushels, 1910-1929

little influence, if it is bad, they do terrible damage. This leaves only climate and cultivation as major factors to be considered in our attempt to find out why the yield of wheat varies from more than 40 bushels per acre around the North Sea to no more than 10 in such regions as Portugal, Greece, and parts of Russia

332 III *Methods of Cultivation and Stage of Civilization* The methods of cultivating wheat differ from region to region as much as do soils, but their effect on the yield of wheat per acre is much less than one would expect In some unprogressive regions where the yield of wheat is only 10 or 12 bushels per acre, most of the wheat fields are merely scratched to a depth of 4 or 5 inches with a wooden plow Parts of Turkey are of this kind The wheat is harvested in the primitive back-breaking way by men and women who bend over from the hips with sickles It is carried to the threshing floor in creaking carts, perhaps with solid wooden wheels, or on the backs of donkeys, or even of men and women In the remoter parts of the country, where modern methods have not yet been introduced, the wheat is threshed by driving cattle and donkeys around and around over it on a floor of dried mud. Finally the grain that has been trampled out by the feet of the animals is winnowed by a man who throws the mixture of finely broken straw and golden grain into the air with a shovel When the wind dies down and no longer blows the light chaff away from the heavier grain, the man may stop work and whistle, thinking thus to call the wind back again Strange as it may seem, a yield per acre practically the same as that of Turkey is also obtained in some of the most advanced parts of the world where the modern methods are used Examples of this are found in the Dakotas, the interior valley of California, large sections of Australia, and the plains of southeastern Russia In some of these places great effort has been expended in developing improved varieties of wheat. The methods of cultivation are ultra-modern with tractors pulling huge plows that bite deeply into the soil and turn the old straw under for fertilizer There the combine cuts, threshes, and loads 500 bushels of grain with the same amount of human labor that is needed to cut, stack, thresh, and bag 1 bushel on the more primitive farms of Turkey Thus we have the most extreme difference in methods of cultivation, but the yield per acre in the advanced regions is scarcely more than in those that are most primitive

333. Large yields per acre as well as small are found where both primitive and advanced methods of cultivation are employed Because of the effect of the best wheat climates on man, as well as for other reasons, the contrast in methods in these cases is by no means

so great as in the areas of low yield In Japan, which has a high yield per acre like that of Maine, the methods of cultivation are almost as primitive as in Turkey, except that the fields are well fertilized In British Columbia, where the yield per acre is still large, the methods of cultivation are of the most advanced type except that only a little fertilizer is used Around the North Sea where the yield is highest of all, the wheat is well fertilized and the methods of culture are well advanced, except that no great amount of machinery is employed Much of the wheat is cut with horse-drawn reapers, which lay the stalks in piles ready to be bound into sheaves A considerable amount, however, is still harvested by hand with cradles, that is, with scythes to which are attached wooden fingers that stick up a foot or more and gather the stalks into piles that can easily be made into sheaves Farther south and east in Europe the amount of hand work increases, the use of fertilizer diminishes, and the yield per acre falls off sharply Nevertheless, the yield per acre in Spain, Italy, and Rumania is about the same as in West Virginia, Illinois, and Indiana Thus the yield of wheat per acre has surprisingly little connection with the extent to which modern methods are employed, and agrees only imperfectly with the extent to which fertilizers are added to the soil



A334—Climograph of Wheat Regions

334 IV. *Climate and Wheat* The climates in which wheat is raised are almost as diverse as the soils and the methods of cultivation. Three diverse types of wheat climate are shown in A334 The heavy solid line represents Brussels, in the world's best wheat region The Maritime Cyclonic climate of Belgium has abundant but not excessive rain at all seasons It enjoys mild winters with only a little

frost and snow, and delightful summers just warm enough to be comfortable. The climograph for Irkutsk in Siberia illustrates a Continental mid-latitude type with mid-summer conditions not much different from those of Belgium, but with a very brief spring and fall, and a long and very severe winter. The climograph for Algiers is typical of a Mediterranean climate with warm, rainy winters, a short spring and fall, and a long, hot, rainless summer.

335 Diverse as these three climates are, they all have one feature in common. Their climographs come together, which means that the weather in all three is similar, at the time when wheat is making its most rapid growth, namely, April in Algiers, and June in Brussels and Irkutsk. In Brussels essentially this same kind of weather continues through July, August, and September; at Irkutsk it continues through July and August, but September becomes quite cool; at Algiers both March and May can be grouped with April so far as temperature is concerned, but March is more rainy and May much drier. The significant point about these three climographs is that in all of them the average temperature and total rainfall of the three months before the wheat is harvested are almost alike. The preceding months, however, when the wheat is making its early growth, differ greatly in the three regions. In Belgium, where 37 bushels of wheat are produced per acre, they are rainy and cool, but not cold. In Algiers, where the yield drops to only 10 bushels or less, they are rainy and warm. In Irkutsk, where the yield is a little better than in Algeria but far below that of Belgium, the climate is so cold that winter wheat cannot be raised, and wheat must be sown in late April or early May. Examination of other wheat regions shows that wheat grows only in climates which at some time during the year have two months of weather similar to that which prevails when the three climographs of A334 come together. The yield per acre depends largely on how far the weather during the rest of the year departs from the optimum which is discussed in the next paragraph.

336 THE CLIMATIC OPTIMUM FOR WHEAT. In order to find out still more about the optimum climate for wheat, let us follow the same method as with corn. Data from 18 European countries and 21 American states for each month during many years show that the optimum climate for wheat differs very little from that of Denmark. In summer the climate of that country is practically identical with that of Belgium, and even in winter it is only a degree or two colder and a little drier. Hence, the climograph for Brussels in A334 comes close to showing the optimum climate for wheat, just as that of Hartford does for corn. When we examine the yield of wheat per acre

in the rest of the world we find that it varies in harmony with the extent to which the climate differs from that of Denmark B330 shows that Denmark with a yield of 43 bushels outranks all other countries It is closely followed by Holland (40) and Belgium (37) A large neighboring area around the North Sea harvests at least 28 bushels per acre, as does Switzerland Farther away, as we have seen, the yield falls off until it reaches a level of only about 10 bushels in southeastern Russia, Greece, and Portugal. In the United States, a yield of more than 20 bushels is found only in the high plateau states of the West, where the crop is often helped by irrigation, and in New York and New England (D330) In British Columbia, where the climate of the wheat-raising section approaches the North Sea type more nearly than anywhere else in North America,' the yield rises to about 26 bushels

337 *The Value of Cool Summers and Mild Winters* All this is interesting because it shows the value of cool summers and mild winters, a value which we shall see again when we study human health and efficiency Even with corn, the yield per acre declines where the warmest month has an average temperature above 72° Wheat gives its highest yields only where the temperature of the warmest month is not far from 63° F and that of the coldest month about 32° to 35° In the United States the nearest approach to such cool summers is found where D330 is heavily shaded Only around Puget Sound, however, are such cool summers associated with mild winters, and there we find the greatest yield of wheat per acre ever recorded in America In 1915 on a soil of only moderate quality a 15-acre field in Island County, Washington, produced 117 bushels per acre In Europe cool, moist summers of the kind best for wheat prevail along the coast from northwestern Spain to Finland, as well as in Switzerland They also extend well inland in Germany The central part of this coastal area, around the North Sea, is especially good because there the winters, as well as the summers are mild.

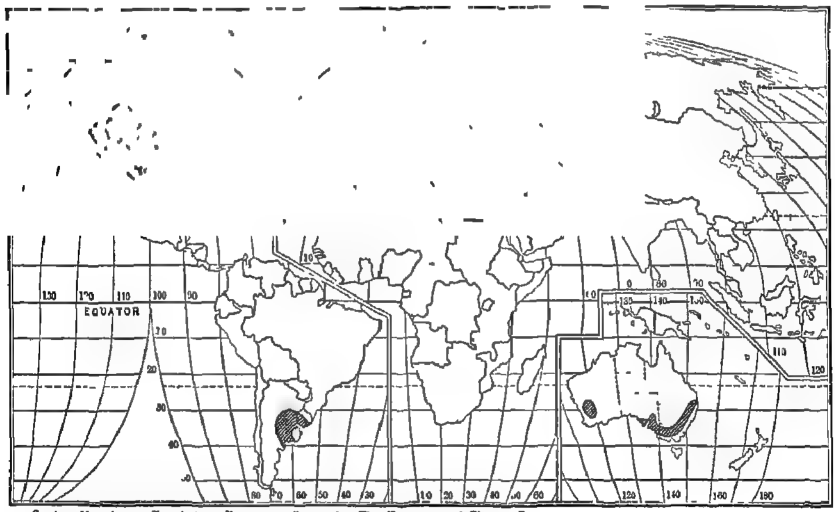
338 Climatic optima play so large a part in economic geography that it is desirable to inquire exactly how the mild climate of regions near the North Sea and Puget Sound helps the growth of wheat. One of the first things is that in such coastal areas, where west winds constantly bring to the land the influence of the sea, variations in the yield of crops from year to year are comparatively insignificant. There is more than this to the matter, however When the average temperature in winter falls below 32°, as it does in most of the great wheat regions of the world, winter wheat that has been sown the previous fall must stop growing. Aside from Australia, Argentina, and

India, most of the regions that are great exporters of wheat are so cold that wheat can scarcely grow at all for three months, more or less Meanwhile, in the regions near the North Sea and Puget Sound the wheat is enjoying temperatures which average above freezing most of the time Hence not only is it able to grow a little, but it is encouraged to tiller or stool This means that it divides into several stalks so that each plant produces several heads of wheat instead of only one Naturally the yield is greatly influenced in this way. Tillering occurs only in the early stages of growth and only when growth is slow Therefore it occurs only under certain special conditions

339 So important is tillering that in regions such as Mesopotamia, Palestine, Oklahoma, and Texas, where the winter is warm and the spring soon becomes dry, the people allow cattle and sheep to graze on the growing wheat during the cooler months This not only provides excellent pasturage, but also causes the wheat to send up more shoots from each root If the winters are cold enough to kill wheat which is planted in the fall, spring wheat must be planted Since it must grow fast in order to ripen during the short summers which prevail in such climates, it cannot tiller so much as winter wheat, or produce such abundant grain, unless the early summer days are exceptionally long and growth is little checked at night Such is the case in the Peace River Valley of Canada, where the yield of spring wheat often rises to 40 bushels per acre Denmark, in about the same latitude (56° N), owes its high yield of winter wheat in part to the long days It is interesting to see how systematically the yield per acre in B and D330 falls off as the climate departs more and more from the North Sea optimum In this respect, wheat behaves like corn, and indeed like every other agricultural product that has been investigated Nevertheless, we must study the matter further before deciding how much of the agreement between climate and yield per acre is due directly to climate and how much to other influences

340 WHERE WHEAT IS RAISED I *Main Regions of Surplus and Export.* One of the problems in respect to every crop is to discover in what regions it pays best to raise it The utilization of land for wheat differs from its utilization for corn because of several important conditions Wheat can be raised in so great a variety of geographic regions and can be so easily transported that the price is greatly influenced by the crops of the world as a whole, as well as by local supply and demand Wheat is used almost wholly for human food and only rarely for animals Again, the methods of cultivating wheat vary far more than those for corn The most heavily shaded parts of A340 produce a surplus of wheat that can be steadily relied upon.

The areas that are lightly shaded with lines produce more wheat than is needed for local consumption, but the exportable surplus is small compared with the population and often disappears in bad years. A comparison of this map with A50 shows that, although certain areas, such as Manchukuo, India, France, and Italy, produce wheat sometimes in large quantities, they have little or no surplus. In fact, a *large* surplus which can be relied on year after year is found in only three regions. The most important of these extends from Oklahoma on the south to central Alberta on the north, and reaches westward to Washington and Oregon. It is divided between the United



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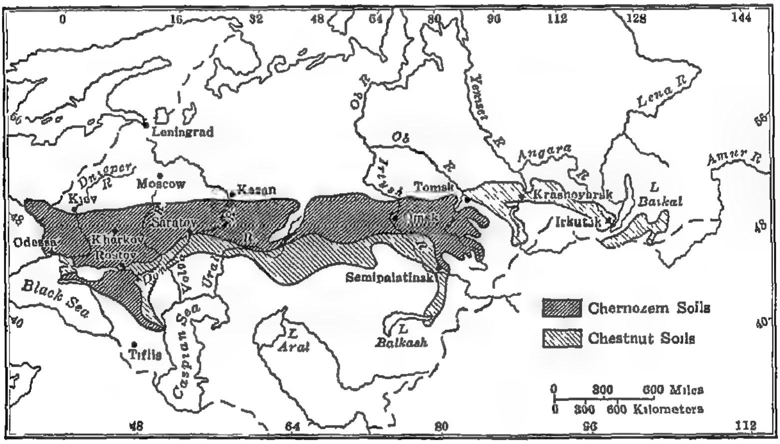
A340—Regions of Wheat Production

States and Canada, winter wheat being raised in the south and spring wheat in the north. It ships its wheat mainly eastward via the Great Lakes to the eastern United States and Canada and to Europe, but some goes west to Puget Sound, some south by way of the Mississippi River, and a little has begun to go to Europe via Hudson Bay. Nevertheless, this North American wheat area is distinctly a unit. Its surplus is so huge that it not only feeds the United States and Canada, but also furnishes about half of all the wheat that enters into foreign commerce.

341 Second in importance among wheat-exporting areas comes the South American wheat region, mainly in central Argentina. Its surplus for export is often more than half as great as that of North

America. This is partly because the number of people to whom wheat is supplied locally is only about a tenth as great as in the United States and Canada The third large and reliable exporter of wheat is Australia, although scanty rainfall is there especially likely to cut off much of the export. Most of the Australian wheat comes from a relatively narrow strip in the southeast on the edge of the plateau a little way back from the coast, and from a similar but much smaller strip in the southwest In New South Wales the surplus of wheat per capita averages about 12 bushels while in South Australia it rises to 48

342 *Other Export Regions* The southeastern edge of the great Russian wheat region, not far from the Caspian Sea, may almost be



A342—Map of the Chernozem, or Black Earths of Soviet Russia

classed with the North American, South American, and Australian wheat regions. This is the section where the Russians have established their huge state farms The surplus of wheat in a section as large as several of our states rises approximately as high as in North Dakota, Saskatchewan, or West Australia This section, however, grades off into the main wheat area of Russia, that is, into the vast Black Earth or "chernozem" region and its extension of dark chestnut soils extending from Ukraine to Irkutsk (A342) In that area, although a huge amount of wheat is raised, the surplus is relatively small, and disappears in bad years If Russian industry and prosperity continue to grow, it is doubtful whether wheat production, in normal years, can more than keep pace with the increase in the demand for wheat within the country.

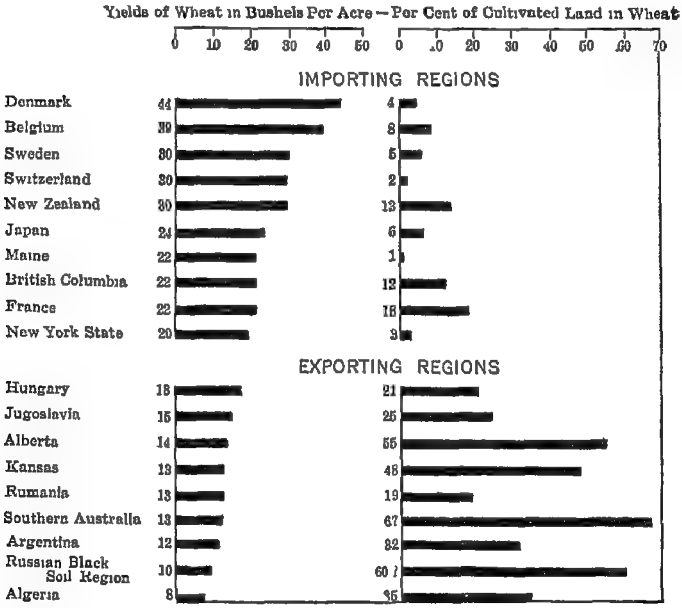
343 Two other areas produce a small but fairly steady surplus of wheat for export One lies in French North Africa, and includes Tunisia, Algeria, and Morocco. The other, in southeastern Europe, includes Rumania, Yugoslavia, and Hungary Until a few years ago, Manchukuo was also supposed to be a great reservoir from which wheat could be drawn for export, but this is fast ceasing to be true Not only is the local population increasing more rapidly than the wheat supply, but the farms are small and the methods of cultivation primitive, so that the surplus raised by any individual farmer is small Moreover, farmers are finding that soybeans pay better than wheat This is not surprising when one notes how far the heavy summer rains and cold, dry winters of Mukden depart from the evenly distributed rainfall and mild winters of Brussels in A334

344 CONTRAST BETWEEN EXPORTING AND IMPORTING WHEAT COUNTRIES I. *Quantity and Quality* In A340 large areas show the light dots which indicate that a country raises some wheat, but not enough for export By far the most important such region is Europe, omitting Russia and the southeast There about a billion bushels, or about a quarter of the world's supply, is raised India, China, Japan, Turkey, Iran, and certain lesser countries raise another quarter, and export relatively little Most of them ought not to export any at all, for their people are underfed much of the time In a general way the countries that raise wheat but have no surplus are more humid than those which have a surplus They are also more densely populated Their wheat tends to be softer and less valuable than that of the drier and colder export regions Although regions with warm winters and cool summers, such as prevail around the North Sea, raise the largest crops per acre, their wheat is comparatively soft and does not make the best bread In a general way the wheat becomes harder as one goes to regions that are either drier or colder The hardest wheat, except the macaroni or durum wheat of Italy and North Africa, comes from cold and fairly dry regions such as North Dakota, the Canadian Northwest, and southwestern Siberia In northwestern Europe it is mixed with softer wheat to make bread that rises well Hence it brings a somewhat higher price than the homegrown European wheat, but not high enough to make it anywhere nearly so profitable acre for acre Nevertheless, the quality of the wheat is one reason for raising it in regions where the yield per acre is low

345 II *Yield per Acre* The yield of wheat per acre in regions that export wheat averages low Compare the yields in such regions during a recent five-year period as shown on the left in the lower part of A345 with those shown in the upper part for regions which do not

raise enough to feed their own people Then look at the percentages of the cultivated area devoted to wheat in the two groups of countries as shown on the right. The exporting countries have yields of 8 to 18 bushels per acre Yet they devote 19 to 67 per cent of their harvested land to wheat The importing countries have yields of 20 to 44 bushels per acre Yet they devote only 1 to 18 per cent of their land to wheat The reason for this is economic, as appears in the next paragraph

346 III *Competition of Wheat with Other Crops* Wheat, like corn, is often pushed out of the regions where it grows best because



A345—Contrasts between Wheat Production in Importing and Exporting Regions

other crops pay better or are more desirable The regions that are best for wheat are also good for a variety of other crops such as potatoes, oats, apples, green vegetables, and sometimes corn Corn pays so well in Iowa that it would be foolish to devote much space to wheat. In Denmark, oats and barley drive out wheat because they furnish feed for animals, just as corn does in Iowa. The Danish farmers have found nothing that pays them better than to raise these crops and others as food for hogs, chickens, and cattle, the products of which can be shipped to Great Britain or elsewhere for good prices Germany and the former Czechoslovakia and Austria find it profitable to

devote at least as much land to potatoes as to wheat. In Yugoslavia, however, the yield of potatoes is only one third as great as in Germany. Therefore potatoes are not very profitable and the farmers fall back on wheat. Another important factor is that the climate and other conditions in the areas where wheat grows best are so favorable to man that the population tends to be healthy, energetic, and therefore both prosperous and dense. Such people can afford to pay well for a good diet which includes a large amount of vegetables, fruit, meat, milk, and eggs. The British farmer finds it profitable to raise these products and eat bread made of wheat from Canada or Australia. Even in some new regions, such as New Zealand and British Columbia, this is fast becoming the case.

347. POSITIVE FACTORS IN LOCATION OF WHEAT CULTURE. I. *Soil*. Certain definite geographical advantages make it worth while to raise a great deal of wheat in regions where the yield per acre is low. One of these is rich, black, or dark chestnut soil (Plate II). The great wheat-exporting regions of North and South America, Eurasia, and Australia are all located in relatively dry climates which favor the growth of grass rather than trees. Accordingly all of them have gradually acquired a deep covering of rich, dark soil. A342 and Plate II show the vast extent of this kind of soil. Its distribution agrees quite closely with that of the main areas that have a surplus of wheat for export. This is true even in India and China, the drier parts of which export wheat in good years, even though other parts are close to starvation. Such soil has some advantages which partially counteract the ill effects of aridity. It holds water unusually well, it is easy to cultivate, for it breaks up readily under the plow and harrow and is free from stones, and it is so rich that it suffers relatively little from repeated cropping without the addition of fertilizers.

348. II. *Climate of Wheat-Exporting Regions*. The same dry climate which reduces the yield per acre also makes it possible for each farmer in the three great exporting regions to cultivate a large area. The dryness frees the farmer from the delays which arise elsewhere from wet soggy fields for several weeks in the spring and fall, and after rain at other seasons. Hence the period during which he can prepare the soil and sow it is longer than in moister climates. A dry climate also lengthens the period during which harvesting is possible. It does this by reducing the danger that the wheat will be beaten down and ruined by rain and wind, or will have the ripe kernels shaken out or rendered moldy. These conditions lessen the actual work of harvesting because in dry climates there is not much need of tying

the wheat into sheaves and stacking the sheaves in shocks in order to protect the grain and let it dry more fully before threshing. Another important item is that a dry climate makes it possible to use machinery on a large scale This is true of plowing and harrowing because the soil in semi-arid regions is not only dry, but usually friable and easily worked At harvest time, since the grain can safely be left to ripen on the stalk, it is possible to use combines In all these ways the cost of raising wheat is reduced In producing wheat the use of labor-saving machinery becomes more and more feasible and profitable as one goes from a moist to a dry climate. Hence in dry climates one man's work will cultivate several times as many acres as in more humid regions

349 Dry Farming The world's growing demand for wheat, the development of agricultural machinery and the fact that semi-arid land was available for settlement have led to the peculiar form of agriculture known as "dry farming" According to this method the land must be harrowed several times for each crop before the seed is sown There is no assurance of a crop unless the ground is moistened to a depth of 2 or 3 feet. In dry climates the rain of a single season is often not sufficient for this unless evaporation from the surface is prevented. One method of doing this consists of shallow harrowing so that a layer of dry dust lies on top of the field and there are no weeds to suck up moisture and pass it out into the air In this way the rainfall of two years can be stored in the ground, thus giving a crop every other year. Such dry farming is too expensive except where the following conditions prevail. (1) land can be procured cheaply in large amounts; (2) the land is fairly level, (3) the soil is rich, easily worked, and good at retaining moisture, and (4) the work can be done cheaply These conditions prevail in the drier parts of the heavily shaded areas of Plate II in North America, South America, Australia, and Asia, between latitudes 25° and 55°. These, then, are the places where wheat can be raised most cheaply. It was because of such conditions that the Soviet Union established its huge state farms in the region north of the Caspian Sea where the population had previously consisted largely of a few nomadic keepers of cattle, sheep, camels, and horses

350. The Cost of Wheat-Raising The gain that comes through using the most modern methods in semi-arid plains is illustrated by the following figures, based on government reports and showing the number of hours of work by one man required to cultivate an acre of wheat under various conditions·

Hours	Country	Rainfall in inches	Yield in Bushels per acre
100 or more	Primitive parts of Turkey, Iran, etc	15–20	10–14
60 or more	Poland, Yugoslavia	20–30	15–18
20–25	Advanced European countries such as England, eastern United States	25–45	17–42
15	Prairie states, Indiana, Illinois	30–40	15–20
5	Dry plains, Dakotas, southeastern Russia	15–25	10–15
2	Same regions as last	15–25	10–15

Among the 100-hour people the primitive wooden plows are drawn by oxen, the crop is reaped with sickles, tied in sheaves with strings of straw, and placed in shocks Then it is carried on the backs of donkeys to a floor of dried mud, where it is threshed by driving donkeys and oxen around on it Finally it is winnowed by throwing the mixed chaff and grain up into wind. In the more advanced countries where 60 hours are needed small plows are drawn by one horse After the grain has been harvested by hand with scythes, it is tied in sheaves, carried home in carts, threshed with flails by hand, and then winnowed by small hand machines Where the time devoted to an acre of wheat falls to 25 hours the farmers use good two-horse plows and reaping machines, and thresh the grain with machines run by engines Large types of machinery, however, are not practicable because the fields are generally small and rain is frequent at all seasons A further reduction to 15 hours is possible where methods like those just described for the eastern United States are employed, but larger machines can be used because of larger and more level fields, as in our prairie states In the drier parts of the prairies it became possible to reduce the time to 5 hours by means of large gang plows, combines, and other types of large machinery A reduction to 2 hours becomes possible only on the biggest farms of relatively dry plains where tractors and combines sometimes travel miles without stopping or turning. On one such farm of about 100,000 acres 30 combines worked day and night, each threshing 1,000 bushels a day The tractors worked 2,800 hours per year instead of only 250 as on the average farm in Iowa. Two men on tractors and one on a combine did a job which formerly required 28 men and 32 horses Wheat was raised for 25 cents a bushel at a time when it cost a dollar to raise it in the East

351 The preceding figures show that there are great advantages in raising wheat in comparatively dry regions, even though the yield

per acre is small. When we take account of this fact, as well as of the competition of more valuable crops in regions of larger yield, it is not surprising that most of the wheat that enters into world trade is raised on dry plains where the yield per acre is small Nevertheless, there are dangers in this The big farm described in the preceding paragraph was put out of business by a combination of two disasters One was the low prices of agricultural products after the post-war boom collapsed in 1929. The other was the great droughts of the 1930's These conditions, especially the droughts, gave dry farming a terrible setback A large part of the dry farmers had to abandon their farms The rich dusty soil on the tops of their fields was blown away in many places to a depth of several inches by dry winds that swept over the country year after year.

352 UNCERTAINTY VERSUS CERTAINTY This brings us back to our discussion of uncertain agriculture in an earlier chapter The great regions that export wheat belong to the Realm of Uncertain Agriculture, or at least lie near its borders They are able to maintain a prosperous population only when they have two main advantages, namely, plenty of space in comparison with the population, and conditions of climate, relief, and soil that make it possible to practice modern methods which deserve to be called marvelous Hence in periods of good rainfall they give the farmer a rich reward Unfortunately they also have two serious disadvantages One is extreme unreliability from year to year, the other is a limited range of possibilities. Barley might be raised, and it would resist drought better than wheat, but there is little market for it Most other crops will not grow without more moisture In the dry plains of the United States, Canada, the Soviet Union, Argentina, Australia, and to a less degree northern China and northwestern India people raise wheat mainly because they are confronted by a choice between that crop and cattle They can make a greater profit from wheat, especially if the most modern methods are used Nevertheless, they run great risks of failure This leads to political discontent and migration in the more advanced and less densely populated regions, and to famine in those that are backward and densely populated in proportion to their resources

353 The extent to which the wheat crop enters into commerce and hence is talked about is by no means proportional to the total amount that is raised or to the amount per square mile, but to the surplus beyond the needs of the local population We hear little about France as a producer of wheat, but in late years it has produced twice as much as Australia or Argentina, and as much as Canada,

Only the Soviet Union, the United States, China, and India produce more Their wheat-raising sections are many times as large as those of France (A50). France simply goes its own way, producing a good but not an extreme yield on about the same number of acres every year Some years it feeds itself entirely, in others it has to supplement the wheat harvest somewhat by imports Normally it raises about 1,300 bushels for every square mile of territory In proportion to its total area France raises more wheat than any states or provinces in North America except Kansas, North Dakota, and Saskatchewan Aside from Egypt, which raises about 3,000 bushels for every square mile of its habitable area, Saskatchewan with about 2,000 leads the world Other countries that raise a great deal per square mile include Hungary, Italy, Belgium, and England. The wheat of these countries attracts attention chiefly when war or the danger of war leads their governments to make efforts to increase the homegrown wheat supply and thus become self-supporting This brings us again to a principle which will be discussed more fully later *the geographical distribution of any occupation or business depends on at least four factors* (1) *the physical optimum for the products concerned,* (2) *economic factors, such as markets and competing products,* (3) *political factors, and* (4) *the energy and skill of the people*

Chapter XV

HUMAN SKILL AND AGRICULTURAL PROSPERITY

354 Nature's Part Compared with Man's In this chapter we shall consider a problem raised by the preceding discussion of corn and wheat, namely, the relative parts played by physical environment and man in determining agricultural productivity Two examples will illustrate two phases of the matter, namely, local differences, which we shall deal with very briefly, and regional differences, which are the main subject of this chapter In central Illinois, as explained on page 133, an average yield of 41 bushels of corn per acre gives place to 21 bushels only 70 miles further south These diverse yields occur so close together that any possible difference in climate could not cause the productivity in one to be nearly twice as great as in the other In both places the relief of the earth's surface is so gentle that it cannot account for the difference On the other hand, differences in the soil are fully adequate to account for the contrast in the crops The high yields of central Illinois, it will be remembered, are obtained from a young, dark, unleached, glacial soil This contains plenty of calcium because it has not been leached, and it is full of nitrates because it was formed under a cover of grass The low yields a little farther south are obtained on an old, pale, gray soil of forest origin, which has been a good deal leached, and has only a small supply of both nitrates and lime A270 provides a similar illustration in Kentucky except that here the rugged relief, as well as poor soil, causes the farm land in the southeastern plateau to be worth only $12 to $20 per acre, while that of the Blue Grass Region close by, where excellent soil and gentle relief help the farmer to get good crops, is worth about ten times as much

355 In both Illinois and Kentucky there is a difference in the methods of cultivation on the good land and on the poor Here, just as in Iowa, the good soil is better cultivated than the poor soil. It gets more fertilizer, is plowed more deeply, and, as a rule, is cultivated and weeded more thoroughly Our problem is to discover what connection, if any, there is between the poor soil and the poor cultivation

356. The problem becomes much more pronounced, but is of the same nature, when we compare the productivity of large regions which differ in climate. In B313, showing the yield of corn per acre in the United States, and in B330, showing the yield of wheat per acre in Europe, local differences such as those which have just been mentioned in Illinois, Kentucky, and Iowa disappear. The general pattern of productivity which forms the most striking feature of these maps must be due to some factor which varies gradually from region to region without the sudden transitions which arise from soil and relief. We know that the pattern of these maps does not arise from the relief of the earth's surface because great variations in productivity are noted even when one confines his attention to regions of similar relief, such as Belgium and Russia. The hypothesis that soil is the controlling factor is likewise untenable. If that were so, the yield per acre ought to be greater in Russia than in Belgium, whereas, as a matter of fact, in both Europe and the United States the yields of corn and wheat are highest in regions of relatively poor soil. Those regions, however, as well as most regions of high productivity all over the world are places where both the climate and the methods of cultivation are favorable. One of these, or else the combination of the two, is responsible for a large part of the great regional differences observable in B313 and B330. Our problem, then, is to discover the relation between climate and methods of agriculture where large regional differences are concerned, and also to discover the similar relation between methods of agriculture and other physical factors such as soil and relief in places where these other factors are evidently influential.

357. HOW CLIMATE INFLUENCES METHODS OF CULTIVATION. I. *Basic Influence of Climate.* Climate and methods of cultivation vary together because the best methods of cultivation are usually developed where a crop finds its optimum climate, or on the cooler or drier borders of areas of optimum climate. This seems unreasonable at first. In fact, it would seem reasonable to suppose that, where the climate or soil is not fully favorable, people would make special efforts to adopt improved methods. In almost any part of the world the crops can be greatly increased, and often doubled, by proper care. Nevertheless, as we shall soon see, the general principle is that *the methods used in cultivating a crop are usually at their best where the climate is best for the crop, or else in regions that are somewhat cooler or drier than the climatic optimum.*

358. This principle is so little understood that it needs careful study. One of its chief foundations is that people make the most rapid progress where their efforts are rewarded by at least reasonable success.

Another foundation is the fact that where the climate is favorable to a large yield per acre the farmer who makes extra efforts is more certain than elsewhere to get a return in proportion to his work. This encourages further efforts. Climate has more effect than soil or relief in encouraging such efforts because it cannot be changed by man, whereas the other conditions can readily be improved. Poor soil, no matter what kind it may be, can be improved and even made good by deep plowing, by raising nitrate-producing crops such as legumes, by adding lime, by employing other means of fertilization, or by draining off the water so that the soil has a chance to become aerated. Rugged land, even though the slopes are steep, can be made cultivable by building terraces. In China, if we include the gently sloping rice fields, about one fourth of all cultivated land is terraced. If the climate is favorable, people find that it pays to spend much work on improving the soil or the relief.

359. If the climate is unfavorable, on the contrary, there is little that can be done to improve it. The harmful effects of excessive rainfall, to be sure, can be partly obviated by drainage. Those of drought can be partly neutralized by irrigation, but this is possible on only a small percentage of the land. In California, where the opportunities for irrigation average the best in the United States, more than 70 per cent of the harvested land is irrigated, but this is only 16 per cent of the farm land and 5 per cent of the total area. Moreover, a large part of the American irrigation projects have been so expensive that it is almost impossible to make enough off the land to pay for their cost and at the same time get a decent living. If the temperature is too high for a crop, nothing can be done to change it, or at least nothing that amounts to anything ever has been done. If the temperature is too low, a little change can be made by very expensive equipment such as greenhouses or smudge-pots in orchards during cold spells. It is evident, therefore, that among the physical conditions which control the growth of crops temperature is by far the hardest for man to change. The quality of the soil can easily be changed, land with suitable relief can be found or made almost everywhere, and in some places the amount of water can be regulated. Nowhere, however, can much be done to change the temperature.

360. In spite of all this there is one way in which man can overcome the effect of temperature. In regions where the optimum climate for a crop does not prevail he can select seed from plants which happen to grow especially well in that particular climate. He has done this unconsciously for thousands of years, and now he does it consciously on a large scale. In climates cooler than the optimum he

selects plants that do well with a growing season shorter or cooler than the optimum In dry regions the same sort of thing can be done in order to get drought-resistant varieties Similar selection is also possible in order to get varieties adapted to high temperature or high humidity In actual practice, the main attempt has been to find or develop varieties that have an optimum temperature lower than the normal optimum of the crop in question Some attempt has also been made to develop varieties adapted to dry climates, but adaptation to heat and humidity has been largely neglected This is highly significant in relation to the present location of the highest yields of crops, as we shall soon see more fully.

361 Before we discuss new varieties of crops let us inquire what happens when farmers try to make improvements in different kinds of geographical environments Suppose that two ordinary farmers of equal ability have farms just alike except that they are located in different climates When both farmers use the same simple methods of cultivation, one gets 20 bushels of corn per acre and the other gets only 10. Now suppose that by spending a certain amount of time and money the farmer in the good climate can raise the yield per acre to 25 bushels where formerly he got 20 By spending 2 bushels' worth of time or money, let us say, he gets 5 extra bushels, or a profit of 3 bushels per acre Suppose also that the 10-bushel man in the poor climate makes a similar expenditure He will be lucky if he gets as great a percentage of increase as the other man, that is, if he gets 12½ bushels instead of 10 That means a profit of only half a bushel over and above the 2 bushels of extra work and expense to which he is put Unfortunately, as we shall see in a moment, the 10-bushel farmer in the poor climate is not so sure of his half bushel as is the 20-bushel man of his 3 bushels Even if he does get it, his reward for a given effort is only one sixth as great as that of the farmer in the more-favored climate. That amount is scarcely enough to encourage him to try again next year, and it has still less effect in persuading his neighbors to do likewise The 20-bushel man, on the contrary, is encouraged by his success and makes greater efforts next season His neighbors note his success and follow suit The principle which we are illustrating is very important It applies not only to climate but to soil, relief, distance from markets, and other conditions which influence production. It may be stated thus *A geographical environment which fosters success is essential if people are to make continued efforts to improve their methods*

362 II *Surplus of Products and Time* The amount of work that a man must do in order to get a living and gain a little capital or

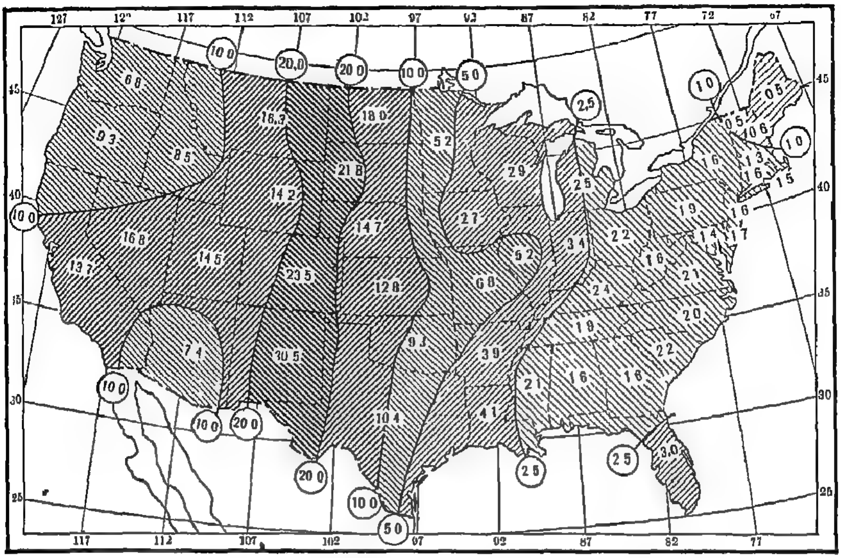
leisure for making improvements is another factor in causing people to progress faster in some geographical environments than in others. Let us suppose that in each of two simple farming communities a farmer needs the equivalent of 400 bushels of wheat per year in order to support his family One lives, however, in a climate where the normal yield of wheat is 20 bushels an acre so that 20 acres of land are needed to support a family, but the other lives where the normal yield is only 10 bushels per acre, so that 40 acres are needed It takes twice as much work to plow and sow 40 acres as 20, and at harvest time the work of reaping 400 bushels by primitive methods is at least one and one half times as great when 40 acres have to be cut as when the same amount can be harvested on 20 acres Now suppose that two farmers—one in each community—have farms of 40 acres each that are exactly alike except in climate The two men are equally intelligent and work equally hard and long The 20-bushel-an-acre man will cultivate 30 acres, let us say, and harvest 600 bushels, whereas the 10-bushel man must cultivate his entire 40 acres, and even then has only 400 bushels, just enough to support his family The advantage of the 20-bushel man is obvious He does not work so hard as the other, and yet he has a surplus of 200 bushels By working as hard as the other man he can increase his surplus And he can use the surplus to buy implements, fertilizer, or better seed He can employ part of it to hire labor, and thereby either cultivate 10 acres which he has not hitherto used, or gain time to make improvements in his tools or his methods The excellence of his geographical environment, no matter whether it be due to climate, soil, or relief, gives him many opportunities which are denied to the man in the less favorable environment

363 III *Variety of Resources and Diet* The farmer with the better geographical environment is encouraged in still other ways to improve his methods He can raise cattle, horses, pigs, and poultry more easily than the one in the poor environment Not only does he have surplus grain with which to buy animals or feed them, but his extra land over and above the 20 acres needed for 400 bushels of wheat enables him to raise grass and forage This gives him less costly farm animals which do his work, furnish manure for fertilizer, and supply milk and meat for his family Moreover, a climate that raises the production of wheat to 20 bushels per acre is also better for vegetables and fruits and even for cows and hens than is one where the average yield is 10 bushels Not only is this profitable to the farmer, but it also gives him and his family a better diet and hence better health than are possible in the poorer climate

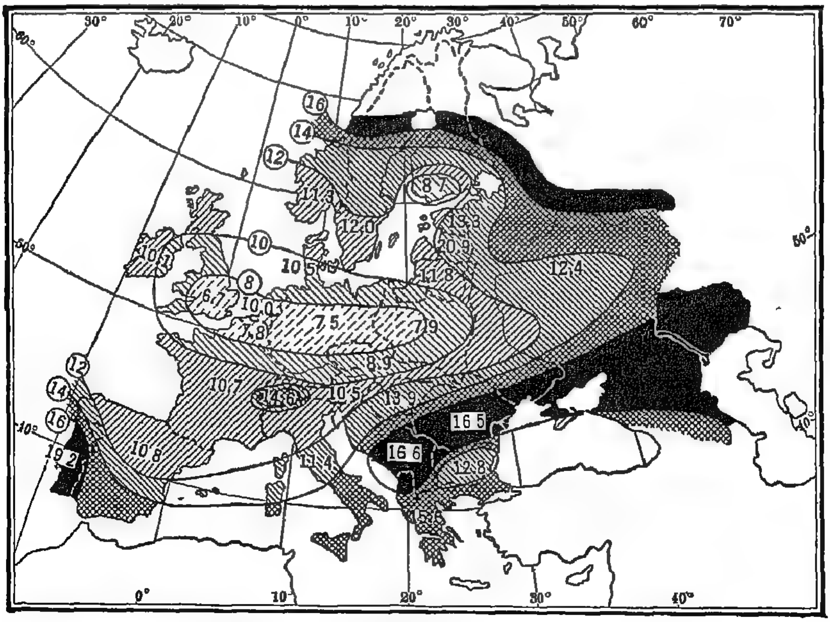
364. IV *Use and Value of Fertilizers* Still another point is that in climates which are too dry, too cold, or too wet, barnyard manure and other fertilizers do less good than where the climate is more nearly ideal If the soil is too dry, the fertilizer is not dissolved or broken up chemically in sufficient quantities to supply the plants If the weather is too cold, the soil is frozen a long time, and that, too, prevents the fertilizer from decomposing and becoming ready to serve as food for the plants Excessive heat and moisture cause the fertilizer to break up too rapidly If there is too much rain, another trouble arises because the soluble materials, which are the only parts that the plant can use, are leached out Most crops need steady but not excessive moisture They need this not only in summer, but at other seasons as well, so that the fertilizers will be broken down and spread through the soil, thus putting the plant foods in such form that they are soluble in the late spring and early summer when they are most needed This is one reason why the steady but not excessive rainfall at all seasons around the North Sea and in the eastern United States is so valuable. The steadiness means sufficient rain at all seasons, relatively slight variations from year to year, and few long spells of either very dry weather or excessive rain. The dry southeastern part of Russia illustrates the opposite extreme

365 V *Reliability of Crops* Comparative freedom from crop failure is still another condition which helps to cause cultivation to be better near the optimum of a crop than near the limits (A and B365) Such failures may arise from droughts and floods, as in China, from frosts as in northern Russia and Canada, or from the ravages of insects or bacteria, as in tropical Mexico. As a plant gets farther from its optimum of either climate or soil, it becomes weaker, and therefore a more easy prey to insects and parasitic diseases In the warmer parts of the area where a crop can grow it generally suffers from such disasters more than in the cooler part Winter frost checks the ravages of many pests, but the frost must not come too early

366 VI *Psychological Effect* The degree to which an agricultural community prospers and develops improved methods is greatly influenced by the psychological effect of conditions such as have been described in the last paragraphs Repeated failures are very effective in discouraging people Contrast the experience of the farmer who lives near the optimum of a crop and that of the one who lives halfway or more toward the limit They both, we will say, make an equal effort to get better seed, use more fertilizer, and cultivate more carefully As a result the first gets a larger crop than ever before, the second gets less than usual, because drought or insects attack his crop The first



A365—Percentage of Crop Failure in the United States, 1934



B365—Variability of Crops in Europe

Expressed as Average Percentage of Departure from Normal, 1927-33

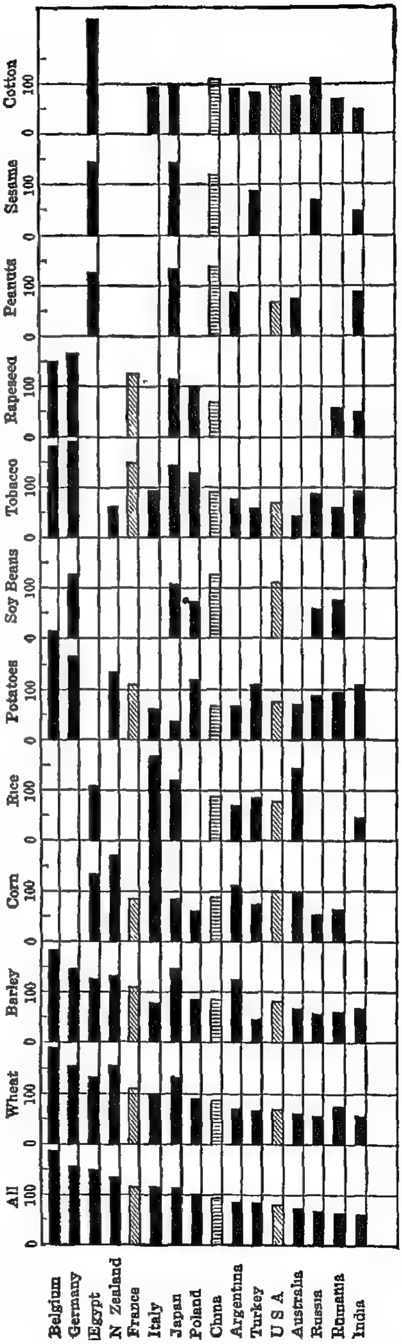
is encouraged to do still better next year, the second is likely to say, "What's the use?" The fatalistic attitude of the Turks, Persians, and many other orientals is probably to a considerable degree the result of repeated disappointments in the harvest Such an attitude of shrugging the shoulders and yielding to misfortune rather than getting up and fighting it has much to do with the low economic productivity of many oriental and tropical countries

367. THE AGRICULTURE OF CHINA China affords an interesting example of a country where high fertility much of the time has encouraged agricultural progress, but where the uncertainty due to floods and droughts has for a long time helped to check further progress. Many people suppose that in China the fields are cultivated better than almost anywhere else in the world. In some respects this is true, but not in all So far as digging, weeding, and the use of human excreta for fertilization are concerned, China is unrivaled. Nor does any country, except perhaps Japan, utilize the land more nearly to its full capacity by starting a new crop before the old one is reaped, thereby getting two crops from the same land in a single summer On the other hand, in large sections of China the dung of animals is burned for fuel instead of being given back to the land as fertilizer. Straw, which might be fed to animals, is burned because wood is scarce and both tea and food require fires even when the people themselves suffer from cold Thus the soil gets only the ashes instead of manure Practically no machinery is used, the draft animals are small, the plows are primitive, and much of the land is dug up by hand Hence deep cultivation of the kind which is best for the soil is costly and rare.

368 One of the other marked defects of Chinese agriculture is that scarcity of animals and undue concentration upon a few crops aggravate the common tendency of farm work to pile up during a few months, leaving little to do during the rest of the year At wheat harvest in North China not only does every man, woman, and child work from dawn to dark, but many people come out from the city to help It is hard to avoid this, but the difficulty would be much lessened if good harvesting tools were available Most of the grain is still cut with sickles, a scythe is a rarity, a cradle is almost unknown The fact that on an average each farm consists of six separate parcels of land in different spots around the village and that each parcel is usually divided into several sections makes it still more difficult to use machinery It also causes much time to be wasted in coming and going at the very season when work is most pressing Much of the year, however, both time and labor are a drug on the market The great abundance of labor destroys the incentive to use machinery and even to

invent better implements for plowing, weeding, harvesting, and so forth Other forms of farm work are equally backward Although 5 per cent of the farm land in China is drained, practically no tiles are used for this purpose Again, according to J. L Buck, the outstanding authority on Chinese agriculture, the Chinese have done almost nothing to control plant diseases and insect pests, which each year destroy from 10 to 20 per cent of the crops Nor has anything appreciable been done to get rid of widespread animal diseases, such as rinderpest Still more significant is the fact that in China there has been little effort to select the best seed, or to develop new varieties of either plants or animals Such conditions suggest that China stands only midway in the scale of methods of agriculture

369 Contrary to what most people suppose, the net result of this kind of agriculture, plus the effect of the Chinese climate and soil, is a comparatively small yield per acre of most crops in China This is evident in A369 In each column there the yield per acre is shown as a percentage of the average yield in all the countries named on the left. India stands lowest in the majority of crops; Belgium



A369—Yield of Crops per Acre in Typical Countries, 1927-1937 The yield of each crop is shown as a percentage of the unweighted average yield of all the countries for which data appear in the diagram

usually stands at the top The United States stands surprisingly low It owes its agricultural prosperity not to large yields per acre, but to a large number of acres per farmer China is about in the middle It gets only a little more than half as much per acre as Belgium, but a little more than the United States and Argentina Its general position is evident in the left-hand column of the diagram where the percentages for all crops have been averaged together without regard to the total amount of each Of course, density of population and the effect of famine must be taken into account in studying A369, but that does not alter the general relation between the geographical environment and progress in agriculture. Up to a certain point the favorable features of the Chinese geographical surroundings encouraged the development of agricultural methods Now, however, the unfavorable features, together with the density of population, greatly handicap further progress

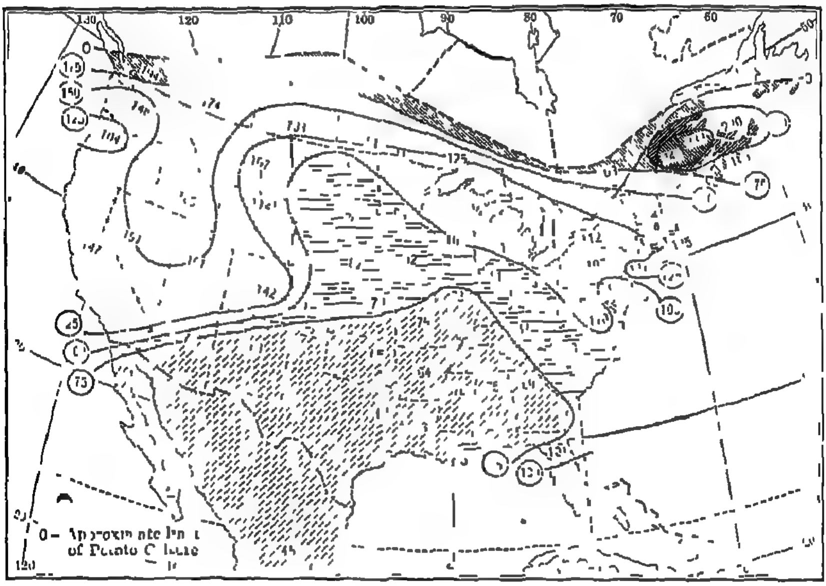
370 The Chinese handicap works both through intense poverty and through conservatism which arises in part from repeated disappointment in the crops The present backwardness of China is evident in the fact that, in spite of fertile plains and warm summer rains, an average day's work of a farmer produces a distressingly small return According to Professor Buck the plowing, planting, cultivating, and harvesting of an acre of corn take a Chinese farmer's full time for 23 days in comparison with only 2 5 in the United States With wheat the contrast is still greater—26 against 1 2 Such figures help to explain why the Chinese have practically no surplus to use in developing better methods They have gone as far as anyone can go in applying labor to the land, for labor is a drug on the Chinese market But they have done practically nothing along other lines where some expense is required. In a country such as India the handicaps to the development of good methods of agriculture are even greater than in China In New Zealand, on the contrary, almost everything except remoteness from markets encourages people to try new methods From all this it seems evident that in regions where the climate and other geographical conditions are near the optimum for a crop there is far more likelihood of improvements in methods of cultivation than in regions far from the optimum Therefore in a general way the geographical distribution of the optimum climate for a crop is usually much the same as the distribution of good methods of cultivating it *Intensive cultivation and fertilization produce remarkable results almost everywhere, but they pay best and are most likely to be practiced where the soil, relief, and especially climate are best for the crops.*

371 BEST CROPS NEAR COLDWARD LIMITS. I *Cereals.* Let us return now to the method by which people overcome the handicap of a temperature lower than the optimum for a crop, as described in Paragraph 360. The facts there set forth lead to a principle which may be stated thus *Most crops are of the best quality, or give the greatest yield, or both, near limits set by low temperature* Sometimes the limit is set by the temperatures of summer and sometimes of winter. Connecticut with its maximum yield of corn lies only about 250 miles south of the extreme northern limit of this crop near Montreal. Southern Maine and southern Wisconsin, still nearer the northern limit, are almost unrivaled for the excellence of their sweet corn This kind of corn cannot easily be raised in good quality at any great distance south of the wedge of maximum yield of field corn in B313 That map shows how sharply the yield declines at the northern border of the wedge Farther south the yield drops off more slowly It continues to drop beyond the United States until a minimum is reached in equatorial latitudes some 3,000 miles from the wedge Yet in all that distance corn is raised, and is often the most important crop

372 Wheat, too, as we have seen, is most productive near the limit set by low temperature in summer, but not winter Where the warmest summer month has an average temperature of less than 63° F the average yield on all kinds of soil and with all kinds of cultivation is about 25 bushels As the midsummer temperature increases, the yield falls off until regions with an average of above 80° in the warmest month get less than 8 bushels Rice (A586) behaves much like wheat. Even in the cool regions north of latitude 35° the yield is twice as high as in most of the regions near the equator where rice is the dominant crop Of course, the yield in any given latitude fluctuates according to rainfall, soil, and method of cultivation, as well as temperature, but all these other conditions together cause only a little irregularity Similar conditions prevail in respect to other cereals. The yield per acre of oats is 10 or 15 bushels in latitude 30°, but rises to 40 or 50 bushels in latitudes 45° to 60°. Barley ranges from 10 bushels in Portugal and similar latitudes to 50 or more in Belgium The important thing seems to be to have a cool summer and a mild winter Thus the crop gets a sufficiently long growing season, but does not suffer from heat and drought

373 II. *Vegetables and Fruits* Garden and orchard products also do best close to their cooler limit. Omitting irrigated areas, the yield of potatoes in the United States is largest and the quality best near the northern border Idaho is famous for its large "baking" potatoes, often 6 or 8 inches long, with smooth, well-rounded surfaces and firm

interior Northern Maine, and likewise New York, Wisconsin, Minnesota, Michigan, and North Dakota, supply seed potatoes for thousands of farmers farther south A373 shows that except near Maine every province in Canada raises more potatoes per acre than the part of the United States just south of it. Maine holds its high position because Aroostook County, in the state's northern portion, has a soil as well as a climate remarkably well adapted to potatoes The Canadian provinces, however, have no special advantages of soil, nor is there any reason to think that methods of cultivation there are better than in the United States. Hence the high yields seem to arise directly or

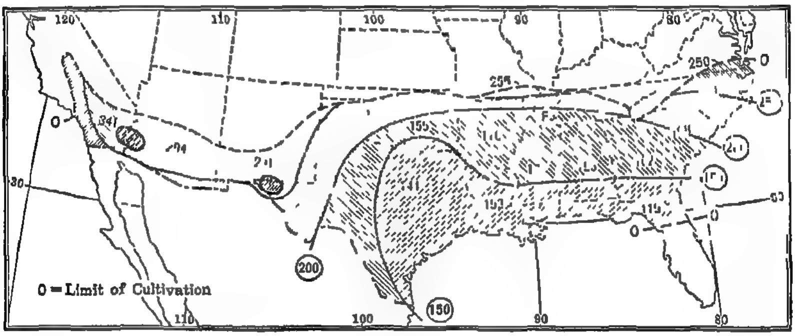


A373—Average Yield of Potatoes in the United States and Canada, 1910-1929 (Bushels per acre)

indirectly from climate In Aroostook County prosperous villages, great potato barns half underground, and up-to-date methods of fertilizing the fields, spraying harmful insects, and digging the potatoes illustrate the way in which methods of living as well as of agriculture tend to rise where a crop finds its optimum in soil and relief, as well as in climate

374 The intimate relation between high yields per acre and cool climates is suggested also by the fact that the yield of sweet potatoes in the United States increases with considerable regularity from south to north. It is highest in New Jersey, Maryland, and Delaware, pro-

vided that we disregard a few high yields due to irrigation in places such as Arizona Again, the cool northern states of Maine, Vermont, Idaho, and Wyoming produce exceptionally high yields of beans per acre Virginia, North Carolina, and Tennessee, which are near the northern limit of peanut production, are also the largest producers of that crop per acre The excellence of the northern apples of Nova Scotia, Maine, New York, and Washington is well known The lemons of northern Italy and Florida are unsurpassed The best oranges are raised in Florida, California, New South Wales, Spain, and Palestine, not in the tropics All these regions are within a few hundred miles of the coldward limits of citrus fruits Such fruits are grown at all latitudes nearer the equator, but in steadily warm regions they tend to be small, greenish, and sour—not at all like the sweet, juicy yellow



A374—Average Yield of Cotton per Acre, in Pounds, in the United States, 1910-1929.

globes raised in valleys near the northern limit of orange culture in central California Many industrial crops and plantation products also grow best near their coldward limit A374 shows that in the United States the yield of cotton per acre increases regularly toward the north Except for the irrigated areas of the Southwest it is highest in southern Missouri The same is true of tobacco The average yield in Massachusetts, Connecticut, New York, Pennsylvania, Wisconsin, and Minnesota is above 1,000 pounds per acre, whereas it drops to less than 500 in Louisiana In tropical countries the best tea and coffee are raised on plateaus so cool that the plantations sometimes suffer from frost.

375 Scores of other crops follow the same general law, no matter whether they are cereals, root crops, fruits, fibers, or stimulants They can be raised in large areas which are too warm to give the best results. Nevertheless, as the limit set by cool summers, or in some cases by

frost in the spring and fall, is approached, they improve in quality and in yield until the optimum climate is reached. The optimum is close to the coldward limit. Beyond it the crop generally declines drastically in both quality and quantity, and may disappear entirely within a few hundred miles of the optimum. All this, it should be noted, is different from what we find in the majority of wild plants. With them, as a rule, the finest growth and largest yield are found more nearly in the center of their range. The size and vigor of birch trees, oaks, pines, or wild grasses, so the foresters say, decline no more rapidly toward the poles than toward the equator.

376. CAUSES OF EXCELLENCE NEAR COLD MARGIN. I. *Cultivation.* Although the geographic principle of relatively abundant production and high quality near the coldward margin has wide application, it has been recognized so recently that its causes are not fully understood. Undoubtedly the good cultivation given to the crops by the energetic people of the colder regions is a factor. With proper care the yield of tobacco as far south as Florida, for example, can be raised almost to the northern level. Good cultivation, however, does not explain such phenomena as the much higher yield of potatoes in Canada than in Pennsylvania, Ohio, and Missouri. Nor does it explain why China (A369) with its extraordinarily intensive cultivation, raises such small crops. It produces only one half as much wheat per acre as Denmark, Belgium, Great Britain, and the other North Sea countries, less corn per acre than the United States, less rice than Japan, and less cotton than the desert oases of Asiatic Russia, east of the Caspian Sea. Only in sweet potatoes and peanuts, crops for which China seems to be especially well adapted, does that country get a yield per acre far in excess of that of the United States. An alleged but unproved cause of the high production near the northern margin is that only the best land is there given to crops. This is true in some places, but in the Scandinavian and North Sea countries, and likewise in Ireland, Canada, and our northern states, practically all crops show high yields per acre. Chile and especially New Zealand also excel the other countries of the southern hemisphere in the same way. Since this applies to practically all kinds of crops it must include those grown on all kinds of soil.

377. II. *Selection.* The most important reason for large and excellent crops near coldward margins is probably that there the process of selection has gone much farther than elsewhere. We have already seen that this is the only practicable method by which the handicap of a temperature below the optimum can be overcome on a large scale. What happens is that varieties having a new optimum are created

This has been done on a large scale mainly on the cooler margin of many crops. The people there generally have more energy than in warmer regions, and their marginal position makes them feel the necessity of selecting not only the best seed but also types that ripen quickly and hence can stand low temperature and a short growing season. When they select such seed they find that it pays. Today the northern margins of active agriculture are in general the regions where agricultural experiment stations are most numerous and active, and where the farmers are most eager to get seed of new and improved varieties. A new kind of sweet corn, for example, ripens a week earlier than any hitherto known. A new kind of cotton is said to do well with a growing season a month shorter than is required for the old kind. A new, hard, red spring wheat that grows very rapidly was produced some decades ago by crossing old varieties and selecting the best strains year after year. This advanced the limits of wheat cultivation scores of miles northward in Canada. At Winnipeg, more than a century ago, the first settlers were almost in despair because frost so often prevented their wheat from maturing, but a hardy Russian wheat proved their salvation. Similar improvement and development of varieties that mature quickly have taken place with almost every crop. Such new varieties push the limit of the crop into colder regions. They are generally selected for quality and productiveness as well as for early ripening. Thus man himself is one of the greatest reasons for the excellence of the crops near their coldward limits.

378. III. *Variability.* Another reason must not be overlooked, although it is not yet fully understood. Many experiments have shown that plants grow best under variable temperatures. For every crop there is a certain optimum temperature for germination of the seed, for the growth of leaves and stalks, for the production of flower buds, and for the ripening of the seeds or fruits. All these optima may be the same, but outside of equatorial regions they are generally different. The optima for flowering and fruiting are usually higher than for germination and vegetative growth, although the witch-hazel flowers in winter, and the elm and peach before any leaves appear. But no matter what the optimum may be, the plant generally grows better if the temperature varies instead of staying always at the optimum. For example, Longwood Gardens in southeastern Pennsylvania, north of Wilmington, Delaware, contain one of the world's most superb greenhouses. The greenhouse is famous for the size, deep color, and general beauty of its flowers, and for the excellence of its grapes and melons. Visitors there in cold weather are surprised to find how cool the greenhouses are at night, or even by day when the sky is cloudy.

Azaleas, amaryllis, and dozens of other flowers are blossoming superbly in a temperature of only 50°. Even the banana room may be only 60°. But on sunny days the temperatures may run from 70° to 80°, or even 90°. Long experience has shown that the majority of plants grow best when the temperature varies a good deal from night to day, and also from day to day Each plant wants an optimum appropriate to its own needs and to its stage of development But if the optimum is 70°, the temperature ought usually to vary up and down from perhaps 60° to 75°, giving an *average* of 70° but introducing plenty of variability Near the coldward margins of their growth crops generally experience more variability of temperature than in warmer regions. In addition to this, the coldward margins are less likely to suffer from drought, or from excessively hot days, which are among the worst elements in lessening the yield of crops such as corn

379 IV *Length of Day* The long duration of sunlight is another reason for the better growth of many crops as one goes farther from the equator Even in middle latitudes this is important. In high latitudes it often leads to marvelous crops of vegetables, provided that frost does not intervene This is the main reason for occasional yields of 400 to 800 bushels of potatoes per acre on certain small tracts, as described by A D Albright, for example, north of latitude 60° in the Mackenzie region of Canada Of course the average would be far less if we take account of the many years when there is only a small crop or none at all because of frost Nevertheless, there can be no doubt that the length of the day joins with other climatic conditions, as well as with man's activity, in making many crops grow best well toward their coldward margins

380 V *Blights and Insects* Another important factor in favor of the cooler margin of crops is comparative freedom from insects and other pests. Plagues of locusts in the drier parts of regions as far apart as Kansas, India, Argentina, and Australia sometimes destroy the crops on thousands of square miles. A swarm which crossed the Red Sea in 1889 was estimated to have been 2,000 square miles in extent Others have been found at sea 1,200 miles from land The author has seen thousands of acres of young wheat eaten completely by swarms of young, hopping locusts in the oases of Transcaspia (Turkmenistan) As long as their crops were good the Turkoman farmers there were satisfied with the Russian government The ravages of the locusts, by destroying the food supply, aroused serious political discontent As soon as the insects had done their work the farmers behaved like the American farmers in our dry plains, and blamed the government for troubles due to nature The localities named thus far in this paragraph

have all been on the dry and relatively warm margins of agriculture, but some locusts are found elsewhere. Swarms that probably were hatched on the west coast of Africa occasionally migrate to Europe. Some are found permanently in most parts of the United States. Nevertheless, the very harmful kinds that migrate in huge swarms are largely confined to the drier or warmer borders of agriculture. One of the worst species breeds in the vast reed beds of the deltas of the rivers flowing into the many salt lakes of Central Asia from the Caspian Sea to China. Few things seem more terrible to the farmers of northwestern China than a host of locusts darkening the sky like a rapidly spreading cloud.

381. Even in our own best agricultural regions we are by no means free from insect pests and blights. The chinchbug illustrates the relation of insect pests to crops and affords an excellent illustration of the fact that insects have climatic optima and limits in the same way as plants. The chinchbug is a little black-and-white insect about a sixth of an inch long. It often damages the crops of the central United States to the extent of millions or hundreds of millions of dollars in a single year. Its bright red young suck the sap of young grain. They grow rapidly, shedding their skins five times as they grow. Often they move along the ground in vast numbers from wheatfields where the plants are becoming hard to cornfields where the leaves are still soft and juicy. E. V. Shelford has shown that the chinchbug thrives best in warm weather with a temperature of about 85°, and little rain. Yet it must have moist air in order to do well. In air at a temperature of 85° and a relative humidity of over 80 per cent the bugs hatch and pass through all five of the so-called nymphal, or skin-shedding, stages in about 30 days. Hence they increase very rapidly, for several generations can complete their life cycle in a season. If, however, there is much rain, the creatures are washed away and drowned. Thus an abundance of atmospheric moisture affects them quite differently from an abundance of rain. If the temperature is either higher or lower than about 85°, or if the air is dry, the bugs grow less rapidly and take longer to become adult and ready to lay eggs. If the temperature rises above 120°, or falls below about 60°, the bugs will not grow at all, no matter how great the atmospheric humidity. In dry weather the cold limit of growth rises to 70°. Thus in regions having cool summers, such as most of Canada, the chinchbug ceases to be a danger.

382. The fact that the chinchbug and many other harmful insects find their optima in warm, moist air, and do not thrive where it is cool or dry, is one of the important reasons why crops tend to be at their best near their coldward limits. The boll weevil, which does so

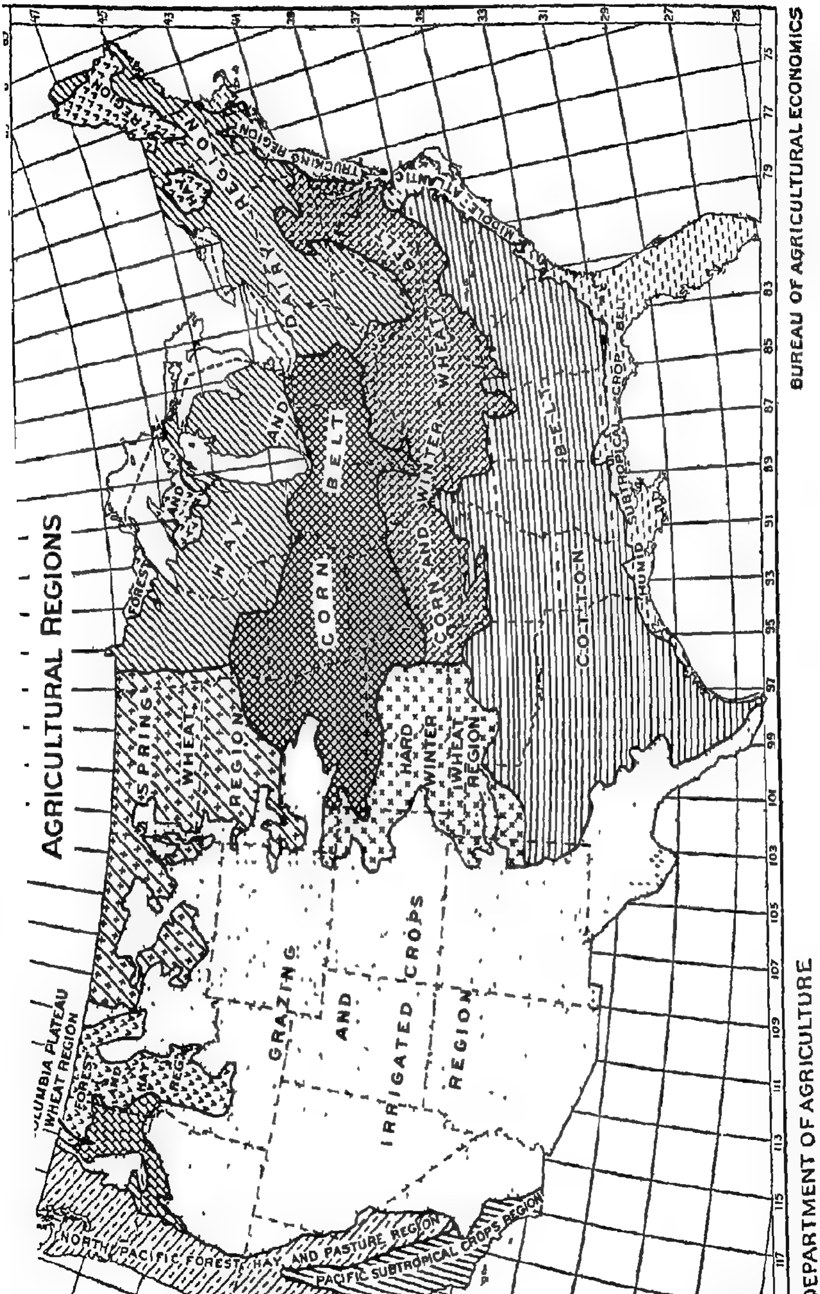
much harm to cotton, illustrates this same point Its reaction to climate is like that of the chinchbug This helps to explain why the largest yields of cotton are found near the northern limit of cultivation The voracious, but beautiful, Japanese beetle is killed by cold winters In general the ravages of insect pests become greater as one goes from cool to warm climates, and from well-watered to somewhat drier ones On the other hand, if the climate is excessively humid, especially if it is also warm, bacterial and fungous pests such as the rusts and smuts of grain become a great menace A final point to be emphasized is that attempts to make useful crops grow in climates cooler or drier than their normal habitat are common, but similar attempts to foster growth in warmer or wetter climates have been rare until recently This is partly because of the relatively poor soil and other difficulties of agriculture in warm regions, and partly because of man's own disinclination to exert himself there In developing a crop along its cool margin man endeavors not only to help the crop, but also to check its pests In a state of nature the pests of a plant generally have the same climatic optimum as the plant itself Hence, when man develops a new variety with an optimum cooler than that of the original crop, he puts the crop at its best in an area too cool for the pests From all this we see that in general the geographical location in which a crop yields the largest return per acre and is of the best quality depends partly on the climatic optimum of the new varieties, and partly on the somewhat different optimum of its enemies Soil and relief, of course, cause local differences So, too, do methods of cultivation In general, however, the crop's own climatic optimum, the best cultivation, and the greatest freedom from pests are all found together in regions not far from limits set by low temperature.

CHAPTER XVI

THE ECONOMIC FACTOR IN LAND UTILIZATION

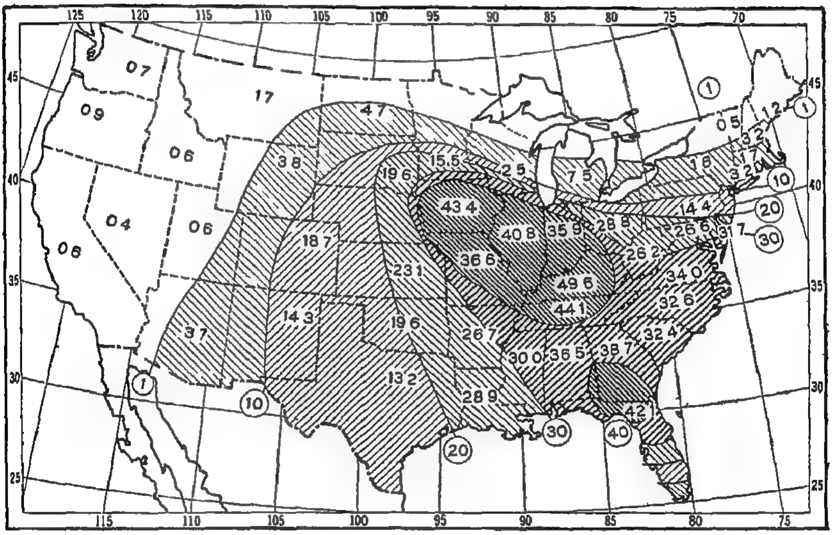
383 THE BALANCE BETWEEN ECONOMIC AND GEOGRAPHIC FACTORS. In Chapter XIII we saw that regions such as Southern New England, where the yield of corn per acre is exceptionally high, are by no means the ones where the largest percentage of cultivated land is devoted to that crop We have also seen that a large part of the world's wheat is grown in relatively unfavorable climates, while some of the best areas, such as the Middle Atlantic States, raise almost none This same sort of disagreement between possible and actual productivity is evident in many other crops, such as cotton, flax, rye, and rice A similar condition applies to animals In southeastern England, where the yield of both wool and meat per sheep is high, the farmers do not depend upon sheep nearly so much as do those of Turkey and Iran, where the sheep are far less productive In all these examples the general principle is the same *Economic conditions often make it less profitable to raise certain products under optimum conditions of climate, soil, and relief than under less favorable natural conditions* Corn and wheat trate the matter so well that we shall discuss them in further detai the general principles which apply to them apply also to many products

384 CONTRASTED DISTRIBUTION OF ACREAGE AND YIELD PER A39 shows that corn is most abundant in the Corn Belt This (A384) extends from western Ohio across Indiana and Illinois it widens to include Iowa and parts of South Dakota and Missouri finally extends well into Kansas and Nebraska It covers the we half of the region where B313 shows an average yield of more 30 bushels per acre, but entirely omits the eastern half which ext into the "Dairy Region" and North Atlantic "Trucking Region A384 The disagreement between yield per acre and acreage is more evident when we examine the percentage of harvested land devoted to corn as shown in B384 From Connecticut, with only 3 per cent, the percentage increases westward to 43 in Iowa, but the yield per acre declines from 46 to 38 Southward the disagreement between yield and acreage is even more notable In most of the "Corn and Winter Wheat Belt" (A384) the yield per acre is only 20 to 30 bushels,



A384—Agricultural Regions of the United States.

but the percentage of harvested land in corn is 32 to 50. Farther south, in the eastern half of the "Cotton Belt," corn yields only 14 to 18 bushels per acre, but the farmers give it 30 or 40 per cent of their cropland. It certainly seems strange that, although Georgia farmers get only 14 bushels per acre, they plant corn on 40 acres out of every 100, while Massachusetts farmers who get 44 bushels per acre give less than 2 per cent of their cropland to corn. A similar condition prevails elsewhere, however, as is evident when we compare the yield of wheat per acre in Europe (B330) with the percentage of the cropped land devoted to that crop there (A330).



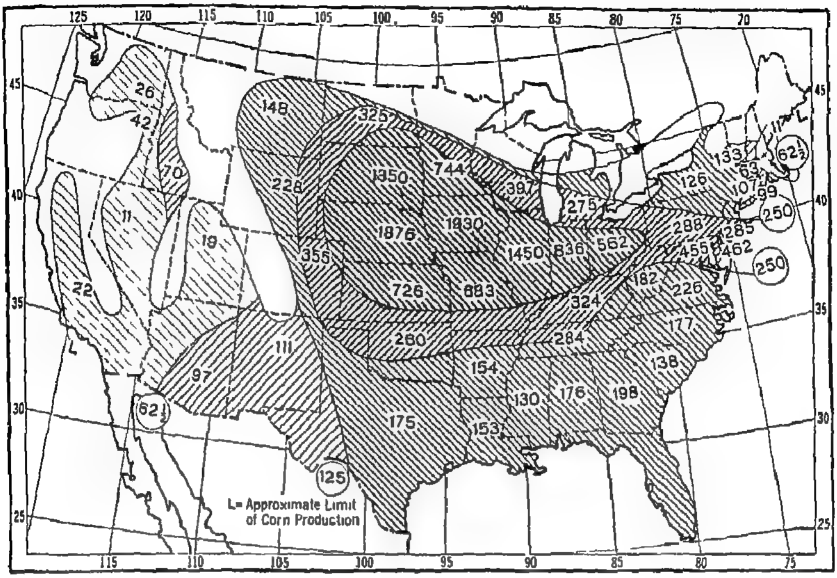
B384—Percentage of Harvested Land devoted to Corn in the United States

385. The way in which farmers devote much land to a crop even when the yield is small is especially astonishing when we investigate counties instead of states. We then find the maximum use of land for corn in mountainous sections such as the Appalachian highlands of eastern Kentucky and Tennessee, and in certain lowland southern areas such as northern Florida. Three fifths of the harvested land is there used for corn. This seems still more remarkable when we recall that land devoted to hay forms part of the harvested area. If hay is excluded, the farmers of 10 mountain counties of eastern Kentucky have long given corn 70 to 75 per cent of the land that they plow each year, whereas 10 counties in the Blue Grass lowland a few miles away have given only 45 per cent. To sum the matter up, corn is the pre-

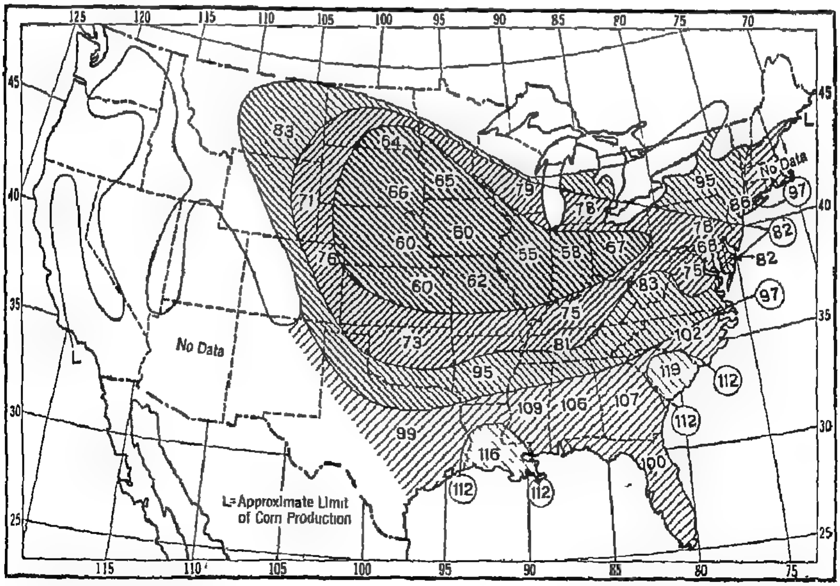
dominating interest of the farmers of the United States in three main regions (1) the prosperous, level, central Corn Belt where the soil is fertile, the climate favorable, and the yield per acre high, (2) the more rugged sections of the Corn and Winter Wheat Belt (A384) where the soil is thin and only moderately fertile and the climate somewhat too warm for the highest yields, and (3) the rolling southeastern part of the Atlantic Coastal Plain (much like Raleigh in A318) where the Cotton Belt of A384 meets the Humid Subtropical Crop Belt There the soil is deficient in nitrogen, and both temperature and rainfall are above the optimum for corn.

386 REASONS FOR CONTRAST BETWEEN ACREAGE AND YIELD PER ACRE I. *Amount of Production and Cost per Unit* We must now inquire into the reasons why acreage and yield per acre have so little correspondence. The principles that apply to corn apply equally well to other crops and also to manufacturing, mining, lumbering, and fishing The general principle is that people try to get as much profit as possible, but the profit depends on cost of production, prices, markets, competing products, and various other factors, as well as upon yield An example will make this clear We want to understand the difference between Illinois and Pennsylvania Illinois raises 35 bushels of corn per acre (B313) and 1,450 per farm (A386), Pennsylvania raises 20 per cent more per acre (42 bushels), but only one fifth as much per farm (288 bushels) One obvious reason for this is that the average farm has 89 acres of harvested land in Illinois and only 39 in Pennsylvania Another is that corn is one of the most profitable crops in Illinois, but stands quite far down the list in Pennsylvania This seems queer when the yield per acre is 20 per cent larger in Pennsylvania When the United States Department of Agriculture investigated the matter, however, it found that the cost of raising a bushel of corn is only 55 cents in Illinois—less than anywhere else in the country—but rises to 78 cents in Pennsylvania and to a dollar or more in the South * The reasons for the low figure in Illinois include (1) the levelness of the prairies which reduces the cost of plowing, transportation, etc; (2) nearness to Chicago, where farm machinery, fertilizer, and other supplies can be bought without much expense for transportation, (3) the large size of the farms In order to estimate the effect of size, remember that, when transportation from the factory is taken into account, the Pennsylvania farmer has to spend more per acre than the

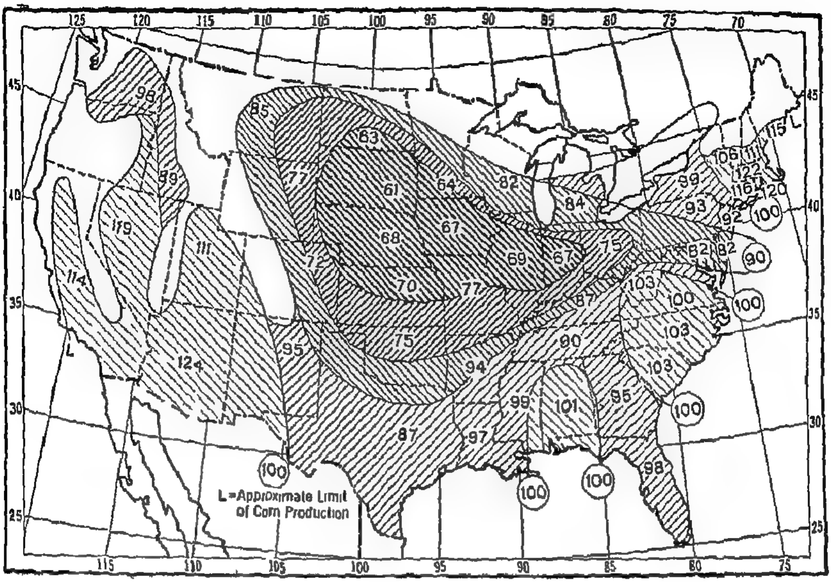
* These figures are too high both because they were compiled during a period of high prices and because the farmers, like other people, have a tendency to overestimate the cost of their work Nevertheless, they give a fairly reliable picture of the *differences* between the various parts of the country



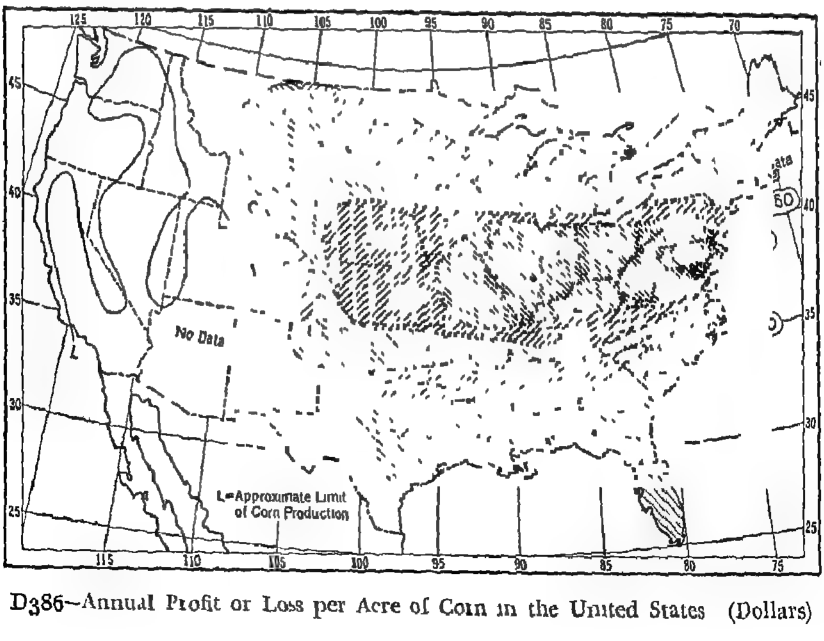
A386—Annual Yield of Corn per Farm in the United States (Bushels)



B386—Cost of Producing a Bushel of Corn in the United States (Cents)



C386—Average Farm Price of Corn per Bushel in the United States (Cents)



D386—Annual Profit or Loss per Acre of Corn in the United States (Dollars)

one in Illinois for a tractor, truck, mowing machine, hay tedder, hay rake, harvester, feed cutter, gasoline motor, and other equipment, or else he must do his work with poorer equipment which takes more time and prevents him from cultivating as much land. Suppose that $1,000 is spent for machinery during a series of years. That would mean $26 per acre in Pennsylvania and only $11 in Illinois.

387. This makes it evident that the cost of raising crops varies from place to place in agreement with one of those principles that we keep coming across. The principle is illustrated by the fact that A and B386 look almost alike, although heavy shading means high amounts in one and low amounts in the other. This resemblance shows that where the average farmer raises a large amount of corn the cost per bushel is low. The general principle is that *the cost of production tends to be low where geographical and economic conditions favor the production of an article in large quantities, high where they lead to small production.* This is true when one farm, factory, or mine is compared with another in the same neighborhood, it is equally true when the production of larger areas such as states and countries is compared. Note how evident it is in A and B386. In all states where the production of corn averages more than 500 bushels per farm, and only in those states, the cost of production is less than 70 cents per bushel. In practically all states where the production averages less than 250 bushels per farm, the cost rises above 80 cents. Of course the two things work together. Low cost of production encourages large production, and large production in turn tends to lower the cost.

388. II. *Selling Price.* A knowledge of the cost of production is not enough to show whether it pays to raise a crop. We must also know how much the crop is worth, that is, the price at which it can be sold. May not the price of corn be so high in Pennsylvania that the farmers there make more profit than those in Illinois? The price that the farmers received at the time when these maps were made is shown in C386. It was 69 cents in Illinois and 93 in Pennsylvania—a considerable difference. The reason for the difference is mainly that there is a large supply of corn in Illinois and only a small supply in Pennsylvania. In fact a comparison of C and A386 shows that except in Missouri the price received by the farmers does not rise above 70 cents per bushel in any state where more than 600 bushels are raised per farm. The price in Missouri is raised by the southern market, which is easily accessible down the Mississippi Valley. On the other hand, in every state where the production is less than 200 bushels per farm the price shown in C386 is at least 85 cents, and in many cases more than a dollar. Here again we come upon a principle. *Under normal con-*

ditions prices are low in regions where an article is produced abundantly and cheaply This is why C386 appears so much like A and B386.

389 III *Profit* In trying to find out why people raise crops in one region rather than another, neither the cost of production nor the sales price is so important as the profit The figures in the last paragraph show that the profit on a bushel of corn is 14 cents in Illinois and 15 in Pennsylvania * For the farmer, however, the profit *per acre* is more important than the profit per bushel This is because the work of raising 20 bushels of corn, let us say, on an acre is nearly the same as that of raising 40 The only difference is that it takes more time to harvest the larger amount That does not count for much because farmers are glad to put in a little extra time in order to harvest big crops The thing that counts is the work in the spring, and that is the same for each acre, no matter what the yield may be So the Pennsylvania farmer by making more per bushel and raising more per acre makes a greater profit per acre than the Illinois farmer The profit per acre as given in D386 is $6.30 in Pennsylvania and $4 90 in Illinois

390 This looks as if there were more reason for raising corn in Pennsylvania than in Illinois, but still other factors must be considered Before inquiring into them, however, let us look again at D386 In many ways this interesting map resembles the three that represent yield per farm (A386), cost of production (B386), and price received by the farmers (C386), but its main part, omitting the West where little corn is grown, is like B313 In D386 there are minus signs in Minnesota, the Dakotas, Colorado, and all the more southerly states This means that, if we accept the liberal allowances made by the Department of Agriculture for interest on the value of the land and for reasonable wages to the farmer himself, the low yields per acre of those regions are not enough to make it pay to raise corn The farmers keep on raising it because they do not make such large allowances in their own estimates Moreover, they do not know how else to get a return for their labor.

391. IV *Competition with Other Products in the Northeast.* In addition to yield per acre, cost of production, selling price, and profit per acre one more element is needed to explain the scarcity of corn in regions where the climate approaches the optimum and the yield per acre is highest. This element is competition with other crops for the use of the land One of the fundamental principles of economic geog-

* These numbers and those in D386 are significant chiefly as showing the *relative* profit in different states The actual profit varies greatly from year to year and depends on the values assigned to land, labor, and so on

raphy is that, *so long as there are no artificial restrictions, a more profitable crop drives out a less profitable one.* Another principle is that *normally the greater the value of a crop per acre, the greater the profit on it, even though the cost of production is also high.* In the whole North Atlantic section good farmland is so scarce and the population so dense that crops compete vigorously with one another. Hills with thin rocky soils occupy a large part of the country, level land occurs mainly in small patches, and even there the soil is often stony because it was laid down directly by the icesheet, or sandy and gravelly because it was laid down in outwash plains formed by streams pouring out from the front of the ice. These conditions are more pronounced in southern New England and southern New York than in Pennsylvania, and the contrast with Illinois is correspondingly great. Therefore, in the rest of this comparison we will substitute southern New England for Pennsylvania.

392. People who come from farther west or from abroad are often amazed that they can travel from Philadelphia to Boston by way of Bear Mountain Bridge near West Point, let us say, and see so few people. They are in the most densely populated section of the whole United States, but the road winds through woods much of the time. A still more impressive example is seen on two different roads between New York and Boston. One is the original "Post Road" through Stamford, Norwalk, Bridgeport, New Haven, Hartford, Springfield, Worcester, and many other busy manufacturing towns. If the automobile driver follows the original road through the centers of these towns he is exasperated by heavy traffic; no sooner does the road open up for a few miles than another town blocks the way. Between the towns one drives not only past uncounted gasoline stations, elaborate roadhouses, and suburban homes, but also past intensively cultivated truck farms, broad fields of hay and forage, fine-looking dairy farms, nurseries full of ornamental trees and bushes, greenhouses, and other evidences of a highly developed and prosperous agriculture. This route, however, as well as the corresponding one along the coast through New London and Providence, is largely abandoned now by the traveler who wishes to make good time.

393. Such a traveler follows quite a different kind of road. He leaves New York City on the Hutchinson River Parkway, follows the Merritt Highway toward New Haven, and then has a choice of several routes which avoid the densely populated strips that lie along the coast on the one hand, and in the Connecticut lowland via Hartford on the other. On this route the traveler avoids not only the towns, but even the farms. One can travel 20 miles at a stretch without seeing more

than a small hamlet and a few none-too-prosperous farms, generally on the plateaulike tops of hills For miles not even farms are in sight—practically nothing but a succession of hills covered with woods It is hard to realize that one is driving through the most densely populated part of the United States, and that each of the many smoothly paved crossroads leads in a few miles to the hum of factories and the din of congested traffic In Illinois, or in such states as Indiana, Minnesota, and Kansas, one cannot find such contrasts Any route which avoids the cities, but keeps within a few miles of them, is bound to be located almost entirely in the midst of an unbroken series of farmsteads and fields These give an impression of nature's bounty instead of the eastern impression that nature loves trees, but hates to provide level land and good soil for farms

394 The contrast between the two kinds of routes illustrates the fact that the eastern manufacturing section provides a wealthy market for farm products and only a small area of good land from which to supply the market's demands Inasmuch as the United States has an excellent transportation system, such conditions have no great effect on products that can be easily transported Therefore it is not necessary to raise all sorts of crops close to this great manufacturing region To bring a bushel of corn from the Mississippi River to New York or Boston may cost several cents, but corn is relatively cheap out there. On the other hand, fresh vegetables and fruits are heavy in proportion to their food value because they contain much water Moreover, many kinds lose their finest quality when kept more than a few days or even hours Even with expensive cold storage they are not so good as when fresh. Accordingly the products which pay the farmers best in the Northeast are those that are so bulky or perishable that it is not advisable to bring them from the Middle West or from other regions where good land is abundant The large and prosperous urban and industrial population among which these northeastern farms are located is able to buy great amounts of milk, eggs, fresh vegetables, berries, orchard fruits, and even flowers, ornamental shrubs, and hothouse products These products command high prices, and the possible profits per acre are far larger than with wheat or corn Here are the approximate average values of different kinds of crops per acre for the entire United States or for certain sections during five years according to recent returns of the United States Department of Agriculture

395. The differences in this table are extraordinary All the cereals stand low, but corn is worth almost twice as much per acre as rye. Cotton is worth still more, but its value is small compared with that of vegetables and fruits. Even the sweet potatoes on an average acre are

TABLE 11

RELATIVE VALUE PER ACRE OF CROPS IN THE UNITED STATES

Rye	$ 11	Tobacco	$150
Wheat	16	Early cantaloupes (Lower Valley of Texas)	170
Corn	20	Late cabbage (Pennsylvania)	200
Cotton	26	Medium early cabbage (Virginia)	220
Sweet potatoes	90	Late strawberries (New York)	420
Potatoes	120	Early strawberries (Florida)	560
Late cantaloupes (New York)	130		

worth $90, while the strawberries are worth from $400 to $600 Of course, it costs more to raise and harvest an acre of cotton than an acre of wheat, and still more for an acre of tobacco, cabbages, or strawberries In spite of this, the chances of making a good profit are enormously greater with the more expensive crops From an acre of land devoted to greenhouses the profit may be as great as from a thousand acres devoted to rye Moreover, the comparatively high cost both of transporting these more expensive products and of keeping them fresh gives the farmers who live close to the industrial centers two great advantages First, their expenses for transportation are less than those of the farmers who live farther away Second, when vegetables, fruits, or flowers which have been transported a long distance compete with those grown near at hand, the local product is often better because fresher It therefore brings a higher price If meat, milk, and eggs were added to this table we should find that where there is plenty of rain and a good market the raising of meat often yields a much larger return per acre than cereals and cotton, but not so large as vegetables and fruits Milk, chickens, and eggs yield still larger returns per acre Thus the kinds of farming that pay best in the Northeast are dairying, market gardening, poultry-raising, and the raising of fruit and other special crops like the onions and tobacco of the Connecticut Valley. Corn is driven out by its competitors

396 V *Supplementary Relation of Corn and Animals in the Corn Belt* The way in which one farm product supplements another is as important as competition in determining what crops are raised in various geographical regions D386 shows that in the Corn Belt the profit on an acre of corn ranges from $2 00 in Nebraska to $4 90 in Illinois In Illinois, during the good times on which the Department of Agriculture based its figures, this would give a profit of $436 per farm each year, provided that the whole 89 acres were equally profitable Fortunately for the Corn Belt, however, the farmers there, especially in Iowa,

tion, on the other hand, makes corn, hay, and fodder very profitable. Therefore it pays to bring the animals to the corn rather than to carry the corn to the animals The location of the Corn Belt is favorable for this because large areas of the Great Plains that are too dry for corn lie to the west, while the warmer areas to the south do not raise nearly so much corn per acre Nevertheless, the areas which are too dry for corn have abundant grass which will support animals while they are young, but will not fatten them Thus the dry plains from the western Dakotas to Texas can raise many cattle and even hogs, but cannot easily get them ready for market Accordingly it pays to ship these animals to Iowa, Illinois, and other parts of the Corn Belt. The animals move toward the eastern markets as well as toward the corn The most profitable thing the Corn Belt farmers can do is to buy them, fatten them, and sell them to be taken East as meat Thus corn and animals cooperate

399 CORN AND COTTON In the South, cotton, and to a less extent tobacco, supplement corn, but in large parts of the states from Kentucky to Florida far more land is devoted to corn than to any other crop (B384) This is the more surprising when we remember that the yield of corn per acre is here much lower than in the North Atlantic States or the Corn Belt (B313) In spite of this low yield the farmers use a third or even half of their cropland for corn Where cotton will grow, most of the remainder is devoted to that crop Before the government undertook a policy of restricting production more than half the harvested land was devoted to cotton in a strip extending along the central part of the Cotton Belt (A384) from North Carolina through Georgia and Alabama to Mississippi There the belt sends a branch up the Mississippi Valley to southern Missouri, while westward it expands into Arkansas, Louisiana, and eastern Texas In the whole state of Mississippi no less than 60 per cent of all the harvested land was devoted to cotton, and more than 90 per cent to corn and cotton together This is because cotton gives a larger return per acre in money than does any other crop that most of the farmers in the South are in the habit of raising Some, to be sure, raise winter vegetables and fruits, but the Subtropical Crop Belt and Middle Atlantic Trucking region where this is done (A384), form only a narrow coastal strip except in Florida

400 For the majority of colored farmers cotton is the only important cash crop, but it needs to be supplemented by a food crop and by forage for animals Corn makes good food for both man and beast Although wheat is generally preferred for human consumption, the warm, moist climate of the South causes the yield of wheat to be so

small that the crop is rarely worth raising. Therefore, in the old days before railroads and automobile roads made transportation easy, corn became the staple food product and the staple feed for animals. The habit thus established still persists. Even among southerners who can afford whatever food they choose, corn is still eaten by choice to an extent unknown in the North. Among vast numbers of the poorer people, especially the Negroes, money is so scarce that the farmers feed themselves with their own corn rather than pay for wheat. Not only is the farm price of wheat higher than that of corn, but the farmer who buys wheat flour has to pay milling and railway charges and the profits of commission merchants and storekeepers. Except near the main roads the conditions of transportation also make it difficult to get a profit from high-priced products like early vegetables and fruit that must be taken to market quickly. This helps to cause the farmers to raise food for both man and beast directly on the farm where it is to be consumed. The main highways in the South, to be sure, are excellent, but many minor roads on which a large part of the farmers live are often muddy or rough. Where transportation is particularly well developed and the farmers are especially progressive, this concentration on corn and cotton does not prevail. Elsewhere, however, the farmers must use their land for something, and they are conservative about making changes. Thus we see that possibilities of profit and old habits, rather than the yield of crops per acre, are the main reason why at least 80 per cent of the harvested land is devoted to corn and cotton in large parts of the southern states. The South illustrates the way in which the choice of crops depends not only on soil and climate, but also on habits, transportation, and a complex series of economic conditions in other regions as well as at home.

Chapter XVII

THE POLITICAL FACTOR IN LAND UTILIZATION

401. Political Factors in Distribution of Production. I. *Security and Protection.* A complete picture of the geographical distribution of land utilization must include political as well as geographical and economic factors. In studying rubber we saw that people invest in tropical plantations much more readily in countries with a well-established and trustworthy government than in those where revolutions and changes of policy are frequent. The experience of the United States with Mexico illustrates the same point. In the past the Mexican government readily granted foreigners the right to buy ranches, develop mines, and drill oil wells. This encouraged the investment of hundreds of millions of dollars of American money, and the building up of large industries run by Americans. A revolution in Mexico led to a new constitution which asserts that Mexico's natural resources must be kept solely for the Mexicans. Hence step by step since about 1925 the Mexican government has taken possession of the property of foreigners, especially the great oil properties. This would be within its rights, provided that it paid for the investment made by the foreigners. It has done little along this line, however. Similar events have occurred in Bolivia and Ecuador.

402. When investments are made unsafe in this way, there is a long train of consequences. (1) Europeans and North Americans hesitate to put money into productive ventures in tropical Latin America or any tropical country, such as Liberia or Siam, which has its own native government. (2) Advanced countries see in such conditions a reason for having tropical colonies of their own. (3) If people from such countries do make investments in self-governing tropical countries, they expect huge profits so that they may get their money back before the government is overthrown or changes its policy. (4) Under such circumstances they sometimes put as much of an industry as possible outside the doubtful country, as has happened in the oil industry of Venezuela. There the American, British, and Dutch oil companies do little in Venezuela except get the oil out of the ground. They carry it at once to neighboring islands such as Aruba and Curaçao which

belong to the Netherlands and therefore have a government which can be relied on. There vast quantities of oil are broken up into gasoline, lubricating oil, and so forth, in some of the world's large refineries. Such conditions naturally hamper the development of poorly governed tropical countries not only by limiting the amount of investment, but by encouraging the investors to skim the cream, so to speak, without trying to build up permanent industries. No one can measure the full effect of the unreliability of the average tropical government, but such unreliability undoubtedly has great influence in determining not only the geographical distribution of plantations and other industrial projects, but also the attitude of the advanced countries toward colonies.

403. II. *Tariffs, Customs Duties, and Protection.* Practically all countries raise part of their national income by means of customs duties upon goods imported from abroad. Sometimes these taxes are designed merely to raise revenue, but often they have other purposes. One such purpose is to raise prices so that an industry which has previously been impossible, unprofitable, or not well developed may make a profit. Some people believe that such protective tariffs raise a country's standard of living, although practically all economists doubt it. According to the economists, protective tariffs may raise profits or wages in the protected industries, but this simply means higher prices for the goods thus produced and no real gain in the standard of living. Whatever may be the truth about this, it is clear that such tariffs sometimes enable new industries to get started. Thus they lead to considerable changes in the geographical distribution of many industries, as we shall see shortly in the case of sugar. Tariffs and other forms of protection of home industries are also one of the reasons why countries try to acquire colonies. They want to be able not only to control the production of tropical foods and raw materials but also to get them cheaply without paying any kind of taxes on them.

404. A still more important influence of tariffs is that they check the flow of trade between countries which would be greatly benefited if commerce were more free. This is notably true in Europe. Before the German invasion of Austria in 1938, Europe, aside from Russia, contained 25 independent countries as well as part of Turkey and such semi-independent little areas as Luxemburg and Monaco. These occupied an area of about 2 million square miles, or only two thirds as much as the United States. Thus the average size of the European countries was only a little greater than that of the states in the United States. Each time goods or travelers pass from one country to another there is generally at least a little delay and annoyance because of custom-house examinations. Often the delay and annoyance are seri-

ous. Moreover, each time any one of the many countries changes its tariff laws people in many of the other countries are annoyed As producers, sellers, or buyers of certain goods they are obliged to alter their prices, change their amount of production, find new markets, or otherwise adapt their business to the new tariff. This leads to two serious effects First, it tends to increase the desire of each country to be self-supporting and to produce many kinds of commodities for which it is not well fitted Thus the distribution of production and industry does not fit the geographic conditions Yugoslavia, for example, tries to manufacture iron goods, which it might much better buy from Germany or England. In the second place, tariffs, custom houses, and the difficulties which go with them tend to accentuate the nationalistic spirit which is one of the greatest dangers in Europe People are continually being made conscious that other nations are doing something which makes life more difficult A country like Germany, which met the custom houses of nine different countries on its land boundaries before Hitler seized Austria, was always being stirred up in this way. In 1930, the Swiss and many other Europeans were quite bitter against the United States because we raised our tariff and thus shut out some of their products When such things happen between neighbors the situation is much worse Thus innumerable annoyances due to tariffs have been one of the great causes of international friction. They played some part in causing the wars which began in 1914 and 1939, and thus in altering the political map and the distribution of people and industries

405 SUGAR AND TARIFFS Sugar provides a good illustration of the influence of tariffs, bounties, quotas, and other forms of governmental interference on the distribution of a commodity Cane sugar can be produced more cheaply than beet sugar This is partly because tropical labor is cheap, and partly because sugar cane not only requires less work in cultivation and harvesting than do beets but also yields more sugar per acre, as appears from the first line of Table 12.

TABLE 12

SUGAR. AVERAGE ANNUAL YIELD PER ACRE IN POUNDS

	Cane sugar		Beet sugar
World	3,500		2,600
Maximum large producer, Hawaii	7,000	Czechoslovakia	4,000
United States	2,150		3,300
Minimum large producer, Madagascar	1,100	Russia	1,500

406. In spite of the cheapness of cane sugar the United States and practically all European countries want to raise their own sugar, and

therefore must get it mainly or wholly from beets Accordingly tariffs are imposed and bounties are given to help raisers of beet sugar. The result is that 10 or 11 million tons of beet sugar are produced each year in addition to about 20 million tons of cane sugar Most of the beet sugar is produced in Europe, but the United States produces about 1¼ million tons In the continental part of the United States beet sugar goes ahead of cane sugar in productivity per acre (3,300 pounds against 2,150) Nevertheless about 300,000 tons of cane sugar are produced in Louisiana each year, in spite of the fact that frost makes it necessary to replant the canes every year Because of the cool climate Louisiana averages only about 15 tons of cane per acre, whereas in Hawaii the cane is allowed to grow 18 to 22 months, and the average yield per acre is about 60 tons Moreover, Louisiana gets only about 145 pounds of sugar from a ton of cane, while Hawaii gets about 235. When allowance is made for the fact that the Hawaiian cane grows for nearly 2 years before it is cut, the *annual* production of sugar per acre in Hawaii is 3 or 4 times as great as in Louisiana.

407 The production of sugar from either beets or cane would not be profitable in the United States unless the government imposed a tariff The tariff increases the price of sugar to everyone within the country, but it enables several thousand people to get a living by raising sugar The tariff also increases the profits on about a million tons of cane sugar raised in Hawaii and another million in Puerto Rico. The Philippine Islands have also been inside the tariff barrier, and that has helped them to market 1¼ million tons of sugar in the United States each year In addition to all this the country still needs 1½ million tons from foreign sources. Where shall this come from? The answer is Cuba Because of its peculiar political relation to the United States that island is allowed to pay a lower tariff than is charged to other nations Plenty of other countries, such as the British West Indies, Java, Brazil, and Peru, would be glad to raise sugar for the United States, but political considerations bar them out

408 This does not end the matter The tropical islands that are wholly or partly within the American tariff wall are able and eager to raise more sugar than is needed within that wall They could easily undersell the mainland sugar-raisers if they had the chance Moreover, the two which are independent or loosely attached to the country, this is, Cuba and the Philippines, could compete very sharply with Puerto Rico which belongs to the United States, and with Hawaii, which is a territory Hence Congress has laid down definite quotas or percentages of the total which each region is to supply Thus a political factor prevents the sugar industry from changing in such a way

as to fit it more perfectly to the geographical environment. Hawaii, to be sure, comes close to having the optimum climate for sugar cane, and Cuba, Puerto Rico, and the west side of the northern Philippines do not fall far behind Within the continental United States, however, Louisiana, which is the only appreciable sugar-raiser of cane sugar, lies close to the climatic limit of sugar cane On the other hand, Michigan, Colorado, and California, which together supply half the beet sugar, come fairly close to the optimum for sugar beets Nevertheless, if the political factor were removed, all production of beet sugar as well as cane sugar within the continental part of the United States would probably be unprofitable Java, Jamaica, and India, which have no political connection with the United States, would compete with Hawaii, Puerto Rico, Cuba, and the Philippines, and the geographical distribution of sugar production would be decidedly different from what it is today

409 III *Physiological Protection.* In practically all countries political methods are used to protect people, plants, and animals against disease. People are protected by quarantine regulations These sometimes interfere with commerce considerably, if there is a severe epidemic, but in our day such interference is rarely necessary In respect to plants, however, the quarantine regulations established by governments often have widespread influence For example, in order to prevent the introduction of new pests such as the Japanese beetle which is now ravaging the eastern states, the United States government has established a quarantine against plants, fresh fruits, and vegetables from foreign countries Hence little fresh tropical fruit, aside from bananas, is imported into the United States Two or three million dollars worth of grapefruit and pineapples, to be sure, come each year from Puerto Rico, and a million dollars worth of pineapples from Cuba These could be raised equally well in Jamaica, Mexico, or other tropical countries, but Puerto Rico and Cuba get the business largely because of their political relationships to the United States Nevertheless, even when products come from these favored islands, they have to meet rigid quarantine requirements and practically everything except bananas, grapefruit, pineapples, and a few oranges is excluded.

410 This gives a great chance to a small section at the tip of Florida where frosts are unknown and tropical fruits can be grown There a most interesting experiment is going on It is an attempt to find or create varieties of tropical fruits which will thrive outside of the real tropics in a place where the temperature occasionally falls quite low Around Miami experimental orchards raise a wide variety of tropical fruits such as the chirimoya, soursop, custard apple, and mango. The

owners are trying not only to raise these tropical fruits at their coldward limit, but also to improve their quality and make them more capable of standing low temperature This is one of the most obvious cases where methods of cultivation as well as the quality of the product tend to be best in regions close to the coldward limit

411 IV *Taxes* The location of industries and even of people is often greatly modified by taxes We have already seen how taxes on forest land induce large owners to sell out very cheaply after the lumber has been cut off, as is well illustrated in Wisconsin In similar fashion high taxes often drive industries and people from one state or city to another The southward migration of the cotton industry from New England to the Carolinas and Georgia was accelerated by the fact that many southern cities granted tax exemption for considerable periods to new factories In another way the same thing is often accomplished by providing a free location for such a factory In one city of North Carolina the city limits were changed in order that a desirable factory site might be placed outside the limits and thus enjoy a low rate of taxation instead of the higher rate prevailing within the city Many wealthy people change their place of residence from cities, where taxes are high, to suburbs, where they are lower, or from states with high taxation, such as Massachusetts, to those with lower rates, such as Florida Closely allied to this is the fact that many corporations are chartered in New Jersey or Delaware because the laws of those states are more lenient than those of such states as New York Another form of taxation which makes a slight alteration in the distribution of wealth is seen in the difference between one state and another in taxes on similar kinds of property For example, at one time Georgia licensed automobiles at the rate of $3 per year when many states were charging $10 or $20 for similar cars This led owners in other states to get licenses in Georgia, thus giving Georgia more money to spend on roads, schools, and so forth, and their own states less If all the varied influences of taxation are added together they have a considerable effect upon the distribution of wealth and industry

412 V *Attempt to be Self-Sufficient* One of the most active political factors in the distribution of crops and industries is the fear of commercial or military war A commercial war means a situation where one country tries by peaceful means to prevent another from buying or selling some commodity. In the 1920's the United States and western Europe were disturbed by the fact that the U S S R. was mercilessly lowering prices in order to sell lumber, coal, and other raw materials The Soviet Republic needed money to buy machinery and saw no way to get it except by underbidding competitors and thus starting

a commercial war. A different kind of commercial interference arises from the fear of war. For example, in 1937 the United States refused to permit Germany to buy helium gas for dirigibles. The American government controls the only available supply of helium, and it feared that Germany would use the gas for military purposes. This refusal stimulated the Germans to search for other places where they can control the production of helium, even though the cost be excessive, but the search has not been successful. Germany has tried all sorts of experiments in order to be able to supply itself with substitutes for rubber, petroleum, leather, and other products which it can produce only in insufficient quantities, or not at all. Other nations are doing the same thing. As a rule the substitutes are neither so good nor so cheap as the originals. Germany's synthetic rubber tires wear out rapidly. Its gasoline made from coal is very expensive. Many countries are also attempting to raise crops which are ill adapted to their soil and climate, or which could at least be raised more cheaply elsewhere. Russia is raising tea in a small warm area at the eastern end of the Black Sea. West winds from the Black Sea and the protection of the high Caucasus Mountains to the north keep the temperature there about like that of Sacramento. There is a rainy season in winter in both places, but Batum then has three or four times as much rain as Sacramento. Even in its driest month, May, it has three inches of rain, or almost as much rain as Sacramento's most humid month. In their effort to be self-sufficing, the Russians are doing just what the Americans are doing in southern Florida and Louisiana, that is, they are trying to make plants thrive at, or even beyond, their normal coldward limits. The Russians are doing the same thing with corn in Siberia, and are immensely proud of little nubbins raised in a relatively warm valley at the base of the Altai Mountains far north of what is usually supposed to be the limit of corn.

413. THE EXAMPLE OF IRELAND. The Irish Free State offers a good example of the way in which political as well as other factors alter the agricultural industries of a country. The main facts are summed up in Table 13.

414. *The Contraction of Irish Agriculture.* Between 1851 and 1931 the rural population of Ireland declined almost half. At the bottom of Column D the population is seen to be only 53 per cent as great in 1931 as in 1851. If the farmers in 1931 had carried on their farms in exactly the same way as did those of 80 years earlier, all the percentages in Column D would be 53. The fact that percentages of 52 are recorded there for potatoes and for "other food crops" shows that the average farm family devoted practically the same area to those

TABLE 13

CHANGES IN AGRICULTURE OF THE IRISH FREE STATE

	A	B	C	D	E
	Thousands of Acres, Animals, or People			Percentages	
	1851	1931	1937	1931 of 1851	1937 of 1931
Wheat	429	21	220	5	1,047
Oats	1,585	623	573	39	92
Potatoes	666	346	327	52	95
Other food crops	829	431	410	52	95
Hay	1,060	2,313	2,087	218	90
Sugar beets	None	5	62		1,240
Dairy cattle	2,185	3,173	3,286	145	104
Beef cattle	608	856	669	141	78
Sheep	3,507	3,575	3,000	102	84
Pigs	1,096	1,227	934	112	76
Poultry	12,000 (est)	22,782	19,491	189	84
Horses	423	450	429	104	95
Rural population in thousands	4,500 (est)	2,388	2,308	53	97

crops in 1931 as in 1851. But notice how far the other percentages in Column D depart from 53 Percentages of 39 and 5 indicate that in 1931 there were only three quarters as many acres of oats and one tenth as many of wheat per family as in 1851 On the other hand, there were more than four times as many acres of hay Sheep, pigs, and horses had doubled in number on the average farm Dairy and beef cattle had reached nearly three times their old number per farm, and poultry had become three or four times as numerous in comparison with the farm population

415. *Expansion of Crops versus Decline in Animals* Turning now to Column E we see that by 1937 the rural population was 97 per cent of what it was in 1931 The acreage of potatoes and of "other food crops" had changed to about the same degree, while oats and hay had declined a trifle more, but not to any significant extent Horses, too, had declined in about the same ratio as the farm population, but were still almost as numerous as at any time in Irish history This is inter-

esting because it illustrates the fact that in Ireland the farmers cannot afford motor vehicles All the other animals except dairy cattle show a decided decline in 1937, bringing them down to not far from 80 per cent of the 1931 level. On the other hand, dairy cattle (104) have gone up compared with the population, while wheat (1,047) has soared tenfold and sugar beets (1,240) twelvefold

416 *Causes of Growth of Irish Animal Industry* The Irish situation presents a pretty problem in land utilization It is typical of what happens in many other countries The changes from 1851 to 1931 were due mainly to (1) improvements in transportation, (2) growth of population in the lands beyond the sea, and (3) increased buying power in England. After 1851 the fertile plains of the United States, and later of Canada, Australia, and Argentina, rapidly filled with settlers who could raise wheat or corn quite cheaply, because they had large as well as level farms, and used improved machinery During the same period railroads were built and steamship services to Europe were multiplied This made it easy for Ireland to import wheat for its people and corn for its animals Wheat could be imported so cheaply that it no longer paid the Irish farmers to raise it except in a few specially favored spots Imported corn also became so cheap that it was often substituted for oats as food for animals These facts, together with the steady decline in the Irish farm population, cut the acreage of wheat to only 5 per cent of what it had been in 1851 and that of oats to 39 per cent Since potatoes, turnips, and other foods raised in Ireland cannot be so cheaply transported, their acreage fell off only at the same rate as the farm population

417 A change such as this always makes trouble and distress for a country Ireland is no exception, and its troubles with absentee landlords and national aspirations were greatly increased by the uncomfortable process of being forced to adopt a new type of farming The result would have been still worse had it not been that the steam engine not only brought cheap grain to Ireland, but also enabled England to develop manufactures and commerce with extraordinary rapidity Hence great numbers of people in England had to buy food instead of raising it Many of them were so prosperous that they could buy plenty of meat, butter, and eggs The humidity of Ireland makes it a fine region for both dairy and beef cattle, and it is not bad for sheep and poultry So the Irish found that by raising animal products and selling them to Great Britain they could more than make up for the losses arising because their grain was undersold by that of America This, then, is the reason why all kinds of animals increased in Ireland from 1851 to 1931, even though the farm population was cut in half. Thus

in 1931 the Irish farmers had become adjusted to a method of land utilization well adapted to modern methods of trade and transportation and to a climate which is generally too wet and cool in summer to be safe for wheat, but is almost unexcelled for grass, as is implied in the popular name, "The Emerald Isle."

418 *The Political Factor* The next thing that happened was a political change In 1933 Ireland insisted on being considered wholly free from Great Britain, and refused to pay certain sums which had been part of its contribution toward the expenses of the British Empire Great Britain retaliated by imposing customs duties on Irish meat, wool, butter, eggs, and so forth. It thus became harder for the Irish to export their animal products, because the price that they could get from British importers was lower This hurt the Irish trade so much that, whereas 49 per cent of the products of Irish farms were exported in 1929, only 34 per cent were exported in 1935 Hence in a few years the number of pigs fell off by one quarter The Irish farmers also gave up a fifth or a sixth of their beef cattle, sheep, and poultry, as appears from the percentages in Column E To meet this situation the Free State Government, or that of Eire, to use the Irish name, took a hand As a means of encouraging Irish agriculture and balancing the limitation of exports to England it gave the farmers a subsidy for butter Hence in Table 13 dairy cattle are more numerous in 1937 than in 1931 The government also obliged the Irish cities to use Irish bacon by prohibiting the importation of any other kind, but this did not prevent a decline of 24 per cent in pigs. The government likewise guaranteed the farmers a definite price for wheat, no matter how low the import price might be Although large amounts of wheat and smaller amounts of corn must still be imported, the government obliges the millers to use a certain amount of homegrown cereals

419 Another effort of the government has been to supply the Irish Free State with homegrown sugar No sugar beets were raised in Ireland in 1851, and there were only 5,000 acres in 1931. By means of government bounties the acreage was raised to 62,000 in 1937 It is still increasing, so that Ireland will soon be self-sufficient The aim of the government is to make the country supply its own needs for food in every possible respect The object is not so much to make the country safe in time of war, as to make it safe against political changes such as still higher tariffs in England, a combination of the great wheat-exporting countries to limit the amount of wheat and raise its price, or an agreement whereby the countries that now buy Irish beef or mutton might get their supplies elsewhere

420. *Political Shifting of Production Away from the Optimum.*

The effect of such political action on farmers and other producers is often unfavorable In the first place, it leaves the farmer in great uncertainty as to the future He may give up dairying, for example, and invest what little money he can scrape together in equipment for wheat-raising He has no assurance, however, against a change of policy which will favor the raising of sheep and poultry and be ruinous to the wheat farmer In the second place, such changes tend to increase the cost of living both in the countries that raise farm products and in those that buy them If the Irish farmer can buy Australian wheat and American corn cheaply, the cost of his own food and that of his animals is reduced Hence he can produce butter and eggs more cheaply If there is no tariff against butter and eggs, the factory people in England can get them more cheaply than at present, and will buy more Thus in the absence of tariff barriers and bonuses the Australian wheat-raiser, the American corn-planter, the Irish farmer, and the British factory hand would all get a better living than they do now The third disadvantage of the political manipulation of production is that it works real hardship to some people and favors others For example, wheat is a practical crop only in the drier, warmer, southeastern part of Ireland It will not grow at all on the west coast because the summers are too cool and wet When a political squabble cuts down Irish exports of beef and butter and leads the home government to adopt measures that raise the price of wheat, corn, and oats, the poor farmer in the west finds increased difficulty in selling beef cattle or dairy products At the same time he must pay more than before for the food of his family and for grain for his livestock This leads to increased poverty and discontent

421 GOVERNMENTAL ACTION ELSEWHERE The effect of governmental action in Great Britain is evident in Table 14, which shows what percentage of certain main food products was raised in Great Britain in 1931 before the recent attempt to increase home production, and what percentage was raised in 1935 after the governmental program for the encouragement of agriculture had been in effect a few years Similar figures might be given for Germany, Italy, Japan, and several other countries

422 *Preparation for War* The most wasteful type of political interference with the distribution of production and industry is that which is aimed directly at preparation for war This includes the making of all sorts of munitions and implements of war such as guns, shells, tanks, military airplanes, war vessels, and gas masks As a rule such munitions are manufactured in the main industrial areas of a country and thus do not change the general geographical pattern of

TABLE 14

PERCENTAGE OF BRITISH CONSUMPTION OF MAJOR FOOD PRODUCTS GROWN AT HOME

	1931 %	1935 %	Increase %
Pork products	33	50	17
Wheat	15	28	13
Eggs	57	68	11
Sugar	13	23	10
Beef and veal	48	53	5
Cheese	26	30	4
Mutton and lamb	41	44	3
Butter	11	9	− 2

its industry In many cases, however, iron foundries, railway yards, naval bases, and other centers of certain types of industrial activity are established in places where people would never think of locating them except for military reasons Germany has moved part of the great Krupp iron works away from the Essen region in order to locate them farther from the French border where they might easily be bombed. This means extra expense for the transportation of coal and iron, and often for hauling the finished product back to where the raw materials started Airplane factories are located where they will be best concealed and safest, rather than where they will be most efficient The Soviet Union has gone even further than Germany in placing its factories in locations that are favorable from the standpoint of defense It has established some of its main iron works far off in the Kuznetsk region in the heart of Siberia 2,000 miles more or less from where the products are used in large quantities It has done so largely because the Kuznetsk furnishes a supply of coal in a region so remote that it seems safe from all enemies no matter from where they come

423 The same sort of unremunerative expenditure of labor and capital is seen in many railways and motor highways of Germany They are planned primarily to enable troops to be shifted as rapidly as possible along the border or across country from one frontier to another The expense and taxation involved in such roads make it impossible to build equally good roads in other locations where they would be much more useful economically All this lowers the buying power of the people, so that fewer motor cars, for instance, are purchased in spite of the fine roads This example resembles most of the cases where political considerations cause human activities to move

away from the locations which they would occupy if only geographical and economic factors were in operation. Rightly directed, the political factor is a help in getting all sorts of crops and industrial activities into the optimum locations, but in actual practice it often serves merely to increase the costliness of production.

PART VI

ECONOMIC GEOGRAPHY OF ANIMALS

Chapter XVIII

ANIMALS AND THEIR OPTIMA

424 Kinds of Domestic Animals. Most of the principles which we have discussed in previous chapters in respect to plants are equally applicable to animals. According to the statisticians of the League of Nations, milk and meat are the two most valuable of all commodities. In the United States, ever since 1926, the value of the animals and animal products produced on the farms has been greater than that of all the crops, including those raised not only for human food, but also for raw materials and as feed for the animals themselves In some years the animal products are worth 50 per cent more than the crops In countries such as China and Japan the importance of animals as sources of food and raw materials is not so great as in this country, but in places like Arabia animals are the main source of livelihood outside the oases. Their importance in transportation and farm work in most parts of the world is still very great. If fish, game, and furs are included, the value of animal products becomes still greater, although all the fish caught by American ships and in American waters are worth only about 2 per cent as much as the animal products on the farms In 1926 animal products had reached as great a value as all the mineral products of the country, including coal, metals, stone, brick, etc In more recent years they have gone ahead of the mineral products by as much as one fourth.

425 By far the greater part of this huge value is due to four species of animals—cattle, sheep, pigs and chickens Horses, too, are of primary importance, but their value lies in work, not in products As a matter of fact, there are only twenty-four species of animals all told which are of much economic significance, and six of these are cattle, three belong to the camel family, and two each to the horse, goat, hen, and duck families The deer, pig, dog, cat, and pigeon families each have one representative The other two species are insects, the bee and

the silkworm Such a list of domestic animals is remarkable for (1) its small size, (2) its limitation to a few orders, (3) the dominance of a single order, and (4) the fact that every one of the animals was domesticated so long ago that we have no certain record of the event, and in most cases not even a tradition. Among the 3,500 known species of mammals our list includes only 17, among 13,000 birds, only 5, and among about 470,000 species of insects only two Reptiles, numbering 3,500 species, amphibia, 1,400 species, and fish, 13,000, have no representatives Among the 17 mammals, no less than 13 belong to the artiodactyla, or even-toed, hoofed order

426 CRITERIA OF USEFULNESS AMONG ANIMALS Let us inquire why man has selected such a small list of animals for his own special use In a stage of culture such as ours, the main uses of domestic animals are (1) food, (2) transportation, (3) clothing, (4) raw materials, (5) fertilizer, (6) protection, (7) hunting, (8) scavenging, and (9) ornament, music, or other esthetic enjoyment The first four uses are now so much the most important that we will not consider the others further The most useful animal, then, is the one which most fully combines valuable qualities for food, transportation, clothing, and raw materials So far as meat is concerned, thousands of wild mammals and birds are about as good as the ones that have been domesticated Hence other qualities have determined which animals man will use One such quality is the ease with which an animal can be fed It is obviously much easier to feed animals on grass than on almost anything else If the animal is allowed to feed itself, a rabbit can thrive in a small fenced grass plot, but a cat must wander over many acres The same is true of cattle compared with lions This suggests another important quality. A good domestic animal, especially the female whose milk or young may be taken from her, must not be too dangerous to handle Tiger's milk may be good, but a cow can be milked much more safely Again, a good domestic animal must be gregarious, so that by nature it tends to live in herds Otherwise, a herdsman could not collect enough animals to give him a living In the thousands of years before man learned to make fences it would have taken most of a man's time to keep only one or two bears from roaming away, but one man can herd a thousand sheep. Another important quality of a good domestic animal is that it should reproduce rapidly and grow fast Pigs, with their large litters, are unusually good in this respect, and so are hens, turkeys, ducks, and geese, which hatch much larger numbers of eggs than do birds such as pigeons or eagles

427 Although a few kinds of domestic animals are worth keeping for only one purpose, the great majority are kept because they supply

some special need in addition to meat. The pig, to be sure, reproduces so fast and is so easy to feed that it is worth keeping for meat alone. The pigeon, too, is easily fed, but is of little importance. Dogs and cats are worth keeping as pets, and for protection and hunting. Every other domestic mammal mentioned above is used as a source of milk. In the cattle family ordinary cows, humped zebus, the Indian gayal, the Javanese banteng, the Tibetan yak, and the water buffalo all supply milk. Some people think that the Arctic musk ox, which belongs to the same family, might also be used for this purpose. The cow and the goat, however, are such good milkers that they are gradually supplanting the others, although the nomads of Asia still drink the milk of sheep, mares, camels, and asses. The milk of both the one-humped Arabian camel and the two-humped Bactrian camel has a somewhat acid taste, that of the water buffalo has a nutlike flavor, sheeps' milk and goats' milk are much like that of cows, but are richer in fats. One of the chief reasons why the cattle family is so predominant among domestic animals is that its members are especially good milkers. Considerable size is highly desirable in an animal that is to be milked. Even the sheep is rather small for profitable milking. In the same way, aside from the pigeon, the birds that man has chosen—hens, turkeys, geese, ducks—are large enough not only to make a good meal for at least one person, but to have eggs large enough to be worth utilizing. They also live in flocks, and the number of their eggs is large. Hens are by far the most common kind of domestic birds both because they are good eating and because they lay many eggs of a convenient size and do not readily fly away. Goose eggs are too big.

428. Aside from supplying meat, milk, and eggs the important functions of domestic animals are to supply raw materials and to assist in transportation. The main use of raw materials derived from animals is for clothing, but many minor uses are familiar to everyone. Furs furnish admirable clothing, and leather is of great use for shoes, but to get either of these it is necessary to kill the animal. One of the chief reasons why sheep are so numerous is that, in addition to supplying good meat and some milk, they supply wool which can be clipped from the animal without killing it. The same is true to a less degree of the hair of the goat, camel, and llama, but these cannot be made into felt or thread so easily as wool. For purposes of transportation the ideal animal must be large enough to carry a man, but not so large that it is unduly hard for one man to manage it, or that it requires an undue amount of food. The elephant is too big to be of much use. Even the camel is larger than the ideal. The donkey and the reindeer, on the other hand, although good, are too small for many purposes. The

horse and ox are about right. That is one reason why they are so widely used. Americans have little realization of how widely the ox is the work animal of countries such as India, Bulgaria, and even France. As soon as mankind learned to use animals not only to ride on, but to draw carts or plows, still another quality in addition to weight, size, and docility assumed high importance. Hard hoofs became essential, for when an animal pulls hard, it must dig its hoofs into the ground. The camel is not good for this purpose because its feet are too soft and cannot easily be shod with iron. The sheep and goat are too small. Cattle are good, but the fact that their hoofs are split is a disadvantage. Thus the horse and donkey are left as by far the best animals so far as their feet are concerned, but the donkey is rather small.

129. It is evident from all this that a good domestic animal must satisfy many requirements. The main things we require in domestic animals may be summed up under three headings. A. *Materials*: (1) meat of good flavor, (2) palatable milk, (3) eggs that are not too large or too small, (4) wool or hair which can be taken from the animal without hurting it and can easily be made into felt or thread, (5) strong, pliable leather. B. *Physical qualities*: (6) rapid rate of reproduction, (7) rapid growth, (8) capacity to live on grass or similar plants so that many can easily be supported on a small area, (9) sufficient size to be worth milking and to be large enough to carry a man or pull a load, (10) hoofs that can get a firm grip on the ground and can be shod. C. *Mental qualities*: (11) gregariousness so that the animals can be herded or kept in flocks, (12) docility so that they can be handled, ridden, and milked, (13) intelligence and spirit so that they can be trained. Other qualities might also be named, but these are enough to show how complicated has been the problem of choosing the right animals for domestication.

130. Of course, primitive man did not think of all this, but nevertheless, little by little, the preceding requirements have governed man's relation to domestic animals. The result is that in the world today the number of domestic animals is approximately as in Table 15.

The significant fact here is that after thousands of years of experimenting mankind has decided that six species of animals are the ones that chiefly meet his needs. Even among these there is a strong tendency to replace goats by sheep wherever possible, and horses are being displaced by motor vehicles, as are donkeys, camels, and llamas. Reindeer may increase in importance, but they are sure to remain a minor factor. It all comes down to this: the best domestic animals are cattle together with buffaloes, horses with asses and mules, sheep with goats in some places, swine, and hens. Cattle are by far the most valuable, while

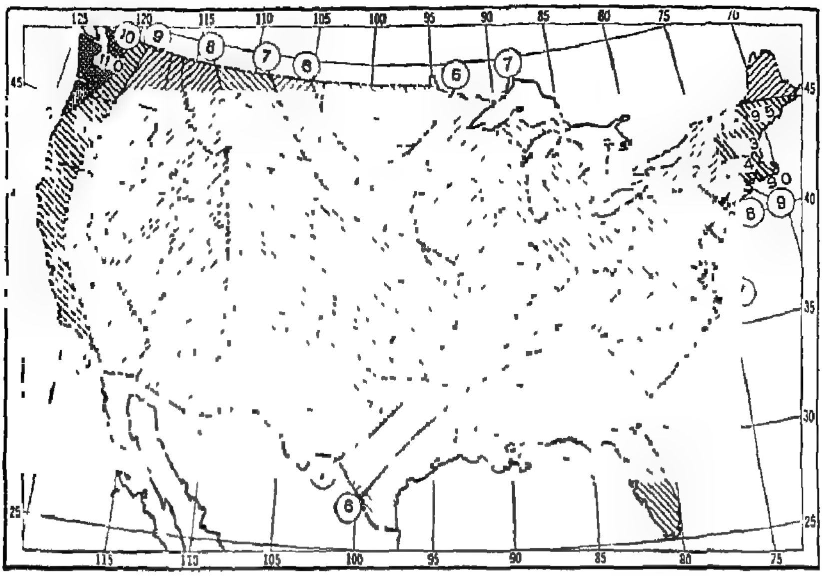
TABLE 15

TOTAL NUMBER OF DOMESTIC ANIMALS (IN MILLIONS)

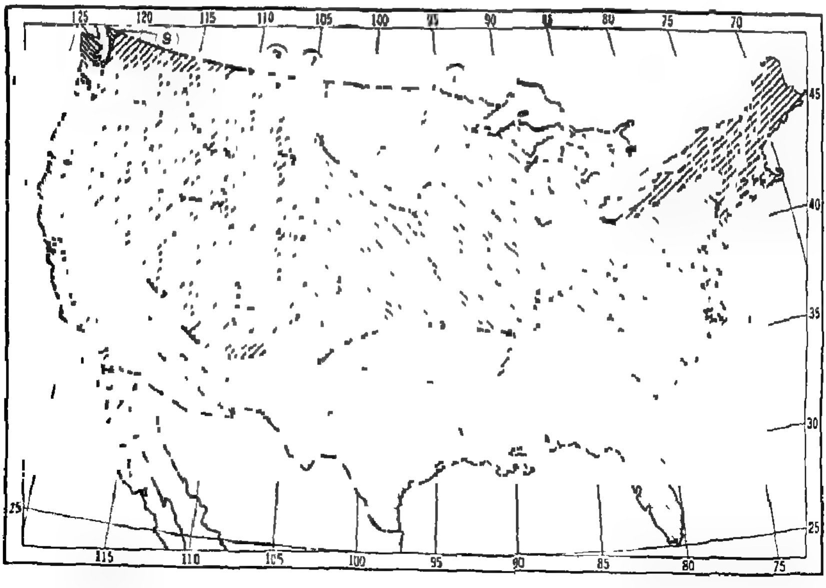
Sheep	700	Buffaloes	73	Hens	1,900
Cattle	650	Asses	33	Ducks	110
Swine	350	Mules	16	Geese	75
Goats	110	Camels	6	Turkeys	23
Horses	96	Reindeer	2		
		Llamas and alpacas	2		

in the past, as a means of helping man to make progress, horses have been extremely useful. Today the prosperity of approximately two thirds of the world's people, the ones who depend on agriculture, is largely bound up with the number and excellence of the animals that they can raise. Therefore it is of the first importance to understand the factors which determine the geographical distribution of these animals. Fish also form a large item in the world's food supply, and insects are important because a few of them furnish valuable products and many do much harm. It will pay us to study animals in the same way that we have studied crops.

431. ECONOMIC GEOGRAPHY OF HENS. I. *Variations in Egg-Laying.* The greatest contribution of birds to the world's economic life comes through hens' eggs. These are worth from $600,000,000 to $800,000,000 in the United States alone every year. Since the chickens and other poultry raised each year are worth four fifths as much as the eggs, the economic contribution of birds amounts to $8 or $10 per year for every man, woman, and child in the United States. The hens in different parts of the country by no means take an equal share in making this contribution. This is evident in A431, which shows how many dozen eggs are laid each year per "chicken 3 months or more of age," as the census puts it. The numbers would be appreciably larger if confined to hens old enough to lay. There are great differences in the various parts of A431. In the state of Washington the average rises to 11 dozen. The rest of the Pacific Coast and most of New England show averages above 9. The Corn Belt falls to 6 or 7, while North Dakota and the southeastern states except Florida range from 5 to 6. Our present problem is to discover why hens in Washington lay almost twice as many eggs as in North Dakota or South Carolina, and at least 50 per cent more than in the states from Iowa to Pennsylvania. In Europe there are similar variations, but the figures there are not directly comparable with those of the United States because of differences in the degree to



A431—Dozens of Eggs per Chicken per Year.



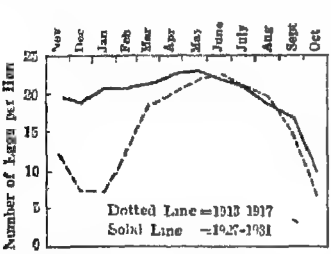
B431—Dozens of Eggs Expected per Hen per Year on the Basis of Temperature

which non-laying poultry are included Taking the figures as they stand, Sweden gets 100 eggs per hen, Denmark farther south gets 110, and Holland, 125 This is about the same as the 11 dozen (132) of Washington, but probably means less because it is based on mature hens instead of those over 3 months of age Farther east and south the figures fall to 75 or 80 in Bulgaria and Spain, and only about 50 in Portugal. In each continent the general map of the distribution of eggs shows considerable resemblance to that of the yield of wheat per acre.

432 II *Reasons for Variations in Egg-Laying* Hens may lay more eggs in one region than in another (1) because they belong to better breeds, (2) because of better feed and care, (3) because rich markets make it worth while to devote much attention to them, (4) because some condition such as climate makes them vigorous It is hard to see how any one of the first three reasons, or all of them together, can account for more than certain minor features of A431 Farmers seek the best breeds of hens in Massachusetts, Michigan, California, and Iowa just as much as in Washington They give equally good care and feed in all these states Moreover, the market for both eggs and poultry is much more extensive in the North Atlantic States and westward to Chicago than in the state of Washington Nevertheless, Washington outranks all other states Moreover, Maine and Oregon, which rank no higher than many other states in nearness to market and in the pains which they take to get good breeds and to feed and care for them properly, stand next to Washington, and far outrank such states as Wisconsin and Illinois In Florida, on the other hand, a production of 7 4 dozen eggs per hen each year, in contrast to 5 5 in neighboring states, seems to be definitely associated with the introduction of good breeds and good methods of care, and with a large and profitable market because of winter visitors and recent migrants from other states farther north These conditions, however, by no means explain why Florida falls so far below New England and the Pacific Coast. If only the poultry belonging to white people in the South are compared with those of the North, the figures for Florida and its neighbors are raised somewhat Such a limitation, however, would by no means bring the figures for the southern states up to the New England level, nor would it account for the low figure in North Dakota, or the high figures in the Northwest Thus, although some of the minor features of A431 can be explained on the basis of breeds, care, and markets, the main features remain unexplained unless we appeal to climate

433 III. *Climate and Egg-Laying.* Egg-laying contests, such as those of the Agricultural Experiment Station at Storrs, Connecticut,

throw light on the relation between egg-laying and weather. Young hens are brought to Storrs from all over the United States and even from Canada and England. The fact that some of them lay 300 eggs per year, while practically all lay 200 in contrast to about 100 for ordinary hens, shows what wonderful results can be obtained by selective breeding and proper care The daily record of the eggs laid by each hen for a full year shows that hens respond to the weather in the same way as plants. Spring and summer are the season for laying eggs The dotted line in A433 shows the average number of eggs laid per month by each of 50 white Leghorns from 1913 to 1917 In December and January the number fell to only 7, but in May, June, and July it rose above 20, and then fell off in the autumn This is the normal course of events In the wild state birds in climates like that of Connecticut lay no eggs at all in winter or fall Then egg-laying shows a sharp maximum in the spring, and then tapers off The reason for this is obvious Chickens hatched in cold weather might freeze to death if not cared for by man, and even if they did not freeze, they would probably die for lack of food Hence in a state of nature birds which fail to be stimulated in some way so that they lay eggs in the spring, or which are stimulated by conditions prevailing at unfavorable seasons, leave few offspring. Their stock soon dies out Thus a tendency toward egg-laying under certain definite conditions of weather has become an essential part of the heredity of hens This fact seems at first to belong to biology rather than economic geography, but further thought shows that it represents a principle which is a major factor in the geographical distribution not only of egg-laying, but of every phase of agriculture and of a vast number of human activities The principle is that *most plants and animals have a definite seasonal rhythm of reproduction, and thrive best where the weather fits most perfectly into this rhythm.*



A433—Seasonal Variations in Egg laying (White Leghorns)

434 IV *The Influence of Light.* Seasonal variations in the amount of light play an interesting part in egg-laying, as well as in the production of flowers and seeds In A433 the solid line shows that during the 5-year period from 1927 to 1931 the seasonal trend of the production of eggs by white Leghorns had become different from what it was from 1913 to 1917 (the dotted line). In the later period the young hens began by laying far more eggs from November to February

than did the hens of the earlier period, but from May onward the two groups behaved the same. The difference is due partly to the fact that the later hens represent greater selection and breeding in order to get strains that mature early and lay in the fall Much of the difference, however, is due to the fact that during the later period the henhouses were artificially lighted for an hour or two before sunrise and after sunset throughout the period of short days. It was formerly supposed that additional eggs were laid under such circumstances because the artificial light gave the hens more time to feed We now know, however, that among many birds an increase in the duration of light, after the days have become short, stimulates reproduction, even if the food supply does not change The increasing length of the day in spring is a primary factor in causing birds to migrate to cooler latitudes It stimulates the reproductive system, and induces a restlessness which is transformed into migration and then into nest-building Confirmation of the effect of light is seen in the fact that in cool weather when the days are short, but not in warm weather when they are long, the hens at Storrs lay more eggs in dry, sunny periods than in those with heavy rainfall The rainfall itself probably has little to do with the matter, but in rainy weather clouds greatly diminish the amount of light, especially in winter In high latitudes the number of eggs per hen tends to be small It is diminished not only by low temperature and long nights, but also by the length of the days in summer The long days stimulate egg-laying at first, but the hens are said to become exhausted because they do not get time enough to rest at night.

435 V *Temperature and Production of Eggs* The egg-laying contests at Storrs and elsewhere show that temperature as well as light has a pronounced effect on hens In cold or hot weather, regardless of the amount of light, the number of eggs falls off More eggs are laid when the temperature averages from 40° to 65° than when the weather is either warmer or colder It is difficult to separate the effects of temperature and light, for the two vary together We can, however, construct a map (B431) showing how the production of eggs would vary from one part of the United States to another if the relation between egg-laying and temperature in all parts of the country from season to season were the same as at Storrs The main features of this map and of A431, showing the actual number of eggs, are similar Both show high areas in the Northeast and along the Pacific Coast, low areas in North Dakota and the Southeast An annual production of 11 dozen eggs per hen in the state of Washington agrees with the fact that at Seattle, for example, the summers are delightfully cool (averaging 65° in July), and the winters are mild with a January

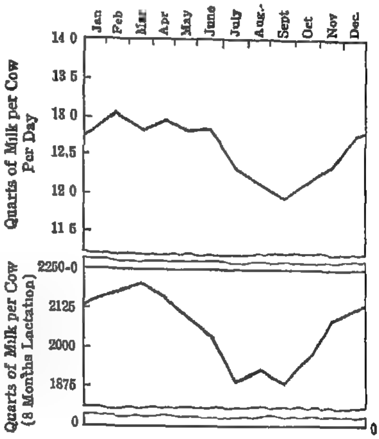
average of 40°. A similarly close approach to the temperature under which the most eggs are laid, that is, to the optimum for egg-laying as indicated by the records at Storrs, is found in a narrow strip along the whole Pacific coast. Its effect is evident in the dark shading of both A and B431.

436. In Maine, where both maps are heavily shaded, the winters are quite cool (average 22° in January at Portland), but the summers (July 68° at Portland) are favorable. North Dakota, on the other hand, is far less favorable in winter (January 8° at Bismarck) and not so good in summer (70°). In the Southeast the winters are favorable, but the long hot summers are distinctly unfavorable to egg-laying In Europe, Holland with its productive hens, enjoys a climate much like that of the Seattle region, whereas Russia, where the hens lay comparatively few eggs, corresponds to North Dakota Bulgaria and Spain, with a moderate production, correspond to Missouri, and Portugal, with a low production, to our Southeast From all this it appears that in general the geographical distribution of egg-laying varies in harmony with the degree to which the climate departs from the optimum for hens, but other conditions often prevent perfect agreement, as we saw in Florida Such disagreements sometimes obliterate the climatic pattern

437 FACTORS IN THE DISTRIBUTION OF CATTLE The principle of climatic optima applies to domestic animals even more clearly than to hens. Inasmuch as meat and milk are the most valuable of all commodities we may well study cattle carefully In Chapter XVI we saw that the average weight of cattle sold for slaughter (A397) is over 900 pounds in Illinois, Iowa, and Wisconsin, and that this is generally attributed to the abundance and cheapness of corn New Jersey, Oregon, and California, however, produce equally heavy cattle, although they are by no means corn states In fact, the weight averages above 800 pounds in the whole northeastern quarter of the United States In the dry sections farther west it falls lower, below 700 in North Dakota In the entire tier of most southerly states the average is still less, 450 to 660. This can be due only in part to poor methods of breeding and care, for Florida, which excels its neighbors in many phases of agriculture, has the smallest, thinnest cattle of any state except Georgia Farther south in Cuba and Venezuela the cattle are still smaller In Europe there is a similar decline in weight from the North Sea to North Africa. Evidently, the weight of cattle varies according to the climate It follows the same general geographical pattern that we have seen for corn and wheat, and shall see again in many other cases. The question is whether this pattern arises because

of direct climatic influences or through vegetation, pests such as ticks, and the care and feed given to animals by their owners

438. *Season of Most Abundant Milk* Before attempting to answer this question let us examine the seasonal and the geographical variations in the production of milk. At a large Walker-Gordon farm, near Princeton, N J, a rotolactor, or revolving platform on which 50 cows are milked at once by machines, is used to milk 1,300 cows three times a day This costs no more than milking twice a day in the ordinary way and gives 15 per cent more milk. In order to maintain a steady supply of milk at all seasons, the birth of calves is spaced almost uniformly throughout the year and the cows are never pastured, but are fed essentially the same kind of green fodder or ensilage all the time Therefore, seasonal differences in the yield of milk must be due mainly to the weather The upper curve of A438 * shows that the average daily yield of milk per cow on this farm during 17 years varied systematically with the seasons More than 13 quarts was produced per day in February, but from January to June the amount was almost as high as this. Then the milk declined rapidly to less than 12 quarts in September, but afterwards rose steadily until February As long as the temperature remained below 60° the amount of milk was large



A438—Seasonal Variations in Milk Production

439 Other kinds of evidence show that the seasons exert a great influence on the amount of milk given by cows The lower curve of A438, for example, is based on John A. Gowan's records of the average amount of milk produced per cow during the 8 months after the birth of calves Cows whose calves were born in March, which appears to be the normal month, gave about 16 per cent more milk than those whose calves were born in summer Experiments made by the United States Department of Agriculture show that in winter cows give the

* In each section of A438 a large part of the vertical scale is omitted because the entire height from zero upward would take too much space The student should pay attention to this point in every such diagram in order not to get an exaggerated idea of the importance of the ups and downs in curves

most milk if the barns are kept at a temperature not higher than about 50°. Still other experiments show that cows cannot stand hot weather because they have no mechanism for sweating. They can cool themselves only by breathing more rapidly. For example, cows kept by Regal and Richardson in rooms where the temperature was accurately controlled and the relative humidity was kept constantly at 60 per cent showed the following differences according to the temperature of the room.

	Breathing rate	Body temperature	Pulse rate	Pounds of milk per day per cow
At 40°	12	101.0	72	29
At 80°	56	101.8	61	25
At 95°	106	103.7	57	17

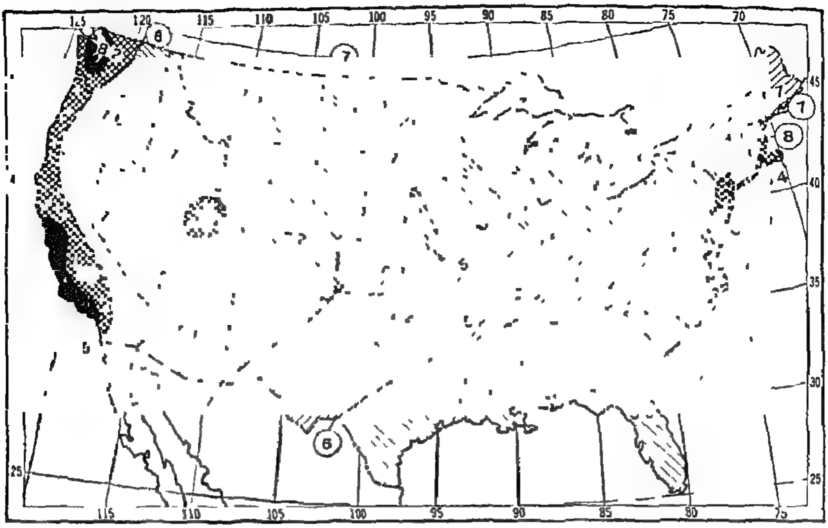
When the cows became thoroughly hot they panted at the rate of almost two breaths a second, and had a genuine fever. Their milk dropped to little more than half the normal amount. Again at Beltville, Md., close to Washington, D. C., 10 Holstein cows which were fed the same food at all times began the month of August with an average production of 45 pounds of milk per day. After 10 exceptionally hot, humid days this had fallen to 38. A cool spell raised it to 42½, then 3 days well up in the nineties brought the daily production back toward 38. Jersey cows showed similar but less marked fluctuations in the amount of their milk.

440. Hot weather makes cows feel so miserable that they do not eat well. For example, at New Iberia, La., a herd of the European breed called Angus was compared with Indian cattle—the kind called zebus or Brahmans. One day the temperature started at 76° at 6 A.M., rose to 92° at noon, and fell to 82° at 5 P.M. The zebus grazed 84 per cent of the time and never sought the shade. The Angus cattle, on the other hand, failed to graze during a quarter or even half the time, and went into the shade trying to keep cool. Animals cannot grow well and put on fat when they feel too hot to eat. What happens to cows in hot weather is evident from the experience of another day when the temperature was 82° at 8 A.M. The poor Angus cattle were already breathing at the rate of 100 per minute, and had an average body temperature of 102° instead of 101°. The rate for the zebus, however, which are adapted to the hot climate of India, was only about 25 and the body temperature 101°. At 10 o'clock the temperature of the air had risen 102° out in the sun, and the poor Angus cattle were

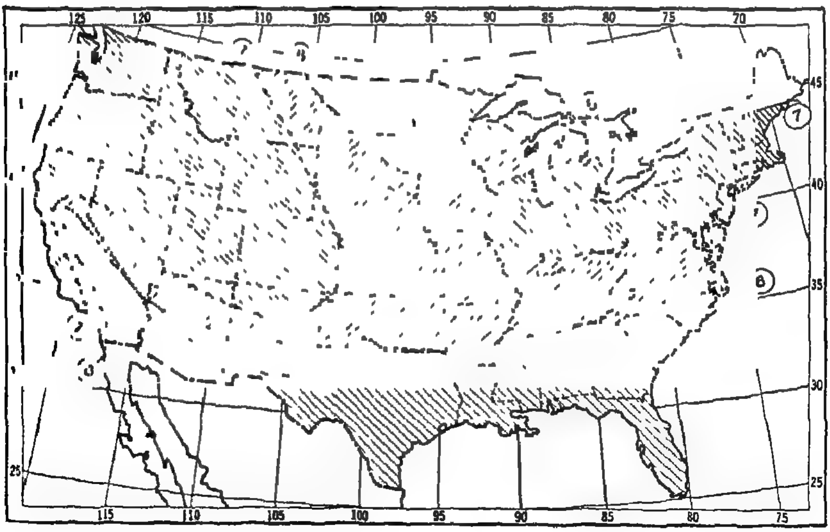
brought in and put in the shade. Their breathing had risen to 140 per minute, and their body temperature to 104.4°. After several hours in the shade they were still breathing at the rate of 120 per minute. The zebus' rate of breathing went up only to 42, and their body temperature remained at 101° all the time.

441. *Best Climate for Milk Production.* These facts and many others make it clear that the climatic optima of cattle are as well defined and important as those of crops. Moreover, they vary from one species, or even one variety, to another. European cattle in general give the most milk when the temperature averages a little above freezing, perhaps 35° or 40° as is common in March in the northern United States. A fairly good rainfall, or at least a fair degree of atmospheric humidity, is also desirable. The best climate, therefore, is one with a winter averaging not lower than freezing and a summer which rarely has temperatures above 80° and has an average of perhaps 65°. This is much like the climate of Denmark, which is the world's most remarkable dairy country. Thus cows behave like hens, except that their optimum temperature is lower and artificial light does not enter into the matter. The lowest production is found at high temperatures regardless of whether the air is dry or humid. Hays, Davidson, and others have found that at high temperatures cows give milk with a low percentage of butterfat. The milk appears to be not only most abundant but also richest in fat at temperatures a little above freezing, and then declines again in both respects at lower temperatures.

442. These facts as to climatic optima explain why ordinary cattle produce not only the most but the best milk in regions such as Holland and Ireland, with cool, moist, fairly rainy summers, and winters that are cool enough to have frost, but not to freeze the ground very hard. Swiss cattle fed on Alpine pastures in summer and kept in the lower valleys in winter are favored by similarly good climatic conditions. It has generally been supposed that cattle thrive in such places because the climate is especially good for grass. The grass in such places is undoubtedly short, sweet, succulent, and nutritious, and hence a great help in producing healthy cattle and abundant milk. We now realize, however, that the climatic conditions which produce such grass also provide the optimum for the cattle themselves. A very delicate adaptation to climate has doubtless been acquired during the long process by which the world's best breeds of cattle were originally developed in the North Sea region. It will be very interesting to see whether animal breeders can cause the ability of these breeds to produce milk and meat to be combined with the ability of the humped India zebu to withstand heat, drought, and coarse food.



A443—Milk per Day per Cow on Dairy Farms in the United States (Quarts)



B443—Milk per Day per Cow Expected on Basis of Temperature

443 *Distribution of Milk Production in the United States* Let us next examine the geographical distribution of milk production. Compare A443, showing the actual daily production of milk per dairy cow in various parts of the United States, with B443, showing the production that would be expected on the basis of temperature alone Here, just as in the corresponding maps for eggs, we find that the major features of the actual geographical distribution are what we should expect on the basis of climate, but certain features are due to other conditions The main points in which the maps of actual conditions (A443) and of expectation on the basis of temperature (B443) are alike are as follows (1) high yield of milk per cow from Iowa eastward to the Atlantic Coast, (2) high yield in a large western section, (3) decidedly greater yield along the coast of California than a short distance inland, (4) a low yield in the Southeast The outstanding features which do not seem to be climatic are seen in the dry Southwest, and near the cities of the Pacific and northeastern coasts where the productivity of both cows and hens is higher than would be expected on a purely climatic basis The proximity of the large cities causes the development of fancy farms where highly bred animals give enormous yields of milk For example, the cows of A438 average 12 to 13 quarts per day, whereas the average American cow gives little more than half as much From such farms many high-grade animals are sold to smaller farms, thus raising the general average of the surrounding region In the Southwest the yield of milk per dairy cow is high in A443 not merely because California has many fine farms like those in the East, but also because irrigated areas raise alfalfa, which is highly nourishing in its fresh state and also retains its vitamin content when dried, especially if it dries quickly as is possible in these warm, sunny regions Then, too, in the dry Southwest dairy cows are scarce in proportion to the whole number of cattle Outside the oases, most of the cattle are of the beef type, and even when milked are not counted in the statistics on which A443 is based Thus the statistics apply only to the better herds and not to practically all herds, such as are counted in the East and South On the whole, then, the general pattern of productivity of cows as well as of hens is set by climate, but other factors modify it The kind of climate which is best for cattle is also very good for nutritious grasses It is not greatly different from the optima for corn, wheat, and chickens, but is decidedly different from that of chinchbugs and boll weevils.

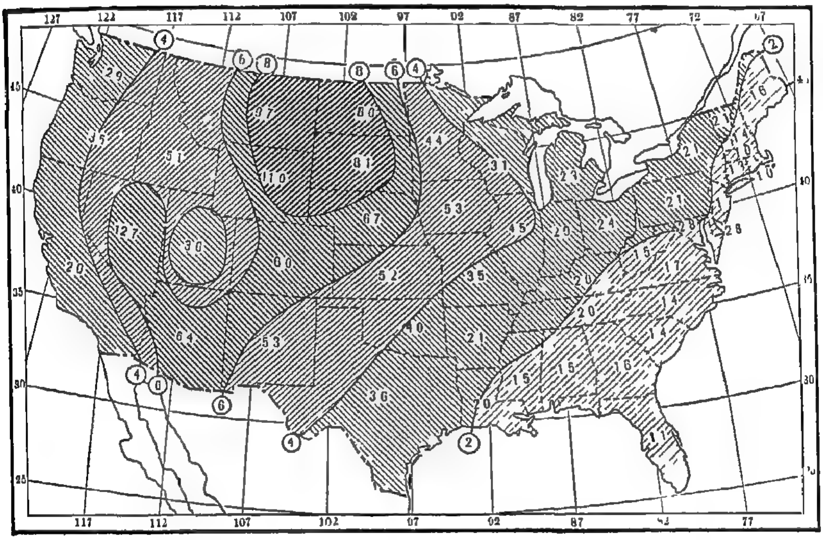
CHAPTER XIX

ANIMALS AND SOURCES OF POWER ON THE FARMS

414 IMPORTANCE OF WORK ANIMALS As a means of promoting human progress the use of animals for carrying people and goods, and especially for pulling plows and hauling loads, may have been more important than their use for food Without beasts of burden, human beings might never have progressed beyond the stage of carrying loads on their own backs or shoulders, as is common in China and among the highland Indians from Guatemala to Peru Unless animals had been used to pull the plow, the main centers of agriculture and civilization might never have migrated into regions of forests and prairies where the climate is especially stimulating and the highest civilizations now center In most such regions a cover of sod soon smothers a field which is not kept free from grass If the fields are not plowed, but are spaded by hand, the labor is so excessive that only a small area can be cultivated Thus people are kept in poverty, and a high standard of living is practically impossible Without draft animals, mankind might never have realized the wonderful possibilities of the wheel and of modern machinery Modern machinery is largely based on wheels connected by gears or bands Another essential element in the growth of civilization has been intercourse between diverse regions Camels, donkeys, and oxen as well as horses, have been of inestimable importance in increasing such intercourse Human progress has been greatly helped by exchanging ideas and inventions with people in other places Thus beasts of burden and draft animals have played an essential part in the evolution of civilization.

415. FARM WORK BY OXEN, HORSES, AND MOTORS It is sometimes supposed that the horse has made the ox no longer a necessity for farm work, and that motor transportation will soon put the horse completely out of business on the farms Each of these statements is true in limited regions, but not in the world as a whole Oxen still draw the plow and carry home the harvest not only in India, Java, and Indo-China, but also in practically all of Turkey, Greece, Spain, and many other parts of Europe Even in countries as advanced as France, oxcarts and automobiles often share the roads, while in Nova

Scotia, Quebec, and the southern Appalachians there are more oxen than tractors Oxen are cheap and therefore are still used in poorer parts of the world, except where the farm work is largely done by man instead of beasts In most of the world the displacement of horses by trucks and tractors is not proceeding any faster than the displacement of oxen by horses In such countries as England and Germany, and even in most of our own country, the plowing and hauling on the farms are still done largely by horses and mules In the United States, the number of horses diminished from 21½ million in 1919 to about 9 million in 1945, and is still falling Mules, which are used mainly



A446—Horses and Mules per Farm, 1930.

in the South, where automobiles, as well as tractors and trucks, are still too expensive for many of the small farms, diminished from a maximum of 6 million in 1925 to about 3½ million in 1945. Their number continues to decline, but less rapidly than that of horses

446 Much of the diminution in horses is due to the substitution of automobiles for carriage horses, and not to the substitution of trucks and tractors for work horses This is evident in A446, which shows that in every state there is still at least one horse or mule per farm, and in most states two or more In the great wheat states of the northern plains, where the farms are large, the number rises to 8 or 10 Farms can raise grass and oats for work animals much more cheaply than they can buy gasoline and oil Moreover they can raise

colts but not automobiles These facts are bound to make the substitution of motor vehicles for work animals a slow process in all except the richest parts of the world In a broad way, the world may be divided into regions of low productivity where camels, yaks, reindeer, and dogs are used for transportation, and productive regions where the farm work is done predominantly in one of four ways (1) by men (and women), (2) by oxen or buffaloes, (3) by horses or mules, and (4) by motor vehicles All four of these sources of power are frequently utilized in the same region, but generally one of them is the main method of turning up the soil in the fields and carrying the farm products to the home and market.

417 MAN-POWER ON FARMS The parts of the world where transportation and the upturning of the soil are performed mainly by human labor contain vastly more people than the sections where motor driven machines do most of this work Only a small percentage of the agricultural population keeps any work animals among three or four hundred million people in China and Japan Among many others in southeastern Asia, the East Indies, and large sections of tropical Africa and Latin America a considerable percentage of the farmers dig up the fields by hand, and carry the products home on their backs A fifth, or possibly a quarter, of all the earth's inhabitants do this

418 Such dependence on human labor arises from a combination of many factors One of these is overpopulation, by reason of which there is no room to raise food for work animals Overpopulation almost invariably means that the farms are small It also means that, having been passed from father to son by inheritance for many generations, they have been much subdivided Hence many farms consist of tiny fields which are often widely scattered among fields belonging to other owners Moreover, in order to get more land for cultivation many hillsides have been terraced, and the fields on the terraces are very small Many fields on terraced hillsides in Japan, China, Syria, Peru, Spain, and even Italy are so small that a yoke of oxen fastened to a plow can scarcely turn around Such conditions make it peculiarly hard to use work animals Another deterrent to the use of animals is that rice straw and the stalks of other products of low latitudes, such as manioc, do not make good fodder Moreover, the native grasses are coarse and tough, in many places they become dry and hard during a hot rainless season, and fodder of better kinds is difficult to raise. Again, in many of the "man-power" regions the climate, parasitic diseases, and insect pests, such as the tsetse fly of Africa, almost prohibit the use of cattle and horses. In some places, such as the Andes, the

steepness of the slopes makes it well-nigh impossible to plow with animals, and difficult even to use pack trains Again, in large sections of tropical regions the soil is wet so much of the time that plowing is difficult although the soil can be dug up with a spade

449 In most of the man-power regions the methods of agriculture are crude As a rule, the soil is turned up with heavy hoes instead of plows This "hoe agriculture," however, varies from the well-known and skillful horticulture of the Chinese and Japanese to the slipshod methods of many tropical people who practice *milpa* agriculture in parts of Thailand, Mexico, and tropical Latin America in general. Having cleared small patches of forest, and burnt the brush, the milpa farmers plant their crops in the ashes among the half-burned stumps They may dig the ground up with hoes, but often they merely make a hole with a pointed stick, drop in a seed, and push soil over it with the bare foot In regions where there are few or no animals for farm work, the people perforce carry loads on their own backs, shoulders, or heads Few things are more characteristic of China than the coolie carrying a bucket, bundle, or other load at each end of a pole that balances on his shoulder In the high plateaus of Central America and especially the Andes an equally characteristic scene is Indians of both sexes and all ages running along mountain trails at a little jog trot with heavy loads resting on their backs and supported in part by a band over the forehead

450 CATTLE AS WORK ANIMALS. One of the outstanding facts in the world's use of animals is the vast size and huge population of the areas where the farm work is done mainly by oxen, or at least by some form of cattle such as the Indian zebu, the water buffalo, or the yak. All the way from Spain and Morocco on the west, through Italy, Greece, Egypt, Palestine, Turkey, Persia, and Afghanistan to India, Siam, Java, Indo-China, and southern China, horses and mules as a rule give place to some kind of oxen In all these regions cattle are kept mainly, or even wholly, as draft animals rather than as a source of meat or milk In Turkey the people are amazed that westerners are fond of anything so tough and tasteless as beef They think beef is poor food because their breeds of cattle are small and thin, and few animals are killed until they have grown wiry and stringy from years of hard work In India the cows give so little milk that the foreigners sometimes speak of them as "teacup" cows—able to give only a teacupful of milk. The use of oxen permits a man to cultivate more land than he can dig up by hand Nevertheless, oxen are slow, stupid, and inefficient compared with horses Their use tends to keep the farms small, and hence to keep the farmers poor If an ox moves only 2 miles

per hour while plowing, and a horse 3 miles, while pulling the same plow, it is evident that a man who uses oxen can plow only 2 acres for every 3 plowed by the man who uses horses. Moreover, in many regions where oxen are used one sees two men for each plow—one to hold the plow and one to drive the animals.

451. In spite of these disadvantages, it seems probable that, in view of the climate, the topography, and the density of population, oxen are probably the best available animals for the countries where they supply most of the power on farms. We have seen that the types of European cattle that have been most carefully bred for milk and meat cannot stand tropical climates, rough, coarse forage, and biting, disease-bringing insects such as the tick, which kills millions of cattle each year. Other breeds, however, and distinct species, such as the humped zebu and the unreliable water buffalo, can endure these conditions. Here again we come back to the problem of limits and optima. Oxen are used instead of horses not because people are stupid and inefficient, as is sometimes said, but because climate, vegetation, and insects, together with density of population, put vast regions beyond the limits where the horse can be used profitably. On the other hand, for certain types of cattle those regions supply conditions that are not far from the optimum.

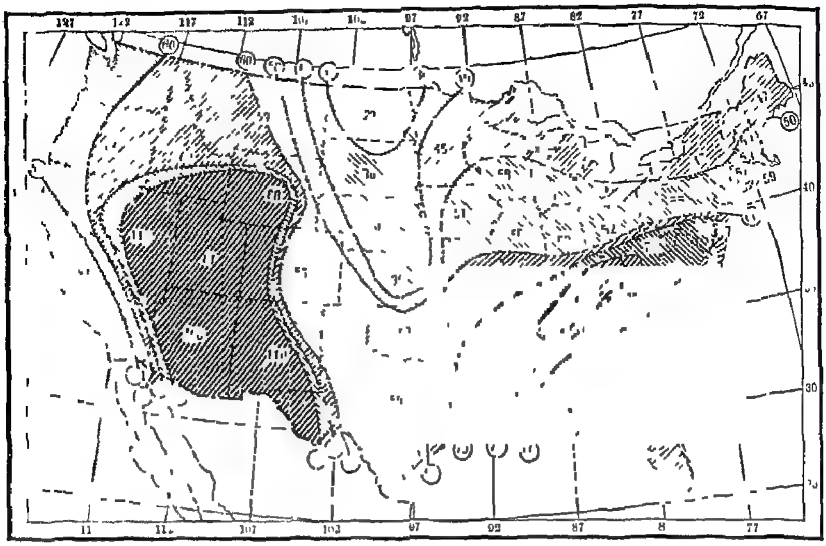
452. Horses and Mules for Farm Work. Although horses and mules are found elsewhere, they reach their greatest importance in regions with the following characteristics: (1) a climate sufficiently cool and moist to give the animals both good food and good health; (2) farms large enough and fertile enough so that farmers can afford to feed their animals with grain and hay as well as to provide pasturage; (3) a topography sufficiently level to make it feasible for horses to pull plows, carts, mowing machines, and havesting machinery; and (4) a fair degree of prosperity among the farmers, but not enough to permit them to buy tractors unless they cooperate so that one tractor serves two or more farms. Where the climate is warm, as in our South or in Spain, horses tend to give place to mules, and even to donkeys. In dry areas such as the borders of Palestine and Algeria camels are sometimes used both for plowing and hauling, but this is rare. Moist heat, such as prevails much of the year in many parts of the tropics and during the summer in monsoon lands like China and Japan, is bad for horses. This is partly because it undermines their health, and partly because it promotes a rank, tough growth of grass, renders haymaking difficult, and forces the farmer to do a great amount of work if he attempts to raise the proper kind of feed. Great density of population also hinders the use of horses because it necessitates small farms, and

therefore leaves little room for pastures, and for fodder crops and grain, which are almost essential for horses during the season of hard work

453 Because of all these conditions the climatic regions that best meet the requirements of horses are the Mediterranean, Temperate Grassland, Cyclonic, and Cool Continental types There horses and mules are the main animals for farm work in the United States and Canada (aside from certain tractor areas), in most of Europe, except the ox regions mentioned in Paragraph 450, in the agricultural parts of the Soviet Republic, in the temperate parts of South America, mainly Uruguay, Argentina, and Chile, and in Australia and New Zealand Australia has about 300 horses for every 1,000 people, whereas southern Asia has only about 10 The European region from northern France through Belgium and Holland to Denmark is especially favorable to horses because of its combination of a mild, cool climate with admirable grass, abundant grain, and people in a high stage of civilization Hence this general region, including England, has developed the finest breeds of horses, among them the massive Percheron, Clydesdale, Shire, and Belgian types, on the one hand, and swift "thoroughbred" racers on the other Arab breeds are famous, but the speediest of them are the result of generations of breeding in England or America The value of good grass is evident in the fame of the Blue Grass Region of Kentucky as a place for raising racehorses The limestone soil there, together with the climate, favors unusually good grass

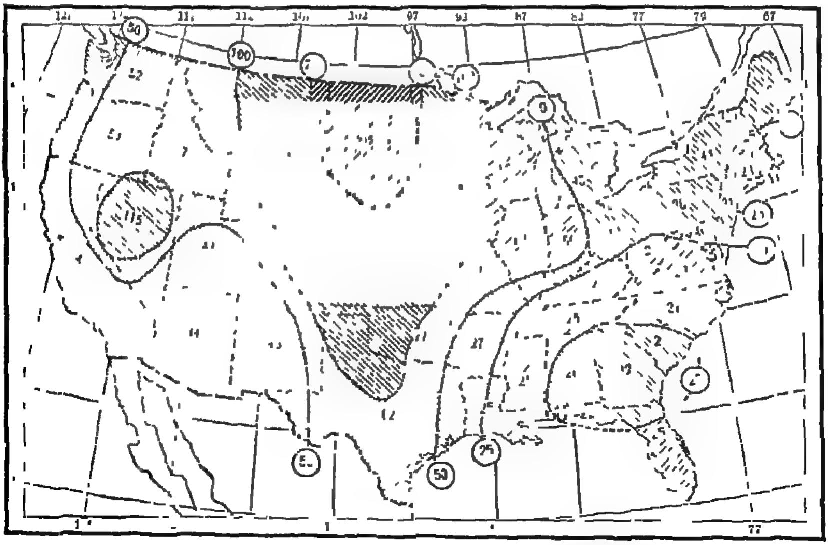
454 Horses and Mules in the United States Although horses and mules have been largely replaced by motor vehicles on the good roads of the United States, they are still very useful on the poorer roads, and almost indispensable on the farms A454 shows that in comparison with the area of cultivated land the number of horses and mules is conspicuously small in the western part of the Great Plains from the Dakotas to Texas It is also small in three corners of the country, namely, the northern part of New England and New York, Florida and Georgia, and California. On the other hand, in comparison with the harvested land, horses are especially numerous in two of the most rugged parts of the country One is the Rocky Mountain states from Arizona and New Mexico to Idaho, the other is the Appalachian states from West Virginia to Alabama and Mississippi, with an extension into Arkansas and Louisiana

455 This curious distribution of work animals arises mainly from the combined influence of the size of the farms, the nature of the farm products, and the wealth and progressiveness of the farmers as shown by their use of trucks and tractors. A comparison of A454 and



A454—Horses and Mules per One Thousand Acres of Harvested Land, 1930

A455 brings out the effect of the size of the farms In A454 we see that North Dakota has the smallest number of horses in proportion to the



A455—Acres of Harvested Land per Farm

harvested land, only 29 horses for every 1,000 acres. In A455 we find that it has the largest area of cultivated land per farm (263 acres).

Similar conditions, but not quite so extreme, prevail throughout the great central plains, especially in their western parts The explanation lies mainly in three facts First, on a big farm there are less likely to be unused horses than on a small farm Many small farms need only part of a horse's time, or they need one horse and a little help from another Nevertheless, each farm of the first kind generally keeps 1 horse, and each of the second kind 2 horses This means that on an average about 3 horses are kept where only 2 are needed. On the other hand, a big farm that needs the work of between 10 and 11 horses, for example, will keep 11 Even if it should keep 12, it would have less than 20 per cent more horses than it needs, whereas the small farms have about 50 per cent more This same principle applies to all sorts of farm equipment A man with only 10 acres needs 1 plow, just as does his neighbor with 30 or 50 acres A fodder cutter is convenient for a man with 5 cows, but if he has 50 the cutter costs no more, and does 10 times as much work. Small farms always suffer not only because they cannot afford much equipment, but because they cannot make full use of what they have, and therefore the little that they are able to buy often represents an unduly large share of the farm income

456 The nature of the farm products also makes a great difference in the number of horses On a wheat farm almost the only work for horses is plowing, harrowing, and seeding at one season, and harvesting long afterward On a corn farm, horses are needed for cultivating between the rows On a truck farm they are needed not only for cultivating but also for hauling the product to market, or at least to the barn at frequent intervals The number of horses needed for a given number of acres increases as the cultivation becomes more intensive If farmers are progressive and prosperous, the number of horses is reduced by the use of tractors and trucks It must be remembered, however, that the great reduction in the number of horses in the United States is due in relatively small part to a reduction in the actual farm work done by animals It arises mainly from the disappearance of carriage horses In the old days farmers and village people kept millions of horses that were used mainly for driving in the same way that we now use automobiles Now there are very few such horses

457 We have already seen some of the reasons why the small farms in A455 are the ones which have many horses in proportion to their acreage according to A454 Farms averaging less than 25 acres of harvested land are a conspicuous feature of the Appalachian belt from West Virginia to Mississippi, and horses and mules are there unusually numerous in proportion to the amount of cultivated land.

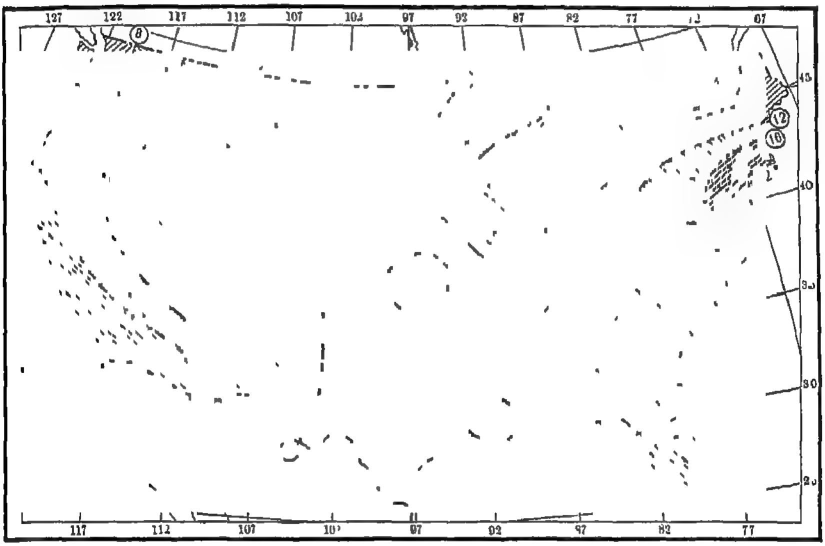
One reason for small farms is ruggedness That is why they are so evident in the Appalachian region. Because the farmland there often lies on slopes one might expect that little machinery and hence few horses would be used Other conditions, however, make the number of horses large compared with the amount of cultivated land For one thing, there is plenty of pastureland unfit for crops, but not bad for colts Another thing is that both the roads and the people are poor, so that automobiles are not common, and trucks and tractors are rare The Appalachians are one of the few parts of the country where the farmers still use horses quite commonly for driving and even for riding In the more rugged sections where people live far apart and are often separated by wooded tracts, high hills, or even mountains, there is a good deal of travel on horseback on trails In the great mountainous area extending from the Rocky Mountains to the Sierra Nevada a similar situation calls for many horses In addition there is need for many in taking care of the large herds of cattle and sheep which spread out over great areas where there is little cultivation

458 In the nonmountainous parts of the southeastern states the nature of the crops leads to small farms, but not to many horses or mules In these gently rolling lowlands the chief crops are tobacco in the north, cotton farther south, and corn almost everywhere The tobacco and cotton require much hand work, and therefore a single farm family can handle only a small area "Thirty acres and a mule" is often spoken of as a normal farm Animals are, of course, needed for plowing and cultivating, but there is little hauling, and animals are of slight help in harvesting In the market gardens and orchards of the northeastern manufacturing region and in the irrigated regions of the Southwest and Pacific Coast, there is plenty of hard work and plenty of hauling, but motor vehicles enter into the problem there, as we shall soon see

459 TRUCKS AND TRACTORS ON FARMS Many people have the mistaken idea that the greatest use of trucks and tractors for farm work is found in the great plains of the United States A459 indicates that this is far from being true In that map, trucks and tractors owned on farms are counted as of equal importance Their number is compared with the total acreage of harvested land, including the land devoted to hay as well as other crops When trucks and tractors are mapped separately the resulting maps are almost alike, and differ from A459 only in being less regular The outstanding fact about A459 is its close resemblance to many other maps which we shall see later In general the Great Plains, especially their drier parts, have few trucks and tractors in comparison with the harvested area, only 3 for every

1,000 acres More surprising than this is the fact that the South, in spite of its small farms, has equally many Of course, this does not mean that the average farmer is as likely to have a truck or tractor in the South as in the Dakotas In North Dakota the farms are so large that 3 tractors or trucks for every 1,000 acres means either a truck or a tractor for 3 farms out of 4 In the South the farms are so small that 3 trucks or tractors per 1,000 acres means that only 1 farmer out of every 10 or 15 owns one

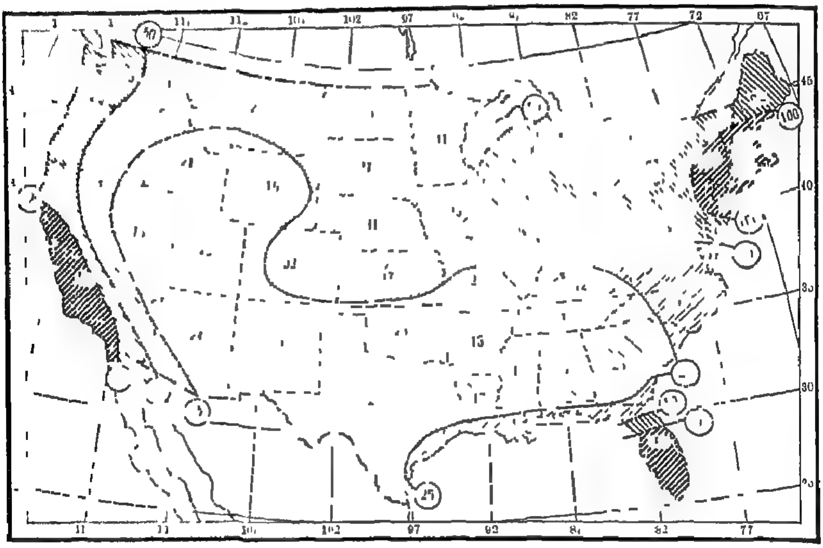
460 Another prominent feature of A459 is the large number of trucks and tractors on the farms in California, Florida, and especially



A459—Trucks and Tractors per One Thousand Acres of Harvested Land, 1930

southern New England and New Jersey In those states the average farmer is about as likely to have one of these motor vehicles as in the Dakotas, and the percentage of all the motor power furnished by them on the farms is far greater This percentage cannot be stated absolutely, because it depends upon how many horses are displaced by the average truck or tractor According to the United States Department of Agriculture the average is about 3, but there are differences of opinion Calculating on the basis of 3 horses as equal to one truck or tractor, and allowing for young horses and mares that are producing colts, we see from A460 that the most motorized farms are in (1) southern New England, New Jersey, and southern New York, (2) southern Florida, and (3) the agricultural part of California The only

other large regions where half the farm work is done by motor vehicles are a strip from New England along the Great Lakes to Illinois and Wisconsin and the coastal part of Washington and Oregon In most of the South less than one fifth of the work is thus done Even in the Corn Belt and the great wheat-raising states, such as Kansas and the Dakotas, only locally does one find places where motor vehicles do much more than one third of the farm work In the rest of the world it is doubtful whether any large region does half of its farm work with trucks and tractors The Russians claim that they do this



A460—Farm Work Done by Trucks and Tractors, 1931

(Figures represent approximate work of trucks and tractors as percentage of work done by horses and mules)

on their wheat farms in the southeastern part of European Russia, but the statistics are not clear

461 We must find out why by far the greatest displacement of farm animals by motor vehicles has taken place in three sections of the United States where the farms are small Their small size in Florida and southern New England is conspicuous in A455 In California the farms in the coastal portion from a little north of San Francisco southward are smaller than the average for the state as a whole, and the use of motor vehicles is especially great Most of these highly mechanized small farms raise fruit, vegetables, milk, poultry, and eggs Trucks and tractors are profitable to the farmer only if used with

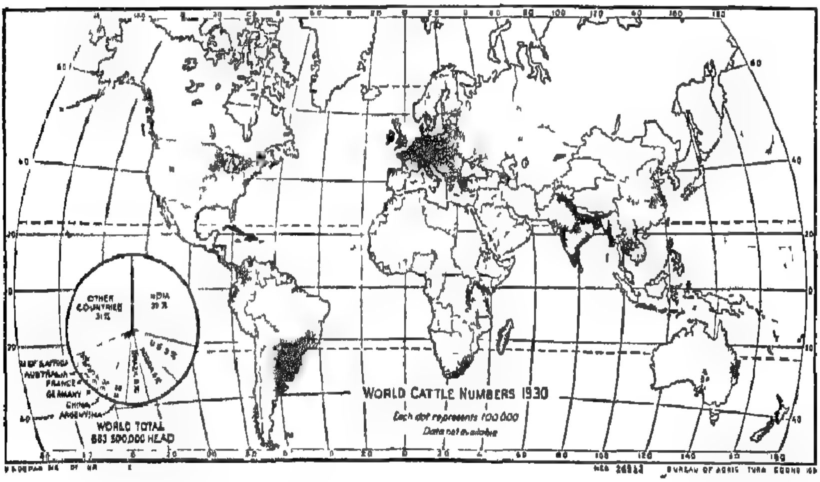
sufficient frequency. A wheat farmer in Kansas uses a tractor a few weeks in the fall for plowing and planting and a few weeks in the summer for harvesting. He needs a truck mainly to haul the seed wheat to the field and to haul the wheat after it is threshed. The rest of the time the machines have little to do, but they deteriorate even when standing idle.

462. With horses the situation is different. When not needed they can be pastured with little expense, and when worked they require cheap grain raised by the farmer himself instead of expensive gasoline. They grow old, to be sure, but new animals can be raised on the farm at slight cost. Hence many wheat farmers do not think it pays to own tractors and trucks, although practically all have passenger cars. But suppose a farmer has a market garden. His season for plowing and planting is much longer than that of the wheat farmer, because he has many different kinds of crops. He can also use his tractor for cultivating the crops and keeping them free from weeds, which is not necessary with small grains such as wheat and oats. Moreover, a large part of his success depends on being able to take his vegetables and fruit to market every few days while they are fresh. If he runs a dairy or poultry farm, the milk must be taken to market every day and the eggs frequently. All this makes it desirable for the farmer to own both a tractor and a truck. The larger and richer the market which gardeners, dairymen, and poultrymen supply, the greater is their average income and the more they need motor vehicles. Thus the greatest use of motor vehicles on farms is found among such people in areas which supply the needs of the great manufacturing cities of the Atlantic Coast from Boston to Baltimore, and of the band of country skirting the Great Lakes. Local farmers supply these needs to a large extent, and hence have many trucks and tractors. Florida and California supply fruits and vegetables to this richest of the world's markets as well as to their own local market. Hence they, too, use many motor vehicles.

Chapter XX

THE DISTRIBUTION OF USEFUL ANIMALS

463 Economic Geography of Cattle. The worldwide distribution of cattle (A463) is a good deal like that of horses, but North America and Europe have a relatively large number of horses per square mile, while India, Java, tropical Africa, and other low latitudes have more cattle. The fact that cattle are more widespread than horses does not mean that cattle of any one type can live in a wide variety of climates. It means, as we have seen, that man has domesticated several species, including not only the European type, but also

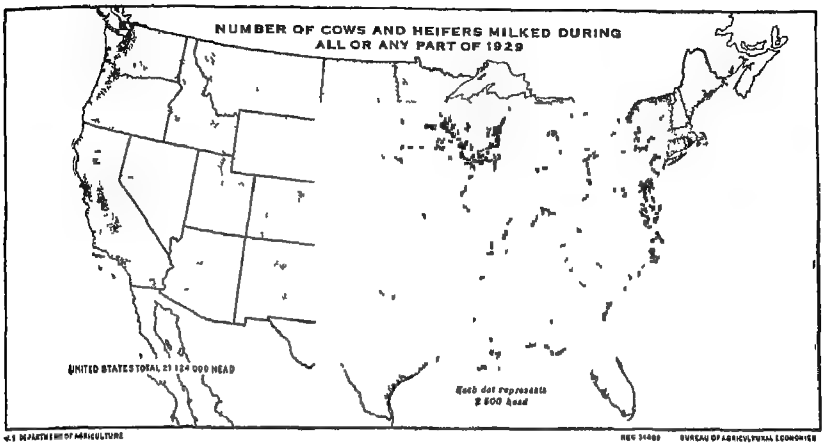


A463—World Distribution of Cattle

the yak of cold Tibet, and the Indian zebu, the banteng of Java, and the almost hairless water buffalo, which are natives of low latitudes.

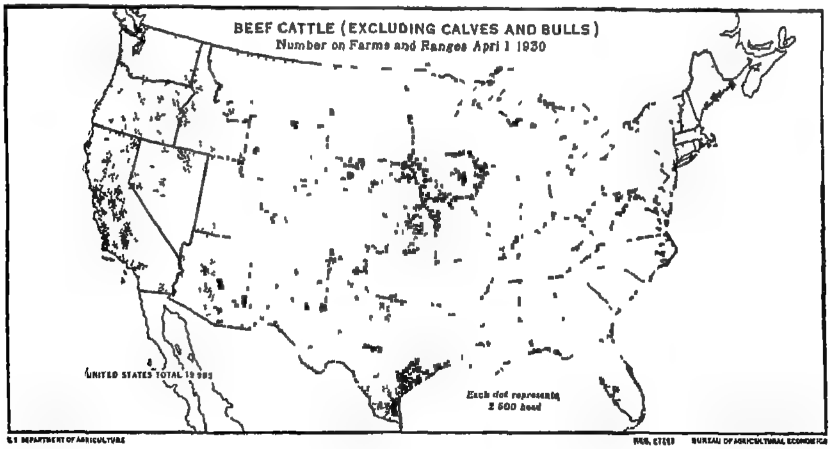
464 In the comparatively cool regions where both horses and European cattle are at their best the distribution of these two animals shows an interesting difference. The distribution of horses depends on the amount and kind of farm work to be done. That of dairy cattle depends partly on the location of the population and partly on the degree to which the climate, soil, and vegetation are favorable to milk production. Even with all the advantages of modern transportation so perishable a product as milk is raised as near as possible to the

places where it is consumed Hence every great city is surrounded by dairy farms, and the distribution of dairy cows (A464) is very different from that of beef cattle (B464) Even New York draws



A464—Dairy Cows in the United States.

only a small share of its milk from distances greater than 250 miles. Hence A464 is heavily shaded from the Missouri River eastward. East of Ohio and farther west, near the belt of cities from Cleveland to

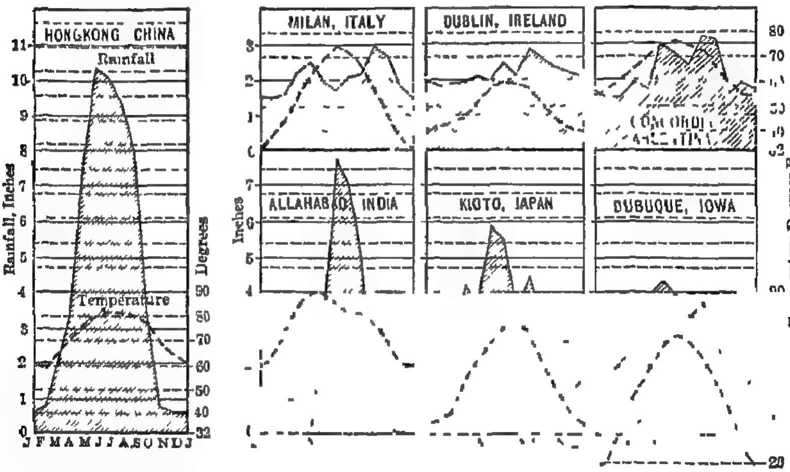


B464—Beef Cattle in the United States

Chicago, most of the milk is sold fresh, although northern New York produces some cheese Farther west Wisconsin alone makes well over half the cheese in the United States, while Minnesota, Iowa, Wiscon-

sin, Michigan, and northern Illinois make about half of the butter that comes from factories. In B464, on the other hand, the greatest concentration of beef cattle is seen in the western grasslands from South Dakota and Iowa southward to Texas. This illustrates the well-known fact that beef cattle are raised on the dry plains west and south of the Corn Belt, but are sent in large numbers to the Corn Belt, especially Iowa and northern Missouri, for fattening. It is cheaper to take the animals to supplies of corn than to transport corn with which to feed the animals. Moreover, the relative coolness of Iowa compared with the plains farther south helps in fattening them.

§465. The Optima for Dairy Cattle. Cattle, like other animals, do best when they live in the optimum climate and are fed with the



A465—Climate of Regions Especially Good or Poor for Cattle

optimum kind of food. In a general way the two kinds of optima are found together. The general optimum for cattle of the European type includes fairly cool, moist summers, and mild winters with so little frost and such abundant rain that the grass is green at all seasons. This kind of climate is illustrated by Dublin in A465. It is by no means an accident that milk and potatoes have long been the staples of the Irish diet, or that Ireland's exports consist largely of beef cattle and butter. The cool summers discourage most kinds of agriculture, but foster an abundance of nutritious, tender grasses. England, the Netherlands, and Denmark have climates similar to that of Ireland, although a little warmer in summer and cooler in winter. Hence they are notable cattle regions and their farmers turn largely to dairying

Denmark and the Netherlands have about 200 cattle per square mile

466 Although the region of Puget Sound and the Willamette Valley in Washington and Oregon form the only part of the United States with a climate as moist and equable as that of Ireland, the country has other climates which are excellent for dairy cattle Vermont, northern New York, Wisconsin, and Minnesota have cool rainy summers which produce pasturage as good as that of Ireland They have long, severe winters, to be sure, but these seem to do little harm so long as the cattle are well fed and are protected from extreme changes of temperature The fact that these regions are also too cool for corn, except as a fodder crop, and are located at some distance from the main manufacturing areas of the country makes it pay the farmers to devote their attention mainly to producing milk, butter, and cheese The coastal part of California is also an excellent dairy region, provided there is irrigation The coolness of the summers is just right for the cows The prolonged dryness is bad for vegetation, but irrigation makes up for this At San Francisco the average temperature of July is only 57°, and the rainfall from June to September only an inch

467. Beef Cattle If the summers are warm enough so that corn grows well, as in Iowa (Dubuque in A465), milk may still be important, but beef cattle are also likely to come into prominence The Po Valley in northern Italy, illustrated by Milan in A465, is like Iowa in being a region where the farmer divides his attention between cattle for milk and for beef Note how the dotted line for temperature falls lower than the Dublin line in winter, but not so low as that of Iowa, where a thoroughly continental climate prevails Note also how it rises higher than at either Dublin or Dubuque in summer. From October to May the rainfall at Milan is almost the same as at Dubuque, and not much different from Dublin Then, however, there comes a drop in summer because the Po Valley lies at the northern edge of the Mediterranean type of climate which has rainy winters and dry summers A rainfall as low as 2 inches per month when the temperature averages close to 75° is by no means a good thing for either cattle or crops It means that the pasturage becomes dry, or at least falls off in quality, the cows suffer from the heat, as we have seen, and the yield of milk diminishes In Iowa, on the contrary, the rainfall rises to its highest level in summer, thus keeping the pasturage good and promoting the growth of corn This illustrates the fact that comparatively small differences in climate may make a pronounced difference in the availability of a region for cattle or any other domestic animal. Soil, too, enters into the matter, for the black soils of Iowa

favor abundant crops both by their richness and by their capacity to hold water and not dry out when rain is scarce

468 If the summers are drier and warmer than those of Iowa a region may still be good for cattle, and may even maintain a great dairy industry. Nevertheless, it is much more likely to raise beef cattle This is one of the reasons why Argentina, Uruguay, Australia, and New Zealand are large producers of beef cattle In A465 notice how high the temperature rises at Concordia, which is centrally located between the cattle regions of Argentina and Uruguay, and how much less rain there is than in Iowa. The average even in the most rainy months (February and March, which are plotted to correspond with August and September of the northern hemisphere) is only 3 inches, while the winter and spring months have less than 2 Such a rainfall adds to the difficulty of dairying, but makes it possible to raise vast numbers of beef cattle so long as the land is not taken over for crops Hence in these countries the cattle have increased rapidly In Argentina they formerly roamed the plains much as the buffalo (bison) roamed the Great Plains of North America For years, during the nineteenth century, these cattle were killed for their hides and tallow alone, and the carcasses were allowed to rot upon the pampa Higher prices for meat, however, plus the cold-storage plant, the refrigerator car, and the refrigerator ship, have made it profitable to ship meat to the markets of northwestern Europe and the northeastern United States With the increased desire for quantity there has come also a demand for a finer quality of meat This has led to improved breeding, and also to better pastures through the introduction of alfalfa American and European firms have established packing plants in Argentina, thus contributing to the further expansion of the industry so important to the prosperity of these countries

469 PHYSICAL ENVIRONMENT AND TYPES OF CATTLE Different types of environment support different breeds, as well as different species of cattle This arises partly from the choice of the breeders, partly from the selective action of nature A familiar example is the contrast between Holsteins and Jerseys The Holsteins originated on the northern coast of Holland, where the moist, cool summers combine with a heavy clay soil and level land to make the grass luxuriant, but watery Cows with large, bony frames and correspondingly large abdomens are apparently better able than others to support themselves and their calves on such vegetation The watery food apparently tends to make the milk abundant, but relatively low in butterfat and other solids. The island of Jersey, on the contrary, although as rainy as Holland, averages about 4° F warmer in summer, and has light loamy

or sandy soil and many hills. Hence the herbage is shorter, finer, and less watery than in Holland. The Jersey cows do not need large stomachs or large frames, and the selection of nature and of the breeders has preserved a type less massive than the Holsteins, and notable for the production of milk which though moderate in quantity is rich and creamy.

470. NON-EUROPEAN SPECIES OF CATTLE. The differences between species of cattle are like those between breeds, but more pronounced. Just as the environments of the Dutch coast and Jersey have led to the selection of special types by man and nature, so the environment of southern Asia has cooperated with other agencies in producing distinct species of cattle such as the Brahman or zebu type, the gayal, yak, water buffalo, and banteng. It is these which give southern Asia and the East Indies their heavy shading in A463. The Brahmans are the main type of cattle in India, especially in the north where the Indo-Gangetic Valley is almost black in A463. The climate where they are most numerous is represented by Allahabad in A465. It is warm at all seasons and excessively hot in the spring before the summer rains begin. The rainfall comes almost entirely in summer. The native cattle of such a region must eat relatively coarse, watery forage during the rains and the same kind in its tough, dry state during the long dry season. They must be able to resist drought and heat, which are often severe, and must not be sensitive to insect pests such as the ticks which cause Texas fever.

471. As long ago as 1849, the knowledge that the Brahman cattle possess such powers of resistance led to the introduction of a few into South Carolina. During the present century a considerable number have been extensively crossed with European cattle on the Texas coast. So well do the crossbred animals endure drought, ticks, flies, and other pests that Brahman blood is being introduced widely in the South. In Brazil, huge prices are paid for choice Brahman bulls.

472. Even the Brahman cattle cannot stand all climates. Neither a cool, moist climate nor a steadily moist tropical climate is good for them. Hence among the mountains north of the Indo-Gangetic plain in India, and also in other moist mountains of south India, the gayal replaces the Brahman. In the lofty, snowy Himalayas the gayal in turn gives place to the yak. Toward the south, likewise, the Brahman declines in relative importance, and the water buffalo increases, until the two species are about equally numerous in Ceylon. Farther east in the moist regions of Burma, Siam, Indo-China, and the East Indies, still another species, the banteng, largely replaces the Brahman type, but not the water buffalo.

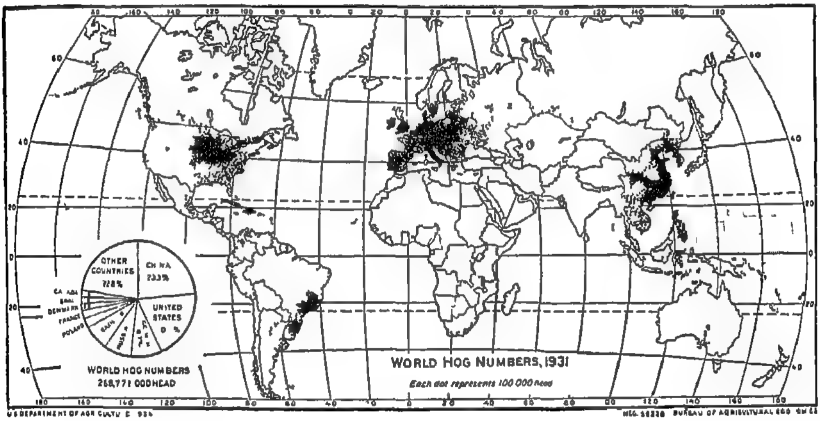
473. The relation of the Brahman, gayal, banteng, water buffalo, and yak to European cattle is important because it illustrates one of the chief principles in respect to the distribution of domestic animals. The same principle is illustrated by the relation of the ass, camel, ox, llama, reindeer, and dog to the horse *Wherever man is able to raise the best type of animal for any purpose he does so, but where the more valuable type cannot thrive, others replace it* Nevertheless man constantly strives to extend the range of the best species, partly by selecting certain types and thus establishing new breeds, partly by crosses with other species, and partly by providing special kinds of food and shelter In general, the number of breeds and their degree of diversity are a rough measure of the extent to which man values an animal Few people have seen more than one variety of peacock, although a pure white type is in existence On the other hand, almost everyone is familiar with several kinds of hens such as Leghorns, Plymouth Rocks, Bantams, Rhode Island Reds, Wyandottes, and Buff Cochins, and with cows of a great variety of colors and builds

474 A HANDICAP OF THE FAR EAST The region where there is most need of breeds of cattle with peculiar adaptations to climate and food is the southeastern coast of Asia There dwells by far the greatest population that suffers for lack of domestic animals Hongkong, with its heavy rains and high temperature throughout most of the year, represents the extreme of this type (A465) Among the dairy cows kept there by the British, the yield of milk declines systematically as the weather becomes hotter and damper The native grasses are so tough that horses, cattle, and sheep do not thrive on them Hence the few dairy cattle are fed on guinea grass, a species introduced from more tropical regions and raised as a cultivated crop Even this is so unsatisfactory that the cows receive special rations Nevertheless, the cattle imported from Europe die at an alarming rate Because domestic animals thrive so poorly in South China, millions of acres of rugged land are left unused

475. In Japan, although the conditions are not so bad as in southern China, attempt after attempt to raise sheep on a large scale has failed, and in the most populous parts of the islands neither horses, cattle, nor swine really thrive The rainfall (Kyoto in A465) does not greatly exceed that of Iowa, but abundance of moisture throughout the year, and the absence of weather cold enough to check growth effectively, seem to cause the Japanese vegetation to be poor as forage. Japan, with its dense, damp forests and coarse reedlike grasses proves a poor environment. Hence Japan has only 12 cattle per square mile and 23 per 1,000 people, in contrast with 74 per square mile and 1,674

per 1,000 people in Iowa Most Japanese families use no milk at all. The entire country of Japan, with more than 70,000,000 inhabitants, produces about 4 per cent as much milk and 1½ per cent as much butter as Denmark with 3,800,000 people In proportion to the population Denmark produces 400 times as much milk as Japan.

476. DISTRIBUTION OF SWINE Domestic swine possess qualities which seem to favor a wide distribution, like that of man himself In the first place, swine breed so freely and grow so rapidly that they furnish a large supply of meat and fat in proportion to their food Second, they eat many kinds of food, such as acorns, potatoes, and garbage, which are of little use for most of the domestic animals Third, although the exact facts are not known, domestic swine appear



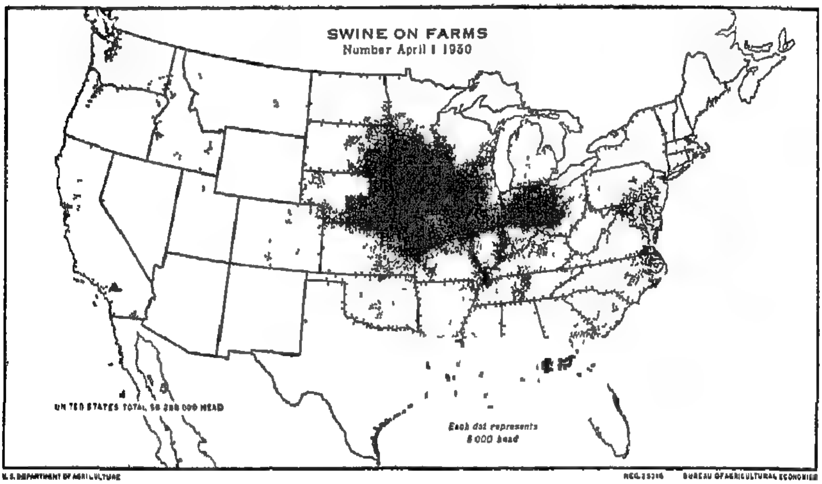
A477—World Distribution of Swine.

to be descended from several wild species, sufficiently alike to breed freely together, but diverse enough to include varieties adapted to a wide range of environment

477 In spite of these advantages, swine (A477) are not nearly so widely distributed as horses, cattle, hens, or even sheep Even in the United States (B477) the swine show a marked concentration in the states from Ohio to Nebraska, while in Europe they abound in the Low Countries, Denmark, and western Germany South America, especially Brazil, has a fair number distributed much like the people, the Philippines nearly as many in proportion to the population, Australia, and South Africa about one fourth as many proportionally as in the United States, and Japan a few—half as many as Norway, but only one fiftieth as many per capita as Brazil. Vast areas in Asia

and Africa are utterly blank in A477, in the United States such densely settled sections as the northeastern states have surprisingly few

478 The explanation of these conditions is found partly in the fact that, although the pig eats nearly everything, it becomes commercially profitable chiefly in regions where one of the following types of food is cheap and abundant: corn, as in our Corn Belt and Brazil, potatoes, as in Belgium, the Netherlands, and Germany, and barley and skim milk, as in Denmark. Relatively high prices for these commodities, even where they are abundant, may reduce the production of swine. This happens near Chicago where corn is relatively more costly than in Iowa.



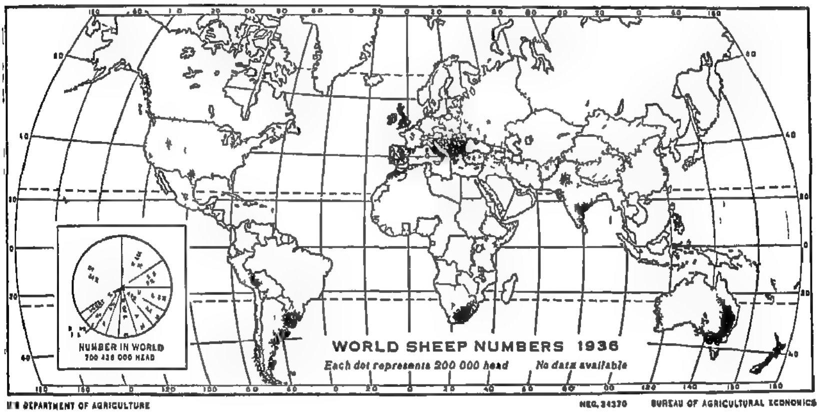
B477—Swine in the United States

479 The disagreeableness of the pig is another reason why its distribution differs from that of man. In cities and towns all over the civilized world the disagreeable smell of pigpens is one reason why vast amounts of garbage, which might profitably be employed for feeding swine, are taken to rendering plants where the fats are extracted. Although no exact information is available, the religious prejudice against swine may have arisen in part because the animal is disagreeable, and partly because it does not thrive in dry regions. The fact that swine are tabooed by Mohammedanism and Judaism accounts largely for their absence throughout most of Asia and Africa.

480 In China there is little prejudice against swine, and they are kept in almost every village. Nevertheless, the number is not so large as is often supposed. According to the best available estimates China

has only about 14 pigs for every 100 people, whereas the United States has 33 and Denmark 100. Unlike draft animals, which rely on grass, the pig, so to speak, competes with man for food A dense and poverty-stricken population cannot afford to feed swine on anything that people can eat In China it would be almost criminal to use corn, barley, potatoes, and skim milk for such a purpose Hence swine must be content with the scanty materials which even the poorest Chinese cannot eat, or else with what they can grub up in fallow gardens and fields

481. Distribution of Sheep Sheep rank with horses, cattle, and swine as one of the four domestic animals which would probably be chosen if the world's choice were limited to four Nevertheless, sheep (A481) are generally crowded out of regions where population is

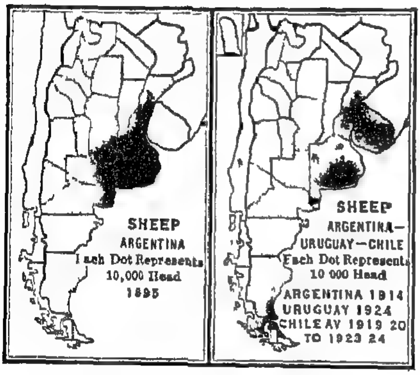


A481—World Distribution of Sheep

numerous and intensive farming is practiced, whereas horses and cattle are most numerous in such regions One reason is that sheep are better adapted than horses or cattle to dry and rugged regions They can thrive on grass so short that the other two animals cannot bite it off, and they can graze on slopes more comfortably than other domestic animals except the goat Another important reason is that, although wool is valuable, it is not so essential as are horses and cattle for draft purposes, milk, and meat Again sheep can be raised and fattened on grass alone Moreover, wool is one of the easiest of animal products to prepare and transport Sheep require only a few people to care for them, their wool can be clipped, and then transported vast distances without further treatment and without special precautions such as are needed for meat The sheep of Australia and South America

were long kept almost entirely for their wool Even now the 400,000 tons of wool exported by Australia weigh 4 or 5 times as much as the exported mutton The helplessness of sheep likewise causes their distribution to differ from that of people Such helplessness makes the animals need shepherds, but it is unduly expensive to provide shepherds unless large numbers of sheep are kept This in turn demands large open areas such as exist chiefly in countries that are sparsely populated because of newness, aridity, or ruggedness

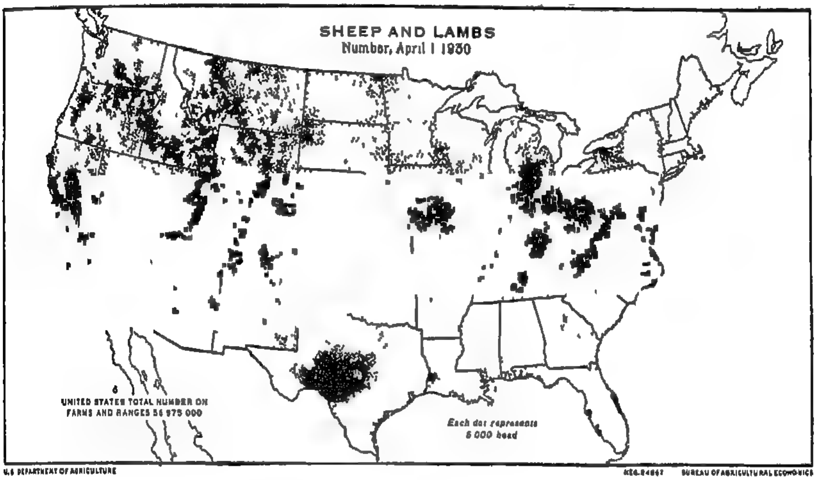
482 For all these reasons sheep tend to be animals of the peripheral marginal areas outside the limits of intensive agriculture Hence they are especially numerous in four southern areas—New Zealand, Australia, South Africa, and the Argentine-Uruguay section In these newly settled regions land is still available for sheep-raising, although this would not pay if the farms were small When the population becomes so dense that the holdings are small, and there is a market for milk, vegetables, and other farm products, sheep give way to cattle and general farming, provided such farming is feasible In Australia and South Africa, as a matter of fact, most of the sheep are raised on land so dry that, if it were devoted to general agriculture, the farmers would be in great danger of frequent and severe crop failures The same is true of considerable parts of Argentina Even in those far-off countries there has been a significant shift of the sheep-raising areas more and more to the outer edges of civilization, as shown in Argentina (A482) On its vast dry grasslands Australia has about 16 or 17 sheep for every inhabitant of the continent Uruguay has nearly as many, and New Zealand more



A482—Sheep in South America

483 In the Balkans, southern Italy, Sicily, and other Mediterranean lands, sheep are numerous because long experience has proved that it does not pay to farm the dry hillsides, or to raise cattle on them The prolonged summer droughts put these regions near the limits so far as other modes of getting a living are concerned, and thus make it profitable to raise sheep In Britain a combination of climatic, economic, and social factors produces a unique condition such that horses, cattle, and sheep are all numerous in almost the same regions

The sheep, however, are kept mainly on the uplands or moors, and the horses and cattle on the lower land where the crops are raised. Great Britain is so cool that an altitude of 1,000 or 1,500 feet makes the climate too cool for crops but excellent for grass. Thus even Britain bears out the idea that sheep are relatively peripheral or marginal animals. Nevertheless, both in Great Britain and the United States considerable numbers are raised in areas of rather intensive agriculture. Southern Michigan, the gently rolling or hilly areas of central and eastern Ohio, and smaller centers in the Middle West are of this



A483—Sheep in the United States

kind (A483). Most of the sheep in such regions are of the dual-purpose type, furnishing both wool and mutton.

484. Goats. Goats bear to sheep much the same relation as asses to horses, and Brahman to European cattle. They supplant the more valuable type in regions that are especially dry or rugged. In one important aspect the relation between goats and sheep is the reverse of that between Brahman and European cattle. As a producer of milk the goat is much better than the sheep. An average cow yields 3,000 or 4,000 pounds of milk per year, the nannygoat from 500 to 1,000, and the average ewe less than 100. Hence in relatively dry countries, such as South Africa, northern India, Mexico, and the Mediterranean lands, especially Greece and the Balkans, where goats reach their greatest density per square mile, they often replace the cow as a source of milk. The distribution of goats then becomes much like that of the people. In the eastern Mediterranean lands, both the goat and sheep are kept

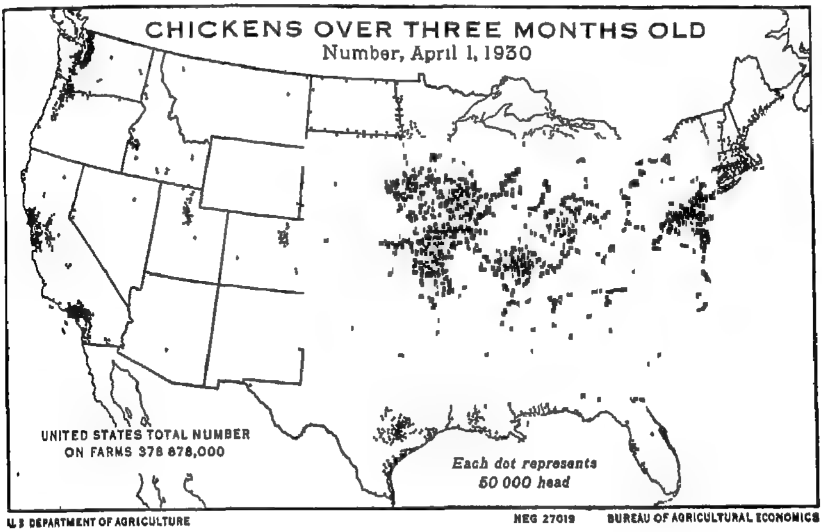
largely in the villages. This is possible because the dryness of the summers and the general ruggedness cause waste land to be abundant. The unfenced grain fields can be used for summer pasturage, for the harvest is usually finished by June Moreover, the standards of living are so low that children do not go to school, and the village boys can be employed to herd the goats and sheep, thus making fences unnecessary.

485 WIDESPREAD DISTRIBUTION OF HENS Hens possess most of the qualities that cause an animal to be widespread The qualities include (1) relative independence of climate, perhaps because hens were derived from several wild ancestors in different parts of the world, (2) a cosmopolitan attitude toward food, so that, like the pig, they can live on many sorts of food, including garbage, and (3) the power to furnish a very useful product The fact that eggs are easily broken helps to cause hens to be widely distributed Only in our own day has there been any extensive attempt to transport eggs long distances China, which today supplies the greatest surplus of eggs for export, ships millions of dozens in the shell In general, however, such transportation is wasteful, and dealers expect to lose 15 per cent by breakage Hence a large share of the Chinese eggs, over a hundred million pounds per year in normal times, are dried or frozen for shipment But freshness is an important quality which is soon lost, and fresh eggs bring much the highest price

486 The net result is that the distribution of hens is much like that of farms both in the United States (A486) and elsewhere Nevertheless, regions with a low stage of civilization, such as the majority of tropical regions, have relatively few hens per capita and per square mile Densely populated and poor countries also have relatively few There is a great contrast between Denmark, with 8 or 10 fowls per capita, and Japan, China, and India, with less than 1 Canada with 5, the United States with 3, and Egypt with 1 hold intermediate positions Far northern regions are unfavorable to poultry. Finland, Estonia, and Iceland have no more per capita than Egypt On the other hand, some tropical regions such as the Philippines have as many as 3 fowls per capita Hens also tend to be especially numerous near great markets, as in eastern Pennsylvania and New Jersey, and in regions with an especially good supply of food, as in the Corn Belt of the United States from Ohio to Iowa

487 LIMITATIONS OF THE SILKWORM The silkworm is peculiarly interesting because it illustrates how the distribution of domestic animals is expanded by man and limited by nature The silkworm was first raised in China, but spread to India, Iran, and finally the Mediter-

ranean lands, where it reached its maximum profitable expansion. Efforts were made to introduce it into England, Mexico, and colonial Virginia. As late as the American Revolution, Benjamin Franklin was engaged in a silk enterprise in Philadelphia, and in 1866 California was offering bounties for the cultivation of silk. All these efforts proved abortive, and the silkworm is still subject to strict geographical limitations. These limitations do not change much. One reason for this is that the high value of raw silk, averaging about six dollars a pound on the New York market in times of good business, makes the cost of transportation almost negligible. Hence there is little incentive to overcome the geographical limitations. The first



A486—Distribution of Hens in the United States

limitation is set by the white mulberry tree. No other easily raised tree provides leaves on which the silkworm can thrive. The various species of mulberry grow in the warmer parts of the temperate zone and in semi-tropical regions, but do not thrive in equatorial regions except on the mountains. With proper care they can be raised in climates as cool as those of Philadelphia and southern England, but they thrive better somewhat farther south. Thus the mulberry tree limits silk cultivation to a zone lying roughly between latitudes 15° and 40° on either side of the equator, although in Europe and western America the limit rises to about 45°.

488. The next limit is set by standards of living and habits of industry. The rearing of silkworms is a painstaking and laborious

occupation, requiring great concentration during a short season Fresh leaves must be cut daily with absolute regularity, the trays on which the worms are kept must be cleaned, the air must not be allowed to become too close and hot, or too cool. Labor must be cheap, or else the cost of production will be greater than the market can stand Such labor is found only among people with low standards of living combined with established habits of regularity and industry In America no such labor has ever been available on a large scale In the United States the labor which most nearly fulfills the necessary requirements is largely devoted to raising cotton, and it is more expensive than that of the regions where silk is actually raised Only in such places as Japan, China, northern India, Persia, and the Mediterranean lands does one find the right labor supply, and those are the regions whence comes our silk

489 Another natural limitation is the degree of moisture during the season when the warms are growing. In regions with the Mediterranean type of climate, unless irrigation is practiced, there is always danger that the rainy season will end too soon, so that fresh mulberry leaves will be scarce In China and Japan, on the contrary, the rains come in summer, so that fresh leaves can be procured during many months instead of only a few weeks In such places the dangers from the many diseases to which the silkworm is subject are indeed increased, but this does not offset the great advantage of China and Japan in their rainy summers, as well as in their labor supply

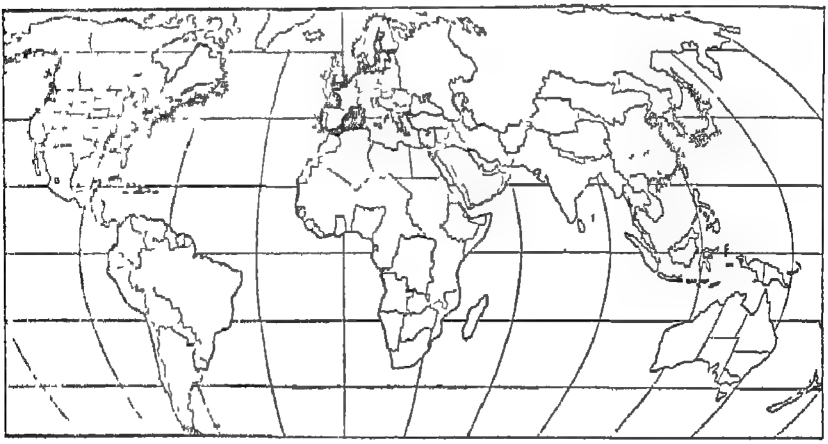
490 Finally, the geographical distribution of silk-raising is likely to be still more limited in the future by the fact that some varieties of silkworms produce only one generation per year, that is, the eggs laid one spring hatch the next Other varieties, however, are bivoltine, that is, the eggs laid in the spring hatch in a few weeks, and another set of worms is raised the same year. Still others are multivoltine, so that several generations are raised in a year Already modern science appears to be able to produce varieties whose eggs can be hatched whenever desired, so that worms may be available at any season Thus silk-raising is changing from an occupation requiring many unskilled people for a few weeks in the spring to one where high skill is required among the technical workers who use the microscope to examine the eggs while the unskilled workers can be kept busy as long as fresh mulberry leaves are available Hence a region like south China, so warm and moist that the mulberry tree sends forth leaves much of the year, has a great advantage over Mediterranean lands where the growth of the trees is limited both by low temperature in winter and drought in summer, or even over Japan with its cool, but not cold, winters

Naturally the silk industry is diminishing in the Mediterranean countries, and growing in Japan and China, especially south China The great danger in south China is the diseases to which the worms are subject As these are conquered, as methods of transportation and storage become still better, and as trade between countries becomes more free, the distribution of all sorts of products, including plants as well as animals, tends to go through the same process as silk *First, the area where the product is raised tends to expand, for people try experiments Then it contracts, and production becomes intensified in the most favorable areas.* One of the interesting problems of the future is the degree to which silk production will decline because of still another stage of development New inventions often cause great changes in the location and volume of production Silk has already lost ground through competition with rayon It is likely to lose still more through the development of still other natural fibers, including even those made of glass.

Chapter XXI

FISHERIES

491 Distribution of Fisheries Fish stand high among the undomesticated animals which are important to man Fishing rivals hunting as man's oldest industry. It still survives as an important commercial industry, whereas hunting is now little more than a pastime Fish add proteins as well as vitamins to man's food, and furnish important substitutes for other forms of meat The amounts used in different countries vary greatly It has been estimated that in the world as a whole fish form less than 3 per cent of the animal food



Denoyer's Semi-elliptical Projection Drawn by Denoyer-Geppert Co, Chicago, Illinois

A492—World Distribution of Fisheries

consumed by man, but in places such as Iceland, Newfoundland, and parts of Norway and Japan they form a far higher percentage The main factor in determining the consumption of fish is the ease with which these animals can be obtained in comparison with other kinds of food, but local customs and religious beliefs also have an important influence

492 Although an abundant local supply of fish is obtained from some rivers and lakes, this is of little importance compared with the supply from the ocean The important fishing areas are found near the coast, or at most within a few hundred miles of it (A492) They

lie partly in the littoral, or shore, belt of shallow water which covers the continental shelf, or submerged platform surrounding the continents Others are located in the shallow water overlying banks, which are elevated parts of the seafloor some distance from the shore The shallowness of the water encourages the abundance of fish because it conserves the microscopic algae which form the basis of the food of the fish. The algae are eaten by the microscopic animals called plankton and the plankton in turn supply food for larger fish, and so on up to the biggest fish When the algae and plankton die, they gradually drop to the seafloor If this lies far down beyond the limits of light and of waves, the plankton is largely lost as food for other creatures If the water is shallow, however, that which falls to the seafloor is stirred up again by storms even at a depth of several hundred feet, and thus becomes available again as food. Low temperature also helps to increase the food supply because it retards decay Hence algae, plankton, and so forth are available for food long after they are dead, whereas in warm water they soon turn largely into gases

493 The world's four outstanding fishing areas are as follows (A492):

1 The coasts of eastern Siberia, Chosen, Japan, China, and Taiwan (Formosa)

2 The coasts of northwest Europe from France to northern Norway, including the North Sea and its arms

3 The west Atlantic area from the Carolinas to Labrador, but especially the coastal and offshore banks from New England to Newfoundland

4 The Pacific region along the coast of California, Oregon, Washington, British Columbia, and Alaska.

494 The first three areas have many types of fish in common The haddock and herring are taken in large numbers, and the cod is especially important in the two Atlantic fishing grounds On the west coast of North America the outstanding commercial fish is the salmon, of lesser importance are the sardines and tuna of southern California The statistics of fisheries include not only these fish, but various other marine animals Of these the best known are oysters, lobsters, and clams

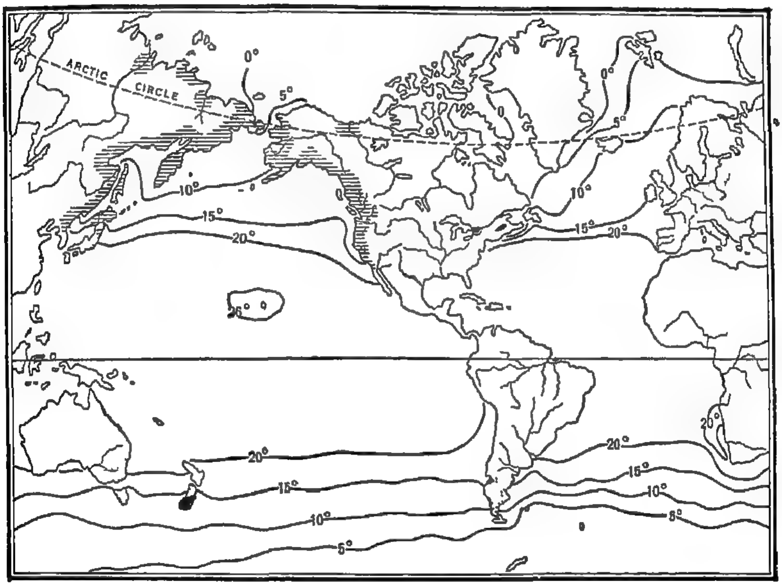
495 Importance of Fisheries Although fishing is an important industry in localities such as Norway, Newfoundland, Alaska, and Japan, the total value of its products does not give it high rank among the main branches of modern production In the entire United States only 1 man out of every 600 who are actively engaged in primary production is a fisherman. Furthermore, the total annual value of the

world's fish products is less than a billion dollars, which is less than the value of the poultry and eggs of the United States alone. Nevertheless, fish are worth twice as much as the world's rubber, and almost as much as either the wine, the tobacco, or the tea, coffee, and cocoa combined. Moreover, the fishing industry has had great importance in the rise of maritime nations. The fast sailing vessels for which New England was noted were an outgrowth of the fishing industry. Drake's seadogs had their training in the mackerel fleets of the North Sea. The merchant marine of many countries, especially Britain and Norway, has depended on the fishing grounds as a training school for seamen. The same has been true of the navies of the world in many countries, but this has become less and less true as the use of machinery has increased. The charm and romance of the ocean, as well as the picturesqueness of fishing and the dangers that often go with it, catch our fancy. All these tend to cause us to overemphasize the economic importance of fisheries in the world as a whole.

496. THE OPTIMA FOR SALMON. A brief study of salmon will illustrate the environmental conditions which cause fishing to be carried on chiefly in cool regions where the temperature, salinity, and aeration of the water, on the one hand, and the food supply, on the other, provide the optimum conditions for especially valuable species of fish. For example, in a good year Alaska may ship out 40 or 50 million dollars worth of canned or fresh salmon, the state of Washington may ship 5 or 10 million dollars worth, and Oregon and California 4 or 5. More than a billion pounds of salmon may be caught on the Pacific Coast of North America. In a bad year the figures may sink to far lower levels. One of the most noteworthy and difficult features of fisheries is that the number of fish varies greatly from year to year in more or less regular cycles. Again and again a particular species almost disappears, and is sometimes supposed to have been permanently depleted by overfishing, but in time it usually reappears in large numbers. Such fluctuations have been especially studied in the lower parts of the rivers of eastern Canada and northwestern Europe. All the salmon regions are much alike in being located in high latitudes where clear rivers of cold water come tumbling down from the highlands into a cool sea. These are the only conditions under which salmon will thrive.

497. Attempts to introduce the salmon of the North Pacific into other regions illustrate how intimately fisheries are related to the geographical environment. This species lives naturally in the areas marked with bars in A497. Because of its great economic value strenuous efforts have been made to introduce it elsewhere. Since

1871 more than 100 million eggs and young have been transplanted to 17 Atlantic states of the United States and to 16 other countries. Where A497 is dotted the salmon have failed to establish themselves in spite of repeated trials and every possible assistance. In most of these regions the eggs hatched without difficulty, but the fish did not thrive and reproduce themselves. Only in the small solidly black areas of southern New Zealand, and southern Chile, and on the coasts of Maine and New Brunswick, with the Lake Ontario region as a spawning ground, have they succeeded in surviving. Two biologists, F. A.



Courtesy of F. A. Davidson and U. S. Bureau of Fisheries

A497—Distribution of North Pacific Salmon.

Davidson and S. J. Hutchinson, have shown that this has occurred only where the latitude, ocean currents, and form of the seacoast cause both the temperature and salinity of the ocean water to be comparatively low, and where the temperature, purity, dissolved oxygen, and clean gravelly bottoms of the fresh-water streams permit salmon eggs to hatch and the young fry to find food. In a broad way climate is the main factor which limits the spread of the Pacific salmon and determines where salmon fishing is profitable. Locally, however, the relief of the land also has a strong influence through its effect on mud and gravel carried by the streams. Climate and relief together provide an optimum environment both for the fish and for the industry.

498 WHERE FISH GROW BEST. The optima for cod, haddock, hake, halibut, mackerel, herring, and other fish determine where the fisheries for these species shall be located just as for salmon It is so hard to follow the life history of a fish, however, that we are still ignorant of many important factors We know, to be sure, that most fish are very sensitive to the temperature and salinity of the water, and that the amount and kind of food affect their size An experiment in the North Sea illustrates the matter The plaice is a favorite European flat fish something like the sole of America A large number of small plaice caught in the eastern part of the North Sea off the coast of Holland were tagged with metal plates. Part were released where they were caught and part on the Dogger Bank 60 miles east of northern England When they were tagged the plaice averaged 7 or 8 inches long. A year later those that were caught again in their original home had gained 2 inches in length on an average, and had doubled their weight Those on the Dogger Banks had gained 5 inches in length and weighed 4 times as much as before. How much of this difference was due to food and how much to the relative impurity and higher temperature of the water off the coast of Holland near the mouths of the Rhine and other rivers is not yet clear We do know, however, that the shallow water of the Dogger Bank, only 6 to 20 fathoms (36 to 120 feet) deep, is one of the best of the world's fishing grounds The fact that it is so easily accessible to England, Germany, France, and other progressive countries makes this an even greater asset than the Newfoundland Banks

499 CODFISH AND THE GRAND BANKS OF NEWFOUNDLAND The cod illustrates some of the chief problems of the fishing industry Off the eastern coast of America cod are taken in even larger amounts than salmon on the west coast. Fishermen not only from Canada, Newfoundland, and the United States, but also from Spain, France, Norway, Great Britain, and other countries carry away more than a billion pounds per year The location of the cod-fishing grounds, like that of other fisheries, depends on the depth of the water and the nature of the bottom quite as much as upon its temperature and salinity In other words, the optimum for fish depends on climate, relief, and sea-bottom (which corresponds to soil) in much the same way as the optimum for plants Below a depth of 200 fathoms (1,200 feet) there are few fish Inasmuch as light does not penetrate there, the algae, which form the basic food of sea animals, are absent, and the minuter forms of animal life on which fish feed are scarce. Most fish live within 100 fathoms of the surface, and even if there were many farther down the expense of catching them would be excessive Hence the world's fish-

eries, as shown in A492, are limited to shallow waters near coasts and to a few places like the Grand Banks of Newfoundland where the sea-floor rises well toward the surface of the ocean.

500. The Grand Banks of Newfoundland appear to be the top of an old mountain range about 300 miles long, running from northwest to southeast, and covered by 80 to 100 fathoms of water. The broad, flat top of the ridge is covered with sand and fine mud. On the south side the banks are washed by the warm current of the Gulf Stream, and on the north by a cold arctic current from Greenland. The currents bring with them innumerable diatoms and algae, too small to be seen except with a microscope. These form the food of billions upon billions of almost incredibly small lobsterlike crustaceans and tiny molluscs like microscopic oysters. These in turn supply food to small fish such as herrings, which are the favorite food of the cod and other larger fish. Fishermen from Newfoundland, the Maritime Provinces, New England, and also northern Europe catch herring by the million for use in baiting trawls or floating ropes from which hundreds of cod-hooks hang down into the deep water. Frequent storms as well as the length of the fish lines and the coldness of the water make it a difficult task to go out in small rowboats and haul in the cod which have been caught on the hooks. During the frequent fogs there is also danger of losing the mother ship. Warm air from over the Gulf Stream blows over the cold water from the northern current, thus causing dense fogs which may last for days.

501. In May and June enormous shoals of caplin, a small fish of the herring family, swim toward the shores of Newfoundland, Labrador, and the coasts as far south as New England. Sometimes they darken the surface of the ocean for miles, and are cast up on the shores by the million. They are followed by squid, a small kind of octopus that makes excellent food for man as well as cod. The caplin and squid are pursued by millions upon millions of cod, anywhere from 1 to 4 feet in length. Hence at this time the shore fisheries are active. In some years, however, for some still unknown reason, the cod, as well as the other fish fail to arrive. Inasmuch as a single cod lays from 1 million to 10 million eggs, it would seem as if there ought to be an abundance of fish each year. Even in good years, however, the eggs and newly hatched fish are eaten by other fish in such quantities that no more than two or three adult fish are usually produced from a million eggs. In unfavorable years a million eggs may provide no adult fish at all. Epidemics of unknown diseases may be one cause of this and so may unfavorable conditions of temperature, but our knowledge is very deficient.

502. In places like Newfoundland, Iceland, northern Japan, or the west coast of Norway such a scarcity of fish is a major disaster. In former times half the population of Iceland is said to have perished because of the combined effect of scarcity of fish and cold, wet summers that spoiled the hay so that the sheep starved during the following winter. In Newfoundland the years from 1860 to 1867 were marked by a prolonged scarcity of fish. Perhaps the fish went somewhere else because winds, currents, and the temperature and salinity of the water caused a scarcity of food in their usual habitat. Whatever the cause may have been, the result was widespread poverty and destitution in Newfoundland, where four fifths of the men still work as fishermen at least part of the year. The government had to support a large part of the population for many years. The inhabitants got so into the habit of expecting public aid that the country became demoralized. Some people think that the effect of this disaster, and of later ones of the same kind but less severe, is one of the main reasons why Newfoundland as a whole has not been able to pay its own way. Because of this the position of the island as a self-governing dominion was changed in 1933 to that of a colony governed by Great Britain. The failure of the fisheries in the 1860's led to an increase in agriculture. Since that time copper mining and especially the pulpwood industry for paper-making have developed. Nevertheless fishing is still the main industry, and the income from it is about four times as great as from farming.

503. WORLD FISHERIES. A comparison of the two parts of Table 16 gives interesting information as to the regions where fish are abundant. One shows the approximate catch of sea fish per year in millions of pounds, and the other the approximate catch per person.

504. A remarkable fact about this table is that Japan and its dependency, Chosen, which together have 95,000,000 inhabitants, catch as much fish as Russia, Norway, the United States, Great Britain, and Germany, which have 5 times as many people. Equally remarkable is the fact that Norway with less than 3,000,000 people catches more than the United States with about 130,000,000. It is even more remarkable that Alaska (63,000 people) and the islands of Iceland (117,000) and Newfoundland (290,000) catch more fish than such huge countries as India and China, which do not appear at all in our list. In the second part of Table 16 Alaska shows the enormous total of 12,300 pounds per year for every person living within its borders. As a matter of fact, however, a large part of the Alaska fish are caught by fishermen who come from Washington, Oregon, and even Japan. The figures for Iceland, the Faeroe Islands, and Norway, on the other hand, represent the actual catch by people in those countries.

TABLE 16

GEOGRAPHICAL DISTRIBUTION OF FISHING

1. Total annual catch of sea fish in millions of pounds		2 Annual catch of fish per capita in pounds	
1 Japan	7,750	1 Alaska †	12,300
2 Chosen	3,350	2. Iceland †	4,200
3. U S S R *	2,900	3 Faeroe Islands †	1,600
4. Norway †	2,400	4 Norway †	830
5 United States	2,300	5 Newfoundland *	410
6 Great Britain *	2,200	6 Chosen	145
7 Germany *	1,300	7 Japan	112
8. Canada *	920	8 Canada *	93
9. Alaska †	770	9 Portugal	84
10 Spain	750	10 Netherlands *	57
11 France *	690	11 Denmark †	51
12 Iceland †	500	12 Sakhalin *	51
13 Netherlands *	500	13 Great Britain *	51
14 Portugal	475	14 Sweden †	37
15 Sweden †	235	15 Formosa	34
16 Denmark †	190	16 Kwantung	34
17 Formosa	160	17 Estonia †	34
18 Newfoundland †	118	18 Spain	32

* 45–55° N latitude
† Above 55° N latitude

505 If one locates all the regions in the second part of Table 16 on a map of the world, it becomes clear that fish are mainly a product of cool regions. In order to get a correct view we ought to allocate the fish caught in the United States and the Soviet Republic to the coastal states or provinces of those great countries instead of to the entire area Even in a country with so much wealth and such good transportation as the United States about nine tenths of the fresh fish are said to be consumed within 200 miles of the coasts to which they are first brought. In countries such as Russia and China the transportation of fish is even more restricted If, then, we count only the coastal parts of the larger countries, we find that the catch of fish per capita is greatest in the north and declines quite regularly as we go southward Alaska, Iceland, the Faeroe Islands, Norway, Denmark, Sweden, and Estonia, all of which carry a dagger in Table 16, catch their fish mainly north of latitude 55° Newfoundland, Canada, the Netherlands, Sakhalin, the Soviet Republic, Germany, Great Britain, and most of France lie more than 45° from the equator, and go farther north for much of their fishing South of latitude 45° the only places with a large total catch of fish, or even with a large catch per capita, are Japan, Chosen,

Spain, and the United States. These countries tend to go north for their fish, however, and catch them mainly along the coasts that are washed by cool currents from the north. A similar condition prevails in Kwantung and Portugal.

506. This leaves only Formosa and the coasts of China as the main regions where fish form an important part of the food supply in latitudes lower than about 35° N. Places such as the Fiji Islands and other such groups in the South Pacific Ocean, to be sure, depend greatly on fish, but their population is small. In recent decades, however, considerable progress has been made in fisheries in warmer waters, but thus far their total catch is small. Even in the United States, where fisheries in warm waters are most highly developed, the value of all the fishery products, including oysters, clams, lobsters, etc., as well as sea fish, is less than half as much on the long coast from Norfolk to Mexico as on the shorter coast from Norfolk to Canada. In the interior of China and in certain tropical regions, such as Java, fish ponds are a normal part of the equipment of many of the larger farms. Although these ponds swarm with fat fish, they do not supply enough to alter the general fact that at present fish are overwhelmingly a product of cool climates.

507. Reasons for Northern Location of Fisheries. The reasons for the northern location of fisheries are often misunderstood because people do not appreciate the importance of optima for both animals and industries as well as for man. It is often said that in such countries as Iceland and Norway people are driven to the sea by the rugged and inhospitable nature of the land. This is not quite so. Greece, Turkey, Persia, Arabia, Peru, Borneo, and many other parts of the world have seacoasts more rugged or unproductive than those of Japan, Chosen, Great Britain, New England, or even Norway, but they do not do much fishing. It is also said that the people of these regions engage in fishing because of the deeply drowned coasts and many sheltered bays and offshore islands. These are important points, but others may be even more important. One point is that fish are more numerous in northern waters than in those of lower latitudes. The main reason for this, as we have seen, is that food is more abundant because the smaller forms of life do not decay so fast as in warmer water. Another important point is that in warm regions fish decay rapidly. Hence, until the recent development of refrigerating methods, it was difficult to preserve them for any length of time in low latitudes. In such regions it did not pay to catch fish except for immediate consumption close to the point where they were caught.

508 Again, the people of northern climates are on the whole more energetic and adventurous than those of low latitudes, and fishing is a dangerous and difficult business It must not be forgotten, however, that the South Sea Islands, such as Fiji, Samoa, and Marquesas, are inhabited by adventurous fishermen Nor are there any northerners who rely more fully upon the products of the sea for a living Nevertheless, the inhabitants of the warm Pacific islands cannot keep their fish any length of time, and hence cannot build up any extensive business in fish This brings us to the last reason why fisheries have developed so greatly in cool, northern countries Instead of saying that the poverty of the land drives people to the sea, we might better say that the riches of the sea and the ease with which fish can be caught and transported attract the people away from the land The situation is similar to that of farmers in southern New England who raise onions, tobacco, or snap beans instead of corn The farmers are not driven away from corn so much as they are attracted to the other crops because they pay better That is the way it is in a country like Norway, or Japan's northern island of Yezo The drowned coast abounds in deep inlets, the waters off the coasts abound in good fish, the low temperature makes it possible to preserve the fish, the many harbors and the trees that grow along the shores encourage people to make boats and learn the ways of the sea Therefore when the problem of making a living must be solved in regions too cool and rugged for profitable agriculture, the sea is often more attractive than the land

PART VII

THE HUMAN FACTOR

CHAPTER XXII

HUMAN EFFICIENCY

509. WIDESPREAD DIFFERENCES IN EFFICIENCY. The efficiency with which people work in any particular part of the world has quite as much effect upon the problem of getting a living as do the natural resources. Regional differences in efficiency must be due to innate racial traits, cultural inheritance, or physical environment. Although innate racial differences of some sort presumably exist, their effects appear to harmonize so well with those of the two types of environment that we may safely omit them in our present study of economic geography.

510. PHYSICAL VERSUS CULTURAL INFLUENCES. Even if we disregard racial differences, there is still great diversity of opinion as to the effect of environment upon man's capacity and inclination for work. It is hard, for example, to decide whether a given condition of vigor is due to physical causes such as climate, or cultural causes such as the quality of the medical and sanitary services. In warm countries with the wet, monotonous equatorial type of climate, white men practically always leave physical work to the darker native races. This is partly because physical work is much more tiring in such climates than in cooler regions, but it is also due to a distinct social custom. The fact that physical work quickly makes people hot and tired, together with the presence of a more backward race, has caused the growth of a social system which frowns upon physical work for the white man. In the tropics many a white man who wants something done in a hurry refrains from doing it not because it would tire him too much, but because people would say that he was doing work that belonged to natives.

511. Still other conditions make it difficult to explain why people are so much more efficient in some regions than in others. Such differences may be due to health, but health depends upon climate, diet, disease, and conditions of living and working, as well as upon the

degree of progress in civilization. In Puerto Rico it is clear that the average peasant does far less work than the average farmer in Kentucky or Pennsylvania The thing that is hard to determine is how much of this is due to each of the causes just mentioned How much is due to the fact that the Puerto Rican peasant cultivates a very small amount of land? How much to mere disinclination to work? How much to the fact that in a tropical climate one can survive with relatively little food and with a minimum effort to procure clothing, shelter, and heat? Sometimes, too, people fail to work because nothing that they can get seems worth the effort, or because the political and social system is such that, even if they work hard, they get no adequate reward Another vital consideration is that disease has a great deal to do with people's capacity for work Common colds, influenza, malaria, dysentery, and many other diseases often have a greater economic effect through making people feel disinclined to work than through actually making it impossible for them to work A hot day, or a cold one, may make people feel the same way It often happens, also, that when some new motive is introduced—when people are able, for example, to buy new and desirable kinds of goods—they are inclined to work much harder than formerly Thus many different conditions may lead to similar results. The problem of people's capacity and desire for work is extremely complex

512 A Basic Climatic Pattern Such conditions as health, inclination to work, and social progress all show substantially the same geographic distribution Their geographic pattern agrees with that of climate, and appears to be set by climate The reason why climate has such a basic influence is that it not only produces its own direct effects upon the human body, but also influences many other factors It alters the soil, as we have seen, and as a rule tends to make the soil poor in the same regions where it has an adverse direct effect upon man It also alters the vegetation, changing not only its amount and kind, but also the degree to which any particular kind of food contains essential minerals, oils, acids, and vitamins which are essential to a good diet The insects, bacteria, and other parasites which cause disease are similarly influenced Parasitic and infectious diseases as a rule are especially dangerous in climates that are otherwise unfavorable Climate likewise is a main factor in determining the kinds of occupations in different parts of the world, and the ease with which transportation can be carried on No one, for example, is likely to establish a chair factory in a tropical grassland, nor a brass factory run by waterpower in a country where all the rain falls during half the year and is sufficient to support only a sparse and poor population.

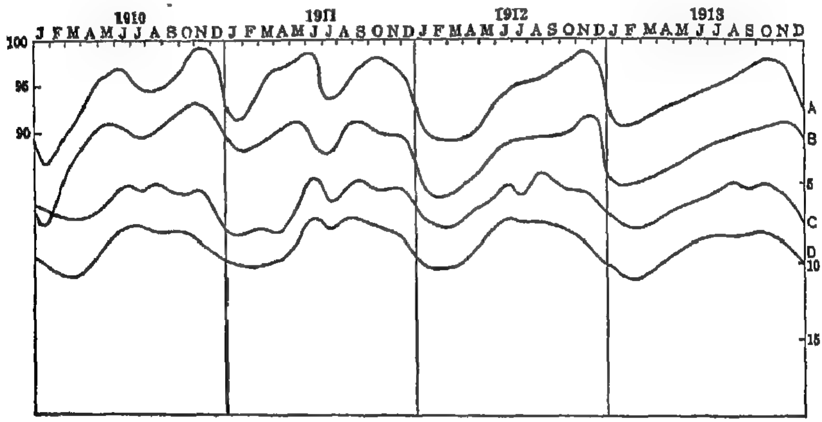
Then, too, climate joins with other factors in creating, or at least increasing or diminishing, the desire for many types of goods. People's attitude toward clothing, houses, fires, and food varies according to where they live—in Greenland or Samoa. Occupations are similarly influenced.

513 It must be clearly understood, however, that, although climate is basic, the pattern that it sets is often changed by other factors. In Florida and Queensland, for example, recent migration causes many phases of life to be different from what they would be on the basis of climate alone. The fact that a region is new and has plenty of space has a similar effect, often raising people's activity to a high level because a given amount of work yields a great result. Hence we must first inquire how climate influences human vigor, either directly or through disease, and then see how social customs and other cultural conditions cause the distribution of economic activity to be distributed over the earth according to the basic pattern set by climate, but with abundant modifications by other conditions. Another phase of the subject, namely, the relation of diet to climate and to such problems as overpopulation, is left for later chapters.

514 FOUR TYPES OF SEASONAL HUMAN VARIATIONS. The effect of climate upon the geographical distribution of human efficiency is closely tied up with the fact that in all parts of the world the march of the seasons causes a regular seasonal variation in births, physical work, mental activity, and deaths. This indicates that human beings are very sensitive to differences in the weather and hence to climate in general. When we compare different geographical regions we find differences like those that come and go with the seasons. We also find that all the main races appear to have almost the same climatic optima. There are differences, to be sure, between tropical and nontropical people, but they are small. No matter where people live, any departure from the optimum appears to be accompanied by lowered efficiency. Each climatic type appears to produce its own distinct effect upon physical vigor, mental activity, and general efficiency. This fact appears to have so much effect upon economic activity that we must examine both seasonal and geographic variations in health and efficiency with much care. This is especially important because it emphasizes certain handicaps which people in general do not properly understand. If the handicaps are ever to be removed, the first step is to know exactly what they are and how they work.

515 I. *Seasonal Variations in Physical Work.* Inasmuch as work is the type of human activity that we are here most interested in, let us see how the work of factory operatives in the northeastern United

States varies from season to season. The data here used represent the largest investigation of this kind that has yet been made. The upper two curves in A515 show the amount of work done by hundreds of factory workers in Connecticut (Curve *A*) and Pittsburgh (Curve *B*) during 4 successive years. These workers were paid by the piece, so that their earnings went up in direct proportion to the amount of work. They were free to earn as much as they could, and most of them wanted to earn considerably more than they actually got. Hence the curves in A515 measure the combined effect of people's desire to work and of their physical ability to work. These two things are the main determinants of a person's activity or efficiency from season to season after he has thoroughly learned his job. Note first that the



A515—Seasonal Variations in Health and Efficiency

two curves fluctuate in harmony. They are based on different kinds of factories 350 miles apart, and there were no fluctuations of general business corresponding to those in the curves. Nevertheless, the amount of work done per hour by these hundreds of people went up and down from season to season in the same way in both regions. This must mean that some common cause was at work.

516. An examination of the special features of A515 gives an idea of the kinds of weather which help or hinder efficiency. Each curve shows a strong dip every year in January when the weather is coldest. In 1912, when cold weather was unusually severe and prolonged, the period of low efficiency lasted through March. Each curve also rises during the spring and is high in May or June. This suggests that a return to warm weather helps people to work effectively. The summers, however, show a good deal of irregularity. In 1910, after a peak

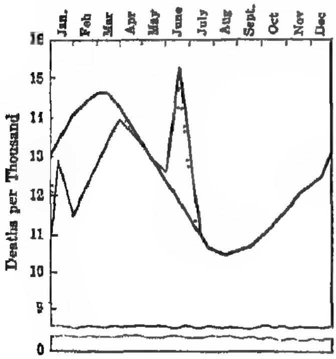
in May and early June, both curves drop during the summer, and the next year they drop still more. During the third summer they merely flatten out instead of dropping, while in 1913 they keep on rising steadily until autumn. This is especially interesting because the dip in the summer varies in direct harmony with the heat. The summer of 1913 was unusually cool and comfortable, and people's ability to work was not diminished. The preceding summer was almost as good, but there was enough hot weather to prevent people from increasing their efficiency for a few weeks in midsummer. The summers of 1910 and especially 1911, on the contrary, were very hot, and people's efficiency fell off badly. Passing on to the autumn, each year shows a maximum in October or November. This suggests that the approach of winter stimulates activity, even though the winter itself is depressing. It is especially noteworthy that after the hot summer of 1911 the onset of cool weather did not bring people's energy back to the level that it had reached the previous May and June. A similar condition has been found in Japan. Such facts suggest that prolonged or extreme hot weather, as well as extremely cold weather, produces an effect which does not wear off for months.

517. Factory workers in England, the southern United States, and Japan show seasonal fluctuations of efficiency like those of Connecticut and Pittsburgh, but with differences that fit the respective climates. As one goes to warmer climates in both the United States and Japan the harmful effect of hot weather becomes worse, while cold weather is less harmful. Experimental studies show that in hot weather, by making extra effort, people can work as hard as in cooler weather, but they become more tired. As a rule, however, people do not feel inclined to make the necessary effort. This *lack of inclination* to work in warm weather is very important in determining the efficiency of workers in various types of climate.

518. II. *Seasonal Fluctuations in Health.* Curves *C* and *D* in A515 indicate the general conditions of health in Connecticut and Pennsylvania during the years covered by the records of the piece-workers in factories. The only good measure of health for which statistics are widely available is the deathrate. Curves *C* and *D* represent the deathrate inverted, so that a low deathrate makes an upward bulge, indicating good health. In general the curves of health in A515 go up and down with the curves of factory efficiency. Deaths are most numerous, however, several weeks after the lowest level of efficiency. This is what we should expect if people are most likely to become ill when their efficiency is lowest. After people become ill several weeks may elapse before they die, and hence there is a lag. On the other hand, in

the fall the increase in deaths begins sooner than the drop in efficiency The onset of cool weather continues to stimulate young, healthy people, such as the factory workers, for a considerable time after it begins to injure the health of the weaker and generally older people among whom most of the deaths occur In general, however, the curves for work and deaths (inverted) vary so closely in harmony that there can be little doubt that the variations in both are due to seasonal changes of weather

519 The closeness of the relation between health and weather is illustrated in A519 The heavy line there with a maximum in early March and a minimum in August shows how the average deathrate varies from season to season in the large cities of the United States Year after year the number of deaths is high and the general level of health low in winter Even when there are no special epidemics this condition is usually altered but little In winter people's health and efficiency are diminished partly by low outdoor temperatures, partly by unduly warm, dry air in their houses, and partly by diseases to which the human body appears to be especially susceptible under winter conditions The lower line in the left-hand part of A519 shows what happens when the winter is unusually warm Unusual warmth in January and February, 1932, in most parts of the United States was accompanied by a deathrate so low that the winter as a whole was the most healthful ever known until 1937-38, when mild and favorable conditions helped to carry the deathrate even lower In 1932 the deaths in the United States numbered about 275,000 less than presumably would have occurred if the temperature had been normal The degree of efficiency at that time was probably correspondingly great, although hard times kept millions of people out of work

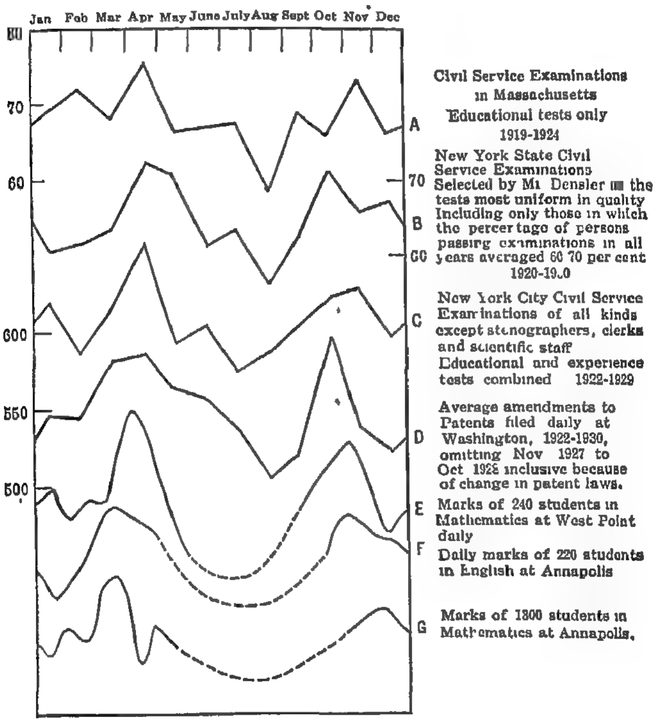


A519—Seasonal Variations in the Deathrate in the United States

520 The peak of A519 in June shows what happens in a short but very hot spell, which breaks records in a large part of the United States Among both children and adults many persons were so weakened that they died because of disorders from which they might otherwise have recovered The deathrate shot up so far that the number of deaths in the entire country appears to have been about 110,000 greater than would have been expected with normal temperatures. Such a hot

period is accompanied not only by illness and death, but also by great discomfort, inefficiency, and disinclination to work Similar evidence of the effect of low or high temperatures can be found on a smaller scale in the records of every year

521 III *Seasonal Fluctuations in Mental Activity.* If production is to be high in any region, and business is to be active, people must be alert mentally as well as efficient physically A521 indicates that



A521—Seasonal Variations in Mental Activity in the United States.

seasonal variations in mental activity are just as pronounced as in factory work or health The three upper curves show the percentage of persons passing civil-service examinations in different months The next shows applications for amendments to previous applications for patents The last three show the marks of students at West Point and Annapolis All these are good measures of people's mental alertness, that is, of their ability to think rapidly, correctly, and constructively In each curve there are maxima of mental efficiency in the spring and fall, and minima in summer and winter In each the spring

maximum tends to be more important than that of autumn; that is, it is higher or broader. In general there is a tendency for the summer level to be lower than that of winter. No cause except the weather appears to be competent to produce such widespread fluctuations of the same sort in such a diverse set of activities. The fact that at Annapolis in Maryland (Curves *F* and *G*) the peaks come earlier in the spring and later in the fall than at West Point in New York (*E*) is exactly what the climate would lead one to expect. In the spring the students at Annapolis win high marks when a certain temperature arrives. When the same average temperature is experienced at West Point a few weeks later the marks reach a maximum there also. In the fall, on the other hand, the temperature at West Point drops back to the most stimulating level earlier than at Annapolis, and the marks follow it.

522. In A521 the curve marked *D*, based on the investigations of J. Rossman of the United States Patent Office, is especially significant. After an inventor has applied for a patent he often has to wait a year or more before it is granted. Meanwhile he may get a new idea as to ways of improving his invention. Immediately he is in a great hurry to send the new idea to Washington, and have it added as an amendment to his original application for a patent. If he does not act promptly, the patent may be granted without it, and he will have to apply for another patent, or somebody else may get ahead of him. Hence Curve *D* is an accurate measure of the highest kind of mental activity. When taken in conjunction with other evidence it means that our ability to do mental work fluctuates constantly in response to the weather. Each spring in March or April we have a period of unusual mental vigor, each fall, a similar but less pronounced period. Winter and especially summer are less favorable times for mental work in the northern United States.

523. IV. *Seasonal Rhythm of Births.* Among the four human rhythms dependent on the season—work, health, mental alertness, and births—we have left that of births till last because it appears to furnish a key to the others. Man, like practically all animals, inherits a tendency toward a seasonal rhythm of births. All parts of western Europe show a distinct maximum of births in the late winter or early spring, with a minimum in October or November. Farther north in cooler climates the maximum comes later, farther south it comes earlier. Social factors, such as seasonal migrations and diet, often modify the rhythm. In the prosperous northeastern quarter of the United States, for example, there is now a double maximum of births in March and

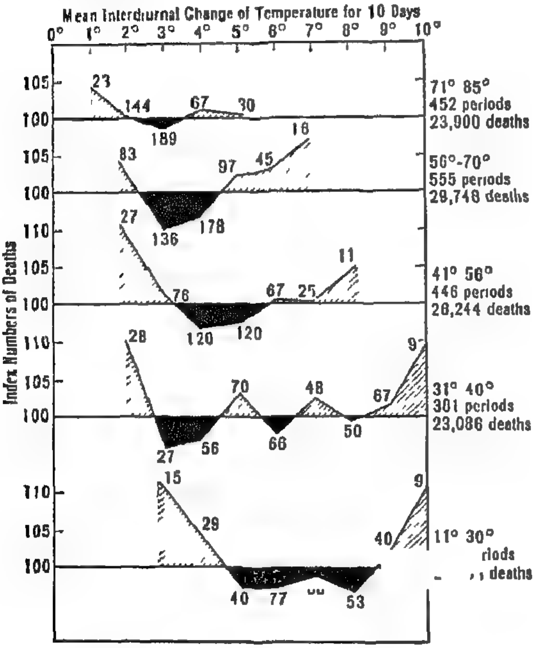
again in August Nevertheless, in some form the rhythm is evident in all parts of the world, and everywhere it is modified in accord with the local climate

524 A strange fact about this rhythm is that it is often more clearly evident in the quality of the births than in their number. People born at its maximum tend to live longer than others For example, among an impartially selected group of 39,000 people born in the eastern United States during the nineteenth century there was a regular annual rhythm in length of life From a maximum among those born in March the duration of life declined to about 4 years less among those born in July, August, and September, and then rose again among people with birthdays later in the year A still more widely evident fact is that people born from February to April stand a decidedly better chance of becoming famous than do those born in other months For instance, suppose the average number of famous people among a given number of births is 100 per month, taking the year as a whole and correcting the figures so as to make the months of equal length Then 110 or 115 famous people will be born in February or March and only 85 or 90 in July or August This apparently has little to do with biological inheritance Its main cause seems to be merely that those born in the late winter or early spring have unusual vigor from birth onward, as is evident from their relatively long lives Hence they have more strength and endurance with which to do the hard work that leads to fame All these facts being taken into account, the climate which helps most in making people physically vigorous and mentally alert appears to be one with a mean temperature of about 60° to 65° F in early June, and approximately 45° to 50° in early March This means that the warmest month averages almost 70° and the coldest about 40° or lower.

525 VARIABILITY AS A FEATURE OF THE OPTIMUM CLIMATE The climate in which mankind appears to be most efficient has two other qualities in addition to the proper range of temperature from season to season. One is sufficient rain and humidity at all seasons. The other is variability from day to day Rain and humidity are necessary for plants and animals as well as for man Variability appears also to be good for crops, but it is especially valuable for man. A525 shows the relation between deaths and changes in the average temperature from one day to the next The deaths are those on the last day of 10-day periods The average change of temperature during the 10 days is shown by the scale at the top, uniform weather with no changes being represented on the left and highly variable weather on the right The

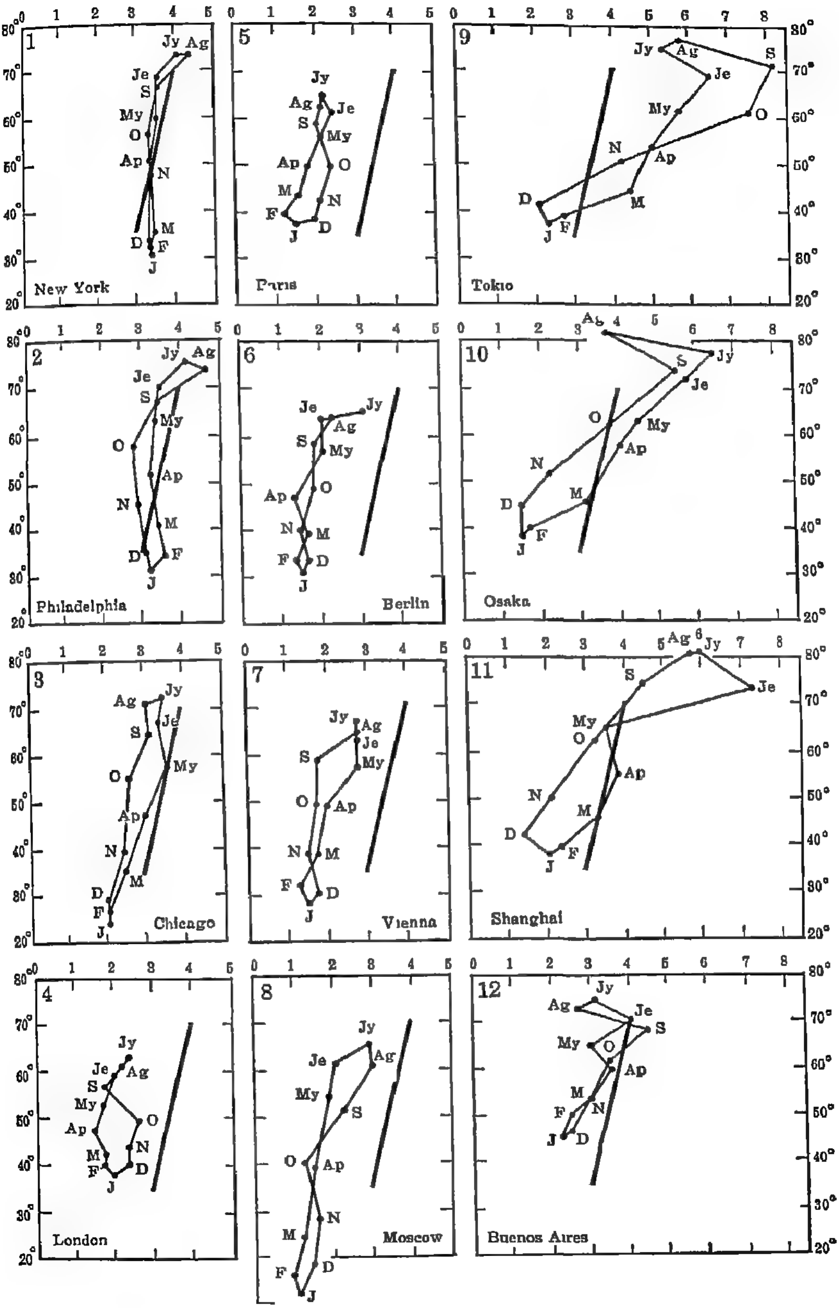
five small diagrams are arranged according to the average temperature of the 10-day periods, hot weather being at the top and cold at the bottom.

526. Every diagram in A525 is high on the left, drops low in the middle, and rises again on the right This means that in summer and winter alike the deathrate goes up at the end of periods when there is little change of temperature from day to day, down when there is a moderate change, and up once more when the changes are great. In winter the most favorable degree of change is greater than in summer, presumably because people are more protected from the weather, and because the greater extremes make them less sensitive to changes The essential point is that at all seasons moderate but frequent changes in the weather are beneficial, whereas unchanging weather or extremely variable weather is harmful. Sea breezes and valley breezes among mountains are valuable means of producing variability Cyclonic storms where masses of warm "tropical" air are forced to rise over masses of cool "polar" air are still more so They not only insure variability of weather, but also bring rain, which is highly important for agriculture The rain helps to keep the air sufficiently humid to prevent the constant dust which is a great menace to health where there are long dry periods Cyclonic storms are most beneficial near seacoasts, especially west coasts, where the severity of their changes in temperature—the cold or hot waves which they bring—is reduced by the influence of the ocean.



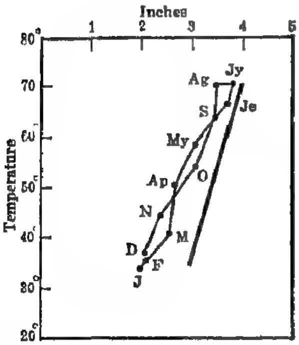
A525—Deaths in New York City in Relation to Changes of Temperature from Day to Day,

527. A Standard Climograph. The preceding discussion shows



A527—Climographs of World's Largest Cities.

that the ideal climate for human efficiency is almost the same as the average of the world's 12 largest cities. The individual climographs of the cities are shown in A527 where the cities are arranged geographically. The average weather of all 12 (with allowance for change of seasons in the southern hemisphere) gives a climograph (B527) so slender and regular that its essential features are correctly represented by a straight line. The end of this line representing July stands at a mean temperature of 71° and a monthly rainfall of 3 8 inches. The other end representing January stands at a temperature of 33 3° and a rainfall of 2 0 inches. A July temperature of 71° is slightly higher than that which appears to be best for human health, a January temperature of 33° is somewhat lower than is best. Agriculturally, however, the higher temperature tends to make the composite climate favorable not only for crops such as wheat, but also for others such as corn which cannot well be grown unless the warmest month (not the three warmest whose isotherm is shown in A39) has a temperature of at least 68°. The rainfall shown by the composite diagram appears to be somewhat less than is best either for human health or for crops, but the difference is not great. In order, therefore, to get an ideal or at least a standard climate we may well modify the average of the 12 largest cities a little. The standard will then be illustrated by a climograph in which a straight line runs from a temperature of 70° and a rainfall of 4 inches in July to 35° and 3 inches in January, as shown in B527.



B527 — Average Climograph of 12 Great Cities

528 CLIMATES OF 12 GREATEST CITIES. The value of our *standard* climograph is evident when we compare it with the individual climographs of A527. At New York (1), the climate comes closer to the standard than does that of any other of the 12 greatest cities. July and August, to be sure, are a trifle too warm for the greatest human vigor, but vacations, playgrounds, air-conditioning and other modern improvements have almost overcome this handicap. The fact that at all seasons New York has approximately the standard amount of rain appears to be an advantage both to agriculture and to human health. The climograph for Philadelphia (2) is much like that of New York, except that the summer is a little warmer and not quite so healthful. The spring and fall tend to have a trifle less rain than either the summer or the winter, as appears from the curve of the Philadelphia

climograph toward the middle. Excellent temperature and well-distributed rainfall join with good soil to make eastern Pennsylvania one of the best agricultural regions in America. Chicago (3), the other North American city in A527, also approaches the standard climate, but has less rainfall and is too cold in winter.

529. Western Europe as well as the northern United States has three of the world's largest cities. It is represented in A527 by London (4), Paris (5), and Berlin (6). These all have climates with an even better range of temperature than the American cities so far as man's health is concerned, but are not quite so good from the standpoint of diversified agriculture. Nevertheless, it must not be overlooked that in Belgium and Holland, where the climate is midway between those of London and Paris or Berlin, the productivity of the land is greater than almost anywhere else in the world. None of these three cities has as much rain as the standard, but the disadvantage thus arising is largely nullified by the coolness of the summers and the comparatively slight degree to which the rainfall varies from year to year. Berlin has a slight handicap in that the winter is about as cold as that of New York and much drier. Abundant statistics show that if winter weather is dry as well as cold the deathrate rises. At Vienna (7) the same handicap appears in slightly stronger form, but probably in neither Berlin nor Vienna does cold weather do any more harm than in Chicago. It is only when we get to Moscow (8), far from the ameliorating influence of oceans or great lakes, that we find one of the 12 greatest cities which has any serious climatic handicap. From May to September the Moscow climate is almost ideal for human health, but the season which has a mean temperature above 50° is much too short for the best agriculture. The severely cold and comparatively dry months from October to April injure the Moscow climate from practically every point of view.

530. Turning to Asia we find another, and apparently more serious, type of handicap. Tokyo (9), Osaka (10), and Shanghai (11) all suffer greatly from long, hot, and very wet, muggy summers like our worst dogdays. The summer heat lasts from July to September in Tokyo and from June to September in Osaka and Shanghai. It hampers the growth of such crops as corn, wheat, cotton, most fruits, and many vegetables. It is even worse for men. By the time September arrives the vigor of the Japanese is so low that deaths are more numerous than conceptions resulting in living births. If the September conditions lasted throughout the year, the population of Japan would diminish instead of increasing. From October to May the climate of Tokyo is almost ideal in temperature, although rather wet. Shanghai and

Osaka also have excellent conditions although a little warmer than the standard In spite of these handicaps Japan and the central Chinese coast around Shanghai are climatically the most-favored parts of Asia, aside, perhaps, from the mountain province of Szechwan and west central Siberia

531. In South America warmth in winter is a handicap to Buenos Aires (12), the only one of our 12 cities in the southern hemisphere There the period when the temperature is near the mental optimum is very limited Moreover, agriculture is handicapped by lack of cold weather to check insect pests A more serious handicap is the summer droughts, which are suggested by the way in which the climograph for Buenos Aires hooks back in July and August, and to a slight degree in May The region around Buenos Aires, or south of it, is the best part of South America except south-central Chile The close association of the great cities of the world with the best climates is astonishing It is very significant that among the 12 cities not only the 2 largest, but also the 7 which are normally most wealthy—New York, London, Berlin, Chicago, Paris, Philadelphia, and Vienna—are the ones having climates most closely approaching the type which seasonal fluctuations in births, physical work, mental work, and deaths all indicate as the optimum for human health

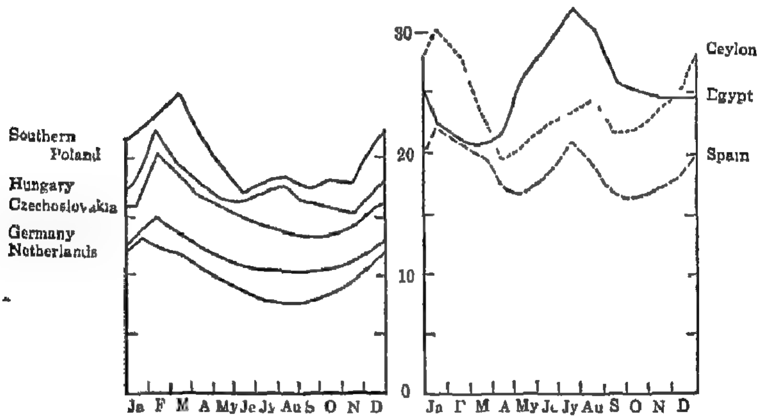
532 In estimating the importance of this chapter two principles should be kept in mind The first is that *human efficiency appears to be at least as important as natural resources in influencing human progress and economic activity* The second is that, *although efficiency depends on many factors, none of the others, unless it be racial inheritance, is so hard to escape from as climate* This is partly because climate plays so large a role in diet, disease, occupations, and social customs It is also because of the extreme persistence and delicacy of our inherited response to the atmosphere A breath of wind, or a drop of 1° in temperature, may make us shiver, or renew our flagging energies, as the case may be Moreover, although all the higher races appear to inherit the same strong tendency to be most vigorous and active in essentially the same definite type of climate, most of mankind live in climates that depart widely from this type The original adjustment of our species to what has become the ideal climate presumably goes back to a time before the various races of mankind became separated, for it seems to be substantially the same in all of them For example, although Cubans and Florida Negroes live in a warm climate, their work in factories shows that they are most efficient at a temperature only about 4° or 5° higher than is best for people in the northern United States or England. The effect of temperature upon the death-

rate is essentially the same in warm Sicily and cold Finland In tropical Java white men, Chinese, and native Javanese all feel most comfortable at practically the same temperatures Moreover, in Java the best health among all alike is found not at sea level, but at an altitude high enough so that the mean temperature approaches the physical optimum Curiously enough the temperature at which people are most comfortable is higher and more uniform than that which most promotes health and efficiency

CHAPTER XXIII

THE BASIC GEOGRAPHIC PATTERN

533. VARIATIONS IN HEALTH AND EFFICIENCY FROM COUNTRY TO COUNTRY. I. *Distance from the Sea* The deathrate varies systematically from country to country. From this we may safely infer that health and efficiency vary similarly The variations may be examined from two points of view, namely, from the coast to the interior, and from north to south In A533 the left-hand set of curves shows seasonal variations in the deathrate in five countries which form a band from west to east in the middle of Europe Each curve has a maximum



A533—Seasonal Variations in Deathrates by Countries

at some time from January to March, and a minimum in the late summer or fall When read from below upward, the five countries are arranged in the order of their position in respect to the Atlantic Ocean. The Netherlands is on the coast Then come Germany, Czechoslovakia, as it was before 1939, Hungary, and old Poland As one goes from the Netherlands eastward toward the continental interior, even though the latitude remains nearly constant, the following changes occur (1) the deathrate increases, and efficiency presumably decreases, as appears from the increasingly high position of the curves in A533, (2) the contrast between the seasons in any one country increases, as appears from the increasing prominence of the maximum in winter, (3) the summers, being warm in the interior, begin to show a minor

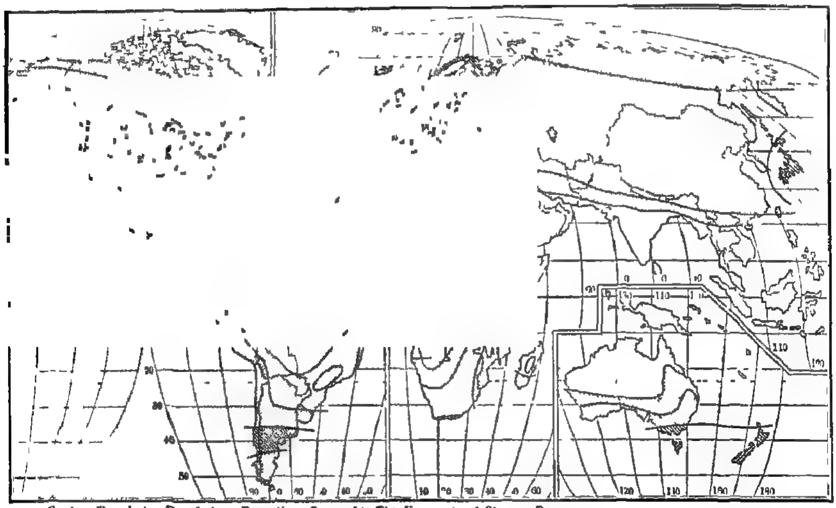
maximum of deaths, as appears in Hungary and Poland These contrasts between the oceanic west and the continental interior correspond both to the climate and to economic activity

534 II. *Variations According to Latitude* The conditions shown on the right of A533 illustrate what happens as one goes southward from the latitude of central Europe In Spain, before the recent period of civil war and reconstruction, the total deathrate was about the same as in Poland The winters, however, are milder and the summers hotter in Spain than in Poland Hence Spain shows nearly equal maxima of deaths in summer and winter Hot summers, however, lower people's inclination to work more than do cold winters even though the total deathrate is the same in both countries We should therefore expect more efficiency in Poland than in Spain Japan, which is not represented in A533, has a mortality curve almost identical with that of Spain except that the maxima come a month or two later

535 In A533 the deathrates in Egypt and Ceylon are even higher than those of Poland and Spain In Egypt the excessively hot, dry summers and the accompanying malaria and other diseases cause that season to have far higher deathrates than the winters, which are comfortably cool In Ceylon the temperature is so nearly constant that it makes little difference in the deathrate. The two maxima of deaths come during the two dry seasons, when the climatic conditions are most favorable to mosquitoes, and deaths from malaria are extremely numerous It is interesting to find that although the Mexican plateau is quite cool, and India quite hot, a high deathrate in both places is lowered by the onset of the wet season with its humidity, cloudiness, and changes of weather

536 Many other statistics show that all over the world people's health, and presumably their efficiency and inclination to work, not only swing back and forth with the seasons, but also vary according to four other factors which influence the climate. One factor is distance from the sea, or more exactly the degree to which winds bring oceanic influences inland The second is latitude, for in general the latitudes between 35° and 60°, or even 40° and 55°, are the best The third is altitude, which may be favorable or unfavorable according to whether the climate at sealevel is too warm or too cool Cyclonic disturbances, such as our ordinary storms, are the fourth factor. Masses of cold polar and warm tropical air meet and produce such storms in certain definite zones The chief zone extends across southern Canada and the northern United States to western Europe Others lie along the Asiatic coast near Japan, and in the southern ocean near New Zealand and southern Australia.

537 MAPS OF CLIMATIC EFFICIENCY On the basis of climatic records all over the world it is possible to make maps showing the degree of efficiency to be expected if efficiency depended on climate alone, and if man's response to temperature, humidity, and storminess were everywhere essentially like that of the people discussed in the last chapter In making the maps of this kind which we shall soon examine, vigor of body and mind are assumed to be equally essential The ideal climate is assumed to be a little cooler than the one shown by our standard climograph based on 12 great cities, because we are here dealing purely with the direct human response to climate, regardless of agriculture The assumption is also made that people have a rather high



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A537—World Map of Climatic Efficiency.

capacity to protect themselves from the weather If the stage of civilization is such that this is not the case, as among the ancient Greeks and Egyptians, the climate of greatest efficiency will be located a little nearer the equator than in A537, which is a world map of climatic efficiency In using this map and others of a similar sort, bear in mind that they are purely climatic Each type of shading represents a definite combination of temperature, humidity, and storminess

538 Four areas in A537 are heavily shaded, indicating the kind of climate that most promotes health and efficiency, but only two are conspicuous One of the two centers around the North Sea in western Europe, the other lies about halfway from north to south in North America Another considerable area of especially favorable climate is

located on the Pacific Coast of North America, and a fourth, of very small size, in Tasmania and New Zealand. In all of these we should expect great activity and progress because the climate is especially favorable to both physical and mental well-being, and such is the case. Nevertheless, even in these areas the climate is not absolutely ideal. Where the temperature is close to the ideal, as on the Pacific Coast of the United States, the degree of variability is not quite great enough and the summers are too dry. Where cyclonic storms provide sufficient variability, as in the northeastern quarter of the United States, the extremes of both heat and cold are too great. The next type of shading, indicating good climates but not the best, surrounds the darkest shading in Europe and North America, and is found also in Japan, central Argentina, part of Chile, a southern section of Australia, and a tiny bit of the southern tip of Cape Colony in South Africa. Japan is too hot and humid in summer, but during the rest of the year the climate is admirable. This is the only part of Asia where cyclonic storms give rise to frequent changes of weather. Siberia has some such changes, but they are not frequent enough, and in winter they are far too severe. In Japan the changes help greatly in making the climate good for both man and agriculture. The parts of Australia, Argentina, Chile, and Cape Colony having the best temperatures and rainfall do not have enough variability because they lie in latitudes lower than those where cyclonic storms are frequent. Moreover, both Chile and Cape Colony suffer from dry, monotonous summers. Only in winter do they have even the edges of cyclonic disturbances, but both are benefited by nearness to great oceans from which come healthful winds.

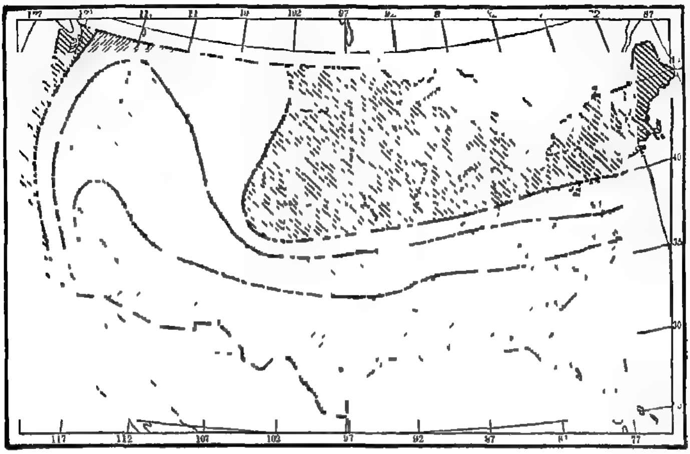
539 The parts of the world with unfavorable climates, that is, the parts where we should expect low energy, high deathrates, and low efficiency, are far larger than those with good climates. All regions that are steadily warm throughout the year, or that have long periods of extreme cold, stand low in A537. This is partly because cyclonic changes of weather are rare or unknown in such regions, especially in the warm parts. Even when the occasional severe cyclonic storms known as hurricanes come to warm regions, they do not bring much change of temperature. In the cold northern regions of Asia and North America, on the contrary, the few cyclonic storms often bring changes of temperature so severe that they do much harm. One of the surprising features of A537 is the way in which the climate deteriorates from the North Sea inland to Siberia. In North America a similar but milder deterioration toward the interior is seen, but the Great Lakes partly neutralize it over a large area, and cyclonic disturbances are much more frequent than in the interior of Asia. The extremely conti-

nental climate arising from the size of Eurasia leads to three handicaps, namely, (1) great seasonal extremes of temperature, (2) long, severe periods of dryness, and (3) scarcity of cyclonic changes of weather The high atmospheric pressure which prevails over Eurasia in winter almost prevents cyclonic disturbances from sweeping across the continent and thus bringing the benefits which they give to Nebraska or Alberta When storms do penetrate to the far interior, especially in the spring and fall, they are likely to be of the blizzard type with terrible winds and bitter cold In summer the conditions are much better Even then, however, the vast size of the continent prevents large parts from getting much rain

540 COMPARISON OF GEOGRAPHIC DISTRIBUTION OF CLIMATE AND HEALTH In both the United States and Europe the geographical distribution of health, as indicated by the deathrate, is almost the same as that of climatic efficiency. The health in the United States is shown in A540 where heavy shading indicates good health and relatively few deaths The map is based on insurance statistics, thus largely eliminating the effect of urbanization and industry Its general resemblance to the map of climatic efficiency is obvious Nevertheless, the influence of urbanization is evident locally It accounts for the fact that, although the states from New Hampshire and New Jersey to Illinois and Wisconsin are among the best parts of the world climatically, they have higher deathrates than the agricultural states farther west The resemblance of the maps of climatic efficiency and health in Europe (C and D540) is even closer than in the United States. Note the heavy shading around the North Sea, and even the small resemblances such as the bulges toward the Baltic and Black Seas and toward Italy

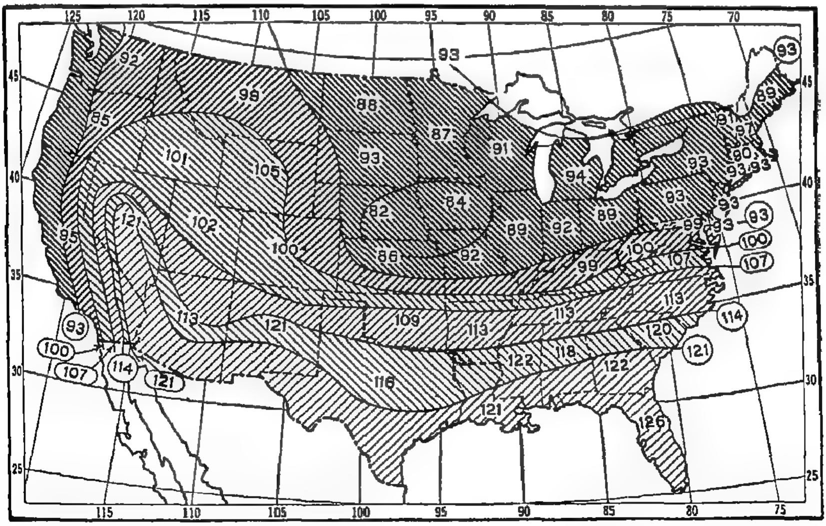
541 Outside of the United States and Europe the agreement between health and climate is also close For the world as a whole, however, the data for deaths are too scanty to permit an accurate map We know, however, that continental interiors tend to have higher deathrates than coastal regions with similar conditions of altitude, latitude, and economic development. We also know that lowland tropical regions and hot deserts generally have deathrates more than twice as high as those in the more-favored climates. In tropical highlands, however, and in places such as Hawaii, on the edge of the tropics, and fully under tradewind influence, the deathrate falls much lower than in the tropics as a whole On the whole the world map of climatic efficiency (A537) is also a fairly good map of health Each of its four darkest parts is a region where the deathrate is relatively low. Among the countries with climates having the second grade of shading in A537, Argentina, Uruguay, and especially Australia rank well in

health The absence of serious congestion, the comparative prosperity, and the relatively slight development of manufacturing all help in this



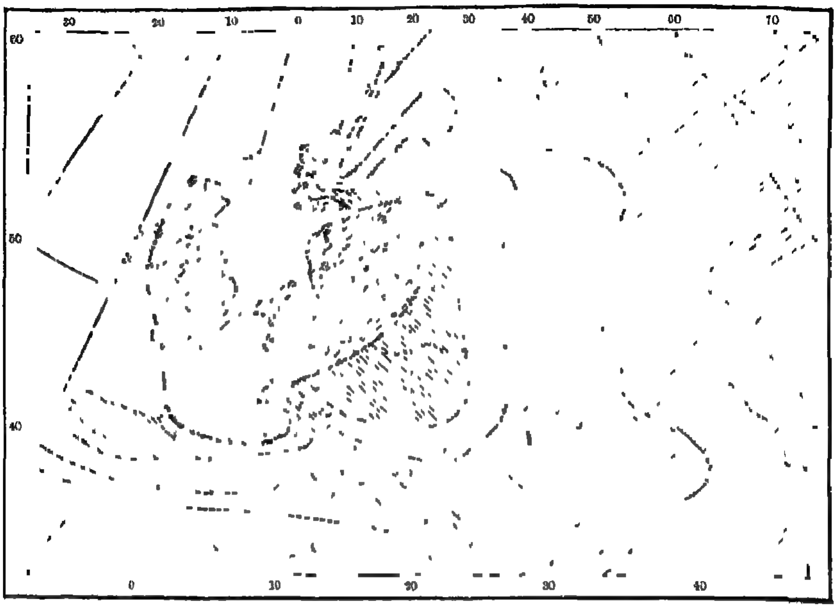
A540—Climatic Efficiency in the United States.

respect. Japan does not stand nearly so well, although it is ahead of its neighbors on the mainland and farther south. The great density

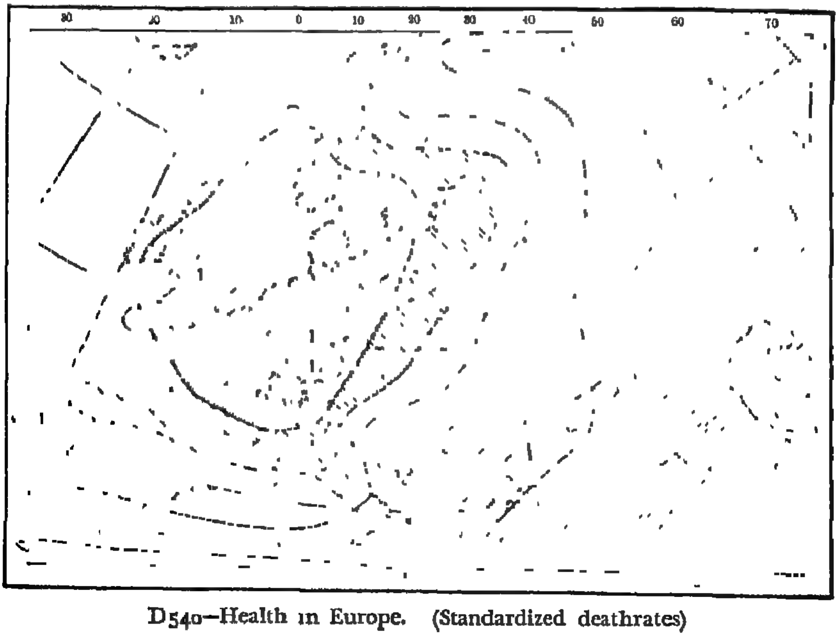


B540—Health in the United States per Life Insurance Statistics
(Percentages of average for whole country)

of population and the poverty of the masses of the people help to keep the deathrate comparatively high. This is presumably still more true



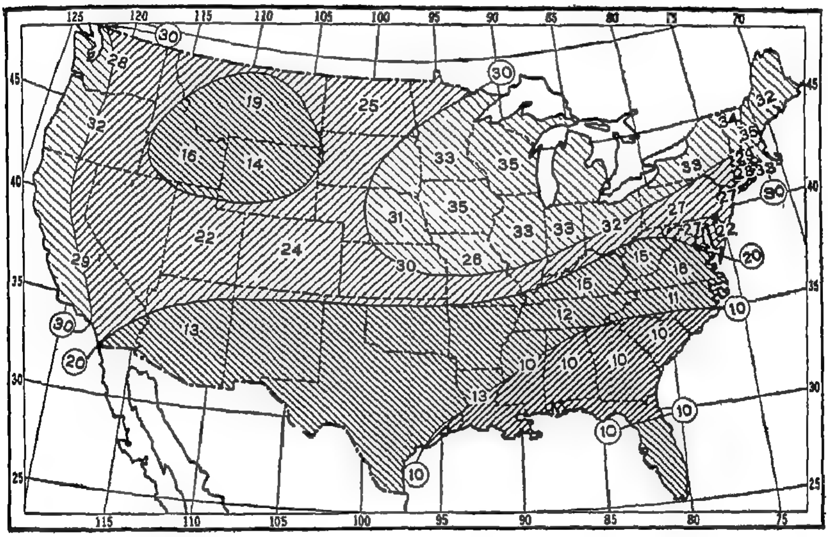
C540—Climatic Efficiency in Europe.



D540—Health in Europe. (Standardized deathrates)

of China, although statistics are scanty. At the other extreme, all the most lightly shaded areas of low climatic efficiency also have high death-rates unless some factor such as the recent immigration of especially vigorous people causes improvement, as in northern Australia

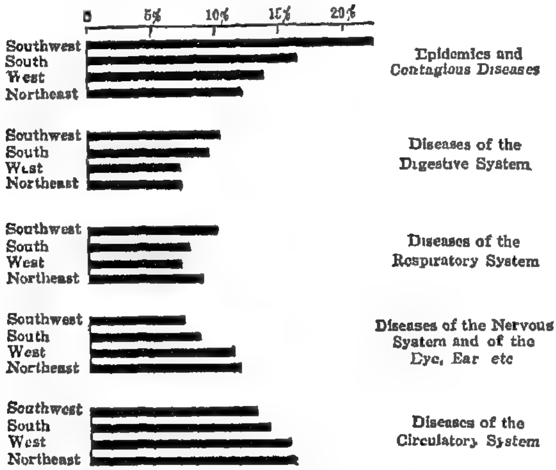
542 DISEASE AND HYGIENE IN RELATION TO CLIMATE. I. *Diseases of Especially Favored Climates* Leaving diet for discussion in a later chapter, let us consider the part played by disease in determining the efficiency with which people work The great majority of diseases display a strong geographic variation in frequency and virulence. Some belong primarily to cold seasons and cool climates, others to hot sea-



A542—Deaths from Metabolic Diseases in the United States

sons and warm climates. In general the most deadly diseases and the ones that most reduce people's ability to work belong especially to warm climates Nevertheless, C A. Mills has shown that deaths from degenerative diseases such as diabetes and many diseases of the heart are most numerous in highly stimulating climates which in general are healthful They have a geographical distribution the reverse of that of deaths from most diseases Therefore A542 has been shaded dark where the deathrate from these diseases is low in order to bring out the fact that the parts of the country which are most favorable for health in general (B540) are unfavorable in respect to these diseases The degenerative diseases arise largely from overexertion of the organs which they affect. A stimulating climate, especially one with many

storms and sudden cold spells, apparently causes such overexcitation. It also favors the development of manufacturing, city life, and an intensely active social system The strain and stress thus induced may be even more important than the direct effect of the climate in producing degenerative diseases Diseases of this kind are less common in the area of high climatic efficiency around the North Sea than in the northeastern and central United States This seems to be associated with the fact that changes of weather and the general intensity of life are not so severe there as in the United States and southern Canada The North Sea type of climate appears on the whole to be the best for prolonged, useful economic activity That of the northern United States favors activity of a more alert but perhaps less persistent type.



A543—Geographical Distribution of Types of Disease in the United States

543 II. *Diseases of Dry Climates* Each season and each main type of climate has its own characteristic diseases which diminish human efficiency. Diseases of the respiratory organs, aside from tuberculosis, are much more frequent in winter than in summer, and in dry months than in moist Diseases of the digestive organs show the opposite condition, being prevalent chiefly in summer The geographical distribution of these diseases accords with the seasonal distribution In the dry, warm Southwest, even among people who have lived there all their lives, epidemic and contagious diseases, including tuberculosis, account for a larger percentage of the deaths than in the cooler Pacific Coast and Northeast (A543). Diseases of the digestive organs show a similarly unequal distribution, but differences between regions are less extreme. Respiratory diseases of the nose, throat, and lungs are like-

wise most common in the southwest, but the far northern states suffer almost as much On the other hand, diseases of the nervous and circulatory systems vary in the opposite way, their percentages being lowest in the less stimulating dry or warm climates, and highest in the stimulating climates where the general deathrate is lowest

544 Insurance statistics agree with A543 in showing that, in spite of the common opinion to the contrary, respiratory diseases are especially characteristic of dry climates The outdoor life there is undoubtedly a great blessing, but the harm done by dust is probably far greater than is commonly realized This is illustrated by Table 17, showing the relative number of deaths from tuberculosis per 100,000 persons among special classes of adult Englishmen, some of whom work in dusty occupations The dust which quarrymen breathe seems to

TABLE 17

DEATHS FROM TUBERCULOSIS AMONG ADULT ENGLISHMEN

Farmers	57	Limestone workers	129
All Englishmen	100	Slate quarry workers	220
Granite workers	127	Sandstone workers	415

explain why they suffer badly from diseases of the lungs This helps to explain why dry regions and deserts have especially high deathrates and tend to be backward in civilization Madrid in Spain, Johannesburg in South Africa, Allahabad in India, and Mexico City are cities with dry climates and peculiarly high deathrates In Chile, Spain, and Palestine the deathrates are exceptionally high in proportion to the average temperature All these have long dry seasons during which the air is repeatedly filled with dust In places like Cairo the dust consists largely of particles of quartz from the desert This is the kind which gives the workers in sandstone quarries an especially high deathrate

545 III *Tropical Diseases and Inefficiency* Diseases appear to reduce man's ability and inclination to work more strongly in tropical countries than elsewhere There the deathrates from epidemic and contagious diseases and from those of the digestive organs often rise to high levels Such countries suffer terribly from malaria, the more malignant types of dysentery, filariasis, hookworm disease, yaws, and various other diseases which are almost unknown in better climates The harm done to human health and efficiency by these diseases and by sores due to insect bites and minor infections is one of the major human problems Malaria is especially potent in reducing man's economic value, and also his comfort and happiness It prevails only

where climate and relief favor mosquitoes of the genus *Anopheles* and certain parasites which they carry. The most dangerous mosquitoes and bacteria are largely confined to low latitudes. Milder forms live in Mediterranean and other subtropical countries, but a few occur farther north. In tropical climates, vast numbers of anopheles mosquitoes breed in stagnant pools and in the damp hollows among the trees or grass. In India, near the end of the rainy season, a great wave of malaria causes the deathrate to rise sharply in November and December when cooler weather would normally cause a decline. It renders millions of people unfit for work for weeks.

546. CLIMATE AND TROPICAL INERTIA. Tropical inefficiency is one of the world's most troublesome economic problems. No one questions its connection with climate, but there is considerable disagreement as to how far the connection depends upon climate directly and how far indirectly through race, diet, parasitic diseases, hygiene, sanitation, and social and political customs. So far as tropical inefficiency is a direct effect of climate, the main remedies may be air-conditioned houses and frequent trips to the seacoast or mountains. So far as the indirect effects of climate are concerned, the best line of attack presumably lies in improving people's diet, eradicating specific diseases, and introducing good sanitation and hygiene. It is hard to say whether direct or indirect effects are more important. The situation is like that of crops. The yield of a crop is certainly influenced greatly by the weather, but it is also influenced by soil, insects, parasitic diseases, and the skill and thoroughness with which the soil is cultivated.

547. Numerous experiments indicate that among healthy people living permanently in the tropics the composition of the blood and the general functioning of the physiological system differ slightly, but systematically, from those of people in cool climates. In general the process of metabolism, whereby food is changed into heat and movement, is less vigorous in the tropics than in more stimulating climates, and the general level of activity is relatively low. This apparently means increased susceptibility to many diseases, at least among white people, while the ability and especially the inclination to work diminish.

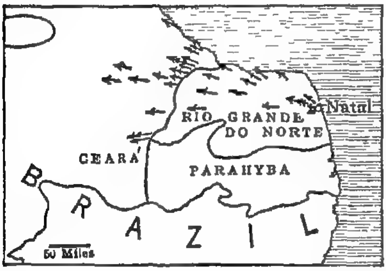
548. Tropical natives are probably less susceptible to tropical disease than white men, but they seem to become tired more quickly. In fact they appear actually to be "born tired." Indirect evidence of this is found in the fact that in the United States, for example, people's length of life varies according to the season at which they are born. Direct evidence is found in the fact that white people born in tropical and semi-tropical Queensland do not live so long as Australians born

in other provinces or in Great Britain. This is true even among Queenslanders who migrate to other parts of Australia and spend much of their lives there. Thus we conclude that the inefficiency of labor, which does so much to keep the tropics backward and unproductive even in regions such as Java, is partly an inborn characteristic due to the climate. Of course, it is also due to the effect of climate later in life and to the diet, diseases, and social customs to which people are subjected throughout life. White people in the tropics can protect themselves against deficiencies of diet and against disease. They cannot, however, escape so easily from the direct effects of the climate, especially if they are born in the tropics. Only in rare cases, such as northern Australia, can they escape from the effect of living among people whose social system is fitted to tropical inertia, rather than to the advanced ideas of more stimulating climates. The importance of all these factors has been set well forth by an Australian, A. Grenfell Price, in *White Settlers in the Tropics*, a book which might well be read as a supplement to this chapter. Much of the last two paragraphs applies with diminished force to the less favorable non-tropical climates such as are found in China, Central Asia, and the far north.

549. Economic Value of Eliminating Disease. The economic harm done by disease becomes especially clear when we examine the results of its elimination. For example, at Baoe-Baoe in the island of Celebes practically every European used to be frequently incapacitated by malaria. In the Dutch military encampment half of the force was often on the sick list. Out of an average of 350 people in the civil infirmary and the prison, 20 died each month. In 1922 the draining of swamps and the building of sewers, gutters, and privies promptly reduced malaria to one fourth its former amount. Previously the constant repetition of malarial infections, especially in childhood, had given all the native children enlarged spleens, and had made practically all the adults anemic and inefficient. Afterward, only 13 per cent of the children had enlarged spleens, and the older people worked with a cheerfulness and vigor previously unknown. Sibolga in Sumatra was made practically free from malaria by keeping cattle between the mosquito-infested area and the city, the cattle attracting the mosquitoes. Panama is a widely known example of tropical sanitary improvement. Sanitary work there has reduced the deathrate by one half in the cities of Panama and Colón. It has made it possible for white people to live in the carefully protected Canal Zone with little fear of the ordinary tropical diseases. But the effect of such work must not be exaggerated. The deathrate in the two cities

at the ends of the Panama Canal is still twice as high as that of the North Sea countries and the United States Americans still find the climate there both enervating and trying Every year large numbers of them are still invalided home

550. Nontropical countries with long dry seasons and much warm weather also suffer from malaria This is true no matter whether the rain comes in winter, as in Spain, Italy, Greece, and Turkey, or in summer, as in China, southern Brazil, and northern Argentina In vast areas from a quarter to a half, or even three fourths, of the inhabitants are infected with the disease even now, or at least have had it during childhood Malaria saps people's energy for a long time It keeps recurring at intervals of months or years, even without a new infection It has a special tendency to cause people to be mentally as well as physically inert In tropical countries it often takes highly virulent forms and is fatal in a short time In the present century great progress has been made in eliminating malaria from the chief tropical seaports and in the areas where Europeans and Americans carry on plantations Nevertheless, both tropical and oriental countries still suffer terribly.



Courtesy of the Rockefeller Foundation

A551—Invasion of Brazil by the Malarial *Anopheles gambiae*

551 The problem of diseases carried by insects is greatly complicated by modern transportation. For example, *Anopheles gambiae* is considered the most dangerous of malarial mosquitoes Its principal home is tropical Africa, where it carries a serious and often fatal type of malaria Until 1930 this mosquito was unknown west of the Atlantic At that time, however, it was carried to the city of Natal in Brazil, apparently by French airplanes or by fast destroyers which were working in connection with the establishment of a French airway between West Africa and Brazil Helped by the prevailing tradewinds the mosquito had spread inland by 1939 to a distance of 300 miles or more (A551) The tradewinds have carried the mosquitoes to the west and northwest, but not to the south As a result severe epidemics of the kind of malaria carried by the *gambiae* mosquito have broken out In one region 200 miles from Natal there were more than 50,000 cases of malaria in 1938 More than 90 per cent of the population were affected, and in certain districts 10 per cent died. So terrible was the epidemic that in some regions crops were not

planted and the production of salt, the chief exported product, was greatly reduced because of lack of labor Some entire villages were abandoned. In 1939, because of this mosquito, nearly every person in the affected areas was on government relief.

552. Another disease of similar baneful character is caused by the hookworm This also is confined largely to fairly warm countries The worm lives in the mud, and enters the foot through the pores in the bare skin The extent to which such diseases reduce human productivity is astonishing In Egypt, more than half the laboring population is infected with hookworm, in the Malay States 60 per cent, in British Honduras 70 per cent, in Sumatra and Java 90 per cent in some regions, and so on for almost all tropical countries The Rockefeller Foundation estimates that well over half of the world's two billion people live in areas where hookworm disease is prevalent Half the people within the tropics probably suffer from the disease at all times, and many others have it occasionally It stunts the growth of children, retards their mental development, and makes adults anemic and incompetent The improvement that follows its eradication is spectacular In Costa Rica, 66 laborers before being treated for hookworm normally cultivated 563 acres of coffee monthly After treatment they cultivated 750 acres without any greater fatigue In India the amount of work increased 20 per cent on one estate and 50 per cent on another, and on both was of better quality than before the laborers were treated Reports from British Guiana indicate that the efficiency of the laborers employed by one company increased from 25 to 50 per cent after measures to eradicate the hookworm were put into operation

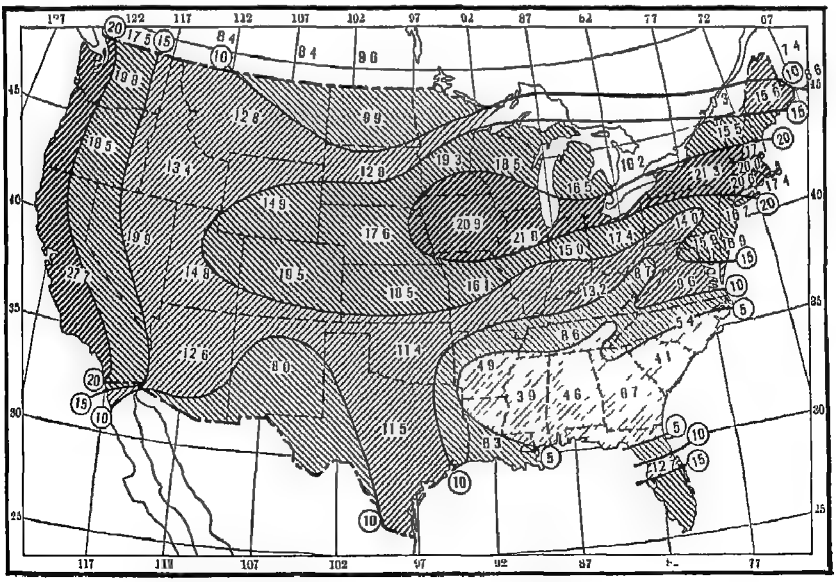
553 Reliance of Tropics on Cooler Climates Such mitigation of tropical diseases occurs only when energetic people from the cyclonic types of climate (Plate I) go to the poorer climates Efforts to overcome disease relax as soon as the influence of these people is removed. Taking the tropical and other unfavorable climates as a whole, the improvement in health during the last two generations has been far less than in the cyclonic climates There is reason to think that not for many generations, if ever, will the diseases of tropical regions be eliminated as fully as those of the better parts of the world It is harder and more expensive to combat disease in poor climates than in good, and the vigor for doing this is less Thus disease joins with diet and social conditions in emphasizing the contrasts in human efficiency which arise as the direct or indirect effect of climate All sorts of efforts at improvement suffer from tropical inertia just as do efforts at better hygiene.

554 TESTS OF THE BASIC GEOGRAPHIC PATTERN. The degree to which the basic pattern of economic geography is set by climatic efficiency, or by other factors with a similar geographic distribution, can be tested by maps of the United States, Europe, and the world Those of Europe show climatic effects with especial clearness because they are based on a comparatively stable, long-settled, and nonmigratory population which has had time to become well adjusted to the local geography In the United States the advantage of uniform statistics covering a large and varied area is partly offset by the newness of the country and the migratory character of the people which prevents the geographic factors from being so dominant as in Europe World maps are at a disadvantage because statistics are scarce, unreliable, or absent in many regions, and the methods of collecting them vary greatly

555 *Records of Disease as a Measure of Mental Alertness* Among the many possible tests of general progress and of the kind of alertness that makes business hum is the care with which causes of death are recorded Although many factors cooperate in this matter, a map (A555) showing the degree of care is closely similar to one of climatic efficiency (A540) or of telephones (B555) The resemblance between A and B555 is remarkable, but neither map represents the cause of the peculiar features of the other The maps are alike because both are influenced by the same basic climatic causes In the most advanced parts of the United States practically all the death certificates sent in by physicians state the cause of death In backward states many certificates give no cause of death, or only an unsatisfactory one The reasons for this include poor education, scarcity of physicians, poverty such that people die without the attendance of physicians, lack of hospitals, and so on A single efficient official may raise the level in his state quite markedly. None of these other factors, however, can influence the climatic map, for that is based on exact weather records On the other hand, climate and weather constantly influence people's feelings, thus altering their ability and inclination to work The degree of efficiency thus arising influences the amount and kind of education, the degree of prosperity, and the way in which medical work is carried on Countries with meager natural resources may stand high in efficiency and general prosperity, as do Norway, Denmark, Switzerland, and especially Iceland, but no country with an unstimulating climate does so

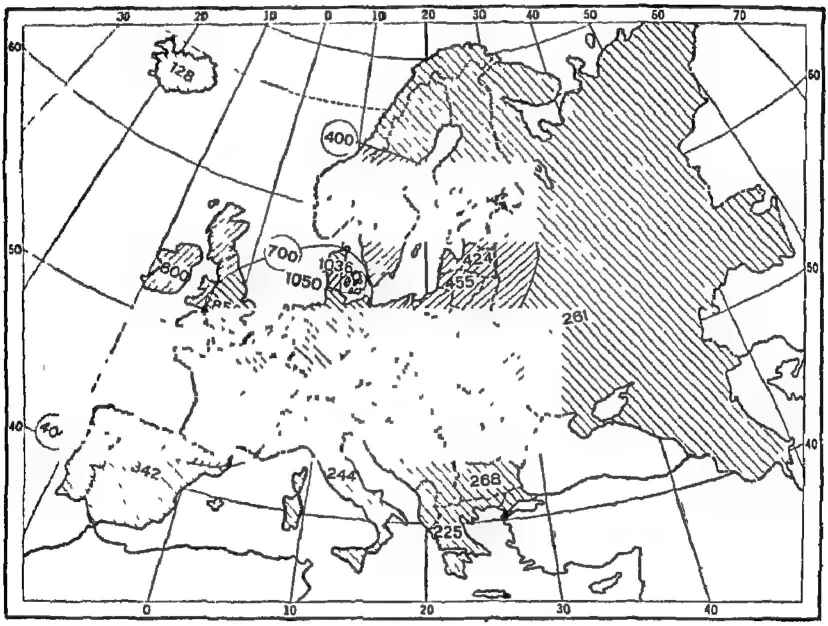
556 *Agricultural Income* The average income among the men at work on the farms is a good measure of agricultural prosperity In

A555—Percentage of Deaths from Ill defined Causes in the United States



B555—Telephones per 100 Persons, 1939

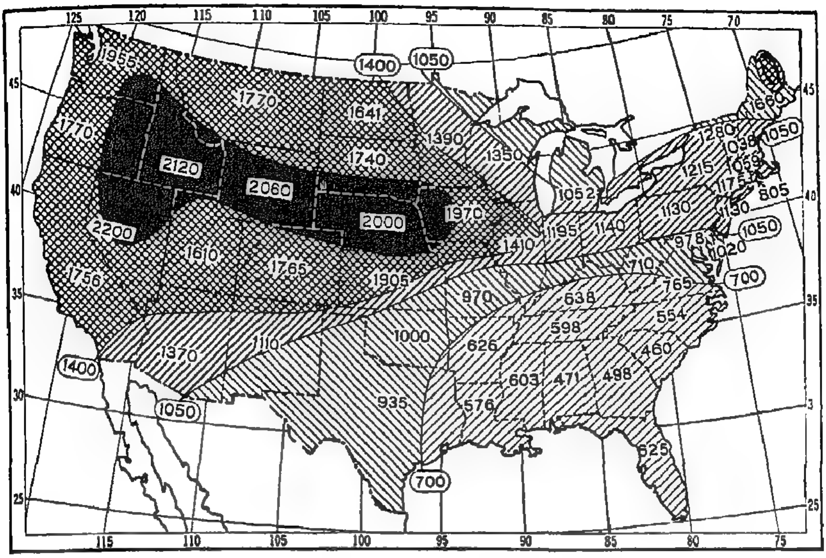
Europe (A556), the geographical distribution of income agrees closely with that of climatic efficiency (C540) The income per man is high around the North Sea and declines outward in all directions Here, too, other factors as well as the direct effect of climate on man must be considered The climate which is best for human efficiency both mentally and physically is moderately good for the soil, and departs only a little from the type that has become best for agriculture Good crops help people to have a good diet They also, as we



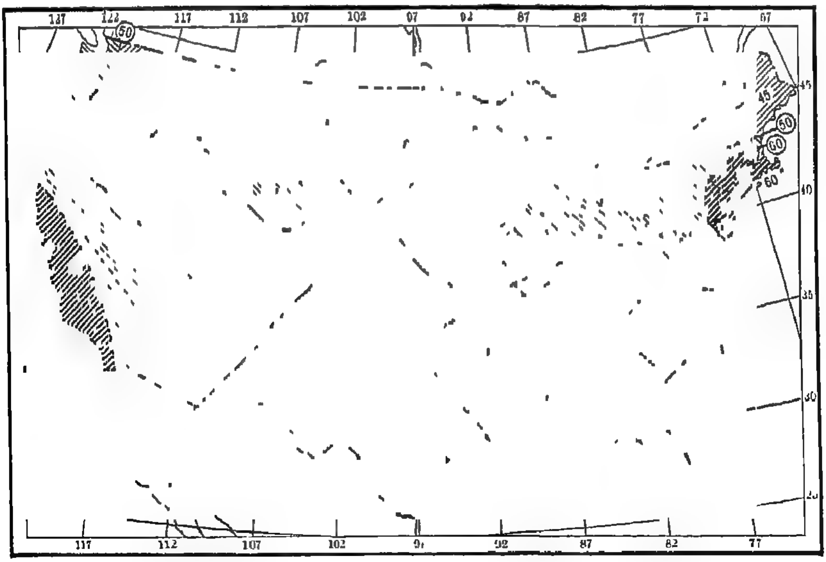
A556—Estimated Annual Farm Income per Man on the Farm in Europe Expressed in Dollars, 1929

have seen, promote good methods of cultivation, which in turn increase productivity still more. Another factor is that more land is cultivated per man around the North Sea than in countries such as Spain, Greece, or Russia This is partly because the people are more efficient, but also because the long dry summers of Spain and Greece and the very cold winters of Russia shorten the season during which crops can be planted, and set limits to the amount of land one man can handle Thus many factors cooperate in producing A556, but the final pattern is strikingly like that of climatic efficiency.

557 In the United States farm income (A557) does not follow the climatic pattern so closely as does the average income from all



A557—Annual Farm Income per Man on the Farm in the United States Expressed in Dollars, 1929



B557—Income per Capita, 1929-35

(Figures represent units of $10.00 National average in 1929 was $652 and in 1933, $353)

sources (B557) The strongest departure of agricultural income from the climatic pattern is seen in the heavy shading from Iowa westward Climatically that part of A557 is not quite so favorable to both man and crops as the more lightly shaded area directly east of it Its farmers get a large income because population is sparse, the farms are large, and vast areas of level plains and rich soil join with a dry climate to encourage the use of efficient machines so that one man's work cultivates a large area Irrigation in the western states also raises the income per man Putting together all kinds of income, and including years of the highest and lowest prosperity, we find that the resultant map (B557) closely resembles the map of climatic efficiency. Southern New England, New York, New Jersey, and Delaware on the Atlantic Coast, and California on the Pacific, have an average income above $600 per person Wyoming and Nevada stand high because of mining industries, as do the states from Pennsylvania to Illinois because of manufacturing. The low income of the Dakotas and the South is noticeable

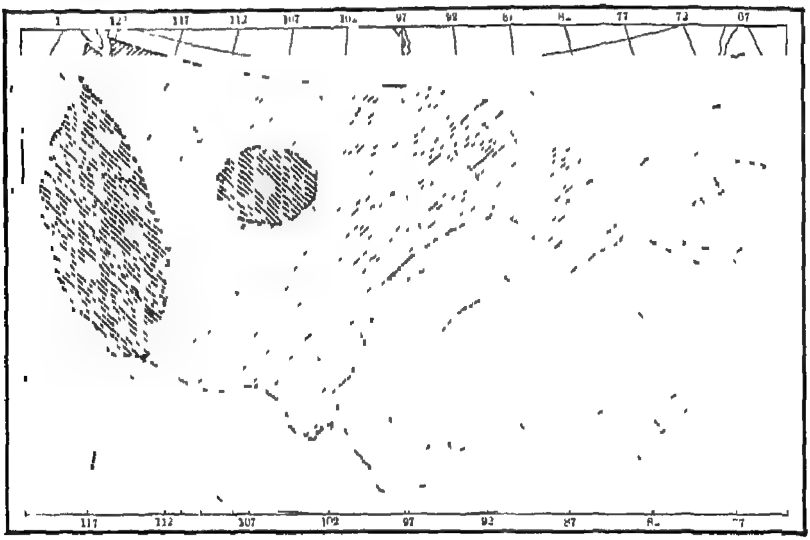
558 *Automobiles, Telephones, and General Prosperity* The number of motor vehicles in a country gives a good idea of the economic status (A560) Telephones give a still better idea Their number in comparison with the population does not stand low in cities in such a way as does the number of automobiles In the United States there are from 10 to 40 motor vehicles for every 100 persons, and the whole population could ride at once In Australia and Canada, if each car, including buses and trucks, carried 4 to 6 people, it would take 2 trips per car to carry everybody, in Cuba, taking 5 people per car, the number of trips would be about 20, in Russia 50, in Japan 100, India 400, and China 2,000. With telephones there is a similar difference Hundreds of millions of people in China, India, Java, and other countries have never used a telephone, whereas in the United States and Canada the *average* person, counting all ages in all parts of the country, holds about 250 telephone conversations each year and many people hold thousands Such conditions agree with what would be expected from the map of climatic efficiency (A537) with its heavy shading in the United States, southern Canada, western Europe, Australia, New Zealand, and the part of South America in the latitude of Buenos Aires and Santiago A telephone map also agrees with the climatic map in having low areas in tropical regions, deserts, and the interiors of all continents except North America

559 On the other hand, automobile and telephone maps disagree with the climatic map in important respects For example (1) the more backward parts of the world show a contrast between the coast

and the interior which is not evident in the climatic map. One reason is that automobiles are brought to the other continents by sea from the United States and Europe Another is that centers of commerce tend to be located on seacoasts, and those are the places where both telephones and automobiles are most used Except in North America and Europe roads fit for automobiles are largely confined to regions within a few hundred miles of the coast (2) Another notable fact is that although Japan has relatively more motor cars and telephones than China, it has far fewer than one would expect from the map of climate Japan is so densely populated that there are two people for every acre of cultivated land The farms are so tiny that the great mass of the people cannot accumulate the large surplus necessary for automobiles or even telephones (3) China has relatively fewer cars and telephones than India, although the climate, and the character and industry of the people, would lead one to expect more. The presence of the British in India accounts for this, they own most of the cars and do much of the telephoning (4) The United States and southern Canada have far more motor cars per capita than western Europe, even in sections where the climate is by no means so good as that around the North Sea The difference in telephones, however, is not so great Several conditions cooperate in producing this result Large farms, abundant natural resources, and the opportunities of a new country not too densely populated have helped make North America so prosperous that a majority of the people can afford cars as well as telephones Moreover, great numbers of people can afford to turn in their old cars for new ones before the old ones are worn out. This provides a great supply of cheap but still efficient cars for poorer people. In foreign lands there are few of these cheap cars, for the original purchasers tend to keep their cars till they are worn out Another important point is that in the United States abundant supplies of petroleum make it relatively inexpensive to run the cars. These conditions and others have stimulated mass production and thus brought down the price of cars The situation is like that of wheat and corn Where the natural conditions are especially favorable, people are most stimulated to develop new methods In a poor country, such as China, with a small market for cars, and with few roads and expensive gasoline, the mass production of cars is not feasible

560 In the United States the number of automobiles in proportion to the population (A560) shows an interesting combination of the general climatic pattern with features due to cities, mountains, race, and migration In countries of this sort, where even the poor

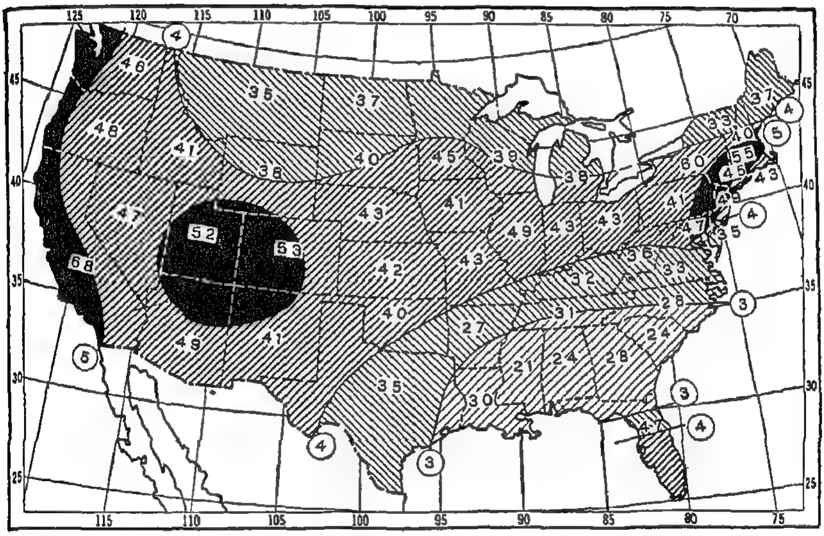
can have automobiles, motor vehicles are decidedly more numerous in rural districts than in cities. New York City, for example, has only 11 cars for every 100 people, but the rest of New York State has 27. The state as a whole is so citified that it has only 20 motor vehicles per 100 people, whereas Kansas, being relatively a rural state, has 31. Many well-to-do city people find that it costs less to hire taxis than to keep cars of their own. Many poor city people who would need cars and be able to keep them in the country do not need them and cannot pay for garage space in the city. In A560 the large cities from Boston to Baltimore and Pittsburgh reduce the number of automobiles, so



A560—Motor Vehicles per One Hundred People in the United States, 1939

that the number of cars per 100 persons in Massachusetts, New York, and Pennsylvania is only 19 or 20—less than in northern New England or the Middle West. In Illinois the presence of Chicago reduces the number of cars to 23 per 100 people, in comparison with 28 in Indiana and 29 in Iowa. The Pacific Coast, however, where distances are great, space is abundant, and the general degree of prosperity is high, has 32 to 43. In poor countries such as Spain, on the contrary, where only the well-to-do can afford automobiles, practically all the motor vehicles are owned in the cities. The effect of mountains with their poverty and poor roads is seen in the northward bulge of the lightly shaded area of few cars in the Appalachians and in a band of relatively fewer cars along the Rockies. Presumably the effect of the Appa-

lachians would be clearer were it not for a racial factor The presence of colored people reduces the number of automobiles in the South. Recent migration, tourists, and winter residents and the prosperous winter vegetable and fruit industries give Florida a remarkably large number of cars in proportion to its population, especially in the far south Level plains, good soil, vigorous people, and the wide prevalence of agriculture raise the number of automobiles in the prairies and Great Plains California leads the world in the number of its cars both because its climate is healthful and pleasant and because of its natural wealth, intensive agriculture, and the kind of people who have migrated there.

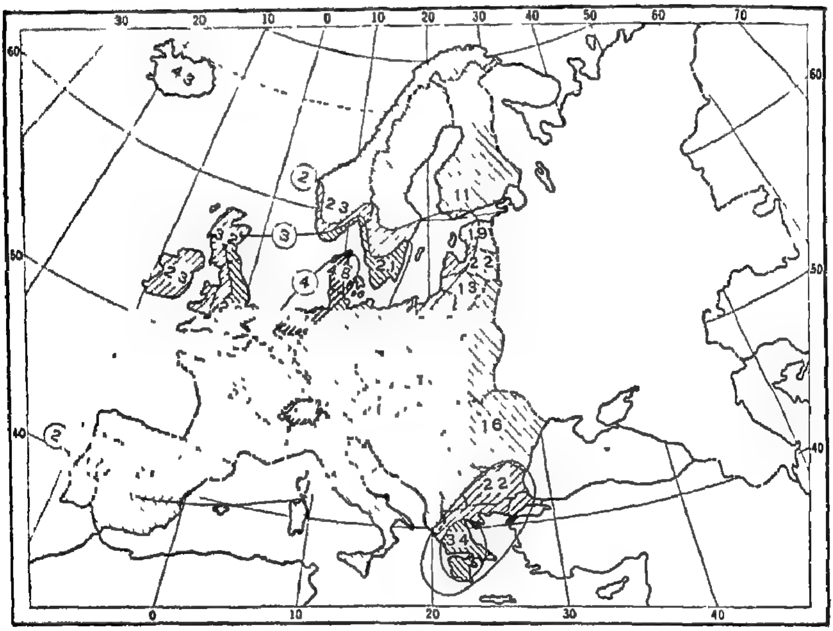


A562—Percentage of Men Occupied in the Liberal Professions in the United States

561 The automobile maps of the world and the United States are significant in that they illustrate the way in which various factors combine to determine geographic distribution The climatic factor determines the main outlines. These are modified, however, in many ways, and are sometimes completely obliterated, as in India's superiority over China The more carefully such maps are studied the more clear they make it that, although the pattern of economic activity all over the world is primarily based on climate, its final form is due to the combined influence of many factors

562 *Distribution of Professional People and General Culture.* Another evidence of the economic activity and general progress of a country is the percentage of professional people. A poor country

cannot support many doctors, lawyers, engineers, clergymen, teachers, artists, musicians and so forth A562 indicates that the distribution of such people in the United States follows the climatic pattern in a general way, but is considerably modified by other factors. Recent migration, for example, has raised the percentage of professional people in southern Florida and the Southwest The high percentage in California is largely due to the attraction exerted by the pleasant climate on persons seeking health, recreation, and a home in old age California's prosperity enables an unusual number of such people to



A563—Percentage of Men Occupied in the Liberal Professions in Europe

find work In Colorado the mining industry and seekers after health swell the number of professional people In Utah, Mormonism may have something to do with the large number The presence of Negroes depresses the proportion of professional people in the Southeast Great cities, on the other hand, tend to raise it in the northeast and California The high figure for New York State, 6 0 per cent, is largely due to New York City

563 In Europe the map of professional people (A563) conforms closely to the basic climatic pattern, with a high center around the North Sea The Greeks and Bulgarians, however, provide exceptions,

for they often report themselves as lawyers, physicians, and teachers even when they do not practice these professions. Switzerland stands higher than its neighbors for quite a different reason, namely, its general attractiveness as a place of residence for liberty-loving people, and its position as headquarters of the League of Nations. Russia is omitted in A563 because its statistics are so unlike those of other countries. The resemblance of the geographic pattern of professional activity to that of climate in Europe is especially interesting because each country there has settled down to an economic condition more fully in harmony with its environment than has yet been possible in the United States. Many other maps, not published in this book, show the same climatic pattern in economic conditions.

564 The General Economic Pattern. In explaining the widespread similarity between economic and climatic maps, the starting point must be the direct influence of climate upon man's physical and mental activity. This is supplemented by the indirect influence of climate upon health through diet and disease. In the especially favored parts of the world the vigor which thus arises enables the farmer to cultivate a fairly large area. The climate also favors agriculture by insuring a reasonably steady return for honest labor. This encourages the farmer to improve his crops and animals. Among people in general all these conditions encourage inventiveness and initiative both by making people alert and active and by providing conditions under which inventions and other new ideas prove profitable. All this gives a great stimulus to industry and helps it to make progress. Cheap and inefficient labor sometimes, to be sure, causes industries to be most profitable in relatively poor climates, but industries requiring much skill have special need of the kind of alertness and activity found in the more stimulating climates and among well-fed, healthy people.

565 *Effect of State of Civilization on the Basic Geographic Pattern.* The geographic pattern of economic activity, it should be most carefully noted, varies according to the stage of civilization. The climatic conditions which are best for the health and activity of naked savages are by no means the best for people like ourselves who have gone far in mastering nature. Even for the ancient Greeks and Egyptians the best climate was not what is now best for us. Although highly civilized, those early people had no stoves, furnaces, window glass, or even fireplaces and chimneys. They lacked the facilities which enable us to have warm, light rooms at all seasons. Nor could they make cheap, warm clothing in any such easy way as we can. Hence the best climate for them was warmer and drier than for us, and the broad geographic

pattern of economic activity was different from what it is today. In the past this pattern has also been different because climate is subject to pulsations which sometimes last many centuries For example, at the time of William the Conqueror and for some centuries thereafter, the summers in England were warm enough so that wine-making was a well-recognized occupation During the nineteenth century no such thing was possible because the summers were so cool that grapes seldom ripened In the same way ancient Egypt and Greece appear to have reached their highest levels of economic activity during centuries when the rainfall was relatively abundant, and the climate more variable than now and more like the type that is most favorable to people with our kind of civilization. Thus the main conclusion of this chapter is that *economic activity tends to be greatest in the parts of the world where the climate most nearly approaches the optimum for the stage of civilization with which we happen to be dealing*

566 In spite of its importance climate is only one of many factors that determine the geographic distribution of economic activities. In any given climatic region the smaller details of distribution are governed by such factors as soil, metals, fuels, waterpower, good harbors, easily traversed valleys, broad plains, high mountains, and purely human actions such as the establishment of an industry in one valley rather than another Moreover, migrations introduce many features which are not what would be expected from climate alone Nevertheless, even with the world's present highly varied types of civilization, the general distribution of human alertness, vigor, and economic activity conforms rather closely to the climatic pattern The final pattern, that is, *the earth's regional geography, is the result of the interplay between the basic climatic pattern and all the patterns set by other factors.*

PART VIII

FOOD AND DIET

CHAPTER XXIV

ECONOMIC GEOGRAPHY OF FOOD THE CEREALS

567 ECONOMIC IMPORTANCE OF FOOD The products with which we deal in economic geography may be classified as foods, fuels, raw materials for manufacture, and manufactured goods other than foodstuffs In the world as a whole the foods are by far the most important Their importance may be judged in various ways For example, the United States Bureau of Home Economics finds that families with living expenses—not incomes—of about $5,000 devote approximately 20 per cent of their expenditures to food, but scarcely 5 per cent of American families live as well as this With lower expenses the percentage allotted to food increases rapidly. Among families whose living expenses are less than $1,000 per year, which means about half the families in the country, approximately 40 per cent of the expenditure goes for food, provided that homegrown products as well as purchases are included In many poorer families the percentage rises to 50, and in the South, among tenants and sharecroppers of both the white and colored races, the value of food often rises well toward 70 per cent of the family expenses Even this, however, is not the limit. In Japan, China, and especially India, as well as in many other countries of Asia and most countries of South America and Africa, food probably represents about three fourths of the value of the entire cost of living.

568 Another evidence of the importance of food is the fact that even in a great agricultural country such as the United States food often accounts for more than 30 per cent of the value of all imports. In Great Britain this percentage rises above 40 Again, in 1933, when financial depression was at its height, the sales in food stores and restaurants in the United States amounted to 33 per cent of all the retail sales, whereas in 1929, a year of great prosperity, the percentage was 26. This illustrates the fact that, the more prosperous a country is,

the larger is the percentage of its income available for other things in addition to food. The importance of food as an element in economic life becomes even more evident when we consider the work of cooking. Taking the world as a whole, women probably devote more time to cooking food than men do to raising it The women have to cook at all seasons, whereas the men do little work in winter or in the dry season In relatively unprogressive countries, such as Iran and Bolivia, it is doubtful whether all other occupations together take one quarter of the working time Moreover, most of the other work is done in order to obtain money wherewith to buy food

569 REQUISITES OF A GOOD DIET Most of the world's people live on a diet which departs widely from the ideal Their health and vigor suffer accordingly Certain physiological facts about food have much to do with the kinds of crops raised in various parts of the world A good diet contains the following ingredients (1) carbohydrates, (2) fats, (3) proteins, (4) vitamins, (5) acids, and (6) minerals such as calcium, phosphorus, potassium, sodium, chlorine, and boron Carbohydrates composed of carbon, hydrogen, and oxygen are represented mainly by starch and sugar Fats contain the same chemical elements, but in a different form They are found both in meat and in vegetable products such as olive oil and the yellowish little embryos which form a small fraction of each grain of corn, oats, or other cereal. The main use of both carbohydrates and fats is to supply the body with fuel. The carbon is ultimately "burned" in such a way that it supplies the body with heat and with the energy which enables us to move and work.

570. Protein is a substance which contains not only carbon, hydrogen, and oxygen, but also nitrogen It is found in almost pure form in lean fish and, with more or less fat, in lean meat and cheese It is also abundant, although in smaller proportions, in vegetable foods like beans and peas Protein is essential to provide the body with new tissues during the period of growth, and to replace exhausted tissues in adults It may also be consumed to furnish heat and energy, or may be converted into fats, if it is more abundant than is necessary for growth and replacement. In proportion to its weight, protein furnishes the same amount of energy as carbohydrates, but only about 45 per cent as much as fats If derived from animals it can be almost completely digested and used, as can both fats and carbohydrates from a similar source If derived from vegetable sources, it is much harder to assimilate, and not much more than 75 to 85 per cent can be utilized An ordinary man engaged in moderately active work requires about 4 to 4½ ounces of protein per day and about 18 ounces

of carbohydrates, or an equivalent in fats. Men who are working hard require more, and women, children, and inactive men less In general, however, the diet of a region is poor unless it contains not only sufficient carbohydrates or fats, but also about 30 per cent of protein so that there may be some to spare if it is hard to assimilate

571 The other three ingredients of a good diet are needed only in small amounts, but are essential to good health Without vitamins neither man nor beast can thrive, the young do not grow properly, and many kinds of disorders appear Vitamins are found in small amounts in practically all foods except sugar, which is a pure hydrocarbon Only in certain kinds, however, such as green vegetables, fruits, milk, and fresh meat, are they abundant. Acids, too, are needed Curiously enough they counteract the harmful acidity which appears in the digestive tract when there is too much starch or sugar. They are found chiefly in fresh fruits

572 Modern science has also shown that many minerals, formerly deemed unimportant, are essential to the best health Milk products, some kinds of meat, and legumes such as peas and beans contain comparatively large amounts of mineral matter in proportion to their dry weight All kinds of food, however, vary greatly in this respect according to the quality of the soil People's health often suffers because they eat bread and other foods raised in poor soils, or because they use milk and meat derived from cattle which have fed on grasses growing in such soils In general the soils of both high and low latitudes tend to be deficient in the more essential minerals Finally, a good diet should be neither too concentrated nor too bulky The stomach functions best when it has a fairly generous but not excessive bulk of material to work on Foods such as bread or beans are so concentrated that they need to be supplemented by others which have more bulk Primitive people such as the Eskimos and South Sea Islanders suffer severely from constipation and other forms of indigestion when they give up their old diet and eat a concentrated diet of imported white flour. In China and India an unduly concentrated diet leads to a great deal of constipation, with consequent headaches and sluggishness On the other hand, an unduly bulky diet, such as the bananas or plantains of many tropical regions, is equally unfavorable Our problem in economic geography is to find out to what extent and in what ways various types of regions get a diet which meets all these requirements.

573 COMPOSITION OF TYPICAL FOODS I *The Protein Type* Before we turn to the geographical distribution of food products, it will be helpful to gain some idea of the composition of various types The foods mentioned in Table 18 are arranged in 10 groups These are

TABLE 18

Composition of Foods

Solid Material (Dry Weight)

	A Protein %	B Fat, %	C Carbohydrates %	D Mineral matter, %	E Amount normally purchased to get 1 pound of solid edible material after refuse and water have been removed
1 Fish					
Fresh cod	92	2		6.4	8.6
Fresh mackerel	67	28		4.8	6.7
Canned salmon	60	32		7.1	2.7
Oysters	51	11	28	9.4	8.5
2 Eggs	56	40		3.8	1.3
Cheese	38	51	4	6.0	1.5
3 Meat					
Veal	61	36		3.2	3.8
Beef	49	47		3.0	3.1
Fowl	41	57		2.0	2.8
Mutton	37	61		1.6	2.2
Pork	34	64		2.0	2.5
4 Legumes					
Peas (dry)	27	1	69	3.2	1.1
Beans (dry)	26	2	68	4.0	1.1
5 Milk					
Milk (skimmed)	36	3	54	7.4	10.5
Milk (whole)	25	31	39	5.4	7.7
6 Nuts					
Almonds	22	58	17	2.1	1.9
Brazil nuts	18	71	7	4.2	2.1
Walnuts	17	65	16	1.5	2.4
Chestnuts	11	10	77	2.4	2.2
7 Cereals					
Oatmeal	18	8	72	2.3	1.1
Wheat flour (Graham)	15	3	80	2.0	1.1
Macaroni	15	0	83	1.5	1.1
Rye bread	14	1	83	2.3	1.5
Wheat bread (Graham)	14	3	81	2.3	1.5
Wheat bread (white)	14	2	82	1.7	1.5
Wheat flour (white)	13	1	85	0.6	1.1
Corn meal	11	2	86	1.1	1.1
Crackers	10	13	75	1.8	1.0
Rice	9	[illegible]	90	0.5	1.1
Rye flour	8	1	90	0.8	1.1
Buckwheat flour	7	1	91	1.0	1.2
8 Starchy root crops					
Potatoes	10	1	85	4.6	5.8
Sweet potatoes	4	2	92	2.1	4.1
Beets	13	1	77	8.8	10.0
9 Vegetables and fruits					
Cabbage	19	3	66	12.4	13.7
Tomatoes	14	7	70	8.8	7.1
Strawberries	10	7	77	6.5	11.0
Squash	10	3	61	5.5	17.3
Oranges	6	1	89	4.2	10.4
Grapes	6	7	84	2.3	5.9
Bananas	5	3	88	3.7	6.2
Apples	3	3	92	2.5	8.5
10. Foods with minimum protein					
Butter	1	96	0	3.4	1.0
Sugar	0	0	100	0	1.0

placed in order according to the average percentage of protein contained in the dry edible material which remains after water and refuse, such as bones, shells, and hulls, have been removed The first group consists of fish. In the portion of these which actually supplies nutriment more than 50 per cent consists of protein In codfish this rises as high as 92 per cent. Only 2 per cent is fat, there are no carbohydrates Minerals in the form of calcium, phosphorus, and so forth form a high percentage. In a food of this kind, the protein of which can easily be assimilated by the human body, the correct ratio of protein to fat would be about 20 to 80 instead of 92 to 2 Column E shows that bones, skin, and water form so large a part of the codfish sold in the market that one would have to buy 8 6 pounds in order to get 1 pound of solid, dry food Such a food supplies the bulk which the stomach craves, it gives protein in a desirable form, it furnishes a varied assortment of vitamins, and it is peculiarly good as a source of minerals It lacks acids, however, and its deficiency in fats and carbohydrates necessitates supplementing it by foods of a different sort. People such as the Icelanders sometimes suffer greatly when other supplies are scarce and they have to rely mainly on fish Eggs and meat resemble fish in composition, but are less extreme They are high in protein and contain considerable fat, but not enough to make them approach anywhere near the ideal balance They also contain a fairly large amount of mineral matter as well as a good assortment of vitamins. As bought in the market, meats include so much water and waste that the amount purchased weighs from 2 to 5 times as much as the final dry weight of the food.

574. II *The Well-Balanced Type* Below the meats in Table 18 we come to three types of food which are well balanced The legumes, represented by peas and beans, contain not only 2 or 3 times as much carbohydrate as protein, but also a good quantity of minerals This makes them excellent for use where the diet cannot be greatly varied, as in China Nevertheless, they lack vitamins and acids, and are so concentrated that they do not give the stomach sufficient bulk They contain practically no waste except a little water, which remains even when they are considered dry Milk has much the same composition as the legumes so far as proteins and fats or carbohydrates are concerned. It also contains abundant vitamins and an uncommonly high percentage of mineral matter Thus, if people have to live on a single food, milk is about the best Nevertheless, it consists so largely of water that people who rely on it as their main food crave something solid. At certain seasons the Arab Beduin live almost entirely on milk. After a few months they loathe it, and would give almost

anything for food that supplies the stomach with something to work on Nuts, as well as legumes and milk, fall in the middle part of Table 18 where there is a good balance between the various ingredients.

575 III. *The Carbohydrate Type* The next section of Table 18 includes the cereals. Here we find that only 7 to 15 per cent of the dry weight consists of protein except in the case of oatmeal where the percentage rises to 18. Thus oats are the best balanced of the cereals, and also stand well in minerals and vitamins The Scotch, who eat a great deal of oatmeal, are fortunate compared with many other people. Nevertheless, all the cereals are much too low in protein, especially when we recall that only about 80 per cent of the protein which they contain can be taken out in digestion Although the cereals are of the utmost value as foods they need to be supplemented by materials containing more protein, minerals, vitamins, and acids. Moreover, they are so concentrated that a cereal diet is one cause of a great deal of ill health and suffering in countries such as China and India. That is one reason why those countries raise and consume enormous quantities of castor beans, which are used as a cathartic

576. IV *The Starchy Root Crops* Closely allied to the cereals, with their high percentage of carbohydrates, are the root crops of Section 8 in Table 18. Tubers such as potatoes are the most important representative of this type, but yams, manioc, taro, and such vegetables as beets, carrots, and turnips all belong here When their dry weight alone is considered, their chief characteristics are a low percentage of both protein and fat, a high percentage of carbohydrate in the form of either starch or sugar, and a high percentage of mineral matter In their natural state they contain an excessive amount of water, so that the weight purchased is from 4 to 10 times the weight of actual food Their function in the diet is about the same as that of the cereals, but they are of less value from the standpoint of protein and of more value for minerals They are deficient, however, in vitamins, but are useful in preserving health because they supply bulk which aids digestion and elimination Thus a diet of wheat and potatoes is better than one of wheat alone, although in either diet there is serious lack of protein, acids, and vitamins

577 V *Fruits and Vegetables* Fruits and fresh vegetables are often even more watery and bulky than meat, roots, or even fish Hence, like meat and milk, they are luxury products Where the population is dense and poor, the people cannot afford to raise them, because the land must be used for products such as cereals and legumes

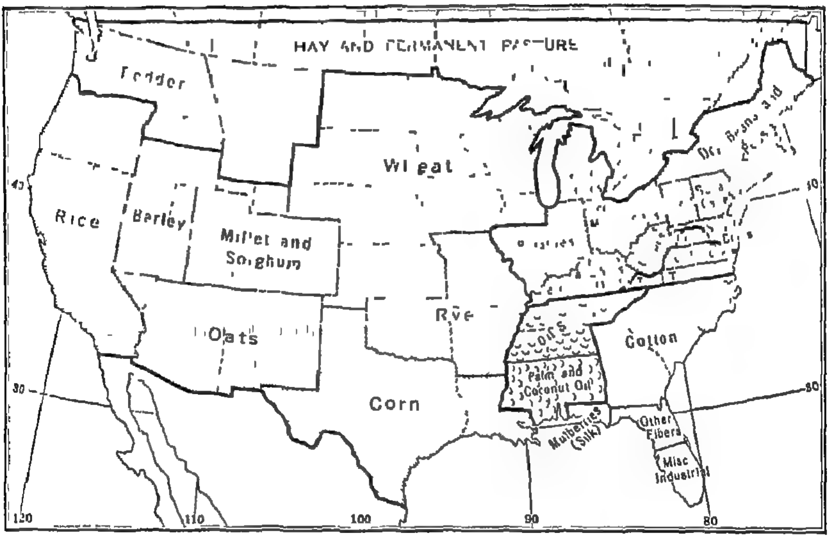
which furnish a large amount of solid food per acre. This is most regrettable, for green vegetables and fruits are the main source of both vitamins and acids. They also serve a useful purpose in giving bulk to a diet. Thus they rank with milk as the kinds of food that put the finishing touches on a diet, and make it approach the ideal, provided there is a good foundation of the main sources of protein and carbohydrates or fats. In some tropical regions, the people eat far too many bananas or plantains, in others, too many fresh coconuts. Such a diet greatly distends the stomach because people eat an inordinate amount of starch in order to satisfy their craving for protein.

578. VI. *Pure Fats and Carbohydrates.* Butter and sugar, at the end of Table 18, are peculiar because they go even further than codfish in being composed of only one of the two major ingredients of diet. Just as codfish is almost pure protein, so butter is almost pure fat and sugar is pure carbohydrate. Butter and sugar are among the best of foods for supplying heat and energy, but they are worthless in building up wasted tissues, or enabling children to grow. Although they stand at the bottom of Table 18 so far as protein is concerned, they rank with dry legumes and cereals as foods with little or no waste. Sugar is the purest of all foods, because every bit of it can be assimilated. Butter ranks almost equally high, but part of its mineral matter is not used by the human body. Both of these are luxury foods. Poor people cannot afford foods which need to be supplemented so largely by other types which furnish protein.

579. THE CEREALS. I. *Their Predominance and Use.* In the following pages we shall discuss the geographical distribution of the various types of food in Table 18. We shall begin with the cereals because they are far and away the most widely used type. In most countries the cereals—wheat, corn, rice, oats, millet, barley, and rye—occupy twice as much space as all other food crops combined. The only exceptions are a few countries such as Uganda where green bananas, or rather plantains, take their place. Even in Ireland, where potatoes are relatively more important than in almost any other country, the cereals occupy two thirds of the land devoted to food crops. The preponderance of cereals in the world's agriculture is evident in A579. We have seen that only about 3,500,000 square miles, or one sixteenth of the earth's land surface, is actually used for crops. This is equivalent to the 3,000,000 square miles of the United States and the southern 500,000 square miles of Canada. A579 shows how much of that area would be needed for each type of crop, regardless of whether the climate, soil, and relief are right or not. A good two

thirds of the United States—2,000,000 square miles, or almost the whole area west of the Mississippi River—would be needed to accommodate all the world's fields of grain

580. Some of the reasons for the predominance of cereals in man's food have already been mentioned in connection with our discussion of corn and wheat, but they may be summed up again here together with others. (1) Cereals will support a large number of people per acre. Potatoes, cassava, or bananas, to be sure, will support still more, but the difference is not so great as appears at first sight In the United States the average yield of potatoes is about 110 bushels per acre, while



A579—World Area of Harvested Land Superimposed on Map of the United States and Part of Canada

that of corn is about 25, and of wheat 14 Three fourths or more of the weight of the potatoes is water. Therefore an average acre of potatoes in this country will supply about 1¼ times as much food as an average acre of corn, and about twice as much an as acre of wheat (2) The quality of the food supplied by cereals is better than that of other foods which give a large yield per acre, especially if the whole grain is used, as we saw in Table 18 (3) Cereals can be raised with less work than crops such as potatoes that supply much food per acre. After the smaller cereals have once been sown they require no care until harvest time Then they can be reaped more easily than crops that have to be picked by hand or dug out of the ground It is far

easier to pick ears of corn than to dig potatoes or pick dry beans. (4) Another main advantage of the cereals is that they keep much better than watery products like potatoes, roots, vegetables, and fruits. Of course they differ in this respect, wheat, barley, and rice being easier to keep than corn, millet, and oats (5) Because of their keeping quality, small bulk, dryness, and ease of handling, the cereals can be easily transported. Such products as beans and peas share this advantage, but they require more work for cultivation and harvesting, and give a smaller yield per acre The combined effect of all these advantages is to cause cereals to be the main food product in practically all parts of the world Where it is impossible to raise cereals the population is almost invariably scanty

581 II. *Geographic Distribution of Kinds of Cereals* It is fortunate that some kind of cereal can be raised in practically all places where agriculture is possible In most regions more than one cereal is raised, and in many the acreage devoted to two different kinds is nearly equal Beginning in high latitudes we find that oats are the dominant crop in all far northern regions from Alaska across the northern part of the habitable portion of North America to Ireland and Finland, and across Europe and Asia to the island of Sakhalin. They are also dominant in Tasmania, southern New Zealand, and southern Chile, which have the coolest summers among the agricultural regions of the southern hemisphere Where the summers are fairly cool and rainy, oats hold the first rank as far south as the northern United States from Minnesota to Maine, as well as in the neighboring eastern provinces of Canada They are usually associated with cattle and potatoes. Next to oats in ability to thrive in cool weather comes rye. It is the dominant grain only in a central European band from Germany across Poland to Russia. Rye often gives a fair yield on soils so poor that the yield of wheat is discouraging It stands drought better than oats These qualities permit its growth on the poor soils of the parts of Europe with a cool continental climate

582. Although wheat can be raised in lower latitudes and in drier climates than either oats or rye, it grows best where the summers are cool It is dominant in the continental climate of the western United States and the adjacent provinces of Canada, and in corresponding areas of Australia and South America, including Uruguay, Argentina, and Chile. It is the chief cereal of Eurasia from France eastward to northern China. It is also dominant in an irregular band where the Mediterranean type of climate prevails from Spain and Portugal eastward to Turkey and Iran, and onward to northern India. Thus wheat is dominant in three kinds of climate, namely, the cool continental

type, the maritime cyclonic type of western Europe, and the Mediterranean type. Wheat grows in many kinds of soil, but on the whole it tends to be most abundant on the richest and darkest soils (see Plate II).

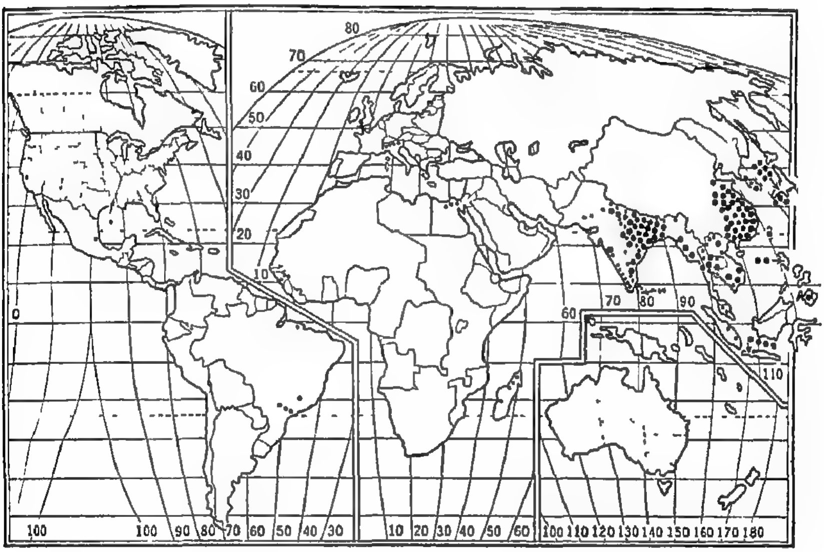
583. Moving still farther toward the equator, we come to the zone where corn is the dominant cereal. Here for the first time we find a cereal which needs warm summers, but we have already seen that warmth no greater than that of Connecticut is best. Corn is primarily an American crop. It occupies more land than any other crop in the central and southeastern parts of the United States, and also in all the rest of both North and South America until the Argentine wheat region is reached. In Europe it is dominant only in Albania, Rumania, and part of Yugoslavia, in Africa only in the south, and in Egypt where it can be irrigated. Neither in Australia nor in Asia is it dominant in any large area. In Australia the reason is the scarcity of summer rain. In Asia the same is true in the west, but little corn is raised in India, Malaysia, China, Manchukuo, and Japan because rice is more profitable where rain is abundant, and millet where the summer rains fail frequently and the heat is great.

584. Following the cereals to still lower latitudes we find that French Morocco and southern Algeria and Tunis are the only areas of good size where barley is the dominant crop. Barley is a sort of poor relation of wheat which does not provide such nutritious and palatable food and can stand poorer soil, a shorter growing season, and greater extremes of heat and drought. Hence a good deal is raised in the marginal portions of many countries such as Turkey which raise wheat as a main reliance, but sow barley in less favorable places. In more-favored regions barley is raised chiefly for use in making malted drinks like beer and ale.

585. The millets are another poor man's crops. Like rye and barley they are the main crop only where some geographic condition is so extreme that more desirable crops, such as wheat, corn, and rice, will not thrive. The relation of the many forms of millet to corn is much like that of barley to wheat. Just as barley replaces wheat in the drier and hotter parts of regions with winter rain and summer drought, so millet replaces corn, and sometimes wheat, in regions where most of the rain comes in summer but where the rainy season is short, irregular, and unreliable. One such region is the Sudan and French West Africa south of the Sahara. Another lies in the central part of India, and a third in north-central China. Wherever either barley or millet is the dominant crop, one may be quite sure that the climate is unreliable and that the people are poor. The many species

of millet differ more than wheat, rye, and barley. Some are short, grasslike plants that suggest a field of wheat or rye, others, such as the sorghums, have tall stalks like corn and supply a sweet sap which is better than that of corn for making syrup All alike bear bunchy heads of small round seeds which are less nutritious, less palatable, and often more difficult to keep than wheat and rice

586 In the lowest latitudes of all, but also ranging almost as far north as corn, rice is the dominant cereal Aside from a few small areas such as Madagascar and British Guiana the rice regions lie in southeastern Asia and the neighbouring islands (A586) Their chief



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A586—World Production of Rice

climatic characteristic is long, warm summers, with abundant rain Although rice has too much starch and too little protein (Table 18) to be as good food as wheat, it probably forms the staple food of more people than any other crop This is partly because so large a portion of the people in rice regions subsist on a scanty diet Rice has the advantage of being probably the surest and most regular in yield of all the cereals, mainly because it depends on irrigation and hence is raised only where the supply of water is large The importance of this in maintaining a dense population is great In southeastern Asia from India to Japan and in the East Indies the heavy monsoon rains of summer or the equatorial rains of the lower latitudes give

plenty of water High mountains lead to great condensation of moisture, thus feeding thousands of small streams as well as great rivers such as the Ganges, Brahmaputra, Si, and Yangtse The waters of these are diverted over terraced slopes, or spread over vast, gently sloping plains covered with dyked fields Thus rice, unlike the other cereals aside from corn, is a crop of rugged regions as well as of plains.

587 Rice exceeds all other cereals in yield per acre, in value per pound even before the husk is removed, and in value per acre Table 19 shows the approximate yield per acre of various cereals in the 10 countries or states where this yield is highest, and the corresponding yield per acre, value per pound, and value per acre in the United States during a recent 10-year period The high yield per acre is not surprising when one considers the amount of labor involved and the fact that the rice crop rarely suffers from lack of water or other climatic handicaps The value per pound, however, is surprising, especially when one recalls that rice is not so good a food as wheat and is the food of countries far poorer than the majority of those that eat wheat Perhaps the high price is a reflection of the large amount of work required in raising rice where it is transplanted by hand It is also

TABLE 19

YIELD OF CEREALS AND OTHER MAIN CROPS

Crop	A Approximate yield per acre in pounds		B Value per 100 pounds, 1926–1935 U. S A	C * Value per acre, 1926–1935 U S A
	World	U S A		
Rye	1,500	650	$1 09	$7 07
Oats	1,600	870	1 03	8 96
Millet †	1,700	750	1 28	9 61
Barley	1,900	1,110	1 01	11 20
Wheat	2,000	820	1 39	11 34
Corn	2,500	1,340	1 14	15 34
Rice	3,000	2,080	1 74	36 20
Flaxseed		350	3 06	10 78
Cotton		180	12 27	22 00
Potatoes		6,700	1 33	79 20
Sweet potatoes		5,000	1 66	82 50
Tobacco		790	16 11	127 30

* These figures are lower than those of Table 11 (§394) because they are based on a period of especially great agricultural depression

† Sorghum in the United States

possible that the great demand for rice in the Orient, where it is considered the most desirable cereal, raises the price all over the world. The price is so high that the United States has become an exporter, even though it raises only a small amount, as is evident in A586.

588. Table 19 is also significant because it shows how far the United States as a whole lags behind the world's best areas in its production per acre of all kinds of cereals. Of equal significance is the small value per acre of the cereal crops, aside from rice, that are raised on the farms of the United States. Potatoes are worth 5 times as much per acre as corn, and corn is worth twice as much as rye. The secret of this lies partly in the ease with which cereals can be raised by machinery, whereas potatoes require much hand work. Another important fact is that cereals can be transported so cheaply that the price is kept down by world competition. The price of cotton, and hence the value per acre, is kept down in the same way, whereas the price of sweet potatoes is relatively high because they are hard to transport. Tobacco, on the other hand, has a very high value per pound and per acre because a great amount of labor is needed for each acre, and even then the number of pounds per acre is small compared with potatoes.

589. *Summary of Cereals.* The main features of the economic geography of cereals are summed up in Table 20. Column A shows how many million acres are devoted to all cereals in each of 7 main divisions of the earth's surface, and columns B to H show the same thing for the individual cereals. In Column B all the figures are fairly high, thus showing that wheat is more generally distributed than any other crop. Four regions, namely, (1) Asia aside from the U.S.S.R., (2) the United States and Canada, (3) the U.S.S.R., and (4) the rest of Europe, each devote from 75 to 100 million acres to that crop. The next column shows that corn is primarily an American crop, almost half the total area being in the United States. Rice is still more fully Asiatic, only 5 or 6 per cent of the land devoted to it is found outside southeastern Asia and the neighboring islands. Oats are equally firmly entrenched in the cool northern regions. Millet, on the other hand, is almost wholly a crop of southeastern Asia. Barley is more widely distributed, but rye is almost limited to northern and eastern Europe.

590. The rest of Table 20 gives an interesting idea as to the relative prosperity of different parts of the world. Column I shows the total production of all grains per year, J shows the amount per acre. Europe, because its western part has such a high yield per acre of all kinds of grain, has an average of 1,225 pounds per acre. South

TABLE 20

PRODUCTION OF CEREALS IN LARGE GEOGRAPHIC UNITS
(MILLIONS OF ACRES)

Population in Millions	A Total	B Wheat	C Corn	D Rice	E Oats	F Millet	G Barley	H Rye
1,125 Asia except U S S R	455	98	29	178	1	111	37	1
137 U S A and Canada	250	75	103	1	51	4	14	3
160 U S S R	240	86	10		41	21	18	64
385 Europe except U S S R	232	88	31	1	42	1	28	41
119 Latin America	66	22	34	4	2		3	1
140 Africa	45	12	16	4	1	3	9	
8 Australia	17	15			1			
2,074 Total	1,305	396	223	188	139	140	109	110

	I	J	K	L	M	N
	Total grain per year in millions of pounds	Pounds of grain per acre	Percentage of population in agriculture	Agricultural Population in thousands	Pounds of grain per annum for each person in	
					Total population	Agricultural population
Asia	420,000	820	70	787,500	373	533
U S A	240,000	960	27	37,000	1,750	6,500
U S S R	168,700	705	70	112,000	1,052	1,505
Europe	284,000	1,225	36	138,500	738	2,055
Latin Am	67,740	1,020	56	65,500	569	1,034
Africa	33,200	740	70	98,000	237	339
Australia	12 890	760	29	2,320	1,610	5,550
Total			60			

America and the United States, with yields of 1,020 and 960, come next because their chief crop is corn, which produces a large yield per acre Nevertheless they are far behind Europe Asia, in spite of the fact that it raises rice, gets only 820 pounds per acre as the average for all grains Finally Australia (760), Africa (740), and Soviet Russia (705) trail far behind This is especially significant in view of the fact that in both Africa and Soviet Russia a large percentage of the population is dependent on agriculture, about 70 per cent as shown in column K

591. Column N, showing how many pounds of grain are raised per year for each person in the purely agricultural population, is one

of the most significant parts of Table 20. The United States and Canada have plenty of land. Therefore, although their yield of grain per acre is only four fifths that of Europe, they get 6,500 pounds per person on the farms as compared with only 2,055 in Europe. Australia (5,550) does almost equally well. On the other hand, in Soviet Russia, although the country is vast, the amount of cultivated land is small in comparison with the huge agricultural population, and the yield per acre is low. Therefore the production falls to only 1,505 pounds per person. In Latin America (1,031 pounds) and Africa (339 pounds) the situation appears to be still worse, so far as Table 20 is concerned. It must be remembered, however, that in both these regions the data are more or less deficient, and the people depend on other crops beside cereals to a considerable extent. In Asia, this is far less true than in Africa and Latin America. In the Soviet Republic only a little food is raised aside from grain and potatoes. In fact, if data for all kinds of food in all parts of the world were available, the Russian food supply in comparison with that of the United States and Europe would appear decidedly less abundant than it does in Table 20 on the basis of cereals alone.

592. On the basis of Table 20 and of rough estimates as to other food, we may say that where a given number of persons on the farms raise 100 million pounds of food in the United States and Canada, the same number raises only about 85 in Australia, 33 in Europe as a whole aside from Russia, 20 in Soviet Russia and Latin America, and 10 in Asia and Africa. These figures are only preliminary estimates, but they are significant because the raising of food is the most important of all human occupations. They show that, even when such huge areas as those of Table 20 are considered, the actual income of the farmers in some regions is 10 times as great as in others. If smaller areas are considered, the differences become still greater. Such differences have a profound effect upon every other phase of economic geography. People as poor as those of Asia cannot possibly behave like those of the United States or Australia in such matters as transportation, manufacturing, and commerce.

CHAPTER XXV

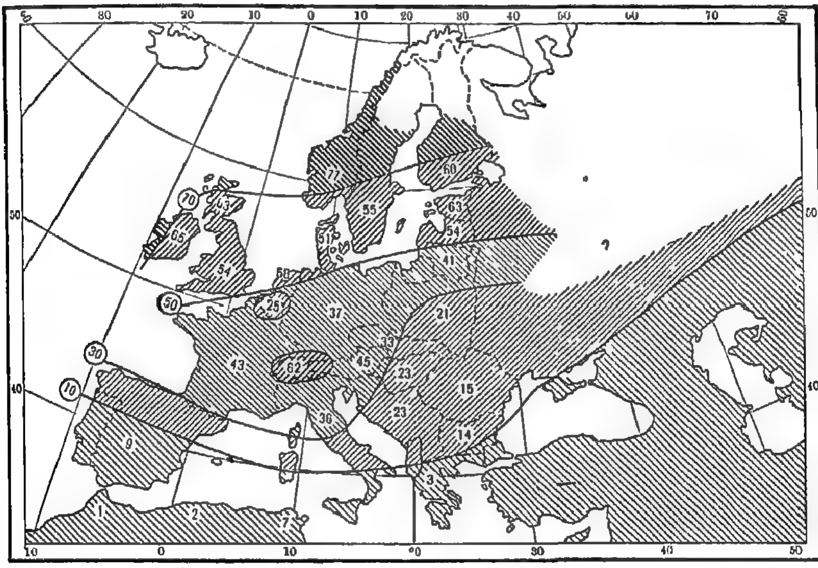
THE SUPPLEMENTARY FOOD SUPPLY

593 ADJUSTMENT OF DIET TO GEOGRAPHICAL CONDITIONS In spite of their dominance the cereals nowhere furnish the entire food supply. Long experience has proved that people must eat something else in order to feel comfortable and to maintain the population from generation to generation. If people felt satisfied with a diet of cereals alone, they would probably follow the line of least resistance and raise little else Cereals require less work than any other kind of food that can be used to support a dense population Potatoes, for example, provide more food per acre, but are by no means such good food when eaten alone They also require more work than cereals in proportion to the final amount of food, excluding waste and water Moreover, they cannot be kept a long time, as can grain They are, to be sure, an excellent supplement to cereals, both because they give variety and bulk and because they permit the farmer to use land and time not needed for raising grain Nevertheless, they surpass grain in importance only where the climate is uncommonly moist with cool summers, as in western Ireland In other regions outside the main agricultural regions such products as the yams, bananas, coconuts, and manioc of tropical countries, or the milk and meat of grassy steppes, replace the cereals as the chief source of food Such places, however, do not support a large population

594 In most parts of the world a long process of trial and error has taught people how to supplement a cereal diet by other products in such a way that the following requirements are met as nearly as possible: (1) the cereals and other crops fit the climate and soil, (2) they provide enough food to support a certain density of population and a certain standard of living, and (3) they come as near as may be expected to providing proteins, fats, and carbohydrates in such proportions that the appetite is satisfied and people can be at least reasonably healthy After these requirements have been met, a population that is not too dense and has enough energy supplements its diet still further by means of fresh vegetables and fruits that provide vitamins, acids, and minerals. Except in progressive countries,

and among the richest of the people in poorer countries, little conscious effort is made to get the right amounts of these rarer ingredients of a good diet Even in China, where we hear so much about horticulture, the ordinary peasants eat few vegetables and little fruit Near the cities they raise some fruit and a good many vegetables for sale to the richer people in the cities, but the peasants themselves seldom eat such expensive articles

595 In most parts of the world the diet consists almost entirely of products raised locally Americans, as well as prosperous city people everywhere, are likely to forget this because their food comes from a



A596—Animal Feed in Europe (Percentage of all harvested land used for animal feed, 1934)

wide variety of sources Yet even in the United States large groups of farmers and village people, especially in the South, depend mainly on food produced locally In Australia, Canada, and the manufacturing countries of western Europe the situation is much the same as in the United States, but even in France and Germany local products provide the main part of the food In the rest of the world, especially among the huge populations of India, China, Japan, and the East Indies, practically all the food is produced within a few miles of the places where it is eaten. Because of the scarcity of good statistics, however, it is impossible to study the problem accurately in these oriental regions. Europe is still the region where the relation between

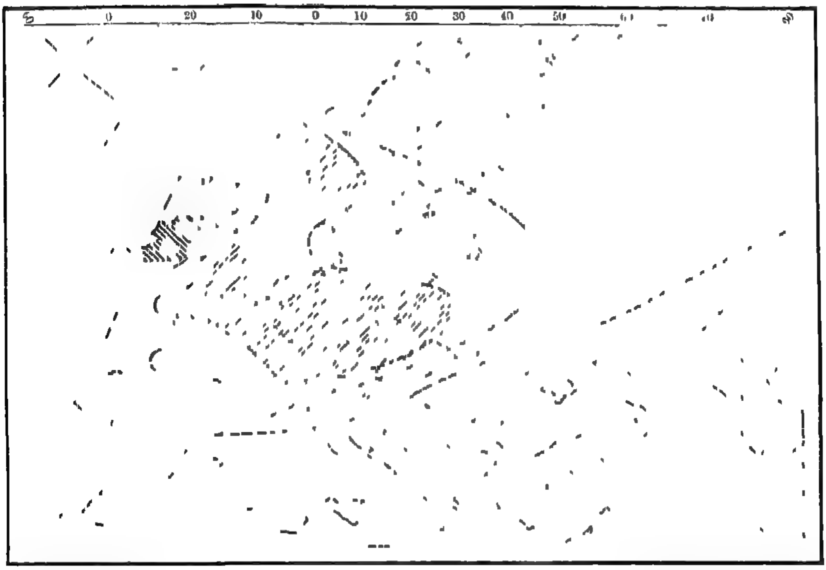
diet, agricultural production, and geographic environment can be most profitably studied The principles illustrated there are exemplified clearly in the less progressive parts of the world, and are applicable to such regions as the United States when modified by the activities of commerce

596 Types of Supplementary Food. I *Meat and Milk.* Animal products, including meat, milk, butter, cheese, and eggs, are one of the best means of supplementing a diet of cereals Table 18 shows that they contain abundant protein as well as fats and minerals They also contain a large and varied supply of vitamins Therefore a diet based on cereal and animal products appears to be good A596 shows what percentage of all the harvested land in Europe is used for hay and forage to be fed to animals If the land devoted to grain that is fed to animals were also included, the high numbers in this map would be still higher, but the low numbers would be little changed The obvious fact is that a large percentage of the cultivated land in the north of Europe is devoted to raising feed for animals There is an almost steady decline in the percentages as one goes from north to south. The extremes are 77 in Norway and 1 in French Morocco This southward decline is not due entirely to a diminution in the number of animals. Part of it arises from the fact that as one goes toward the south the necessity for raising food to feed the animals in winter declines

597 II *Starch and Sugar* Somewhat farther south than the belt where dairy products and meat are the most important types of supplementary diet, but overlapping it considerably, comes a zone where starchy or sugary roots rise to first importance This is evident in potatoes, which reach their greatest productivity in Holland and Belgium and in the limey loams of Aroostook County in northern Maine (A373) They are abundant in the cool part of the United States from Maine to Idaho and Washington, and in the adjacent parts of Canada The sugar beets of relatively cool portions of Michigan, Colorado, and California form a sort of southern fringe of the potato belt Europe (A597) shows a similar belt in which root crops, such as potatoes and beets with their high percentage of carbohydrates, are even more closey associated It reaches its greatest intensity in Ireland, Norway, Belgium, and Switzerland, where the land devoted to potatoes and sugar beets forms at least 20 per cent of all the land from which human food and raw materials are harvested It extends in full force into Poland, and is a strong feature of western Russia The importance of this belt of starchy root crops would be still more evident if turnips, carrots, and other roots were included. Potatoes,

sugar beets, and other roots belong together not only because they grow underground, but because they play the same general role in diet. They all consist mainly of carbohydrates, which take the form of either sugar or starch.

598. The main northern area of white potatoes in western and central Europe is more or less balanced by a main area of yams and sweet potatoes in southern China. In considering the starchy foods of tropical countries, however, certain other products and areas must be taken into account. Yams and cassava are important starchy foods derived from roots in Brazil, central Africa, and the Malay, Indo-



A597—Potatoes and Sugar Beets in Europe. (Area devoted to potatoes and sugar beets as percentage of all land devoted to human food and raw materials, 1931.)

Chinese, East Indian, and West Indian regions. Cassava is the dried flesh of the manioc or mandioca plant. The more common form of this root, when first dug, contains a poison, but when it is pounded and soaked in water, the poison is dissolved and the remainder becomes edible. In these same tropical regions the plantain is another widely grown starchy food. In the more humid tropical countries each little thatched hut is usually surrounded by a small grove of great green stalks and gigantic leaves belonging to bananas or plantains. Tropical people eat bananas fresh as they ripen, and the children often consume a great many. Nevertheless, there is no great enthusiasm for them. Plantains, not bananas, form the main

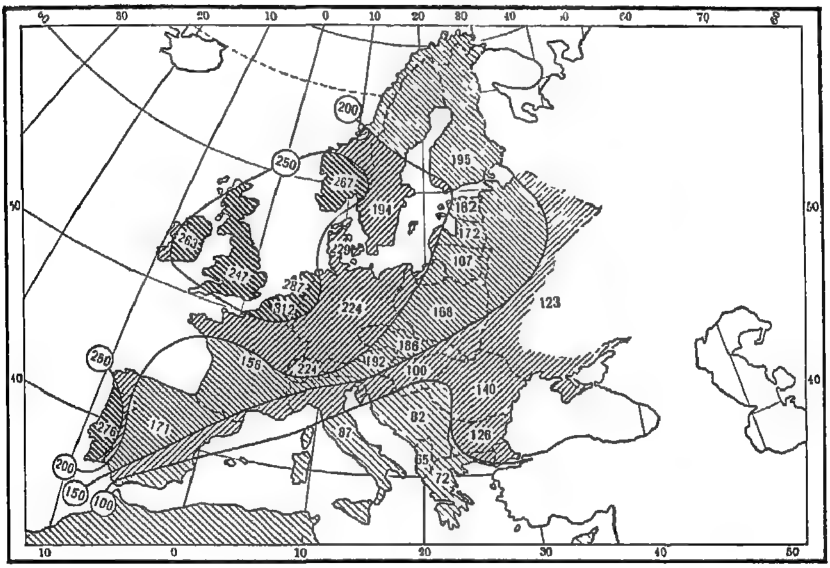
article of diet, but they have to be cooked In Uganda a young married couple consider themselves established for life and fairly sure of a living when they have built a hut and planted a good grove of plantains around it The long fruits, like green bananas, are cooked in kettles with leaves above and below them When well cooked they provide a food suggesting sweet potatoes, but more insipid Taro, a favorite food of the Hawaiians before the coming of the white man, is another starchy food of the tropics It is raised in many tropical regions and even as far north as Japan

599 CARBOHYDRATES IN TROPICAL VERSUS COOL CLIMATES There are interesting differences between the starchy foods of cool regions and warm regions One of these is already apparent. In cool regions potatoes are the only starchy noncereal food of much importance. Among the starchy foods of warm regions sweet potatoes, yams, cassava (manioc), and plantains (bananas) all hold high rank Thus whereas a single species holds sway in cool regions, many species share the honors in warm regions. The reason for this is found partly in another contrast In cool regions potatoes are a substitute for cereals only in marginal sections such as western Ireland, in many tropical regions the starchy roots and fruits replace cereals and are the main source of food This illustrates one of the many handicaps of such regions Sweet potatoes, bananas (Table 18), yams, and cassava contain only about half as much protein as white potatoes in proportion to their dry weight, and only one third as much as wheat Therefore, unless supplemented by foods high in protein, they form a poor diet.

600 THE WHITE POTATO Let us return for a moment to the northern belt of carbohydrates as shown in A597 and as suggested in A600 The ordinary white potato, mainly because of its better keeping qualities, higher percentage of protein, and more abundant minerals, is a more valuable crop than sweet potatoes, yams, cassava, or plantains It is primarily a crop of northern Europe For every 1,000 bushels raised in Europe only 65 bushels are raised in North America, 12 in South America, 11 in Asia aside from Siberia, 3 in Australia and New Zealand, and 2 in Africa Data for sweet potatoes, cassava, and yams are not available because those products are raised in small patches in regions where good statistics are scarce.

601 White potatoes cannot be raised profitably in countries such as China because of the dry spring in the north and the hot wet summers everywhere This is a pity, because few other crops produce so much food per acre The amount of labor required in raising potatoes would be no hindrance if the climate were right, for China

has vastly more labor than it knows what to do with The Chinese could probably cultivate 2 or even 3 times as much land as at present without any more people, or any new tools and machinery The Chinese largely waste their vast labor supply because land is scarce. Potatoes are so sensitive to climate, however, that even in the United States the average yield per acre is only about half as much as in Europe (A600) Although potatoes came originally from the high, cool, tropical plateau of Peru, they are now raised chiefly in northwestern Europe where the climate is similarly cool and humid, but far more variable In Peru, however, they are still raised in relatively

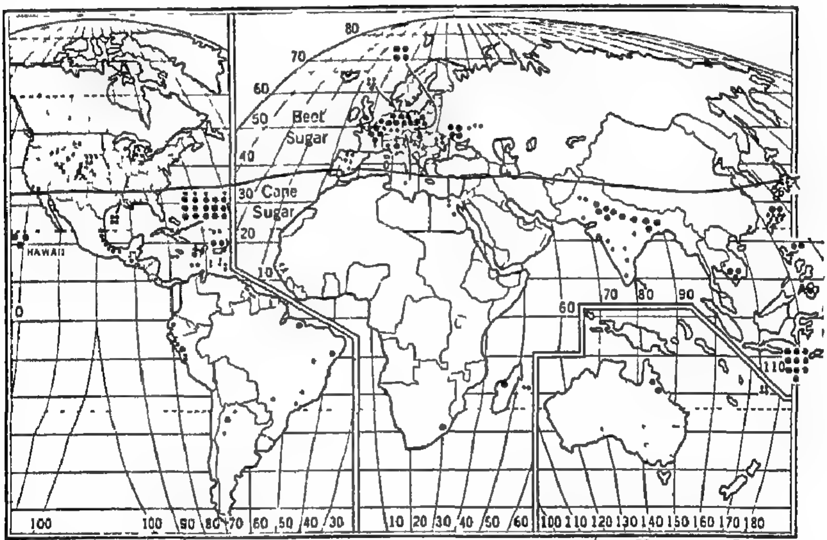


A600—Potatoes in Europe (Yield in bushels per acre, 1926-1936)

large quantities In both that country and Bolivia they occupy about the same amount of space as corn In those two high Andean countries potatoes occupy half as many acres as in the entire United States In proportion to other crops they are there more important than anywhere else, for they occupy about 20 per cent of the harvested land in comparison with 14 in Germany, 9 in Ireland, and only 1 3 in the United States

602. SUGAR The story of sugar (A602), like that of most great products, is learned so well in the elementary school that only a few points which appeal to maturer minds will be pointed out here The first is that sugar is a luxury crop By this we mean that people do

not raise sugar because they need it for food, but because they like the taste of it Nevertheless, as far as the amount of food per acre is concerned, sugar is a good crop In Hawaii, where the highest yields are obtained, the production of sugar and molasses per acre has about the same weight (8 tons) as the average production of potatoes in Maine, which leads North America in that respect Since two thirds of a potato is waste material in the form of water, an acre of sugar cane in Hawaii provides 3 times as much fuel for the body as an acre of potatoes in Maine



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A602—World Production of Sugar

603 In Louisiana the annual yield of sugar is one third as much as in Hawaii,* so even there the sugar cane gives about as much food as the potato On the other hand the work of plowing, planting, and cultivating is at least as great for a cane field as for a potato field, the work of harvesting is far greater, there is a large expense for grinding the cane and converting the sap into sugar, and much of the molasses is used for alcohol or fed to animals instead of being eaten by man In spite of all this, 25 cents spent for 5 pounds of sugar purchases just about the same amount of carbohydrate as 25 cents spent for a peck of potatoes This looks as if sugar were a fairly economical food

* Including molasses, and making allowance for the fact that in Louisiana the crop is harvested after 9 months' growth, and in Hawaii after 18 to 22 months'

such as a dense population might well raise India, as we have seen, produces an enormous amount of sugar, 4 or 5 times as much as the continental part of the United States Nevertheless, it consumes only one third as much in proportion to the population Most of that is in a cheap, unrefined, brown form called "gur " The people of India cannot afford the luxury of eating much sugar, or of refining the small amount that they do eat Nevertheless, it is interesting to note that the per capita production and consumption of sugar in India have doubled since the World War The people of India have so scanty a diet that any kind of addition is acceptable What they most need, however, is not more carbohydrates, but a larger supply of proteins and vitamins, neither of which is supplied by sugar

604 III. *Legumes as Sources of Protein* Aside from dairy products and meat, the chief products for supplementing a starchy diet are legumes, especially beans, peas, chick peas (gram), and lentils Table 18 shows that a quarter of the food in these is protein The use of legumes as a source of protein increases from the most prosperous countries to the poorest tropical countries This is evident in Table 21, which shows how many acres of leguminous crops there are for every 100 acres devoted to the main starchy crops, namely, cereals, potatoes, and yams

605 In the new countries of Section I in Table 21 the population is not yet dense, and there is so much room that dairy products and meat can be raised in abundance Moreover, the abundance of land, the use of farm machinery, and the generally high stage of progress provide so much wealth that the city people can afford to purchase a great variety of foods, especially dairy products and meat The farming population is also so prosperous that it can get money enough for taxes, clothing, tools, and other necessities without selling its most valuable products, as is commonly done in countries like India Inasmuch as both the urban and the rural people get a good supply of protein from meat and milk, there is little need of legumes Hence such products occupy scarcely more than 1 per cent as much land as cereals and potatoes This is not because of any difficulty in raising legumes High yields per acre are found not only in a belt from southern New England and the Middle Atlantic States to Idaho, but along the whole Atlantic Coast and especially in the far southeast from Florida to Louisiana There are so many kinds of legumes that one sort or another grows well in a wide variety of climates

606 Following down the various sections of Table 21 we see that, as the degree of prosperity decreases and as the problem of overpopulation becomes more acute, the percentage of land devoted to legumes

TABLE 21

ACRES OF LEGUMES PER 100 ACRES OF CEREALS, POTATOES, AND YAMS

I New and Prosperous Countries in Temperate Climates	
Canada	0.3
Australia	0.4
U S A	1.3
II Old Prosperous Countries in Cool Temperate Climates	
Sweden	1.3
Finland	1.3
Germany	1.4
France	2.1
III. Old Countries of the Less Prosperous East European Type	
Hungary	2.6
Russia	2.6
Bulgaria	3.5
Rumania	4.4
IV Relatively Poor Countries with Mediterranean Type of Climate	
Greece	7.7
Turkey	8.0
Spain	8.5
Chile	11.6
Syria	13.5
Italy	14.1
V. Poor and Very Densely Populated Countries of Eastern Asia	
Japan	11.7
China	15.0 (estimate, per Buck)
Manchukuo	40.0*
VI. Poor Tropical Countries	
Peru	12.5
Mexico	19.2
India	20.0 or more

* This figure is very high because it includes soybeans, which are raised for oil as well as protein and are exported in large amounts

in comparison with cereals and starchy roots and tubers becomes steadily greater In the most advanced countries of Europe, where animal food is abundant, it amounts to only 1 or 2 per cent (Section II) In the less prosperous countries farther east (Section III), the percentages range from 2 to 5 In the relatively poor Mediterranean countries (Section IV) they run from 7 to 14. Finally in the very poor countries of eastern Asia (Section V) and of the tropics (Section VI) they rise to heights of 12 to 20 This means that in those countries a combination of the following circumstances has compelled the people to find products which yield a large supply of protein in proportion to the land which they occupy (1) The cereals which grow there, chiefly rice and millet, are poor in protein (2) The climate and forage are not well adapted to domestic animals, the animals furnish little milk, they do not readily put on fat, and the density of the population makes it impracticable to feed them properly (3) Both Buddhism and Hinduism forbid their followers to eat the flesh of animals, especially cattle (4) In warm, moist climates meat does not keep well. This has a strong effect in preventing its use. Inasmuch as an animal must be eaten as soon as it is killed, the meat is relatively tough and tasteless according to western standards.

607. Another important reason why people turn to legumes for proteins is that in tropical climates it is usually not easy to sell meat profitably. When an animal is slaughtered by a peasant in a small village of the usual Indian or Chinese type, the number of possible customers who can afford to buy the meat is very small. Hence most of the meat must either be wasted, given away, or used at once for extravagant feasts. Only the rich according to the standards of those lands can afford to kill useful draft animals, or young animals that may become useful. Thus only old, tough, worn-out, or diseased animals are usually available for food, and beef gets a bad reputation. In countries such as Turkey, southern Russia, and even Italy and Spain a somewhat similar situation prevails. The absence of a market for meat is an important reason for the use of legumes. A directly opposite situation prevails in respect to ordinary dry beans, peas, soybeans, lentils, chick peas, and the numerous other kinds of lentils. They can easily be raised, they give a fairly high yield per acre, they can be kept easily, and they can be produced and sold cheaply in any desired quantity. Therefore among much more than half of the world's population they are raised in large quantities for the express, although unrecognized, purpose of providing the protein which is deficient in the cereals.

608. IV. *Oils and Fats.* Although carbohydrates and fats contain the same chemical elements and may be substituted for each other as sources of bodily heat and energy, both are needed because they perform different physiological functions. Fats are needed both to supply variety in the diet and to promote digestion. We have already seen that a diet composed largely of cereals is not sufficiently bulky and leads to indigestion. Even if dried legumes are added, the same trouble is found. A sufficient but not excessive supply of fats or oils obviates this difficulty and makes for better health. The vegetable products which furnish fats belong to three chief types: (1) oily seeds such as sesame, (2) oily fruits such as the olive, and (3) oily nuts such as the peanut and coconut. The seeds appear to give the largest supply of oil per acre, and therefore are the main reliance in the most densely populated countries. The cool countries which rely on oil seeds are located mainly in eastern Europe. There the commonest kinds are rapeseed, hempseed, sunflower seed, colza, and linseed or flaxseed. The oil extracted from rapeseed and sunflower seed is used for salads and cooking in Poland, Russia, Rumania, and other parts of eastern Europe. Sunflower seeds, when roasted, are a favorite delicacy in southern Russia, where they are produced in vast quantities and eaten as Americans eat peanuts. Linseed was formerly used as a food product in the same way as sunflower seed, but linseed oil is now mainly a

raw material in manufacturing. In India and China, however, linseed oil is still used as food Sesame, together with certain other oil seeds in India and soybeans in China, is the most important of the oil seeds.

609 The importance of these sources of oil may be judged from Table 22, showing how many acres are devoted to colza, rape, sunflowers, sesame, peanuts, soybeans, castor beans, and olives for every 100 acres devoted to cereals, roots, tubers, sugar cane, and other sources

TABLE 22

USE OF OIL SEEDS AND OLIVES

Scandinavia	0 0	Java	4 2
France	0 1	Japan	7 8
Germany	0 2	India (British provinces)	10 0
Poland	0 3	Turkey	15 0
U S A	1 0	India (native states)	17 0
Hungary	1 0	Spain, Italy, Greece	20 0
Rumania	2 3	Manchukuo	40 0*
Russia	3 8		

* See note, §606

of sugar The high figures for Turkey, and for Spain, Italy, and Greece are due to the abundance of olive trees, which cover large tracts Among all the sources of edible oil, none seems to be better than olives Although a few olives are raised in California and Chile, this plum-like fruit is almost limited to the countries immediately surrounding the Mediterranean Sea It needs the Mediterranean type of climate with mild, rainy winters and long, hot, dry summers Peanuts, coconuts, and palm nuts are only a few of the many kinds of nuts which furnish oil They are the main sources of oil in warm countries

610. Americans often suppose that the southern states from Virginia southward and westward are the world's great peanut region That is a mistake The southern states do indeed raise about a billion pounds per year—8 pounds apiece for every American man, woman, and child, but two regions, British India and Senegal in French West Africa, export as much as our entire production The United States is not the greatest consumer of peanuts either Our own production is all that we require, but both France and Germany use more than we do In fact, in proportion to the population Germany demands twice as many as the United States, and France 6 times as many Those countries do not eat peanuts at baseball games, but utilize them chiefly to supplement the supply of fats derived from animals and from oil seeds Just as peanuts represent a main source of vegetable fats in subtropical countries, so coconuts and palm nuts represent a main

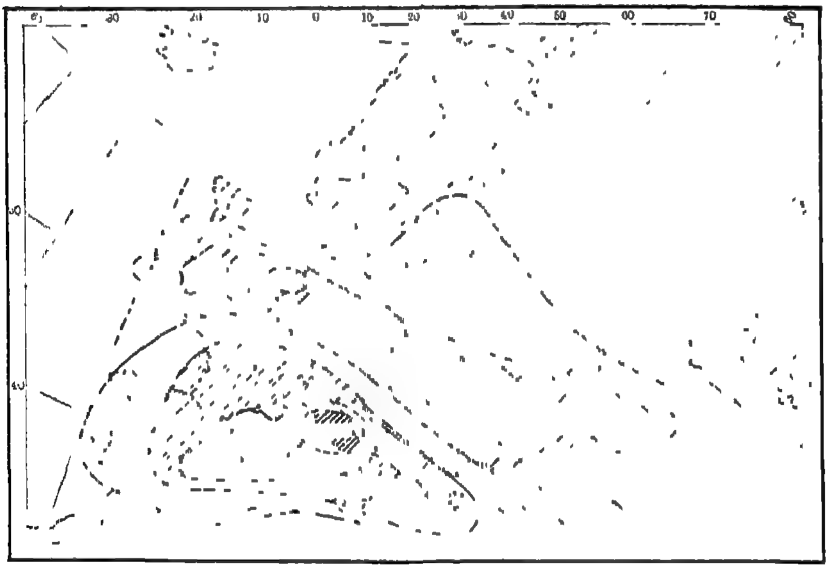
source of such fats in genuinely tropical countries. Their oil, however, is largely used for industrial purposes, but it is also an important article of food. Substitutes, for either butter or the animal fats used in soap, for example, can be made from palm oil.

611 The highest figure in Table 22 is that of Manchukuo. It represents practically nothing except soybeans, and is the same as the 40 per cent for Manchukuo in Table 21. This means that some products, among which soybeans are the most conspicuous, supplement a starchy cereal diet by supplying both protein and fats. This fact, together with the great reliability of the yield per acre, has much to do with the extreme importance of this crop in eastern Asia. In this respect soybeans resemble meat and milk. They are not so valuable, because the animal products not only fulfill the double function of supplying both protein and fats, but also supply more minerals and vitamins than soybeans. The way in which one type of fat can be substituted for another is interesting. In recent years tropical oils have largely replaced animal fats such as lard in Europe. This, as well as the recent hard times in Europe, has largely decreased the exports of lard from the United States. The use of vegetable oils has also been increased by chemical discoveries whereby the same oil can be made suitable for food, soap, or paint.

612 V. *Fruits and Vegetables.* The last important way in which a diet is raised toward the ideal is by means of fruits and green vegetables with their great abundance of vitamins, acids, and minerals. A612 shows where these products are raised in Europe. The supply is evidently deficient in the northwest, but the harm done by such deficiency is largely overcome by the great abundance of milk. There is a similar deficiency in Russia, and it is by no means obviated by milk and meat, for these are relatively scarce in the Russian diet. In England and the Netherlands the people evidently make a special effort to raise fruits and vegetables. In the Netherlands, especially, such products are raised not only on farms, but also in the world's most extensive greenhouses. The reason is, of course, that the area around the North Sea is second only to the English-speaking countries across the seas in wealth, and has a huge urban population that can afford a good diet. Germany and old Poland show an eastward decline in fruits and vegetables due largely to the fact that severe continental winters make it hard to raise fruit and lessen the yield of vegetables by frosts in the late spring and early fall. A city such as Moscow in a thoroughly continental and fairly cold climate is surrounded by market gardens, like almost every other great city. During July and especially August the markets are filled with such a profusion of vegetables

that the uninformed visitor thinks that the supply is unlimited. Two months later, however, the supply has largely disappeared. Fruit is scarce, and most of the people in both city and country are fortunate if they can get cabbage to add to their potatoes, bread, and tea.

613. The most conspicuous feature of A612 is the area of abundant fruit and vegetables around the Mediterranean Sea. This is due much more to fruit than to vegetables. Grapes are the most important fruit, but peaches, apricots, pears, plums, mulberries, figs, and citrus fruits play their important part. The fact that grapevines are often planted far apart with other crops between them raises the figures in A612



A612—Fruits and Vegetables in Europe. (Area in fruits and vegetables as percentage of all land devoted to human food and raw materials, 1934.)

somewhat, especially in Italy. Nevertheless, in no part of the world are fruits more varied, more abundant, and of higher quality than in the countries around the Mediterranean Sea, and in regions such as California, central Chile, and others with the Mediterranean type of climate. In such climates fresh vegetables also grow well in the spring and fall, although they dry up in summer unless irrigated.

614. All things considered, the countries with the Mediterranean type of climate probably are the most-favored parts of the earth from the standpoint of the kinds of food that are readily available. Winter wheat and other cereals thrive there to provide carbohydrates. Abundant hilly or mountainous tracts that cannot easily be cultivated pro-

vide pasture for sheep and goats. Olive trees provide a large and easily harvested supply of oil. An abundance of fruits is available from May or June when the mulberries and apricots ripen until December when the last grapes disappear, but figs, either fresh or dry, and oranges prolong the season of fruit. The chief deficiency is in protein. Cattle thrive only moderately, and the summers are too warm and dry for the best results in dairying, except in especially cool sections such as northern Italy and within a few miles of the coast of California. Legumes can be raised in large quantities, but here again the dry season limits the yield. Nevertheless, there appears to be no part of the earth's surface where so little effort is required to obtain a fairly well-balanced and healthful diet during the greater part of the year. This is probably one of the factors contributing to the early and remarkable development of civilization in the lands around the Mediterranean. Similar and even more favorable conditions on the Pacific Coast of the United States are one cause of the rapid advance of that region.

615. In highly advanced countries, the local supply of food has ceased to be the determining factor in the nature and adequacy of the diet. The thing that counts most now is the ability to procure from elsewhere the right kinds of food at all seasons. In all large cities, as is well known, fruits and vegetables, as well as meat, milk, and all the other ingredients of a good diet, are available at all seasons. Nevertheless, the people who live where that is so, and who can afford to buy products that are raised far away, ought not to forget that the major part of the earth's inhabitants—an overwhelming majority of perhaps 80 or possibly 90 per cent—still depend on local food supplies, and have a diet which departs widely from the ideal. By means of countless experiments through generation after generation they have discovered that by making certain combinations of crops they can get a diet which at least enables them to live without unbearable discomfort. The unconscious skill with which these adjustments have been made is remarkable. Nevertheless, there is still room for great improvement, especially in the overpopulated countries of Asia and the inert countries of the tropics. Even in eastern Europe the diet is far from being as good as it might be, and the same is true of the majority of people even in western Europe and the United States.

PART IX

REGIONS OF HIGH PRODUCTIVITY

CHAPTER XXVI

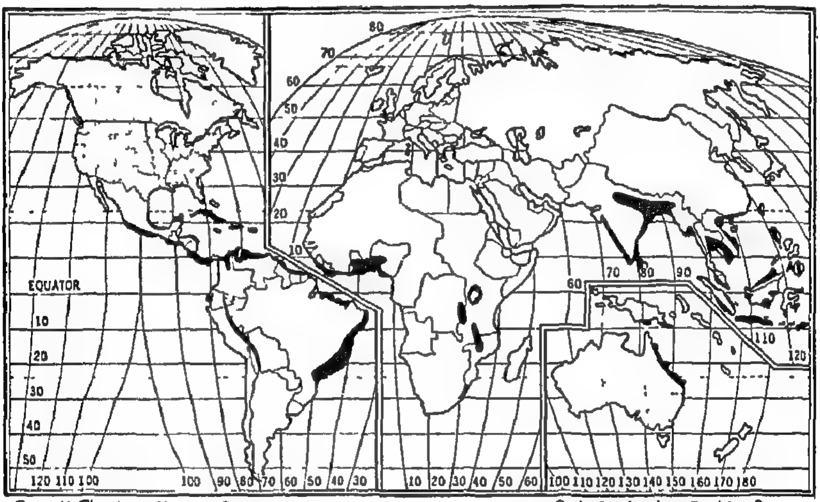
TROPICAL REGIONS OF HIGH PRODUCTIVITY

616 REGIONAL TYPES IN THE REALM OF HIGH PRODUCTIVITY Having gained some idea of the optima of crops, animals, and man, and of their significance in relation to production, we are ready to study the Realm of High Productivity That realm includes the parts of the world which are conspicuous as places where numerous crops find their optima The Realm of High Productivity contains a greater number of natural regions than does either of the three less productive realms, but these regions are relatively small in area They are divided into three types, tropical, transitional, and cyclonic. The tropical type comprises Regions of Wet Tropical Agriculture (No 2 in Plate I) and Cool Tropical Highlands (No 4) The transitional type consists of (A) regions with the Mediterranean Type of climate (No. 7) on the west coasts of continents between 30° and 40° (sometimes 45°) from the equator, and (B) Mild East Coast Regions (No 8) which occupy the east side of the continents in nearly the same latitude as the Mediterranean Regions The cyclonic type lies farther from the equator than the other two, and is almost limited to the northern hemisphere. It is divided into four parts, namely, Marine Cyclonic Regions (No 9), the American and European Continental Cyclonic Regions (Nos. 10 and 10A), and Cool Continental Regions (No 11).

617 The Realm of High Productivity has a total area of about 10,000,000 square miles, or less than one fifth of the earth's land surface Its population, however, numbers nearly 1,500,000,000 or approximately three fourths of the world's total From the standpoint of productivity it ranks still higher It produces at least four fifths of the world's crops and domestic animals, most of the coal and iron ore as well as a large part of the other minerals, and practically all the manufactured goods It produces at least nine tenths of the world's lumber, although its forests are by no means so extensive as

those of the Realm of Uncertain Agriculture and the surrounding Realm of Nonagricultural Production

618 THE REGION OF WET TROPICAL AGRICULTURE *Location.* Within the Realm of High Productivity the Region of Wet Tropical Agriculture is noteworthy for the way in which the production of numerous and highly valuable tropical crops is concentrated in plantations So important is this phase of agriculture that this part of the world might also be called the Plantation Region Plate I and A618 show that this region is of small area and that its various little parts are mostly found on windward seacoasts facing the tradewinds or monsoons, or else on the slopes of interior mountains such as the

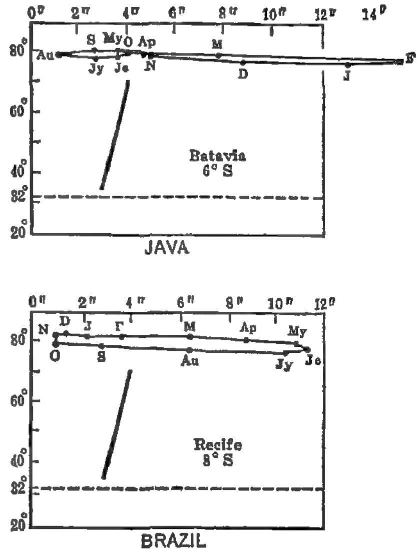


Goode's Semi homolosine Equal Area Projection

A618—Distribution of Regions of Wet Tropical Agriculture

Himalayas and Andes which also face to windward Note how the tradewinds cause narrow bands of Wet Tropical Agricultural country along the east side of Central America, South America (on both sides of the equator), India, the Malay Peninsula, South Africa, and northern Australia Monsoons or allied winds give rise to similar strips in many parts of both the East and West Indies and along the west coast of India and the Malay Peninsula Monsoon winds that blow inland cause a broad band of Wet Tropical Agriculture along the base of the eastern Himalayas in India, while tradewinds blowing still farther inland across Brazil produce a similar band at the eastern foot of the Andes Most of the Pacific islands, such as Hawaii and Fiji, belong to this same type of natural region.

619 In places of this type the population tends to be dense and industrious in comparison with the people in the neighboring Tropical Rainforests, or in the Wet and Dry Low Latitudes. In all of them, likewise, there are at least the beginnings of plantations devoted to raising such crops as bananas, coffee, cacao, manila hemp, sugar, tea, pineapples, jute, and quinine for sale to the people of cooler climates. In many places such plantations have now become the main business and produce major commodities for world trade Even such products as coconuts and palm nuts, which were formerly raised only by natives, are now becoming important plantation products in the regions of Wet Tropical Agriculture. In all these regions people from cooler parts of the earth, especially Europeans and Americans, but also Chinese and people from the Andean highlands, have come to establish plantations and exploit the tropical resources.



A620—Climographs of Regions of Wet Tropical Agriculture

620. *Climate of Regions of Wet Tropical Agriculture* Judging by A620, the Regions of Wet Tropical Agriculture seem at first to have a climate almost like that of the Tropical Rainforests (B102). Other climographs might be drawn which would seem at first to resemble those of the Wet and Dry Low Latitudes (B107). Careful examination, however, shows a real and important difference between the climate of the typical plantation regions and those of the other two types of tropical regions, between which this type often lies The difference is that in Regions of Wet Tropical Agriculture the dry season is more distinct than in the Tropical Rainforests, but not so long or severe as in the Wet and Dry Low Latitudes A moderate difference in this respect makes a great difference in productivity. The difference in climate between rainforests and Regions of Wet Tropical Agriculture is so slight that many geographers do not separate them It has so much effect upon the utilization of the land, however, that it becomes highly important in economic geography.

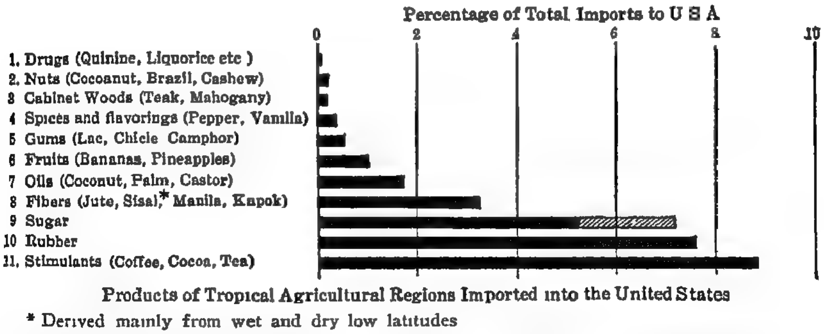
621 The fact that Regions of Wet Tropical Agriculture have a distinct dry season but not a long or very dry one has two important advantages for agriculture First, it gives a reasonably long but not harmful dry season which helps many crops to ripen The dry season also affords an opportunity to get rid of weeds, plow new land, get rid of pests, and do various other jobs which are difficult or impossible when rain falls daily, the ground is always muddy, and weeds grow up almost overnight In the second place, the fact that the dry season is short and not absolutely rainless largely eliminates the danger and uncertainty of agriculture arising from the severe dry season which is so highly developed in the grassy forests and savannas of the Wet and Dry Low Latitudes. Although the Regions of Wet Tropical Agriculture have a distinct dry season, it is rarely dry enough to injure such sensitive crops as bananas and cacao. On the other hand, it is dry enough and long enough to allow crops such as sugar and pineapples to ripen with plenty of the sunshine that is needed in order to make them sweet, or to develop their starch as in corn and rice. Hence crop failures are relatively few in these regions, the quality of the crops is high, and the yield per acre relatively good.

622 *People of Regions of Wet Tropical Agriculture* A human advantage arises from the conditions of agriculture in the Regions of Wet Tropical Agriculture before plantations were established In those days these were the parts of the tropical lowlands where corn was raised in the New World and rice in the Old Rice culture has been a major factor not only in permitting tropical populations to become dense, but also in making such populations comparatively good workers. Rice culture cannot be carried on without at least a fair degree of careful, steady work, painstaking irrigation, and carefully regulated customs as to the sharing of water and control of floods. Thus it has been an important factor in creating habits of steady though slow industry among the people of southeastern Asia and the East Indies It also appears to have been a strong factor in promoting the kind of social organization in which people work steadily and peacefully, and cooperate in public affairs Just how great these effects have been we cannot say, but it is clear that the best tropical workers come from rice-raising parts of the Regions of Wet Tropical Agriculture. Corn culture has a similar effect, but by no means so great

623 Another advantage of the Regions of Wet Tropical Agriculture is that they are the parts of the tropics where the diet is most varied and abundant, and there is least danger of the frequent periods of scarcity which afflict other tropical regions aside from the highlands,

These small agricultural regions are the part of the tropics which has given rise to the popular misapprehension that tropical countries are places where anyone can get a living without doing much more than open his mouth. Moreover, they are the only parts of the tropics which a great many travelers have ever seen Hence they have had far too much influence in shaping our ideas of the tropics as a whole. Regions of Tropical Agriculture, such as the coastal plain of southern India, Bengal, the lowlands of Java, the coastal regions of eastern Brazil, many of the West Indies and the Pacific Islands of Hawaii and Fiji, are garden spots in comparison with tropical regions as a whole The villages are hidden in groves of mango, banana, breadfruit, and papaya trees Rice fields in many places, and corn fields in others, dominate the surrounding landscape. Plantations of many kinds cover the slopes. We who are of European descent in cooler lands owe a great debt to these people It would do us little harm if Tropical Rainforests and the Wet and Dry Low Latitudes were entirely eliminated, but it would cut deeply into our daily habits if the products of wet tropical agriculture were eliminated from our tables and from our motor vehicles

624 *Products of Regions of Wet Tropical Agriculture* Products of wet tropical agriculture represent about 30 per cent of the value of all the imports into the United States A624 gives an idea of the nature and relative importance of these products Some of the commodities



A624—Imports of Tropical Products into the United States Lightly shaded part of sugar bar indicates beet sugar from temperate regions

of this diagram, such as cabinet woods and vanilla, are derived from the rainforest rather than from genuine Regions of Wet Tropical Agriculture Their total value, however, is so small that their omission in A624 could scarcely be detected Bananas and cacao are other products which are raised on the edge of the genuine rainforest They are

raised only, however, in places where a dry, or at least a drier, season is quite definitely developed Therefore, it seems best to include such areas in the Regions of Wet Tropical Agriculture, and to consider that those regions are gradually encroaching on the rainforest How far this encroachment can go is not yet clear, but it is still slight. Another point to note about A624 is that some of the products, especially tea, coffee, and quinine, come from regions of fairly high altitude, and might therefore be counted as products of the Cool Tropical Highlands If an altitude of 2,000 feet should be considered enough to warrant including a region among the highlands a large part of all three of these products would belong there The highlands can scarcely be called cool, however, until an altitude of about 5,000 feet is reached Therefore these three stimulants belong almost wholly to the regions of Wet Tropical Agriculture.

625 *Minor Tropical Products.* Items 1 to 5 in A624 are of such small importance that we may group them all together Among the drugs, quinine is the most interesting Although the cinchona tree, from which it is derived, is a native of the eastern slopes of the Andes at an altitude of 3,000 feet or more, nine tenths of the world's supply come from Java Quinine has a history somewhat like that of rubber It grows naturally in an inaccessible location in South America among a sparse population not well adapted to steady work It has been carried to the East Indies and improved so much that the new varieties produce several times as much as the wild type A Dutch experimental station for quinine at the height of almost a mile in Java is a most fascinating place Not only does the air have a delightful coolness, like that of May in the northern United States, but there are glorious views of volcanoes, and the graceful Javanese people are very picturesque as they cut the weeds among the trees or gather the bark Although quinine is one of the minor tropical products in value, and is imported into the United States in only small quantities, it has played a great part in the economic development of the Regions of Wet Tropical Agriculture For a long time it was practically the only means by which the white man could combat the malaria which naturally afflicts those regions If the white man had not been able to check the ravages of that disease by means of quinine, it is doubtful whether the present huge development of such regions could have taken place

626 Spices (No 4 in A624), as well as quinine, are more important in their historical effect upon economic geography than in their present effect In the old days before Columbus made his famous voyage, the diet of Europe was limited, and the cooking was poor.

Meats were the predominant food of the wealthy, and they were often served in a state of semi-decay because there were no refrigerators in which to keep them. Bread was the main food of the poorer people. Vegetables and fruits were scarce, and so were milk and eggs in comparison with the present. Such conditions made the food unpalatable, monotonous, and unsatisfying. Therefore people craved pepper, cinnamon, cloves, mace, and other spices to make their food taste better and stimulate their appetites. Practically all the spices came overland from India by way of the deserts of western Asia. This made them costly. Therefore spices ranked with gold as an incentive to Ferdinand and Isabella in encouraging Columbus to search for a shorter and more oceanic route to the spice islands of the East Indies. Because he thought that he had found those islands, the islands of the Caribbean region are now called the West Indies.

627. The decline in the importance of quinine and spices illustrates another of the principles of economic geography. The principle is that, *as civilization rises higher, the most advanced countries tend to find substitutes for products which come from a distance, especially if the products are difficult to obtain.* Quinine and other tropical drugs have diminished in importance because other methods of combating malaria, especially the elimination of mosquitoes, now make that disease far less common than formerly, at least on the plantations run by white men. Spices, as we have seen, are needed far less than in the past not only because our food is much more varied and better preserved than formerly, but also because new kinds of catsups, pickles, and jellies have been developed. Flavoring extracts of tropical origin such as vanilla are less necessary than formerly, even though we now use vastly greater quantities of ice cream. The reason is that many nontropical flavoring extracts are now available, some being made from fruits or other products of temperate lands, and others synthetically. Other examples of the substitution of products of the temperate zone for those of the tropics are seen in beet sugar, synthetic rubber, and moth balls made from coal tar, or petroleum to take the place of camphor which comes mainly from Taiwan (Formosa). The gums in item 5 of A624 include not only camphor but also lac (shellac), for which artificial substitutes are being found. Sometimes the best way for a cooler country to replace the products of distant regions is to introduce a new crop. This has been done repeatedly in the United States. The soybean of north China, for example, is fast becoming a major American product, the tung tree of south China is now raised in large groves in Florida and Mississippi. One of the great risks of the Regions of Wet Tropical Agriculture is that they will lose im-

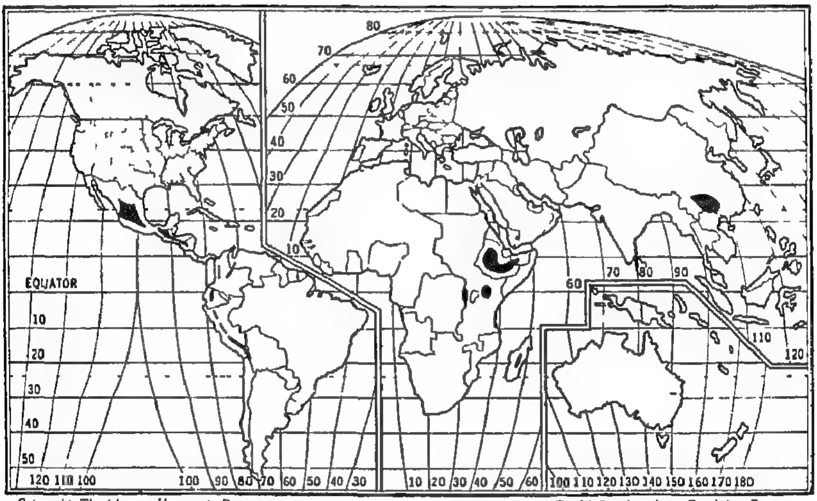
portant kinds of economic activity because the more advanced regions find ways of making or raising substitutes We have seen that this is what is happening to the silk of Japan

628 The effects of the principle which has just been explained are often partly neutralized by those of another principle *When people from a more advanced economy come to a relatively backward economy such as that of the tropics, they tend to discover new uses and markets for hitherto neglected resources* This is illustrated by chicle gum. Our ancestors learned to chew spruce gum which they got from the trees around them In the tropics they found that the abundant gum of the zapote tree is a better base for chewing gum, and now southern Mexico, especially Yucatan, does a large business in tapping zapote trees Cabinet woods are another line in which a similar and greater development has only begun Teak is almost the only wood which is raised to any great extent in tropical plantations, but there are many beautiful and valuable species which might be thus raised It seems highly probable that some day groves of trees like the mahogany may be planted in the Regions of Wet Tropical Agriculture, just as pines are planted in Germany, Sweden, and parts of the United States To-day mahogany trees and many others, such as rosewood, are procured only by searching far and wide among hundreds of varieties in the tropical forest Plantations of nut trees will probably show a much more rapid development The Brazil nut is steadily growing in favor The curious West Indian cashew nut, whose stem is like a juicy, appetizing fruit, was unknown to most Americans 20 or 30 years ago, but now is imported in growing quantities

629 *Tropical Fruits.* We hear a great deal about luscious tropical fruits, but most people who go to the tropics are disappointed in them Not only is it hard to get them, but very few are as good as apples, pears, peaches, plums, grapes, oranges, and grapefruit Oranges, to be sure, grow in many tropical regions, but in quality and abundance they cannot compare with those of California, Florida, Spain, and Palestine One reason for the poor quality of tropical fruits is that they have not been improved by man to any such degree as the fruits of higher latitudes Another reason is that most of them either are too sweet or have a rather insipid flavor Even the banana and mango, which are two of the best, have a less definite character—less of a pleasant kind of acidity—than the orange, apple, and grape Nevertheless, the banana and mango are grown almost universally around the houses of even the poorest natives in the Regions of Wet Tropical Agriculture Other tropical fruits of economic importance include the papaya, breadfruit, pineapple, and avocado. The first

two are rarely shipped any great distance, but the others enter into foreign commerce The mangosteen, with its pure white flesh inside a thick ring of pink-fleshed rind, is often called the best of tropical fruits It is of little economic importance, however, because it does not keep well, and is largely confined within narrow climatic limits in Java and neighboring parts of the East Indies

630 TROPICAL HIGHLANDS Another type of tropical region where productivity is high is the cool tropical highlands Plate I and A630 show that these are found in the Andes, Central America, Mexico, Central Africa, Madagascar, southwestern Arabia, and the plateau of Yunnan in southern China Small sections of India along the western



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Goode's Semi homolosine Equal Area Projection

A630—Location of Tropical Highlands.

side of the peninsula and on the southern slope of the Himalayas in Kashmir, Nepal, and Sikkim might also be included Tropical plateaus exist also in the little-known interior of the huge island of New Guinea Such areas tend to have a more permanent and advanced type of agriculture, a denser population, and a higher native civilization than the neighboring lowlands. Ethiopia, Peru, and Mexico have long ranked far above regions in the same latitude at low altitude This is partly due to the fact that the rainfall is relatively more moderate than lower down, the dry season does less harm because it is not so hot, the variety of crops is greater, and the soil as a rule is of better quality than in the hot, wet lowlands More important than this perhaps is the fact that in tropical highlands

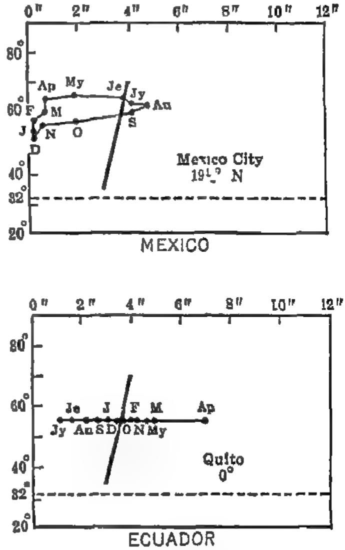
human health and vigor are much better than in lowlands, especially than in lowlands beyond the ameliorating influence of the sea

631. *Great Value of Small Differences in Temperatures.* The contrast between tropical highlands and lowlands is due largely to temperature. This is seen in the high plateaus already mentioned and in lower tropical plateaus like those of southeastern Brazil, Central America, Uganda, Madagascar, and the peninsula of India. Within the tropics the temperature usually declines approximately 1° F for every 300 feet of altitude, although there is much local variation Nowhere else are human health and activity so greatly influenced by differences of altitude. An average temperature of 60° to 70° for day and night with a fairly high humidity is the optimum for health and physical activity Temperatures only 10° higher, averaging 80° or more and running up to 85° or 90° by day, are uncomfortable and debilitating If the humidity is also high, they cause most people to be listless, or at least disinclined to work. Near the equator the temperature along the seacoasts averages approximately 80° the year round, in the interior at low altitudes it runs still higher—from 80° in the coolest month to 83° in the warmest at Manaus on the Amazon, for example. Even at the tropics, 23½° from the equator, the temperature on the coast averages above 70° at least half the year, while at low altitudes in the interior such temperatures may last 9 months and the warmest month may average as high as 95° Such conditions are among the chief causes of tropical inefficiency They not only favor the development of parasites, such as the hookworm and the anopheles mosquito which carries malaria, but also apparently reduce people's power to resist diseases in general, and certainly diminish the desire to work

632. From all this we see why a rise of a few thousand feet in tropical regions is extremely beneficial, even though it brings only a little of the variability which is so important as a stimulant A rise of only 2,000 or 3,000 feet brings the temperature close to the optimum Thus at La Guaira on the coast of Venezuela the monthly averages range from 78° to 83° This is not so bad if one has nothing to do except drink lemonade. But it is debilitating, and greatly slows up people's work, especially when the vertical sun brings calms and high atmospheric humidity Not far away at Caracas, an altitude of 3,300 feet reduces the temperature to 64° in the coolest month and 69° in the warmest Such temperatures are delightful, and they are not uncomfortable even when the air is moist At Quito, close to the equator, but 9,000 feet high, no month averages below 54° or above 55°. At Nairobi, the capital of Kenya, on the equator in central

Africa at an altitude of 5,500, the sun is very hot, but the nights are pleasantly cool. Rio de Janeiro, where the coolest month averages 69° and the six humid months 74° to 79°, does not now grow so fast as São Paulo, 2,600 feet higher on the edge of the plateau. There average temperatures of 58° to 69° join with the coffee industry in producing one of the most energetic and progressive cities in South America.

633 *Rainfall and Vegetation in Cool Tropical Highlands* When the midday sun approaches the zenith a rainy season generally begins in Tropical Highlands. The rains increase in intensity for two or three months (A633) because the equatorial zone of rising air, calms, and showers follows the sun in its movements northward or southward. When the vertical sun is farthest away, however, in June or December according to the hemisphere, one of the tradewind belts, or even the desert belt of descending air, reaches the plateau. Then there is little or no rain. On the windward slopes, to be sure, where the tradewinds are forced to rise and give up their moisture, rain is common at all seasons, the vegetation takes the form of a tropical forest, and a region of wet tropical agriculture is usually found. On the plateaus, a dry season is usually well developed and therefore grassland is often interspersed with forests. In parts of the Guatemalan Plateau one rides among pine trees and blackberry bushes. Around Mexico City, where monthly average temperatures of 54° to 65° are almost ideal, although too monotonous, the surrounding mountains reduce the rainfall so much that dry grasslands prevail, together with irrigation, adobe houses, and miles of hedges made of the desert-loving agave or century plant. In Kenya, at heights of 8,000 feet, broad grassy slopes alternate with thick bamboo jungles.



A633—Climographs of Cool Tropical Highlands

634 Higher still in Ecuador or Colombia the more level parts of the Andean Plateau look much like the eastern United States or western Europe. They afford charming landscapes with fields of green grass, corn, and potatoes, and many green trees, cattle, orchards,

blueberries, and soft, fleecy clouds. In the little parks or squares of the towns, flowers which come with us at various seasons from early spring to autumn—violets, tulips, primroses, daffodils, columbines, roses, and fall asters—are all blooming at once In the orchards the same tree may carry buds, flowers, and both green and ripe peaches, apples, or plums. Unfortunately, the fruits and vegetables of cooler regions are usually of poor quality when raised in the tropics They need a winter in order to do their best

635. *Temperature and Crops* The Andean capitals of Bogotá, Quito, and especially La Paz are so high that they are often uncomfortable. The morning sun is hot, but in the afternoon the wind may chill one to the bone, and the evenings make one shiver Out-of-doors one wants to be like the Indians and wear a poncho, or woolen cape with a hole in the center for the head The low temperature at a height of about 10,000 feet near Quito almost on the equator has a curious effect on the corn crop The temperature at night is so low that this crop needs six months to mature On clear nights when the sun is either farthest north or farthest south frosts may occur. Hence the only way to raise corn is to plant it just before one frost season, and harvest it just before the next The seeds germinate while there is danger of frost, but the sprouts do not appear above the ground Then the plants have just time to mature slowly before frost comes again

636 In Kenya, at altitudes of 7,000 to 9,000 feet, one sees a curious mixture of tropical and temperate conditions Wild African black people dressed in cattle hides and loaded with anklets, armlets, necklaces, and huge earrings of copper wire and beads live near recently arrived white settlers The natives support themselves by means of cattle on the upper grasslands and by raising corn and bananas lower down The white people have plowed up large fields where they raise soft wheat at the higher levels, soft white corn with big kernels lower down, and coffee still lower at altitudes of about 5,000 feet Close to an English home, almost naked black "boys" are driving long teams of oxen which pull the white man's plows. Across a bushy little gully the tracks of an elephant that came last night form big holes among the plum and apple trees of a European orchard.

637 *Highland Civilization* This close juxtaposition of different civilizations is characteristic of Tropical Highlands In the Andes and Mexico, before the coming of the white men, the rulers were the descendants of invaders who had conquered but not displaced the earlier owners of the land The Spaniards followed their example, finding the country good because it was cool and relatively free from

disease But the old civilization still persists beside the new Four centuries of life with the Spaniards have only slightly altered the Indians' mode of life

638. In Africa the contrast between the lowlands and the central plateau is almost startling In the extremely hot lowlands of southern Sudan the Shilluks and Dinkas of the Nile plain are among the most backward people in the world They cake their naked bodies with ashes to keep off insects They rarely plant fields, but wander about with their cattle Houses are replaced by flimsy shelters from the sun, made by tying together the tops of enormous grasses 10 feet or more in height, and spreading the bottoms in an arc At a height of 3,500 to 5,000 feet in the Uganda Plateau a little farther south the relative coolness and the favorable distribution of rainfall throughout the year have helped to develop the most advanced of the native people of the great equatorial belt of Africa Long before the white man came they lived in villages where neat grass huts were hidden among banana patches surrounded by small fields of peanuts, yams, and manioc They had learned to make beautiful cloth out of bark, and to build round huts that are spacious, dry, and pleasing to the eye They had organized a real government with a king surrounded by vassals Their ability is illustrated by the railroad to Kampala, their chief town When this was planned, the British officials intended to use foreign laborers The native chiefs thought this would be bad for their people, and so persuaded the British to let them do the work themselves. They did the work "as well as Chinese would have done it," according to a British official, and finished it three months ahead of time The naked cattle people of the hot lowland would never have the energy to undertake such a thing. At a still higher altitude 5,000 or 10,000 white settlers are holding the highest, coolest parts of Kenya in the midst of 2½ million black people About 25,000 Hindus and others from India have also immigrated thither By serving as artisans, small business men, and clerks the Hindus create great jealousy, for they take the jobs which are wanted by the abler natives on the one hand and by the less able whites on the other

639 The great reason for this mixture of civilization is that the highlands, especially the lower ones, are the best of all tropical regions They supply not only valuable metals, as in Peru, Bolivia, and Colombia, but also most of our tea and coffee In southern Brazil the plateau and the coffee have attracted energetic Italians and Spaniards to São Paulo where they challenge the power of the Portuguese and Negro elements of Rio de Janeiro and the north This has led to frequent revolutions even in the present century. In

similar fashion the people who live high up in Arequipa, Quito, and Bogotá are scornful of their more dusky compatriots down below at Lima, Guayaquil, and Cartagena. Yet in spite of these advantages the Cool Tropical Highlands can boast of only one large center of population, Mexico City. The ruggedness of the plateaus, their remoteness from the sea, and the small size of the territory that is naturally tributary to any one center hamper the growth of cities. Mexico City has attained its present size not only because it is the Mexican capital, but also because it lies in an unusually wide and fertile basin which has easy access to other parts of the plateau farther north and south.

Chapter XXVII

PLANTATION PROBLEMS AND PRODUCTS

640. Tropical Productivity. In Regions of Wet Tropical Agriculture there is a strong contrast between the value per acre of (1) subsistence crops, such as rice, corn, cassava, sweet potatoes, and beans, which supply the people with food, and (2) plantation crops such as sugar, coffee, and cacao, which are raised for export The subsistence crops as a rule are poorly cultivated by the natives, while the plantation crops are usually well cultivated under the direction of people from outside the tropics Plantation products yield the highest value per acre in such islands as Hawaii, Cuba, Mauritius, Puerto Rico, Formosa, Fiji, Ceylon, and Java They also yield high values on tropical east coasts such as those of Brazil and northern Australia. As a rule the most productive regions lie near one of the tropics, not far from 20° from the equator. Even these best regions, however, usually have only a small yield per acre of the subsistence crops. Here are some figures showing the combined yield of corn and rice when the yield per acre in Hawaii is called 100, and each crop is given an importance proportional to its area Hawaii 100, Italy 89, Japan 70, Fiji 50, Formosa (Taiwan) 47, Dutch Guiana (Surinam) 40, Java 36, Philippines 28, Puerto Rico 27, Ceylon 16 The yield of other subsistence crops varies in much the same way Hawaii is evidently exceptional, for it stands above Italy and Japan, and gets twice as much per acre as any other tropical region mentioned above, and about four times as much as the Philippines and Puerto Rico This is due mainly to unusually good volcanic soil and exceptionally intelligent and skillful white settlers The failure of other tropical regions to come up to the Hawaiian level, either in subsistence crops or plantation products, illustrates the fact that within the tropics both people and soil tend to be unproductive

641. *Tropical Subsistence Crops.* The small yield per acre of corn, rice, and other subsistence crops in most of the plantation regions helps to explain why such regions rarely raise their own food in sufficient quantities, and rely to a considerable degree upon food from higher latitudes. Although the wages paid by plantations are almost

invariably low according to our standards, they are generally worth more per year than the subsistence crops which the laborers normally raise on their own land. Therefore, unless there is some restraint by the government, as in Java, the peasants are eager not only to get jobs on the plantations, but to sell their land to the planters. Thus we get conditions such as prevail in Puerto Rico where about 240,000 acres are devoted to sugar, 190,000 to coffee, and over 50,000 to tobacco, but only about 160,000 to corn, beans, and sweet potatoes, or yams, the three main subsistence crops. Taking all the land into account, the Puerto Rican crops raised primarily for export occupy between three and four times as large an area as those devoted to food for home consumption. Under such conditions plantation regions are forced to import food from a distance. In Hawaii and the Virgin Islands the disproportion between subsistence crops and export crops rises still higher. In Hawaii sugar, pineapples, and other export products occupy not far from 30 times as much land as subsistence crops, and in the Virgin Islands almost 100.

642. Such a condition is undesirable in that it puts the plantation regions at the mercy of events which happen far away. The demand for plantation products is unreliable for at least two main reasons. One is that the area where new plantations can be started is still large, and there is a tendency to start more than are needed, thus creating an oversupply of tropical products. This lowers prices, and perhaps puts old plantations out of business. Another trouble is that plantation products are mainly luxuries, or at least necessities, such as sugar and rubber, the use of which can be greatly curtailed in hard times. If the plantation laborers are thrown out of work and go back to cultivating their own land once more, they generally do it in the old slipshod fashion. Their work yields a scantier income than they had when they worked on the plantations, and they cannot live so well as formerly. Often, however, they no longer own any land to which to return, and therefore are left with no means of support. Such conditions naturally create great suffering and discontent. They prevail in Puerto Rico and the Virgin Islands to a dangerous degree.

643. Some Plantation Products. *The Banana.* Let us turn now to some of the individual products which make plantations profitable, but at the same time create a serious economic problem. Among tropical fruits the banana is by far the most important. One evidence of this is the way in which it has spread from its original home in India and southern China to all parts of the tropics. Another is the extraordinary rapidity with which its use has increased in the generation or two since it was introduced into the diet of temperate regions. The

banana probably forms a staple article of food for more people than any other fruit. Outside of the larger cities, nearly every inhabitant of the Regions of Wet Tropical Agriculture and of the more favored parts of the other natural regions in low latitudes has a few banana-like plants as part of the mixture of trees, bushes, and crops that surround his house. Some of these produce bananas which are edible when raw, but more, as we have seen, produce plantains which need to be cooked. In tropical regions the cooked fruit, usually baked or steamed, but sometimes fried, is far more important than the raw. Millions of people in Latin America, Africa, southern Asia, and the East Indies think that something is radically wrong if a bunch of green plantains or bananas is not hanging in the hut. More perhaps than any other plant the banana helps to make life easy in the tropics, while at the same time it ties people down to permanent residence in one place.

644. The banana will grow wherever the climate is always warm and there is rain enough to produce tropical forests. All the important commercial plantations are found at low elevations, but this is mainly because of the need of ocean transportation. Nowhere do people rely on the banana more completely than in the plateaus of Uganda, Urundi, and Kenya in central Africa at elevations of 3,000 to 6,000 feet. After the shoots have been set out the banana needs little care, and will usually produce fruit in 12 to 14 months, although at higher elevations or on the edges of the tropics it may require nearly 3 years. The yield per acre is enormous, varying normally from about 200 to 300 bunches per year. Of course there are hazards. Dry seasons reduce the yield very greatly and sometimes cause famines in central Africa. Commercial plantations suffer both from hurricanes and from diseases, especially banana wilt, or Panama disease. Such conditions are only a few of the many reasons why managers of unusually high ability are required on banana plantations. The difficulties, however, are of minor importance compared with the nutritive qualities and high yield of the fruit, and the fact that bananas can be picked quite green and yet ripen without loss of flavor or nutritive value. This gives time for transportation and allows machinery such as moving belts to be used in handling the bunches.

645. The great area for commercial production of bananas centers around the Caribbean Sea. Here the climate is favorable, and large areas of almost unused land are available for banana production. The United States is close at hand to supply both a large market and men with capital and initiative to start plantations. Hence more than 60,000,000 bunches of bananas come to us annually, about half from

Honduras and Jamaica and the rest from lesser producers of the Caribbean area Banana plantations have not developed on a large scale south of Europe because the areas in the same latitude as the Caribbean are largely desert, and the good banana areas farther south in Africa are much farther from Europe than is the Caribbean section. Fast vessels carry bananas from the Caribbean to Europe as well as to the United States

646 *Vegetable Oils* The value of vegetable oils (No. 7 in A624) imported into the United States is as great as that of all the fruits and gums put together. Vegetable oils have been used for thousands of years. The Bible is full of phrases such as, "Mine head with oil, thou didst anoint." Nevertheless, it is only recently that people have realized the commercial value of the oils contained in a vast number of plants Now, however, the increased demand for oil not only for salads and other foods, but especially for soap, varnish, lubricants, and other purposes has caused an enormous increase in the production of vegetable oils Most of the production comes from seeds or nuts which grow in warm or tropical climates Here is an approximate list of the number of short tons (2,000 pounds) required for the production of oil to be used in the United States

TABLE 23

SOURCES OF VEGETABLE OIL

Cottonseed	1,100,000	Coconuts (copra)	165,000
Soybeans	600,000	Sesame	135,000
Peanuts	450,000	Palm fruit	120,000
Linseed (flaxseed)	320,000	Rapeseed	105,000
Olives	320,000	Hempseed	25,000

647. Most of the oil imported into the United States comes from the coconut palm and the oil palm, and is derived from Regions of Wet Tropical Agriculture As a source of food, drink, thatch, and fiber the coconut palm has long been highly important to dwellers near the seacoast in the moist tropics, and now its fruit enters into general commerce among civilized nations Both the meat and the oil of the coconut are used for human foods; even the poorer oil is good for soap, and the residue left after the oil has been pressed out makes good cattle feed The ill-smelling dried coconut meat, known as copra, comes to us mainly from the Philippines, but the remaining East Indies, Ceylon, and the islands of the Pacific are large producers in proportion to their size. On some of the coral atolls of the Pacific copra is practically the only product that can be sold.

648. The main source of palm oil, and palm kernel oil, which is not the same as coconut oil, is the fruit of a palm found mainly in West Africa, especially Nigeria, but now cultivated also in the East Indies. The fruit suggests a peach with an outer pulp surrounding a nut, but unlike the peach it grows in huge dropping clusters. The oil comes mainly from the pulp, but another sort comes from the kernel. Both kinds are used like coconut oil for soap, candles, lubricants, and in making the substitute for butter known as margarine. Palm nuts are one of the few tropical products whose main production is largely in the hands of natives. In general there is a strong tendency for the production of plantation products to be concentrated in the hands of Europeans and Americans. This is partly because the natives are likely to bring to market small quantities which cannot easily be sold, and also because they are not careful to raise nuts of uniform quality. It will be interesting to see whether the oil palm will continue to be raised by natives or will gradually fall into the hands of plantation owners, as has already happened to a large extent in the Dutch East Indies.

649. *Fibers.* Warm regions are the great producers of fibers as well as of oils. Aside from flax and hemp all the important fibers come from regions at least as warm as the Cotton Belt of the United States. Several of the most important fibers belong entirely to the Regions of Wet Tropical Agriculture. The most important of these is jute, which stands next to cotton as the most widely used fiber derived from plants. Its popularity depends partly on the ease with which it can be raised and prepared for market. It is almost the only material used for burlap, gunnysacks, and other bagging material. Jute can be grown in many countries, but no other region seems to combine so many advantages as eastern Bengal near Calcutta. Tropical heat, periodical floods of the Ganges, cheap and reasonably steady labor, and a location close to a busy port all combine to make jute one of India's big exports. The United States sometimes imports 100,000 tons per year. Jute matures so rapidly that three months after the seedlings have been transplanted like rice, they are 10 feet or more in height, and are ready for harvest. This helps to make jute cheap.

650. Among the other fibers of Regions of Wet Tropical Agriculture, manila hemp, from the bananalike abaca plant, resembles jute in having a long fiber, but is much stronger, and better for cordage. It needs a peculiar combination of warmth, moisture, and deep, loamy, fertile, loose-textured and well-drained soil such as is found in parts of the Philippines. If a good decorticator, or machine for separating the fiber from the surrounding pulp, should be invented for abaca,

it would probably soon put sisal and henequen out of the running. Coir fiber, for brushes, brooms, doormats, stair carpets, and other coarse articles, is derived from a rough husk on the outside of coconuts, and is exported from the same Malaysian regions which export copra. **Kapok**, or *tree cotton*, is obtained from the seed vessels of the so-called silk cotton tree, chiefly in Java. Some is used for filling life preservers, but more for such purposes as mattresses.

651. SUGAR. As we follow the items downward in A624 our conviction of the importance of the Regions of Wet Tropical Agriculture increases. One single product of those regions, namely, cane sugar, supplies more than 7 per cent of the value of imports into the continental part of the United States. This is a huge amount for one single article. Only rubber ranks higher. Sugar is probably the most widely used of all food products. It supplies almost the same needs as potatoes, and produces even more food per acre. Measured in calories, a pound of sugar is equivalent to nearly 6 pounds of potatoes. Although sugar cane is raised in minor subtropical areas such as Louisiana, southern Spain, and southern China, it does best in typical Regions of Wet Tropical Agriculture. Fairly dry weather during the ripening of the cane greatly increases its sugar content. With the exception of irrigated areas the great producing regions have high atmospheric humidity, an annual rainfall of more than 40 inches, and a temperature of at least 80° during the warmest month. Compare A602 with A618, and note how well the distribution of cane sugar agrees with that of Regions of Wet Tropical Agriculture. The only important exceptions are Louisiana and four areas where sugar is raised by means of irrigation, namely, Egypt, northwestern India, Peru, and northern Argentina. Large areas where the climate is right do not have the proper soil. The leading producers are British India, Cuba, Java, the Philippines, Hawaii, Formosa, Puerto Rico, and Australia. The chief exporters were formerly Cuba and Java, but the Javanese sugar industry was largely ruined by the depression between 1930 and 1940. More than two fifths of the world's cane sugar is produced in the western hemisphere, and about one third in the Caribbean area alone. The most important importers are the United States, the United Kingdom, and Japan. Although India is one of the greatest producers of cane sugar, it does not supply it own needs, and is the fourth greatest importer. In Cuba so much of the good corn land is given up to sugar that the island is one of the most conspicuous importers of foodstuffs. Sugar is the one basic kind of food, as distinguished from luxuries such as tea and coffee, that is exported in huge quantities from tropical to nontropical lands.

652. *The Sugar Plantations of Cuba* The sugar industry in Cuba illustrates not only the general principles of plantation agriculture, but also the dangers of one-crop agriculture and of political uncertainty This one island supplies one sixth of the world's cane sugar and one tenth of all the sugar, including both beet and cane Excepting some of the smaller Caribbean islands, such as Barbados, no other area is so largely given over to sugar production The prosperity and political stability of Cuba are bound up in this one crop The laborers who work on the sugar plantations raise so little food, and other Cuban farmers produce so small a surplus, that Cuba imports large quantities of food in exchange for her sugar. Each year, if the price of sugar is fairly high, Cuba sends to the United States $75 or $100 worth per Cuban family. Usually a third of this, and often more, is used to pay for imported food, which the Cubans themselves might raise In estimating the importance of these figures it must be remembered that half the Cuban families probably live on an income of less than $200 per year, and many on less than $100 Because Cuba is so dependent on imports, she is vitally interested not only in the price of corn, wheat, and pork in the United States, but also in the success of the Newfoundland fisheries, which supply the Cuban tables, especially on Fridays The Cubans are also concerned about such matters as troubles in American oil fields whence comes fuel for tractors, and in the strikes of the Pittsburgh district whence comes heavy machinery for the centrals, or sugar factories, and rails for the tramways that ramify widely in the cane fields She is still more vitally interested in changes in the American tariff and in the recent revival of the sugar-beet industry in Europe. Such changes, together with quota allotments and financial depression, reduced the sugar production of Cuba so that in 1933 it was only 37 per cent as great as in 1929, and even in 1939 the production was far below that of 10 years earlier Since prices, as well as production, fell off greatly from 1932 to 1934 the Cubans were in a very bad plight, from which they have not yet wholly recovered.

653 Variations in the price of sugar from year to year have a great effect on Cuba During the World War, competition from the beet sugar of Europe and from the sugar regions of southern Asia was largely cut off This caused a period of high prices in Cuba, with resultant prosperity and great expansion of the industry Money was spent lavishly, many fine houses were built, and others were started only to be left uncompleted when the slump finally arrived This happened when beet sugar came back into the market, when other cane-sugar areas such as Java became serious competitors, and when

the United States tariff was raised The people of Cuba suddenly found that from a standard of living that was high for tropical people they were thrown down to a condition of poverty and misery Political discontent and rebellion broke out. Such conditions are likely to occur in areas which depend upon a single crop If only a quarter, instead of three quarters, of Cuba's exports consisted of sugar, that island, presumably, would be much happier Nevertheless, if the demand for sugar should again increase, as it probably will, Cuba is likely to continue to raise sugar, for few places are better adapted to this crop not only in climate and soil, but also in location near a good market

TABLE 24

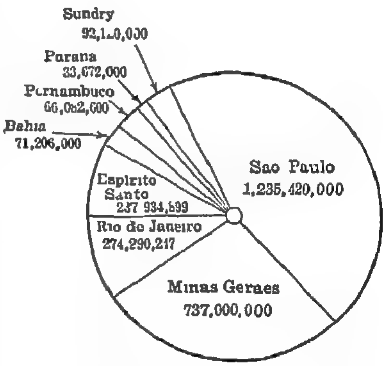
PER CAPITA CONSUMPTION OF SUGAR

Country	Pounds per capita per annum	Country	Pounds per capita per annum
Australia	128	Switzerland	86
Hawaii	121	Italy	20
Denmark	112	Egypt	18
United States	112	U S S R	18
Cuba	97	Haiti and Santo Domingo	9
Great Britain	92	China	6
Canada	90		

654 *The Consumption of Sugar* Table 24 gives samples of some of the greatest and some of the smallest consumers of sugar per capita under normal conditions The wide differences there shown depend mainly on economic conditions but also in part on the abundance of local supplies of sugar and on local customs or industries such as the canning of pineapples in Hawaii Consumption is high in western Europe and in all English-speaking countries It is also high in Hawaii and Cuba where much sugar is produced It is low in countries such as Italy and Russia which produce little sugar and cannot afford to import much It is also low in Egypt, Haiti, and Santo Domingo, where people cannot afford it, even though sugar is produced locally Southern China raises a little sugar cane, but northern China, as well as large sections of India, contains tens of millions to whom sugar is a rare luxury From all this it appears that, in spite of the competition of beet sugar, the production of sugar cane makes the Regions of Wet Tropical Agriculture of great importance to many of the most advanced nations That is one reason why the United States has invested so much money in Cuba, and has so strong

a political interest there For similar reasons Great Britain is interested in the West Indies and Japan in Formosa

655. THREE STIMULANTS I *Coffee* We come now to the last two items in A624 Nothing more need be said about rubber, for it has been fully discussed in Chapter I The fact that the three main stimulants, coffee, cocoa, and tea, account for almost a tenth of the value of the imports into the United States illustrates the high buying power of the country It also shows how dependent we are upon Regions of Wet Tropical Agriculture for some of our most comforting luxuries Coffee illustrates a number of geographical principles For example, it illustrates the way in which *the optimum of a crop varies in response not only to the selection of varieties by man, but also to diseases and pests* Coffee was introduced into Europe from Arabia in 1650, but of course it could not be raised there. Accordingly the Europeans took it to Java and Ceylon, which for nearly two hundred years were the main coffee areas of the world "Java coffee" is still famous, although it no longer comes from there About 1875 a leaf blight ruined the plantations of Ceylon, and spread to India and Java These regions thereupon turned to tea and other plantation crops, or to hardier types of coffee, as in East Java. Meanwhile, coffee had been introduced in the Caribbean region and especially in Brazil, which now produces most of the world's supply It grows best in Regions of Wet Tropical Agriculture with a rainfall of 50 to 100 inches.



A656—Coffee Trees in Provinces of Brazil

656 Coffee also illustrates a principle with which we are now familiar, namely, that crops tend to do best near their cooler limits The coffees of the finest flavor are largely produced at considerable altitudes Transportation is often costly in such places, but the delicate flavor of the upland coffee of Colombia and Guatemala, for example, raises the price enough to offset this disadvantage Again, southern Brazil is by far the greatest producer of coffee, and the production there is almost entirely upon cool plateaus, where the temperature in July, the coldest month, averages about 60°. Moreover, by far the greatest production is in the relatively southern province of São Paulo (A656), where frosts are not unknown. More significant

than this, however, is the fact that in São Paulo the production per tree is greater than in any other Brazilian province except Bahia, near the equator, where it is about the same, and Paraná, where the production is nearly three times that of São Paulo. Paraná lies south of São Paulo at the coldward margin of coffee culture. Another advantage of the cooler regions is that, since there is a distinct cool season, all the coffee berries ripen at about the same time. Hence there is need of only one picking, instead of several as in the tropical regions.

657. A political factor in coffee-raising is the so-called valorization scheme. For a while the coffee plantations in Brazil were so profitable that far too many trees were set out. Therefore, the supply of coffee became so great that prices dropped to ruinously low levels. Accordingly, at various times the Brazilian government has intervened, buying coffee from the planters and storing it in government warehouses in hope of selling it later at a profit. This profit, however, has often failed to materialize, and sometimes hundreds of thousands of sacks of the poorer coffee have been piled along the railroad tracks and burned, or else thrown into the sea. The Brazilian government has spent millions of dollars in this way without much success—another example of the danger of relying too much on one product.

658. II. *Cacao.* Although cacao was introduced into Europe early in the sixteenth century, it was long a Spanish monopoly and far too expensive for general use. Even in the early nineteenth century the entire European consumption was probably less than 12,000 tons. In recent years, however, there has been an extraordinary increase in output accompanied by a shift in the areas of production from the Caribbean region and Ecuador to Brazil, and especially to the African Gold Coast and Nigeria. In 1900 the world production of cacao was about 150,000 tons per year, now 600,000 is not uncommon. The cacao tree bears well only in warm, humid regions, chiefly within 12° or 15° of the equator. In fact cacao might almost be called a product of the Equatorial Rainforest. The yield is best where there is little wind, for dry winds injure the growth of the heavy green, red, or brown pods which are attached directly to the main branches and trunks, and strong winds often break them. Hence cacao plantations are largely restricted to relatively calm districts free from hurricanes. There they usually occupy protected valleys and small clearings in the equatorial forest. Moreover, they are often protected by taller trees which furnish shade and diminish the dangers from both sun and wind. In Trinidad, for example, the pink blossoms of these trees present a superb aspect when seen above the cliffs that border the northern side of the island.

659. The shift in production of cacao from American to African

regions has been accompanied by a corresponding shift from large plantations owned mainly by foreigners to small holdings owned by natives. This means that in the production of cacao, as well as of palm oil, the natives of the Guinea Coast of Africa hold the unique position of being the main producers of an important crop which is raised almost entirely for export. Whether the natives are more competent than those of other tropical regions, or whether they owe their success to the skillful guidance of their British rulers, is not certain. The Spaniards still drink more cocoa than most people, but the rapidly increasing demand for this delicious product comes largely from the use of chocolate in confectionery. The United States is the greatest consumer, taking more than 50 per cent of the world supply, while western Europe takes most of the balance.

660. III. *Tea*. This stimulant grows not only in the Region of Wet Tropical Agriculture, but also in the Mild East Coast Regions (Plate I). It is borne on a hardy bush, which is probably a native of the highlands of southern China. From there it has spread equatorward as far as Java, westward to Ceylon and Assam, and northward to southern Japan. Nevertheless the greatest production and local use of tea are still found in southern China. Many Australians do indeed drink tea five or even seven times a day, but even to them tea is not quite so great a necessity as to the multitudes of Chinese and Japanese who drink practically nothing else. This condition arises partly from the danger of infection through drinking unboiled water in regions of dense population where sewage is used on the fields. It also arises partly from the ease with which tea can be raised on slopes too steep for other crops. The fact that tea is so universally grown in southern China has had a bad effect on that country's foreign tea trade. A century ago China was the great source of tea for all the western world. There was no room for large plantations in China, however, and no incentive for Europeans to go there as tea-raisers. The Chinese farmers usually produced only small quantities of unsorted tea with no separation according to grades of quality. Accordingly, European planters in Assam were able to cut deeply into the Chinese tea trade. Then, when coffee suffered from disease in Ceylon and Java, tea took its place. There the moist climate is better for tea than for coffee, which needs a dry season. New tea areas were also developed in Sumatra and Formosa, and Japan began to export its unfermented green tea, which is stronger than the fermented black tea. Thus China became a minor factor as an exporter of tea.

661. CITIES. A dense population and valuable products cause the Regions of Wet Tropical Agriculture to be well supplied with

cities. Eighteen have populations above 250,000 As one gets away from the Equatorial Rainforest the size of these cities increases Thus the seven within 12° of the equator (Singapore, Colombo, Batavia, Surabaya, Belém, Recife, and São Salvador) average about 350,000 inhabitants, the four between 12° and 18° (Madras, Rangoon, Bangkok, Manila) average about 700,000, and the seven in higher latitudes (Bombay, Calcutta, Hongkong, Canton, Habana, Rio de Janeiro, and São Paulo) average over a million, and none has much less than 600,000 It is interesting to note that, aside from Bombay in latitude 19°, these larger cities all lie close to the tropics at latitudes of 22° or 23°. The fact that the lowest latitudes do not favor the growth of large cities becomes still more evident when we note that Hongkong, Colombo, and especially Singapore owe much of their growth to the fact that they are way stations on great routes between more active parts of the world in higher latitudes.

662 One of the noteworthy facts about these eighteen tropical cities is that all except São Paulo are seaports This is partly because the Regions of Wet Tropical Agriculture lie mainly along the seacoasts, as is evident in Plate I, but it is also because the cities in this part of the world are almost purely commercial and governmental with little industrial, financial, or cultural activity Many are closely associated with the export of some particular product Calcutta with jute, Bombay with cotton, Bangkok and Rangoon with rice, Manila with manila hemp and sugar, Surabaya, Habana, and Recife with sugar, Singapore and formerly Belém with rubber, and São Paulo and Rio de Janeiro with coffee. Although São Paulo is not a seaport, Santos, its port, may almost be counted as a distant suburb of this rapidly growing coffee city São Salvador in Brazil is quite a cacao port, although sugar and hides are also important Bananas are the only main product of tropical plantations not represented in this discussion of cities They are so perishable that they cannot be collected at any one great port, but must be picked up by ships at many small ports close to the plantations Batavia, Canton, Hongkong, Madras, and Rio de Janeiro do not specialize in definite products so much as most of the other cities in regions of this type All except Hongkong have grown large because they are centers of both trade and government for large numbers of people

663 *Manufacturing* Factories are of minor importance in Regions of Wet Tropical Agriculture. São Paulo, with its textile and other factories, comes nearer than any of the other cities to being a manufacturing center. This is just what would be expected, for São Paulo lies not only about 2,400 feet above the sea, but also on the border of

the tropics so that it has comfortably cool winters and not uncomfortable summers. Bombay comes next as a manufacturing place, but its cotton mills give occupation to only a small part of the population. In the other cities there is a certain amount of simple manufacturing in the way of preparing raw materials for market. Many people likewise engage in primitive industries which they carry on in their homes, or in the little shops of silversmiths, embroiderers, coppersmiths, wood-carvers, ivory-carvers, shoemakers, and the like, who lend to the bazaars of such cities a large part of their charm. Moreover, in modern days the community industries in these tropical cities include not only primitive bread-making, carpentry, and masonwork but the supplying of electricity, gas, and transportation according to European methods. Yet, in spite of all this, there is little real manufacturing. Hence the trade of all these tropical cities consists mainly of the export of plantation products, or in some cases of hides and forest products, and the import of machinery, cloth, motor vehicles, and sometimes cereals from regions in higher latitudes.

664. The White Man. Except in the more temperate regions of southern Brazil and Natal, where the white man forms an important part of the population, the white man's task in wet tropical agriculture is mainly one of supervision. His numbers, therefore, are small compared with the native population. But even for this work of supervising he often needs fresh vitality, for the warm, monotonous climate saps his energy. He can get this temporarily by going up into the mountains where lower temperatures prevail, but to escape the monotony he has to go back to the area of cyclonic storms. So one of the rules for the white employees of private concerns, as well as governments, is that regular and frequent vacations shall be given.

665. An exceptional situation in respect to white men in the Regions of Wet Tropical Agriculture is found in northern Queensland. There about 200,000 white people from the races of northern Europe do all sorts of work under more or less tropical conditions. Protected by law against labor competition from colored races, these people are carrying on a unique experiment in colonization. Although the cost of production along almost every line is considerably higher than in tropical regions with colored labor, the results have been fairly successful. The deathrate, which is generally high in tropical regions, is here remarkably low, and the people work with considerable energy. This is explained partly by the fact that, inasmuch as practically the whole population is white, it has been possible to apply modern hygiene in such a way that the dangerous tropical diseases have been largely kept out. Still more important perhaps is the

fact that people rarely migrate to an area where the climate is known to be a serious handicap unless they have unusual energy and good health Moreover, if other types migrate to such places, the weaker and less successful generally move out again, leaving a group of outstanding energy and enterprise Thus, although the climate continues to be a burden on man's energy, the selection of the population permits a progressive community to maintain itself This does not prove that the white race can retain its full energy in Regions of Wet Tropical Agriculture It does show, however, that if tropical diseases are eliminated, if the right kind of people are chosen, and if the white people live apart from the natives and maintain their own cultural and moral standards, they can get along fairly well in these most productive parts of the tropics

CHAPTER XXVIII

THE MEDITERRANEAN TYPE

666 WEST VERSUS EAST Leaving the warmer parts of the Realm of High Productivity, where wet tropical agriculture prevails and plantations have newly been developed, we pass next to subtropical latitudes Here we find the natural regions which have longest been highly productive and economically active In Plate I they are defined as the Mediterranean Type on west coasts, and Mild East Coast Regions, sometimes called Humid Subtropical Regions, on the east In these regions, just as in those within the tropics, exposure to moisture-bearing winds from the ocean is a primary factor in determining the nature not only of the natural vegetation, but also of agriculture and the general aspects of economic development Aside from temperature, no phase of physical environment has a greater effect upon economic geography than the atmospheric movements which govern rainfall

667 It will be remembered that on the west side of each continent, as one goes away from the equator, the rainforest is gradually replaced by jungle, and this in turn by tropical scrub The scrub forest passes into savanna, which degenerates into steppes, and then into desert After 500 or 1,000 miles of desert, grassy steppes appear once more and then pass into forest at a distance of 30° more or less from the equator This Mediterranean forest consists of small scattered hardwoods like the laurel and olive, many of which are bushy The forest as a whole has hard, small, evergreen leaves and tough knotty wood The main agricultural activity is the raising of winter crops of wheat and barley, but this is supplemented by tree crops such as olives, nuts, and grapes, and by summer agriculture on small but intensively cultivated irrigated tracts.

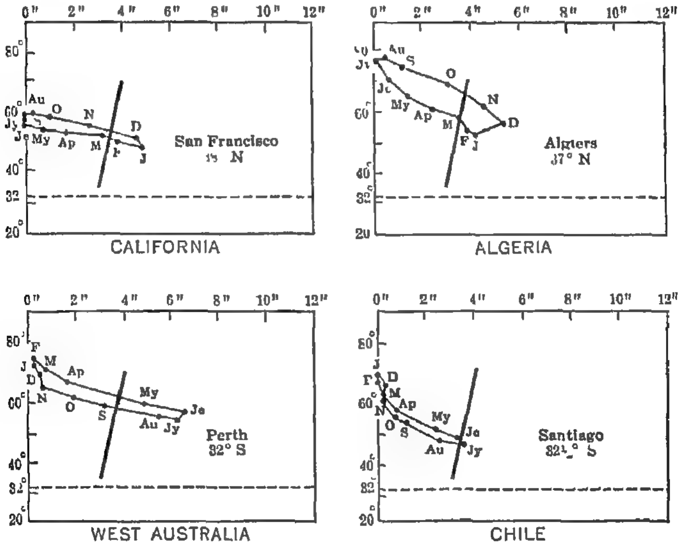
668 On the east side of the continents a different distribution of vegetation prevails Except in East Africa north of the equator, there is a gradual transition from tropical rainforest and wet tropical agriculture to temperate hardwood forests without any interruption by deserts, steppes, savannas, or even tropical scrub An intensive type of summer agriculture is dominant wherever the denser tropical rainforest gives place to jungle or temperate hardwoods Irrigation is

important here as well as on west coasts, but it takes the form of widespread summer irrigation for rice during periods of abundant rain, or else of irrigation in the spring on a small scale to start other crops before the rains arrive. The difference between the east and west coasts is brought out by the fact that, when people in the Mediterranean type of climate are gathering their main harvest from May to July, those in the Mild East Coast Region are either planting seed, setting out rice plants, or weeding beds of young seedlings. On the other hand, in the fall when the east coast people are harvesting many of their crops, the west coast farmers are plowing the land and getting ready to sow their main crop. With this contrast in the seasons of agriculture go many other contrasts in kinds of crops, abundance and nature of domestic animals, and methods of agriculture, architecture, clothing, and eating.

669. Each continent has at least a sample of both the Mediterranean and the Mild East Coast types of natural regions. The samples, however, vary greatly in size, degree of separation, and amount of difference. The larger the land mass and the higher the latitude into which it penetrates, the greater the contrast in practically all respects. In Plate I notice how the black area of the Mediterranean type on the west coast of each continent, counting Eurasia, of course, as a single continent, is balanced by a more lightly shaded area indicating Mild East Coast Regions. In the southern continents, and even in North America, one or both types of natural region is reduced to small dimensions. In South Africa both types are included in Cape Colony. They are very small, not sharply distinguished, and not separated by deserts or steppes. The economic as well as the climatic differences between them are so slight that most people scarcely know that they exist. In Australia the contrast between the Mediterranean climate, as found in West Australia and South Australia, and the Mild East Coast climate, as found in Victoria and New South Wales, is well known to Australians and has local importance, but the world as a whole thinks little about it. On the other hand, the majority of well-informed Americans realize that climatic differences make Chile and Argentina quite different. Few, however, realize how largely Chile owes its permanent strength to the inconspicuous little strip of unusually cool Mediterranean climate shown in Plate I. In the same way Argentina and Uruguay owe theirs mainly to the relatively small section with a Mild East Coast climate. This section dominates both countries politically, industrially, commercially, and agriculturally. The separation of the west coast and east coast types of climate by deserts and steppes as well as mountains is clearly evident in South America

This is a characteristic feature of latitudes 30° to 40° provided a continent extends into latitudes above 40°

670 In North America the contrast between California with its oranges, grapes, prunes, vegetables, and Hollywood, and the South Atlantic and Gulf states with their corn, cotton, tobacco, and Mardi Gras, is too well known to need comment Notice the width of the grassland and semi-desert which separate them in Plate I Finally the Eurasian countries of Spain, Italy, Greece, and Turkey are so different from China and Japan that few people would think of classing them together Their separation by thousands of miles of desert, steppe, and mountains has helped to make them highly different in race,



A671—Climographs of Regions with Mediterranean Type of Climate

customs, and historical development Nevertheless, even if they were next door to one another, the difference in their seasons of rainfall and in the plants, animals, and methods of agriculture which fit the climate would make them different, even if inhabited by people of the same race, language, and culture

671 THE MEDITERRANEAN TYPE OF CLIMATE *Summer Drought and Winter Rains* The primary difference between west and east coasts in latitudes 30° to 40° arises from the contrast in rainfall (A671 and A694) West coasts get abundant rain in winter and little or none in summer, east coasts get a great deal in summer and less in winter. In summer the Mediterranean or Dry Subtropical sections of all continents are under the influence either of the belt of subtropical high

pressure which causes a general downward movement of the air with little wind, or of the tradewinds which blow equatorward on the border of the high-pressure belt. Thus they have clear skies, and little or no rain. The rainless period lasts six months or more on the equatorward side of such regions, but declines to only a month or two on the poleward side. In fact northern Italy gets enough summer rain so that its climate is not typically Mediterranean. It is classified in this book as having a cyclonic climate, although it has a rather dry summer, and lies south of mountains which form a distinct climatic barrier.

672. In winter the Mediterranean type of climate gets rain because the belt of cyclonic storms migrates equatorward following the sun. Cyclonic storms are great eddies of air several hundred miles in diameter. They develop where bodies of relatively cool, dry air from higher latitudes (polar air masses) meet similar bodies of warm moist air (tropical air masses) which move from relatively warm oceans in lower latitudes. When such masses of air meet one another, rain ensues because the cold air, being relatively heavy, acts as a barrier over which the warm air rises almost as if it had encountered a plateau. As the tropical air rises, it becomes cool, thus often causing rain. Meanwhile the cool air mass, which is usually moving toward lower latitudes, gradually pushes its way under the warm mass. Hence toward the end of a storm the warm air usually disappears. The cold air which displaces it may cause snow flurries at the end of a rain in winter. Its arrival accounts for the fact that most storms in middle latitudes end with cooler weather, and sometimes with cold waves. Thus storms provide variability in addition to the benefits arising from rain and humidity. Sometimes the opposing masses of air meet in such a way that their meeting place remains almost stationary, and rain keeps falling day after day. Such conditions often lead to floods. Far more commonly both masses of air have an obliquely eastward movement. Hence their meeting place moves forward toward the east in a more or less curved path. Beyond the Atlantic some storms travel across central and northern Europe, but tend to fade away toward Asia. Others swing across France to northern Italy and then toward the east, but with diminishing vigor. The intensification and southward shifting of the storm belt brings at least a little winter rain to the whole region around the Mediterranean Sea, including even the Nile Delta. A similar equatorward shifting of the meeting place of polar and tropical air masses gives rise to winter rains in regions having the Mediterranean type of climate in the other continents.

673. *Characteristics of the Mediterranean Type of Climate* When cyclonic storms shift equatorward during the winter in this climate where so many great nations have arisen, frequent rains alternate with blue sky and warm sun to make nature look fresh and charming Average monthly temperatures of 45° to 55° with noonday temperatures of 50° to 70° are cool enough to be invigorating, but are seldom accompanied by killing frost In summer, however, the poleward migration of the belts of tropical high pressure brings heat and drought, and for months the sun burns down from a pale blue sky through a dust-laden atmosphere Only near the coast do sea breezes temper these conditions Where the ocean temperature is abnormally low, however, because of the rising of the water from great depths, as happens in California, Chile, and Morocco, the onshore winds may make the summer delightful The coolness of the water is due partly to cool currents which flow equatorward from higher latitudes on west coasts that are exposed to the open ocean It is also due in no small degree to the fact that the rotation of the earth deflects all moving objects to the right in the northern hemisphere and to the left in the southern Hence the currents tend to swing away from the coast, and this permits cold water to well up from the depths of the ocean close to the coast Even on coasts that are cooled in these two ways the dryness of the summer continues to be a handicap, though tempered slightly by heavy coastal dews, or still more by fogs On the Pacific Coast of the United States from about latitude 35° northward the coast for a few miles inland is shrouded with fog much of the time in summer As the sea breezes induced by the heating of the land cross the strip of cool water near the coast they are chilled and their vapor is condensed into fog In the morning, especially, this blows over the land, thus lowering the temperature both because the inblowing air is cool and because the sunlight is kept out

674 The coolness, humidity, and variability thus introduced are important elements in making this coastal climate healthful and invigorating In Chile the city of Valparaiso, in latitude 33°, lies about 1,500 feet lower and half a degree nearer the equator than Santiago, the capital If the temperature depended only on these two conditions, one would expect January, the warmest month, to be about 5° warmer at Valparaiso than at Santiago. Another condition, however, enters into the matter Valparaiso lies on the seacoast, and is cooled by sea breezes, whereas Santiago lies 60 miles inland As a result Valparaiso is cooled so much that its January temperature (64°) is lower than that of Santiago (69°) instead of being 5° higher Now compare these Chilean cities with Beirut, a little farther from the

fall, it is ready to harvest at the beginning of the dry season It is grown intensively on alluvial plains such as that of the Po Basin where high yields can be obtained because of the relatively favorable rainfall and temperature and the consequent efficacy of fertilizers It is grown extensively without much preparation of the soil, or else is replaced by barley, in dry sections where the yield is lower and few other crops can be raised. Wheat fields are found in the valleys of the Coast Range and other parts of California, in the coastal zone of Morocco, in the valleys and uplands of Palestine and Syria, and on the basin-shaped plateau of Anatolia A wheat belt runs through southwestern Australia between the isohyets of 25 and 15 inches of rain, and wheat is the main crop of Chile Corn partly replaces wheat where the Mediterranean type of climate merges into the cyclonic type so that there is some summer rain and plenty in spring and fall, as in the Po Basin Where irrigation can be practiced, the Mediterranean type of climate permits a great variety of crops such as rice in the Po Basin and California, cotton in the Cilician Plain of southeastern Asia Minor, and even sugar cane on the south coast of Spain. Irrigation is also practiced to a great extent in raising fruit It has been common since ancient times in spots where water and level land are available

678. Land Utilization In most of the regions where the Mediterranean type of winter rain prevails, only a small percentage of the land can be profitably cultivated. Mountains occupy a great deal of space. Then, too, the long, hot summers make the growing season unduly short in many places where the soil dries quickly because it is thin or sandy or slopes toward the noonday sun. Moreover, the sparsity of vegetation in such places during the dry summers has allowed a great deal of soil to be washed away by the heavy winter showers, thus leaving only scattered pockets of soil, as on many of the hills of Palestine The abundance of uncultivated land, together with the nutritious, winter-grown grass, and the comparative freedom of the winter from frost and snow, have led to the extensive raising of sheep and goats, and to a fairly widespread use of cattle. Throughout the whole Mediterranean area from Spain to Palestine and on to Persia almost every village has flocks and herds which are driven out to graze each day The milk of goats and sheep, as well as cattle, is widely used, but generally in the sour state or as cheese rather than as fresh milk or butter

679 Rugged relief, eroded slopes, and dry summers with a greatly reduced water supply cause the economic life of regions of the Mediterranean type to be concentrated to an unusual degree in val-

leys, mountain-girt basins, and the hillward borders of the larger plains. People are kept away from the more rugged sections not only by the difficulty of cultivating slopes, but also by the fact that winter rains following summer droughts have in many places washed away much of the soil, especially during certain centuries, such as the seventh of our era, which were unusually dry. Away from the mountains large sections, even in plains with fine soil, cannot support many people because of lack of water in the dry season. This is the case in the Syrian plain near Aleppo, on the great central plain of Anatolia, in most parts of Persia, in Greece, Chile, and Australia, and in much of the Great Valley of California. Winter floods cause other sections to be swampy and malarial, as in Boeotia north of Athens, and in the coastal plain of western Italy.

680. Another feature of life in Mediterranean climates arises from the presence of extensive tracts of grassy land too dry, too hilly, or too high and cold for cultivation. This has fostered the raising of livestock, especially sheep and goats, and a great development of seasonal migrations—transhumance. During the winter the flocks and herds find plenty to eat close to the villages. When the dry summer arrives the supply of fresh grass soon disappears and all the dry grass is soon consumed. Meanwhile among the mountains the heavier rainfall and lower temperature have produced abundant grass, which keeps on growing during much of the summer. So from thousands of villages the flocks and herds are driven up to mountain pastures.

681. Cities and Industries. On the whole, the regions with the Mediterranean type of climate are fairly densely populated, but not to such a degree as the Regions of Wet Tropical Agriculture or the Mild East Coast Regions. They supply a considerable amount of material for world trade in the form of fruits, olive oil, wheat, wool, hides, and lumber. Some sections also supply metallic ores and petroleum, but none have much coal. Large cities are fairly numerous. Lisbon in Portugal, Seville, Madrid, Valencia, and Barcelona in Spain, Marseille and Nice in France, Genoa, Florence, Rome, Naples, Palermo, and Catania in Italy, Athens and Saloniki in Greece, Istanbul and Smyrna in Turkey, Tiflis in the Caucasus, Beirut in Syria, Algiers in Algeria, San Francisco, Oakland, and Los Angeles in the United States; Santiago and Valparaiso in Chile, Capetown in South Africa, and Adelaide in Australia. Only six—Seville, Madrid, Florence, Rome, Tiflis, and Santiago—are inland cities. The remaining twenty are seaports, or lie so close to the sea that they have seaport suburbs like those of Athens, Los Angeles, and even Rome. Only two—Rome and Los Angeles—have more than a million inhabitants, but Naples ap-

proaches this size, and the metropolitan district of San Francisco and Oakland much exceeds it. These cities all carry on a considerable amount of complex manufacturing. Yet only a few, such as Barcelona, can be classified as primarily industrial rather than commercial. Both San Francisco and Los Angeles, although originally of the commercial type, have now built up large and varied types of manufacturing. The Los Angeles region, because of the neighboring oil fields, is one of the world's greatest oil-refining centers. It should be noted that the cities which carry on the most manufacturing in Italy, namely, Milan and Turin, belong to the cyclonic type of climate, and are not included in the list given above. On the whole, the regions having the Mediterranean type of climate now import manufactured goods far more than they export them. The present relatively slight development of manufacturing in Mediterranean lands is in part associated with the fact that, although the winters are sufficiently cool, humid, and variable to be highly invigorating, the hot, dry, monotonous summers remain a heavy handicap except in favored locations like the California coast.

682. ANCIENT GREATNESS. Centuries ago these same Mediterranean countries were the world's chief exporters of the simple manufactured goods of those days. To understand the concentration of the highest ancient civilization in this climate rather than farther north we need to recall that even as recently as the time of Christ mankind had not learned to live comfortably in what are now the most stimulating climates. At that time forests covered practically all of the Cyclonic regions. These could not be conquered until man discovered cheap and easy ways of making iron axes and learned to utilize beasts of burden, prepare warm clothing, and build warm, dry, and well-lighted houses. Thus, in those ancient periods, the Mediterranean region offered the best climate among the parts of the world which man had hitherto been able to conquer. Moreover, during ancient times many cenuturies appear to have been periods when the Mediterranean regions were more stormy and rainy than now. Hence they were especially favorable both for agriculture and health.

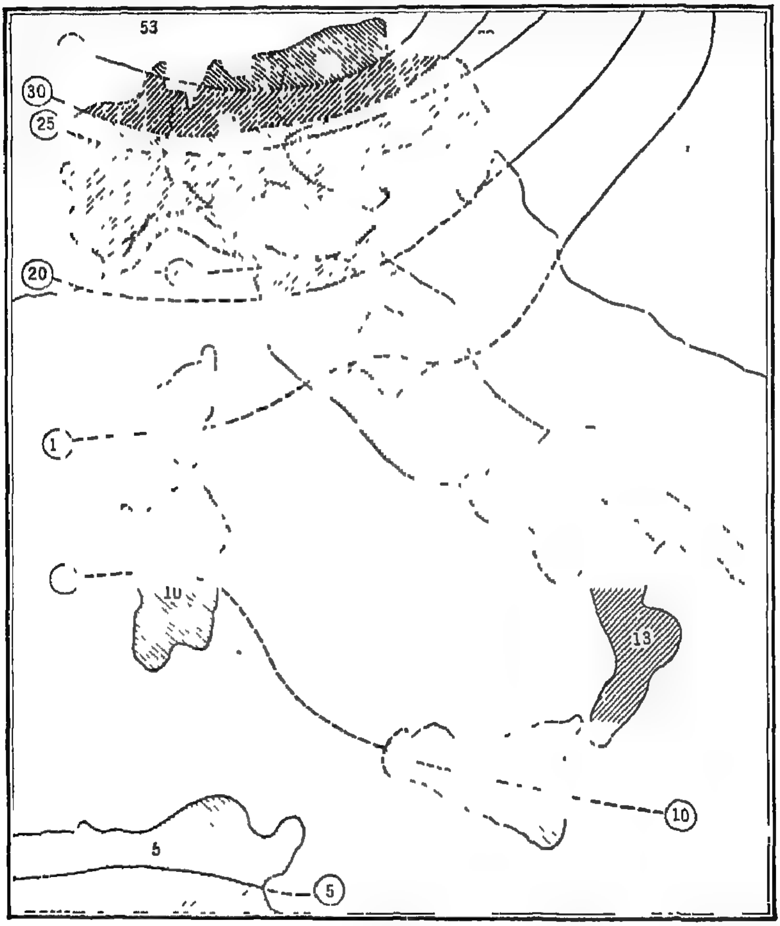
683. MODERN ATTRACTIVENESS. A recent economic asset of many regions with a Mediterranean climate is their attractiveness both for tourists and for people who want to settle down in a mild climate. California offers an interesting example of the way in which climate can thus become an economic asset. Thousands of people go there every year to find a pleasant place in which to spend their declining years. The motion-picture industry has made Hollywood its center largely because southern California affords not only fine scenery on

the seacoast and in the mountains, but also a delightful climate which permits outdoor photography at all seasons. The Riviera coast between Marseille and Genoa has advantages like those of southern California Its proximity to the great manufacturing area of Europe enables hundreds of thousands of people to make visits there in winter "Tourism," as the Europeans call it, is the chief business of this whole coast It is far more important than the specialized agriculture which provides fancy grapes, fruits and vegetables, together with a profusion of flowers for decoration and for the perfume industry Nice is primarily a tourist city, and Monte Carlo has practically no business except gambling and the care of tourists

684 CONTRASTS IN PRODUCTIVITY The magnitude and importance of contrasts in the productivity of the soil can be seen with unusual clearness in subtropical Mediterranean regions such as Italy and California A684 is a map of agricultural productivity per acre in Italy It is based on practically all crops grown there, and on records for many years Each crop has been counted as having the same value per pound regardless of the part of the country where it is raised Italy is especially good for such a study Although it is small and comparatively homogeneous in many respects, it differs sharply from north to south Northern Italy is an active, progressive, prosperous industrial region with a profitable type of agriculture Southern Italy is less active, poorer, and more conservative, and has a far less profitable agriculture This difference can be due only in small part to government, language, or general type of culture, for these are essentially the same all over the country Some people ascribe the difference to race, asserting that in the north an admixture of Nordics has given the Mediterranean stock unusual vigor, whereas in the south Greek and Arab blood has had the reverse effect No one, however, has ever presented convincing scientific evidence of this So far as we yet know it is quite possible that, if the northern and southern Italians were placed for several generations under identical conditions of physical and social environment, there would be no appreciable difference in their average abilities and achievements There is certainly more energy in the north, and agricultural methods are better, but the geographic environment, especially climate, seems to explain the major part of this

685 Before we explain the climatic factor a word should be said about soil and relief Locally the presence of rich volcanic soils, as on the slopes of Aetna, Vesuvius, and the volcanic hills near Rome and Florence, is of great importance So, too, are alluvial plains in the Po Valley and elsewhere. A684, however, bears little resemblance

to a map showing the quality of the soil, except as the soil is influenced by climate. In the same way it shows little resemblance to the relief except as the relief helps to cause relatively cool, rainy summers, and thus increases productivity, as appears in the dark shading of northern Italy. Another reason for thinking that A684 owes its appearance



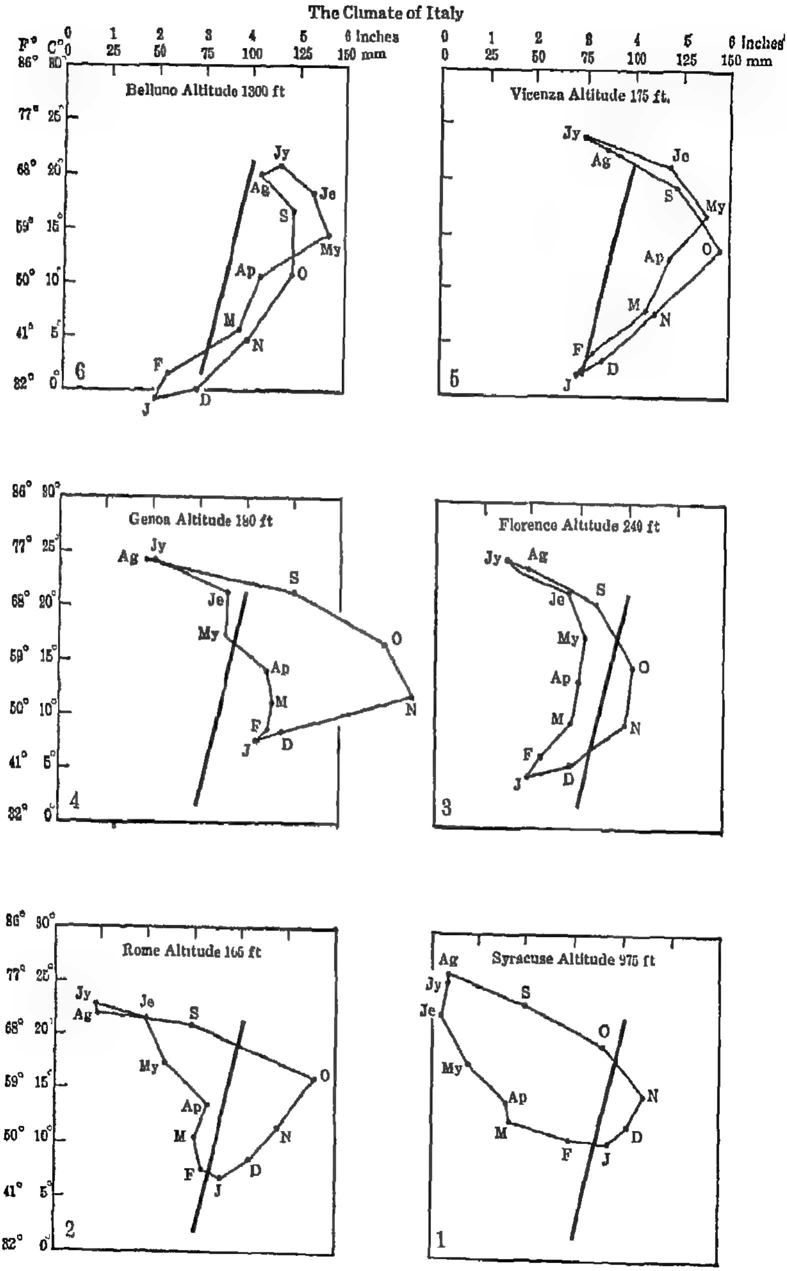
A684—Agricultural Productivity in Italy Based on Annual Crops, 1930-35

only in limited measure to soil, relief, or even methods of cultivation is the fact that there is a strong contrast not only between Sicily and the Alpine foothills, but also between the province of Lazio, where Rome is located, and the Apennine provinces slightly farther north. In A684 Lazio has an index number of only 12, whereas Tuscany, the next coastal province to the north, with Florence as its capital, has 18,

and Emilia, a little farther north, with Bologna as its capital, has 27. Racially, socially, and from the standpoint of soil, relief, and agricultural methods the differences between Lazio and the other provinces appear to be much too small to account for a difference of 50 per cent in productivity in 100 miles and over 100 per cent in 200 miles.

686. *Climate and Productivity.* Turning to climate we find a much clearer picture. The two darkest areas in A684 consist of mountains against which winds from the south deposit fairly abundant rain. They are cool compared with the rest of the country. Much more definite evidence is summed up in the six climographs of A686, which are arranged approximately according to their location on a map. Beginning at the bottom with the most southerly climograph (No. 1, Syracuse in Sicily), we at once note a great departure from the standard climate represented by the straight line. This largely explains the poor crops. The five months from October to February are the only ones having 3 inches or more of rain. March and April each have between 1 and 2, which is far too little for agriculture, especially in the spring. The four months from May to August are practically rainless, and July and August have an average temperature of almost 80°. This makes it almost impossible to raise summer crops without irrigation. It also reduces the yield of winter crops because it makes the ground too dry in both autumn and spring. Irrigation considerably increases the productivity of small areas, but the main crops of cereals cannot be irrigated because there is not water enough. The climate of Rome (No. 2 in A686) is better than that of Sicily. Nevertheless, May and June are so dry that there is great danger of drought, while July and August are hot and almost rainless, so that vegetation dries up badly.

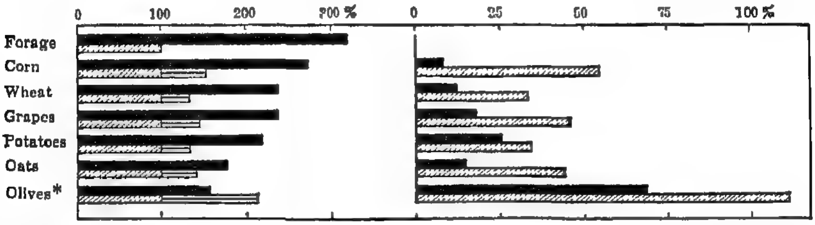
687. Between Rome and Florence (No. 3) a seemingly slight but really significant difference in climate is apparent. At Florence, unlike Rome, the critical months of May and June are as rainy as March and April, while July and August, although rather dry and warm, have frequent showers. Such a difference at the most important season accounts for much of the 50 per cent by which the productivity in Tuscany exceeds that in the region around Rome. The next climograph (No. 4) shows that at Genoa the danger of drought has almost disappeared, although July and August are still dry and warm in comparison with the standard as shown by the straight line. With this further improvement in climate the productivity rises to twice the level of the regions with dry summers farther south. Farther north the rainfall of Vicenza (No. 5) at the base of the Alps exceeds the standard except in midwinter and midsummer. Even in July and



A686—Climates of Italy

August, there are at least 3 inches of rain per month Such conditions account for an average productivity of 24 in the eastern Po Basin as a whole, and of 31 in the province of Lombardy farther west with Milan as its capital Finally at Belluno (No. 6) in an Alpine valley the productivity rises to 38, and the climograph approaches the standard more closely than any of the others The way in which agricultural productivity increases as the climate approaches the standard set by the world's largest cities and by our study of human vigor is extraordinary When the needs of both agriculture and man are taken into account, Belluno appears to have an almost ideal climate

688. *Nature's Part in Producing Good Crops* Before we make up our minds as to the relative parts played by nature and man in the contrasted productivity of the different parts of Italy, let us inquire into the yield of the main crops In A688 the average yield per acre



A688—Yield of Crops per Acre in Northern and Southern Italy Left, the solid bars show yield per acre in Lombardy expressed as percentage of yield in Calabria indicated by the more heavily shaded parts of lighter bars Right, percentage by which largest yield per acre from 1928 1937 exceeded average yield in Tuscany (solid bar) and Calabria (lighter bars)

* Tuscany and Sicily substituted for Lombardy and Calabria, because olives become scarce in North

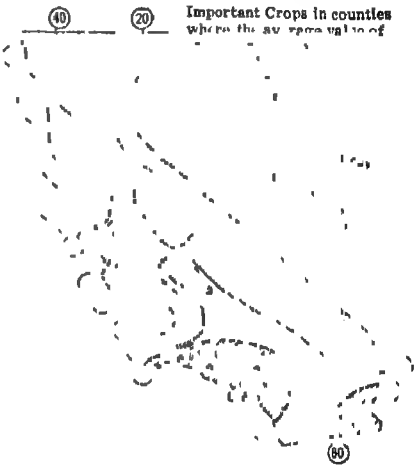
of seven crops during 10 years in Calabria, the southerly province in the "toe" of the Italian "boot," is called 100 per cent, and is represented by a bar shaded diagonally The corresponding yield per acre in Lombardy (Milan) is indicated by a solidly shaded bar The yield of forage per acre is more than 3 times as great in the north as in the south, the northern yields of corn, wheat, potatoes, and even grapes are more than twice as great as the southern, and that of oats reaches almost the same level Olives supply the most surprising feature of A688 In their case the provinces that are compared are Tuscany in the north and Sicily in the south Although olives are preeminently a crop of dry regions, the yield per acre in fairly well-watered Tuscany well to the north is about 60 per cent greater than in relatively dry Sicily in the south.

689 There is little difference in the way the seven crops of A688 are cultivated in the north and south of Italy, except that in the north the fields get better care and more fertilizer. They get more fertilizer largely because the greater rainfall of the north encourages the raising of cattle. When allowance is made for fertilization and cultivation, it seems clear that, if climate were the only variable factor, Lombardy would have a productivity nearly twice that of Calabria. The productivity of Lombardy (31), however, is more than twice that of Calabria (13). This is partly because in northern Italy a greater proportion of the land is devoted to the more valuable kinds of crops. It is a general rule that, *where the climate is favorable, the variety of crops and the percentage of the land devoted to the more valuable crops both tend to increase.*

690 *The Handicap of Unreliable Crops.* The horizontally shaded additions to the lightly shaded bars in the left-hand portion of A688 indicate the yield per acre in Calabria during the best of the years whose average is there shown. Even in the south the yield per acre evidently rises quite high if the weather is favorable, but only in the case of olives was the largest southern yield more than the average northern yield during the 10 years used in preparing A688. In the right-hand section of A688 the largest crop in the 10 years under discussion is shown as a percentage of the average. Except for potatoes, in every instance the diagonally shaded bars for south Italy are much longer than the solid bars for North Italy. This means that in the dry southern climate there are great variations above and below the average yield, while in the moister, cooler northern climate the variations are relatively small. Olives appear to be a highly variable crop no matter where they grow, as appears from their long bars. Variability in yield is evidently a great handicap in southern Italy, as it is in most climates that depart far from the optimum. This has a great influence on the degree to which the farmers improve their methods, for it tends to make people fatalistic, shiftless, and unthrifty. When they get a bumper crop, they feel rich and spend money recklessly, but afterward, when the next bad crop comes along, they feel discouraged.

691. *Indirect or Nonclimatic Factors.* It would be a mistake to leave this discussion of Italy without referring once more to the problem of human ability. There can be little doubt that the large yields per acre in north Italy are due in part to good cultivation. This is in accord with Chapter XV, in which we saw that, where nature helps the crops to grow well, the people are encouraged to make improvements. In addition to this, we saw in Chapter XXIII that where the climate is favorable people enjoy a degree of health and vigor which

aids greatly in making them work hard and employ good methods. Where the human as well as the agricultural conditions are favorable, the general level of progress naturally tends to be high, as happens in northern Italy. Hence although the sharp decline in the productivity of the land from the Alpine foreland southward in Italy is mainly due to the direct effect of climate on the crops, it is also partly due to the effect of both climate and crops on man. The greatest factor, so far as the crops are concerned, is the amount of rain from April to July or August. Almost equally important apparently, although this is rarely recognized, is the temperature. The largest crops are obtained where the summer temperature averages not much above 70° and the winters are cold enough for ice and snow. A strong indirect influence on productivity is exerted through the effect of the weather on human health and activity, and through the psychological influence of good crops in stimulating progress.



A692—Values of All Crops per Acre in California (1919 and 1929). Fruit and nuts have been estimated for 1919, but census figures are available for 1929.

692. CALIFORNIA COMPARED WITH ITALY. Although California and Italy both have the same general type of Mediterranean climate, and do not differ greatly in rainfall, the coolness of the Pacific Ocean and the height of the mountains give California a decided advantage. The nature of this advantage is suggested by a comparison of the relative distribution of productivity per acre in the two regions. A692 shows the average value per acre of all crops in California, including hay as well as fruit and nuts. In those coastal sections where California has little irrigation the average value per acre is relatively high in the north, approximately \$50 to \$70 per acre, and low in the south central region where San Luis Obispo County has no streams that can irrigate large areas. If there were no irrigation, the value of the crops per acre would presumably decrease from the more humid north to the drier south, much as in Italy. It would also tend strongly to decrease in all latitudes from the relatively cool and humid coast to the dry interior, as is clearly evident in the northern part of A692.

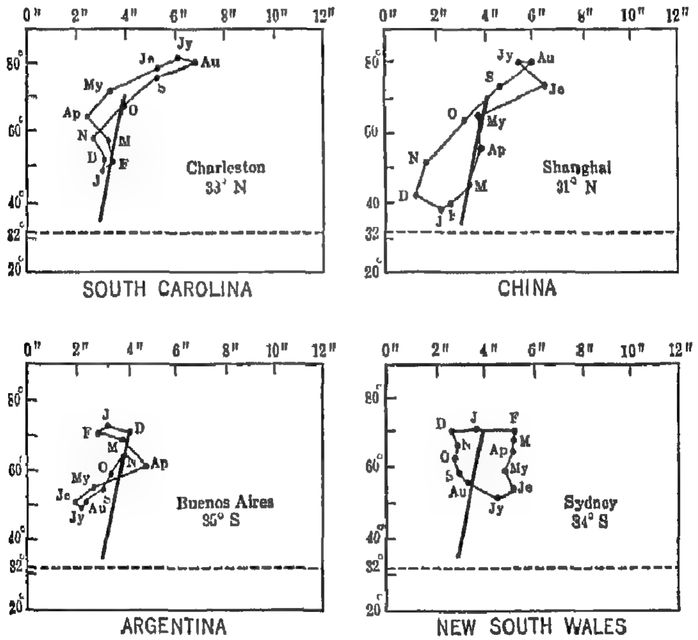
The height and location of the California mountains, however, and the relatively low temperature along the coast alter the distribution of productivity to the great advantage of the southern two thirds of California. The mountains, especially the lofty Sierras with their snows which last far into the summer, make it possible for California to have a higher percentage of irrigated land than any other state in the United States. The low temperature along the coast prevents the dry summer from injuring vegetation as it does in southern Italy. It also enables a given amount of water to irrigate a larger area than in the hotter interior. Moreover, the lower temperature favors the growth of certain valuable crops such as apples, fresh vegetables, beans, and flower seeds, as well as hay and forage.

693. Because of all this a large part of the two heavier types of shading in A692 is located along the coast and in the south. Near the northern coast, hay and apples (H and A in A692) are important crops. Farther south near San Francisco grapes (G), plums or prunes (*P*), and vegetables (*V*) are added to these coastal crops. In lower latitudes, still keeping near the coast, dry beans (*B*) and walnuts (*W*) become important in Santa Barbara and Ventura Counties. Then citrus fruits (C) rise into prominence around Los Angeles and San Diego. Another band of high productivity per acre lies along the western base of the Sierra Nevada in the great inner valley. Here productivity rises especially high in Sacramento and San Joaquin Counties, where grapes and vegetables are important, and farther south in Fresno and Tulare Counties, where grapes become a still more outstanding crop and are often associated with plums and allied fruits. East of the Sierras in the southeastern corner of the state the water of the Colorado River produces crops of high value per acre among which winter vegetables and cotton hold a prominent place. Thus the parts of California corresponding to the regions of lowest productivity in Italy are exceptionally productive. The greatest values per acre are found around Los Angeles and just south of San Francisco. The Fresno region comes third. In the Old World the Mediterranean regions which occupy the same latitude as the most productive parts of California include Algeria, Tunis, Sicily, and Greece, in all of which an average acre produces crops worth only about one third or one fourth as much as in northern Italy. Thus the mountains, with their rain and snow, and the ocean with its cooling breezes, give California a great advantage among regions having the Mediterranean type of climate.

Chapter XXIX

MILD EAST COAST REGIONS

694 Rainfall, Temperature, and Continentality The Mild East Coast Regions shown in Plate I—five of them, if Japan and southern China are counted as one—differ much more than the various regions of the Mediterranean type This is not because of temperature so much as because of rainfall The temperature does indeed vary a good deal, for regions of this type extend from latitude 23° in South



A694—Climograph of Mild East Coast Regions

China to 35°, or even 40°, if we include all of Honshu, the main island of Japan Japan, however, like northern Italy, is a transitional region with a climate that belongs in many ways to the cyclonic type In the main regions of the Mild East Coast type, the average temperature of the hottest months ranges from 70° south of Sydney in Australia to 83° at Hankow (A694), whereas in Mediterranean regions a range from 58° to 82° can be found The comparative uniformity

of temperature in the Mild East Coast Regions is due partly to the fact that in these latitudes the east side of every continent is washed by a warm current from the equator. Moreover, there is no welling up of cold water from the ocean depths to produce a coastal strip with cool summers, such as is found in California, Chile, and Morocco

695 All the Mild East Coast Regions normally have abundant summer rain in contrast to the almost rainless summers of Mediterranean lands All also have a fair amount of winter rain, as may be seen in A691. In the southern hemisphere the very small tract of this kind at the southern tip of Africa, and the larger tracts centering around Sydney in Australia and around Buenos Aires and Montevideo in South America, have almost the same amount of rain at all seasons. In the northern hemisphere, the large Mild East Coast Region in the southeastern quarter of the United States shows a decided excess of summer rain, but also a good amount in winter (Charleston in A691, also Houston) Central China (Shanghai) and Japan (Tokyo) have an excess of summer rain, but tend more toward winter droughts because of the scarcity of cyclonic storms This difference between North America and Asia is mainly due to the size of the continents

696 The effect of continentality is illustrated in Table 25, where six places, all lying on the seacoast and in practically the same latitude, are arranged in the order of their temperatures during the coldest month. Note how the summer temperatures decline as those of winter go up Cold winters are associated with hot summers Thus central China (Shanghai) has fairly cold winters with a good deal

TABLE 25

CLIMATE AND CONTINENTALITY

Place	Country	Latitude	Temperature		Rainfall	
			Coldest month	Warmest month	Total, inches	Percentage in warmer half year
Shanghai	China	31 12N	38°	80°	44	66
Kagoshima	Japan	31 35N	45	80	85	66
Savannah	U S	32 5N	52	82	49	64
Rio Grande do Sul	Brazil	32 10S	55	74	47	47
Port Maquarie	Australia	31 25S	55	73	61	58
East London	South Africa	33 0S	58	70	26	50

of frost, but its summers are very hot. Being also humid they are debilitating. Southern Japan (Kagoshima) is not quite so cold as central China in winter, and gets only a little frost, but its summers are as hot as those of central China and much more humid. Japan, however, gets enough cyclonic storms to relieve the monotony of the climate somewhat, and to be a distinct asset to health and energy. In the southern United States the winter temperature at Savannah, about a degree north of Shanghai, averages higher than in southern Japan, but cyclonic storms are more prevalent and bring more frequent and stimulating changes. In winter they carry frost to northern Florida. In this same latitude in the southern hemisphere, as represented by Rio Grande do Sul in southern Brazil and Port Maquarie north of Sydney in Australia, the seacoasts are free from frost, and yet the summers are not so hot as to bring much discomfort. East London, at the southern end of Africa, is so nearly surrounded by oceans that there is only 12° difference between the coldest and warmest months. Among places located on continents, in contrast to oceanic islands, it has one of the most uniform climates outside the tropics.

697. The rise in winter temperatures and the decline in summer temperatures from top to bottom of the preceding table mean that the places there named are arranged in the order of their continentality. Shanghai and Kagoshima, even though the latter lies on the southern end of an island, Kyushu, are dominated by the alternating monsoon winds which blow outward from the cold fastnesses of inner Asia in winter and inward from warm ocean currents in summer. Savannah, because of the size of North America, is similarly but less markedly influenced by outblowing and inblowing winds. The smaller bulk of South America and Australia, especially of the portions in latitudes higher than the Mild East Coast Regions, has only a slight effect in causing contrasts from season to season. At Rio Grande do Sul and Port Maquarie it permits a close approximation to oceanic uniformity. East London is dominated by the ocean rather than by Africa, which lies wholly to the north.

698. In the preceding table the total amount of rainfall shows the effect of continentality less clearly than does temperature, because rainfall is greatly influenced by local relief. Yet on the whole the precipitation is greatest where the continental indraft is greatest, and diminishes where this almost dies out in South Africa. The percentage of rain falling in summer shows more relation to continentality than the total amount. Among the stations of Table 25 the three in the big continents of Asia and North America get two thirds of the rain in the warmer half year, and over 40 per cent in three of the warmer

months Nevertheless, it is worth noting that all Mild East Coast Regions have the advantage of at least a moderate rainfall at all seasons

699 *Vegetation and Agriculture* This distribution of rain is highly important for vegetation It means that in a state of nature the Mild East Coast Regions are well covered with broadleaved forests (A60) Japan is densely forested, few countries give a greater impression of cloudiness and universal greenness, as is evident in Japanese art, with its trees and mountains characteristically rising out of mist In southern and central China the trees have indeed been largely cut off by man, but many mountains are well forested, and when a tract is protected, as in certain enclosures near Amoy and Swatow, big trees grow rapidly The mountains near Foochow carry pine forests. In Australia the eucalypts in the southern part of the Mild East Coast Region attain enormous size In the United States the southern states contain fine hardwood forests in the clayey or rugged sections and splendid pines on the sandy tracts

700 The abundance of rain at all seasons permits agriculture to thrive and allows a dense population to be supported Southern China and Japan are so densely populated that the farmers are forced not only to carry cultivation far up the slopes of the mountains, but also to be content with extremely small farms In Japan the average amount of cultivated land, not counting grassland, averages only about 2 2 acres for each farm family of 5 persons In the United States, the South contains the densest agricultural population of the country, especially in the sections where the percentage of Negroes is high. In the southern continents the density is not so high, but, in proportion to the level land available, it is higher than in almost any other sections of which the white man has recently taken possession.

701 The kinds of crops raised in the Mild East Coast Regions vary according to density of population and standards of living as well as rainfall and temperature. In southern China and Japan, high summer temperatures, heavy summer rain, and old established customs cause rice to be overwhelmingly the most important crop The large yield of rice per acre, as we have seen, is one reason for the extremely dense population. Sweet potatoes and beans are secondary food crops of considerable importance Most fruits do not grow well because of the excessive summer rain Tea, raised mainly on the hillsides, is the great luxury crop. The most important industrial crop in China and Japan is mulberry leaves for silkworms The long, warm, wet summers are ideal for this tree, which often sends out two or three sets of new leaves after the first crop has been stripped off for the

worms Accordingly most of the world's silk is raised here, and cocoons furnish the main source of ready money Cotton is also raised in considerable quantities in the Yangtze Valley

702 In the Mild East Coast section of the United States, corn takes the place of rice, partly because of a difference in climate, and partly because the old way of raising rice demanded much hard labor Silk and tea are not raised, for similar reasons Perhaps, they might be profitable if machine methods of harvesting them could be devised, as has been done for rice Cotton, however, has been so profitable that other crops have been neglected, and this region has become the world's main source of that crop Hence "Cotton Belt" is the name of one of the chief agricultural regions into which the United States is divided (A384) Cotton requires a great amount of cheap labor but less than silk and tea Many people doubt whether cotton would ever have become of great importance here had it not been that Negroes were introduced from Africa It is noteworthy that on the western edge of the Cotton Belt, where white farmers predominate, great efforts have been made to devise mechanical cotton pickers Success appears to have been attained in the relatively dry sections where the cotton bolls can be left on the plants until all are ripe without danger of becoming discolored, and where the cotton plants are short enough so that machines can drive over them Up to the present, however, it has been difficult to get the cotton clean after it has been picked by machines

703. In the Mild East Coast Region of Argentina and Uruguay, corn, alfalfa, cattle, and sheep are the chief products, although wheat is of great importance in the south In Australia the lower temperature and rainfall of the plateau which forms the main part of the Mild East Coast Region cause wheat and sheep, with some fruit, to be the main standbys.

704 Economic Habits *Australia.* The character of Australia as a whole is well typified in New South Wales, which forms the central part of the Mild East Coast section in this continent Along the coast lies a narrow plain fringed with winding bays where the land has been drowned a little General farming and dairying give this plain a pleasant, fruitful appearance, but agriculture is a small matter compared with the industries of the great city of Sydney and its beautiful suburbs scattered along many miles of winding waterfront. After work each day tens of thousands of workers hasten to the ferry landings to take boats to the suburbs Sydney, and also Brisbane, farther north near the border of the Mild East Coast Region, are not only European but British to an extraordinary degree. Yet the social

and moral tone is lighter and more buoyant than in England. Horse races and betting at all seasons, and innumerable picnics and parties, as well as a high degree of comfort and assertiveness, are some of the evidences of this. These qualities are partly due to the relatively adventurous and unconventional character of the immigrants who have chosen to come to this distant land. They are also due partly to the economic possibilities of a new continent with great mineral wealth, abundant land, and vast opportunities for raising wheat, wool, and cattle. In spite of the recurrent years of drought, people can prosper here because the population is sparse and the farms therefore large enough so that good years tide over the bad ones. Then, too, the scarcity of laborers has permitted the working people to insist on high minimum wages, short hours, old-age pensions, and assurance that their jobs cannot be taken from them suddenly. Hence there are few poor people in Australia, and a large percentage even of the poorer classes have homes of their own.

705. A little way inland from Sydney the deeply dissected escarpment of the plateau forms a rough tract which to this day is almost uninhabited. At its top the relatively cool and bracing plateau stretches westward in a succession of green rolling hills covered with wheat fields interspersed with patches of gray-green eucalyptus trees and occasional orchards and vineyards. Farther west, where the rainfall is less, still larger farms support huge flocks of sheep, like those of the adjacent Temperate Grasslands. This pleasant region of comfortable farms and prosperous villages illustrates the main source of the wealth which enables so many people to earn a living in the cities. If its farms should be broken up into smaller units, as many people desire, the surplus available for the support of Australia's high standards of living would diminish. Yet here, as in many other farming districts, a great many people are eager to get away from the farms and live in the city.

706. *Argentina, Uruguay, and Southern Brazil.* The South American part of the Mild East Coast Region resembles the Australian part in being thoroughly European in racial composition and in being highly urbanized, as appears in the two great cities of Buenos Aires and Montevideo, as well as in smaller cities such as Rosario. Another resemblance is that the wealth which supports the magnificent cities is derived largely from wheat, sheep, and cattle which thrive in outlying regions and are shipped through the great ports. As a supplementary source of wealth, however, the minerals of Australia are replaced by corn in Argentina. This is the natural result of the fact that the

South American plain extends far inland, thus giving warm and fairly moist summers over a large area

707 Although the wealth of Argentina, Uruguay, and the extreme southern tip of Brazil rivals that of Australia, it is distributed far more unequally The farms are even larger than those of Australia, and the owners rarely work them themselves Even more than the Australian landowners they congregate in the great cities In the past this has given the cities a relatively large number of wealthy and idle people whose mode of life has created the feeling that work is undesirable Labor has not been so scarce as in Australia, and the laboring people have never succeeded in organizing themselves and getting privileges like those of the Australians Accordingly, in the cities we find a great number of comparatively poor people and few of the middle class On the farms there is a corresponding body of poor peons who work for the large landowners or get a living on rented farms so small and poorly equipped that it is very difficult to maintain the unequal fight against frequent crop failures This development of an upper and lower class with only a small middle class has arisen partly because most of the earlier settlers here were Spaniards, and the later immigrants have been more largely Italians, both races bringing with them an economic system different from that of England

708 *The Southern United States* Coming to the northern hemisphere we find that the warmer summers of our South (Table 25) have fostered an economic situation quite different from that of either Argentina or Australia They favor corn, tobacco, and cotton, and give people from northern Europe a distaste for outdoor work on the farms Moreover, the opportunities for manual labor in the cooler North have led laboring people to migrate there rather than to the South Hence Negro slaves were imported, and in time became such a force that a white man who worked with his hands was considered inferior Since the Civil War this idea has gradually weakened. Many of the white people of the South are genuine farmers Some live on farms or in villages and run plantations where they employ colored people or rent land to them and supervise their work. Others live in attractive villages, towns, and small cities, and carry on all kinds of work except pure manual labor, which is left to the Negroes In addition to this the South has the problem of "poor whites," people who have not succeeded well in competition with cheap colored labor, or who have been forced onto farms too small or unproductive to support the usual white standard of living The Negroes, too, show a wide range of character Those of one type are self-respecting, indus

trious, and intelligent, although often handicapped by the small size of their farms or by the limited kinds of work in which colored people are welcome Another type comprises shiftless, happy-go-lucky people, who live in small dilapidated shanties and work only when absolutely necessary.

709. The South has another economic feature unknown in the corresponding regions of the southern hemisphere This is a well-developed and growing cotton-manufacturing industry Australia, to be sure, is making an increasing proportion of its own manufactured goods, but it manufactures little for other people The South, on the contrary, does not make any great assortment of manufactured goods, but manufactures a huge amount of cotton cloth, much of which it sends away in exchange for other kinds of manufactures This industry is fully discussed later

710 All these conditions, taken together, have retarded the growth of great cities Although the South has five times as many people as Australia, it has no city of a million inhabitants in contrast to two there New Orleans, the largest southern city, has about half a million people, Atlanta, Birmingham, Dallas, Houston, and Memphis have from 250,000 to 500,000. Aside from Birmingham, all these are primarily centers of commerce, although all carry on some manufacturing Birmingham with its iron, on the other hand, together with smaller cities such as Columbus, Augusta, and Charlotte with their cotton mills, and Durham with its tobacco factories, provide a type of manufacturing city almost unknown elsewhere in the Mild East Coast Regions except in Japan

711 *Japan* Climatically southern Japan resembles the southern United States more closely than any other section of the Mild East Coast Region These two sections differ from the three in the southern hemisphere in having not only warmer summers, averaging about 80° instead of a little above 70°, but also colder winters They differ from these and from South China in having fairly abundant cyclonic storms and abundant rainfall at all seasons Japan, however, is more humid than our South in summer, and its winters are less rainy It is also much more mountainous, and in this respect has more resemblance to South China than to New South Wales with its plateau, or Argentina and our South with their plains As a result of the humidity and of the density of the population on its small tract of tillable land Japan is not a good place for the cotton, corn, wheat, cattle, and sheep which are the standbys of the sections of the Mild East Coast Region thus far discussed It is excellent, however, for rice as the main reliance for food, silk for cash, and tea for drink. The most typical

scene in Japan includes misty mountains swathed in dark forests, watery rice fields in which barefooted men and women are bending over to set out rice plants, and narrow pathways which are too small for carts and are often merely the tops of mud dykes between rice fields. Such a scene also includes villages where houses with flimsy paper walls are topped with a thatch of rice straw The villages usually stand on the drier, poorer, sloping land on the edge of the rice fields The slopes above them usually show innumerable terraces, which often bear mulberry trees that have a low bushy form because they are repeatedly pollarded—cut back to the trunk—so that it will be easy to cut the small new branches and thus get leaves for silkworms In other regions the terraces are beautified by the soft, round green balls of the tea bush, or are devoted to little fields containing green vegetables, as well as wheat and barley which are cultivated almost like beans, being sown, weeded, and reaped by hand

712 Everywhere there is evidence of an enormous density of population and of an immense amount of work expended upon tiny plots of land One wonders whether Australia, Argentina, and our South would become like this if the agricultural population kept on growing denser for centuries In such a country the greatest problem is bound to be poverty Although the Japanese are exceedingly industrious, they find it difficult to supply their 70 million or more people with food, to say nothing of a surplus for sale So the Japanese are looking restlessly for new opportunities They have extended their sway over Chosen, Taiwan, and many islands, and have made themselves masters of Manchuria (Manchukuo, as it now is) and much of China But these places afford little chance for Japanese colonies, for they are already well populated with people whose standards of living are lower than those of the Japanese They do, however, afford some opportunities for trade and for getting raw materials in exchange for the goods which Japan manufactures But the population of Japan is growing so fast that it is difficult to find ways of supporting so many

713 Being an energetic people, the Japanese are meeting this situation by improving their agriculture, developing manufactures, and conquering or finding new markets Their manufacturing development has followed almost the same course as in our South, but has gone further They have developed a huge textile industry which began with silk, but in which cotton imported from the United States, China, and India has now become the chief factor Rayon, too, has recently become important They have also developed a small iron industry, and have begun to make more complex goods. They even send to the United States such articles as toy footballs, electric light

bulbs, and Christmas cards Yet this amounts to relatively little compared with Japan's needs, or with the manufactures of Germany, for example One of the greatest economic problems of Japan is scarcity of fuel and raw materials A still worse handicap is lack of markets Coming late into the industrial field, Japan has to compete both with strong old rivals such as the United States and Great Britain, and with younger rivals such as India

714 In Japan, as elsewhere, the cities give a clue to the activity of manufacturing as compared with trade Tokyo, with over 5,000,000 people in the central city and suburbs, and Yokohama with more than 600,000 are parts of a great metropolitan district which is surpassed only by New York and London as a conurbation, or city group There are many factories here, but trade and minor handicrafts, carried on at home or in tiny shops, occupy the people far more than factory work in our sense of the word Because human labor is still so cheap while machines are expensive, the day of the labor-saving machine has only dawned in Japan Osaka with 2,500,000 people and its neighbor Kobe with 800,000 form another huge conurbation which is decidedly more industrial than Tokyo If Kyoto, not far away, with its commerce and handicrafts is added, we have a conurbation which ranks with the largest of those in the great manufacturing regions of Great Britain and northwestern Germany Nagoya, with more than 900,000 people, is a thoroughgoing manufacturing city which has grown mostly within the last two generations Since the only other city of more than 250,000 is Hiroshima, it is clear that most of Japan has not yet reached a high stage of industrialization

715 *South China* This region, like Japan, is occupied by a relatively homogeneous native population which has been there so long that the land has been divided and subdivided interminably. The plains of the Yangtse in the northern part of the Mild East Coast Region and the great mass of mature mountains extending southward to the Si Valley are alike in one respect. Wherever one goes, all the good land, all that is not too steep, too thinly clad with soil, too waterlogged, or in other ways unfit, is carefully cultivated Even in the mountains, villages of mud or stone with roofs of tile or heavy thatch are extraordinarily numerous In the plains, just as in Japan, innumerable people swarm everywhere, bending over their work, or staggering under heavy loads suspended from the ends of poles Each plot of land is weeded again and again, and a single acre often receives work enough to cultivate five acres Very few domestic animals are kept in proportion to the number of people, for human muscles can dig up the fields, and the land needed to support a single ox will pro-

duce almost enough food for a family Some animals can be fed on the fallow fields, on slopes too steep for cultivation, and on the edges of dykes and paths, but the rice straw which is so abundant is poor fodder, and the hot, humid summers are bad for animals

716 The economy of the Chinese is astounding Human waste, decayed weeds, and every other bit of organic matter that can be made into fertilizer is saved to enrich the fields Marvelous economy is exercised in the use of fuel and raw materials as well as food Wood, for example, is extremely scarce even though the climate favors the growth of trees Wherever people are numerous the slopes have been denuded for generations Hence no wood except small sticks is used for fuel, and when good wood is needed for houses and boxes it is rarely available The scarcity of wood in contrast to the abundance of labor is reflected in the way in which Chinese craftsmen spend their time in beautifully painting and enameling boxes made of wood so thin that it breaks under a slight strain In the same way the women put beautiful embroidery upon the cheapest kind of cloth

717 Although the average person is very poor, the energy and thrift of the Chinese make trade more active than in tropical countries The city markets present scenes of fascinating activity with their open-fronted shops, gay paper goods, curious and innumerable kinds of food, street vendors, beggars, and softly trotting porters Transportation is carried on by man-power even more than in Japan, where jinrikishas and man-drawn carts outnumber automobiles, and horse-drawn vehicles are a rarity. In many Chinese cities numerous streets are so narrow that even jinrikishas have difficulty in passing one another, and practically all the traffic is carried by men Yet many of the larger cities, such as Shanghai, Changsha, and Foochow, have modern quarters and foreign sections, quite different from the main sections These cities, and others such as Chengtu, Chunking, Nanking, Ningpo, and Soochow, are perfect hives of industry and skill They are filled with thousands of craftsmen skilled in weaving silk, working metals, carving wood, and preparing foodstuffs Nevertheless, practically none of the workers use machinery run by anything except human power, and real factories are still scarce Shanghai, to be sure, has begun to follow the example of Japan in establishing large cotton as well as silk factories Hankow is following Birmingham in making the rougher forms of iron goods, and other cities are putting up factories Nevertheless, the typical Chinese city is of the Hangchow type, that is, the commercial center of a rich agricultural section, and perhaps also a center of art and architecture handed down in Buddhist monasteries.

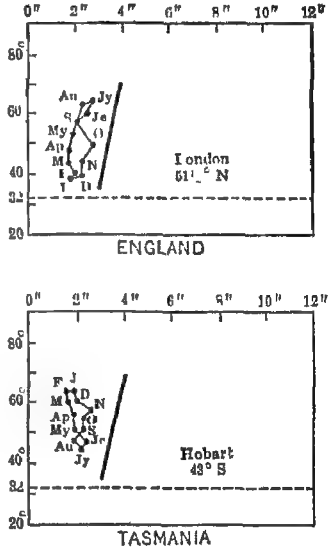
Chapter XXX

REGIONS OF HIGHEST PRODUCTIVITY

718 CONDITIONS OF HIGHEST PRODUCTIVITY. As we go from regions where productivity is low to those where it is high, outstanding differences are evident in the general level of civilization, in the activity of the people, and in the variety of resources which are put to use Regions of low productivity include vast areas where neither crops nor domestic animals are raised, where there is no manufacturing, where there is very little in the way of trade In regions of a somewhat higher type, the variety of crops and animals is small, the industries are of the household type, and only a few goods are brought from a distance. Even in regions that rank as high as China or Turkey, minerals are as yet employed on only a small scale, manufacturing is in its early stages, and trade is mainly local. One of the outstanding features of regions of high productivity is the great use made not only of fibers and other raw materials of vegetable origin, but also of minerals, including metals, fuels, cement, and fertilizers Equally characteristic is the fact that regions of very high productivity bring to their own doors all sorts of raw materials, as well as food, from all over the world. We have already given much space to the geographical distribution of food, because problems of food occupy the attention of two thirds of the world's people most of the time, and of the other third much of the time. In the rest of this book our main subject will be mineral resources and vegetable raw materials, and the way in which these as well as food are transported, manufactured, and used as articles of commerce. First, however, we must get a clear idea of the nature as well as the location of the remaining parts of the Realm of High Productivity not yet discussed, for these are the natural regions in which manufacturing, commerce, and the other aspects of a high civilization are most highly developed All these, it should be noted, are regions of frequent cyclonic changes of weather.

719. THE MARINE CYCLONIC REGION (A720) Among the cyclonic regions (Nos 9, 10, 10A, and 11 in Plate I) the most populous is the Marine Cyclonic type. Mountains limit this region to a small coastal strip in North America, South America, and Australasia, but in Europe

it expands far inland. From latitude 60° on the Norwegian coast it extends through the three Scandinavian countries, the two Low Countries, Germany, Great Britain, Ireland, and France to nearly 40° of latitude in the northern corner of Spain. Bohemia (which was formerly western Czechoslovakia), Austria, and Switzerland lie on its edges, and the Po Valley of northern Italy is a transition region of only a slightly different type. The outstanding human feature of this region is its economic, social, and political leadership. Only the Continental Cyclonic Region in the eastern United States rivals western Europe in these respects. The fact that both regions are called "cyclonic" indicates their most distinctive geographic characteristic. The main climatic difference between the two is their relation to the ocean. One is marine because it lies on the west side of the continents, the other is continental because it lies on the east side, and west winds bring the influence of the continental interior. In Europe the cyclonic disturbances come from the ocean and therefore bring relatively mild, although frequent, changes of weather. In America they come across the continent, with its far greater contrasts of temperature, and therefore are accompanied by more pronounced changes of weather and by extremes of cold and warmth. Furthermore, the Continental Cyclonic region, like all continental areas in middle latitudes, experiences strong changes of temperature from season to season.



A720—Climographs of Marine Cyclonic Regions

720. Inasmuch as the Marine Cyclonic Regions lie on the western edges of the continents, the dominating westerlies carry oceanic influences inland. Mild winters and cool summers are the result. Notice how little space is occupied by the climographs of A720, either vertically or horizontally. Where the coastal waters are exceptionally warm in winter, as on the west coasts of North America and especially Europe, the marine control is correspondingly intense. Hence mild winters prevail on the coast as far north as the Arctic Circle. One reason for this winter warmth is that oceans always cool off more slowly than land. Another is that the ocean surface off the coast of the Marine Cyclonic

Regions consists of water that has drifted poleward and eastward from low latitudes near the equator. As the water flows away from the equator it of course becomes cooler. Nevertheless, it remains *relatively* warm long after it has reached high latitudes Then, too, in the northern hemisphere the fact that the Arctic Ocean is almost enclosed by land largely prevents cold water from mixing with warm These conditions explain the fact that in winter the Japanese current creates an area of exceptional warmth off the Alaskan coast of the Pacific. For similar reasons the Gulf Stream and its continuation in the North Atlantic Drift create an area of still more exceptional winter warmth to the west of Great Britain and Scandinavia The absence of corresponding warm areas in the southern oceans arises from the fact that there the eastward-moving currents travel thousands of miles in the same latitude before reaching Chile or New Zealand

721 In summer the westerly oceanic winds which blow over the Marine Cyclonic Regions cool the land instead of warming it This seems almost like a contradiction of what has just been said about winter warmth Both statements, however, are true In the first place, the water in the oceanic "drifts" moves very slowly, taking the better part of a year to get from low latitudes near the West Indies to high latitudes near Norway, and still longer to go from the East Indies to Alaska Hence when it gets so far its temperature even in summer has been lowered from about 80° in the tropics to 60° or even 50°, depending on the latitude On the other hand, in high latitudes the long summer days cause the land to heat up very rapidly Even in latitude 62° Yakutsk has an average temperature of 66° in July All places with the same latitude and elevation would have approximately that temperature if a broad belt of land extended clear around the world in latitudes 50° to 70° This is much warmer than the ocean water even in the northeastern part of the Japanese and Atlantic Drifts Hence westerly winds from the ocean cool the lands in summer, although they warm them far more in winter

722 Besides causing mild winters and cool summers the westerlies bring oceanic moisture to the land This gives the Marine Cyclonic Regions high humidity, cloudy skies, and heavy precipitation when the air is forced to rise on the westward side of the mountains, or when it encounters land that is cool because the sun is low in winter The mountain ranges running parallel to the coast limit such precipitation to a narrow coastal zone in North America and Chile, and greatly diminish the marine area in Scandinavia, Tasmania, and New Zealand Where there are no barriers, however, as in west central Europe, ocean moisture is carried far inland

723. THE FUNCTIONS OF CYCLONIC DISTURBANCES In this climatic background, cyclonic storms bring two great advantages, as we have already seen First, they cause the moist oceanic air to rise and become cool, thus producing rain at all seasons (A68) The value of this to agriculture can scarcely be overestimated Outside the great equatorial rainforests abundant rain at all seasons is found in only a minor fraction of each continent (A68) In the second place, the cyclonic disturbances supply the variability which we have seen to be so valuable for both agriculture and human health. A brilliant morning may change into a drizzling afternoon, after a day of rain the wind may chase away the clouds, bring out the sun, and cause a refreshing drop in temperature In the Marine Cyclonic Regions, clouds are as typical as is the sun in the Mediterranean Region In their various forms and colors they are an essential part of the landscape But the chief importance of the instability of cyclonic weather is that it increases the energy, vitality, and health of the inhabitants and has much to do with their economic, social, and cultural activity Some factors, such as isolation, lack of innate ability, or the small size of a country, may hamper production, other factors, such as coal, iron, racial ability, and the advantages of past success, may increase it Nevertheless, in the world as we know it today, it is only where the climate is favorable that we find the highest levels of modern development

724 VEGETATION Forests once covered practically all parts of the Marine Cyclonic Regions except the cool, wet, upland moors and the swamps In Europe the ancient Roman legions were checked by the thick German forest In Washington, Oregon, and southern New Zealand much of the forest still persists and is very dense and tall In Europe, however, most of the forest has disappeared because of man's search for arable land It persists only on the mountains, on patches of poor sandy soil, in special reservations or parks, and in parklike groves with rows of trees set out by the hand of man. Lowland countries such as Holland and Denmark have little real forest, although trees are abundant along the roads and canals and around the individual farmhouses, while the villages are hidden in their foliage On the mountains, however, the forest still stands in its glory. Oaks and beeches are the most typical trees in the deciduous type of forest which occupies the warmer sites and better soils On the higher slopes and on sandy soils coniferous trees replace these broadleaved species Forestry is more highly developed in the Marine Cyclonic Region of Europe than anywhere else in the world, but even with its help Europe must import large quantities of lumber Along the upper limits of the forest, alpine meadows with their brilliantly colored flowers extend

to the upper limit of vegetation. Along the west coasts in many places and on some uplands strong winds and frequent, prolonged drizzly rains prevent the growth of trees and cause the grassy type of vegetation so typical of the moors of the British Isles.

725. POOR SOILS. As might be expected from the climate, the Marine Cyclonic soils are constantly leached and are not especially good (Plate II). The gray *podsols* of the north, being acid as well as leached, are infertile. The most typical kind, however, is the brown forest soil. Although this is somewhat leached, its moderate humus content shows the influence of the deciduous forests. Peaty soils on the swampy lowlands and on the grassy, rain-drenched uplands of the western coasts complete the main soil divisions. Locally these unfavorable conditions of the soil are ameliorated by mixed glacial soils produced by the ice-sheet which covered great areas during the Ice Age. Another relatively good kind of soil is the loess which was deposited during dry periods by winds blowing out from the icesheets over great wastes of débris laid down by ice and water. This forms a fertile blanket over the limestone soil of parts of France and Belgium and along the Danube Valley in Germany, especially Austria. Fertile clays also cover coastal plains from which the ocean has receded in recent geological times. The recession is due partly to a slight rising of the land, such as has exposed much of the best soil in Scandinavia. It is also due to human action in the Low Countries where the Dutch have been especially active in dyking and draining the land and thereby converting it into "polders." Alluvial river deposits also provide small areas of good soil not only in ordinary floodplains, but also in structural depressions like the Willamette Valley in Oregon, and the Rhine *Graben*, where a depressed segment of the earth's crust is now divided between Germany and France. In spite of these exceptions, however, the average soil of the Marine Cyclonic Regions is far from ideal.

726. METHODS OF LAND UTILIZATION. Although the soil is generally infertile the Marine Cyclonic Regions show a high type of land utilization. As C. F. Marbut well shows, this takes place in spite of the soils and not because of them. Two geographical advantages make it possible. One is the fact that the dominant climate closely approaches the optimum for many of the most valuable crops. The optimum, it will be remembered, includes not only mild winters and moderately warm summers but likewise rain and humidity at all seasons. The other advantage is that the climate approaches the optimum for both the physical well-being and mental activity of people who have risen to a stage of culture where they can easily protect themselves against cold and rain. Former chapters have shown how

the high yield of grains obtained in these regions is chiefly a response to climate, how the cows give more milk and a greater percentage of butterfat than anywhere else, and how the hens lay more eggs The fact that man here has good health and a surplus of energy of both mind and body has increased these advantages and is in fact one great reason for them

727 *Agricultural Productivity* The results of this mental activity are conspicuous in agriculture Man's keen mind has found out how to improve the soil with fertilizers The principles and practice of crop rotation have been worked out The yield of crops has been increased by the selection of good seed and the crossing of different types, and by detailed studies of what kinds of crops suit the local soil The desire to maintain high standards of living in spite of the growth of population has stimulated the production of crops which yield the greatest value per acre, thus favoring those which have to be raised near the market because they are perishable Along with this has gone a constantly increasing tendency to import the kinds of food which can easily be brought from a distance Thus a prolonged shift away from the grains towards vegetables and fruit proceeded steadily until the growth of nationalism, after the first World War, made many nations try to raise their own food as completely as possible Truck gardens surround all the cities and are also located in other favorable spots Under normal conditions, long trains loaded with vegetables run every day from Holland to the industrial regions of Germany In the Marine Cyclonic Regions millions of acres are planted to potatoes, and a meal in western Europe is scarcely complete without them. Orchards of apples, peaches, pears, plums, and cherries cover large sections of northern France and western Germany, just as in Oregon, Washington, and central Tasmania In May the rounded hills of the Swiss plateau are white with apple and pear blossoms Although grapes are by no means so important as in Mediterranean climates, they are raised far enough north so that France is the world leader in the production of high-grade wines

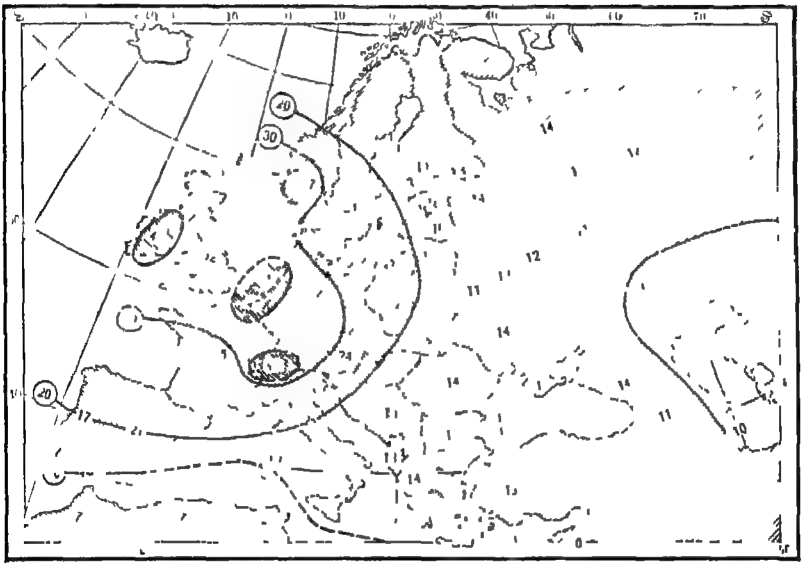
728 In addition to all this the *grains* are still important, especially where the soil is less favorable for more intensive production Although wheat has its highest yields in Denmark (B330), it assumes the greatest importance to farmers in the more sunny fields of France Oats, in spite of the fame of Scotch oatmeal, are mainly raised as feed for horses and other stock Rye, which can withstand low temperatures and poor soil better than wheat, rises to great importance on the humid, sandy plains of northern Germany, the Baltic States, and Poland. There dark rye bread replaces the white bread of the French

and English Barley is much raised for swine, especially in Denmark. With hops it forms the basis of an important beer industry, especially in Germany

729 Industrial crops as well as grains have declined in importance, as the Marine Cyclonic Regions have made their agriculture more intensive Even the large sugar-beet production in the zone extending from France through Belgium and Holland into Germany and Poland has a hard battle against the cane-sugar production of the Region of Wet Tropical Agriculture It might disappear if not helped by tariffs and subsidies, but it has a value apart from the mere production of sugar Beets form a valuable item in some of the systems of crop rotation that best preserve the soil The residue after the sugar has been extracted makes an excellent cattle feed On the other hand, such extremely specialized and valuable crops as the bulbs and nursery products of Holland and of a small area in the similar climate around Puget Sound are of steadily growing importance High-priced products of this kind are like the butter and eggs of Denmark or the fine watches and clocks of Switzerland and the brassware of Connecticut They illustrate a general principle which is also illustrated by tropical plantations. *One of the chief ways in which energetic and progressive people show their superiority is by specializing in products of high quality in regions where the natural resources are scanty or where isolation makes it difficult to market products that are heavy or bulky* Such products can be profitably sold in a wide market all over the world because of their high price in proportion to the cost of transportation

730 The preeminence of the Marine Cyclonic Region in agriculture is illustrated in A730 There, just as in A688, the numbers represent the average value of all the main crops per acre for a series of years The value of each crop per pound is reckoned as being the same in all countries The Marine Cyclonic section around the North Sea and extending as far south as Switzerland stands out conspicuously as a region where the index numbers nowhere fall below 30 This is in strong contrast to other natural regions. Notice the decline in the value of the average acre of crops as one goes southeastward from the Netherlands (51) to Germany (32), old Czechoslovakia (29), Austria (24), Hungary (17), Yugoslavia (15), Greece (14), and Cyprus (10) Eastward and southward from the North Sea the same sort of decline is seen In the countries outside the Marine Cyclonic Region small areas can be found where the productivity rises to a high level, but the average for all farms in large districts is only one half to one fifth as much as around the North Sea This contrast is due partly to high yields

per acre of individual crops, but also to the fact that the agriculture of the Marine Cyclonic Region tends to be highly diversified. Most farmers there raise not only cereals, but also such crops as potatoes, hops, and sugar beets, which have a high value per acre. The more valuable crops grow well in this region, and find a ready market because intelligent and active work causes a large industrial and commercial population to have high buying power. Because of lack of data, A730 does not include some of the most valuable crops, such as garden vegetables, nuts, and fruits (aside from grapes and olives). If



A730—Agricultural Productivity in Europe

The index numbers here indicate the value of all harvested crops per acre with the exception of hay, orchard fruits, and garden vegetables.

these were included, the general outlines of the map would not be changed, but the contrast between the Marine Cyclonic Region and the rest of the continent would be intensified.

731. *Cattle and Dairying.* The Marine Cyclonic Region enjoys the same sort of superiority in animal industries as in crops. The fact that nutritious but not unduly large grasses grow there luxuriantly, and remain green most of the year encourages the raising of stock, especially cattle on the lowlands and sheep on the uplands. This has been particularly true of the more marine western sections of Europe, where the green meadows of Ireland, Great Britain, Brittany, Normandy, Holland, and Denmark are unsurpassed for dairy products. The same

is true of western Oregon and Washington, and of the small section of southern Chile which falls in this climatic zone One sees now, however, a tendency, wherever possible, towards replacing the meadows by fields of clover, turnips, swedes (a kind of beet), and grains like barley and oats which are cut while green These are fed to the cattle in the barns. With crops of this kind the yield of milk is higher per acre than with pasturage, and the milk contains a higher percentage of fat It even pays to import cattle feed such as cottonseed cake and soybean cake from other parts of the world Thus butter and cheese are increasingly important products of all the Marine Cyclonic Regions. Swine grow rapidly on the skimmed milk, while chicken farms round out the picture of intensive animal production The British highlands and New Zealand are among the world's greatest sheep regions, for the animals there excel in both wool and mutton

732 The perishability of dairy products and the alertness of the farmers in the Marine Cyclonic Regions have stimulated the development of farmers' cooperative movements Such movements have spread to other products, especially fruit and early vegetables Denmark is the outstanding pioneer in this respect, but New Zealand and the Pacific Coast of the United States are following close on her heels In Denmark nearly everything connected with the economic life of the farmers is carried on cooperatively The animals of the sea, as well as of the land, are utilized to an unusual degree in the Marine Cyclonic Regions The coasts of Norway and the shallow North Sea form one of the world's greatest fishing regions The Pacific Coast from Oregon to Alaska with its abundant salmon and other fish also ranks very high (A492)

733 The Dominance of Manufacturing Although vegetables, fruit, grain, livestock, and lumber provide most of the economic wealth of the part of the Marine Cyclonic Region located outside Europe, these are overshadowed in Europe itself by manufacturing Smoke from chimneys hanging low over the industrial regions of western Europe often makes the landscape grimy and unpleasant Long freight trains haul heavy loads to and from the factories. Huge, unattractive cities have spread over the land so rapidly that large sections of the industrial parts of Britain, Belgium, and Germany have become great, dirty conglomerations of factories, houses, and people Power, in the form of coal, is the main reason why the activity of the people in these regions is directed toward industry, whereas the equally great activity of the Marine Cyclonic Regions such as Denmark, New Zealand, and Washington is directed mainly toward agriculture. The presence of a dense population which is ready to work in factories has been another

favorable condition for manufacturing, and so have the presence of abundant iron ore and an advantageous position in respect to transportation by land and sea.

731 The driving force in the modern development of manufacturing has been the energy of the population Long before the steam engine was invented this same spirit enabled the Marine Cyclonic Regions to lead the world in home industries, in exploration, in foreign trade, and in military and political prowess. Then when the time was ripe these regions used the existing home industries and the old commerce as a base for the development of a new system founded on machinery In some ways the system was not really new Cotton, wool, and linen had been manufactured before in the same places where the new development took place. Iron tools had for centuries been famous products of local districts, such as Sheffield, which now have become agglomerations of big steel plants In the eighteenth century more ships sailed from the harbors of western Europe than from any others, and more wagons and carriages drove over the roads. Other regions such as China had the necessary supplies of power, raw material, and labor for a similar development Such a development failed to occur not merely because the people were closely bound in an ancient social and political system, but also because the requisite stimulus to activity and inventiveness was not strong enough, even though the necessary inherent ability may have been present. There seems in this way to be a strong relation between high crop yields and complicated factories Japan, for example, stands midway between the North Sea region and China. Its average crop yields are decidedly higher than those of China, but less than those of western Europe In manufacturing, Japan also stands midway between the two, and the same is true of the degree to which the climate has the cyclonic quality which is most stimulating In similar fashion the average value of all crops per acre in the United States is highest in the manufacturing belt from New England and New York westward along the southerly part of the Great Lakes Region, and in a narrow strip on the California coast. Neither high crop yields nor factories necessarily result from the cyclonic type of climate, and both are found in other types of climate. Nevertheless, both are more likely to be found in cyclonic regions than elsewhere because of climatic conditions close to the optimum The great industrial sections of England, France, Germany, and Belgium, and the smaller but more specialized industrial areas of Switzerland, Sweden, northern Italy, and the Netherlands, are examples of what keen, inventive minds can do under favorable combinations of environmental conditions.

735 ABUNDANCE OF CITIES The great number and large size of the cities also bear witness to the dominance of the Marine Cyclonic Regions Among the world's cities of more than 250,000 inhabitants, no less than 56, or approximately 28 per cent, belong to this one type of region Among these are London, Berlin, Paris, Hamburg, Glasgow, Birmingham, and Melbourne, all with more than a million inhabitants Nearly half (24) of these great cities are seaports, but the sea is less important in this region than in many others as a factor in determining which settlements shall grow into great cities The reason is that here for the first time in our study of natural regions we come upon great inland cities which owe their growth chiefly to manufacturing rather than to commercial and political conditions How important and numerous such cities are may be judged from column C in Table 26. All the cities in the other columns, however, no matter whether seaports or inland centers of commerce and government, also possess important manufacturing industries In fact, the outstanding characteristic of the cities in the Marine Cyclonic Regions is the great amount of complex manufacturing which they carry on in order to supply not only their own needs, but also those of regions far away.

736 TRADE In an advanced exchange economy food must be shipped in large quantities from rural districts to manufacturing cities, and manufactured products must be sent back A constant surplus of one kind in one region balances a constant deficit elsewhere A dense railroad net, navigable rivers, numerous canals, and well-built roads, besides airlines and pipelines, provide the routes of traffic But note that ease of traffic by no means depends wholly on favorable conditions of relief and waterways Where people are active they insist on good means of transportation It is vastly easier to travel in the rugged Alps than in the plains of Siberia Portland, Oregon, a hundred miles inland, has dug out its river and is a far better port than it would be if located on the finest harbor along the naturally favorable drowned coast of Celebes In New Zealand, in spite of the small population and high mountains, the roads, railroads, and airplanes make all parts far more accessible than the scantily populated Sinai Peninsula close to the Suez Canal, or the densely inhabited plateaus of Central America The activity which manifests itself in trade leads to many other activities Europe's colonial empire is largely the result of the expansion of trade under the influence of the energy and vitality of the European people Great seaports, especially near the mouths of rivers, are another result of this kind of active trade And the trade, in turn, by its worldwide quality, makes those cities international meeting points

TABLE 26

CITIES OF MORE THAN 250,000 INHABITANTS IN MARINE CYCLONIC REGIONS

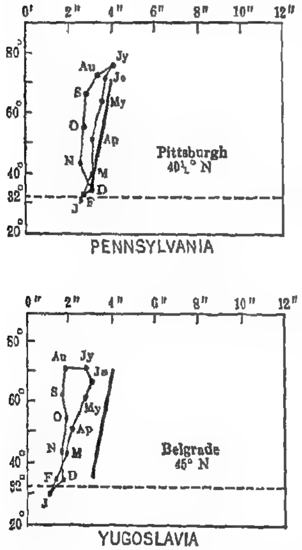
A Seaports	B Inland commercial and political centers	C Cities owing their growth almost wholly to manufacturing
Amsterdam	Berlin	Birmingham (England)
Antwerp	Cologne	Bochum
Belfast	Dresden	Bradford
Bordeaux	Frankfort	Chemnitz
Bremen	The Hague *	Dortmund
Bristol	Hannover	Duisburg-Hamborn
Brussels	Leipzig	Dusseldorf
Copenhagen	Lyon	Essen
Dublin	Magdeburg	Gelsen Kirchen
Edinburgh	Munich	Leeds
Glasgow	Nuremberg	Manchester
Hamburg	Paris	Mannheim
Hull	Prague	Nottingham
Liverpool	Stuttgart	Sheffield
London	Vienna †	Stoke
Melbourne	Zurich	West Ham
Newcastle		Wuppertal (Elberfeld-Barmen)
Oslo		
Portland (Oregon)		
Rotterdam		
Seattle		
Stettin		
Stockholm		
Vancouver		

* Although close to the sea and having a little harbor for fishing boats The Hague can scarcely be called a seaport

† Vienna is on the border between the Marine Cyclonic Region and the Continental Cyclonic Region

737 MODES OF LIFE In the Marine Cyclonic Regions the modes of life reflect not only the great variety of nationalities, but likewise the division of economic development into an agricultural and an industrial phase The straw-thatched homes of the Irish, where acrid peat smoke hangs over the valley, the charming English cottages in gardens of sweet peas and roses, the stately, red-roofed, cleanly washed farmhouses of Holland, the gray, square peasant homes of France, the picturesque architecture of the ancient German villages, and the attractive wooden cottages or châlets of the Swiss with their long balconies

represent only a few of the many different types of rural settlement. In the cities, especially where factories are dominant, there is little of this pleasant variety of types Some of the cities of historical and political fame have still a "soul" of their own, as Count Keyserling says, but others show the regular combination of official buildings, dreary factories, and great, stupid tenement houses far neater than the slums which they replace, but desperately uninteresting Even in England where the many-storied tenements are largely replaced by endless rows of two-story, one-family brick houses packed wall to wall, the effect is little better Factory smoke and grime reduce everything to the same dull color, and the main impression is dreary and ugly Yet among all this one finds many of the world's most beautiful old cathedrals, town halls, and other stately buildings whose beauty and charm vie with those of the castles out in the country Moreover, no part of the world can boast of nobler modern museums, opera houses, churches, government buildings, and homes of wealthy citizens



A738—Climographs of Continental Cyclonic Regions

738 The American Continental Cyclonic Region *Resemblance to Marine Cyclonic Regions* The American Continental Cyclonic Region in the northeastern quarter of the United States (Plate I) resembles the Marine Cyclonic Regions in many respects, both physically and economically The most characteristic physical resemblance is that the weather in both regions is dominated by a fairly regular succession of cyclones and anticyclones These move in general from west to east, bringing a constant alternation of clouds, rain, and variable winds during the cyclones, and sunshine, fair weather, and westerly winds or calms during the anticyclones The cyclones and anticyclones move faster and are more intense and numerous in winter than in summer, and their paths swing farther south. At all seasons the cyclones provide sufficient rain, and in large sections the rainfall in all months is practically the same (A738) Few other parts of the world enjoy such regularity of rainfall both from season to season and from year to year Thus, as we have seen, assures the farmers of a reasonably good yield per acre except in rare instances It thereby encourages them

to make improvements and helps them to produce the surplus which is necessary if commerce and manufacturing are to flourish. For this reason the farm population is well spread over the whole of both the American and the European Cyclonic areas except where the land is too rugged. The storms also provide conditions of atmospheric humidity and variability which are excellent for health and activity among both animals and man. The climograph for Pittsburgh in A738 is almost identical with the standard climograph except that the summers are a little too warm and the winters too cool. Hence these regions are among the most healthful and active parts of the world. Their deathrates average 10 to 12 per thousand compared with twice as much in tropical and oriental countries. Add to all this a favorable location in respect to other productive natural regions, together with abundant local supplies of coal and iron, and it is not surprising that the Cyclonic Regions, taking all of them together, carry on by far the greater part of the world's manufacturing, especially in its more complex forms, and possess a corresponding share of the world's wealth, political power, and culture.

739. The benefits of cyclonic disturbances are by no means confined to the Continental Cyclonic Regions as shown in A738 and Plate I. They extend in greater or less degree to the Cool Continental Regions (A755) as we shall soon see, to the Mediterranean Type of Regions (A671), as we have seen, and also to much of the American Temperate Grasslands (6 in Plate I). In addition to this the Mild East Coast section of the United States (8 in Plate I)—our South Atlantic and South Central States—differs from the similar section in Asia (south and central China) in being much more subject to cyclonic disturbances. In this respect it resembles Japan more than China. Nowhere else do cyclonic low-pressure areas swing so far toward the equator, or bring abundant rain and healthful changes of weather so regularly at all seasons, as in the southeastern United States. Although cyclonic storms sometimes bring floods, or lead to frosts that ruin fruit trees and crops, this is a small matter compared with the good which they do. The people of the Bahamas, for example, say that when they spend the winter in Florida they feel much more vigorous than at home. The average temperature is essentially the same, but the variability in Florida is much greater. North of Florida, in the South Atlantic states, the value of cyclonic storms increases. Thus much of what is here said about the American Continental Cyclonic Region of the northern United States applies in modified form to the South.

740 Differences between Continental and Marine Cyclonic Regions In spite of great similarities the Continental and the Marine Cyclonic Regions differ more than is usually realized We saw this difference concretely in previous chapters where we found that the Marine Cyclonic Regions provide the optimum conditions for wheat, whereas the Continental Cyclonic Regions provide an appreciably different optimum for corn Nowhere in the Marine Cyclonic Regions does wheat come anywhere near the standard set by the Marine Region The difference between these two optima is as important for man as for crops The first factor in causing this difference is latitude The Continental Cyclonic Region in America lies 10° nearer the equator than does the Marine Cyclonic Region in Europe Hence the sunshine is much more intense In four cities near the edges of the American region, July averages about 10° warmer than in four corresponding European cities, as appears in Table 27. When the

TABLE 27

Contrasted Temperatures in Continental and Marine Cyclonic Regions

Continental Cyclonic Region	July	January	Marine Cyclonic Region	July	January
Milwaukee	70°	21°	Oslo	63°	25°
Boston	72	27	London	63	39
Baltimore	77	34	Berlin	66	32
St Louis	79	31	Bordeaux	68	41

temperature rises too high in the American region the deathrate rises ominously and human efficiency is lowered in spite of vacations, fresh-air funds, refrigeration, and modern sanitation and hygiene Even the normal summers of the Continental Cyclonic Region are too warm for the largest output of potatoes, milk, eggs, and other products Their heat, on the other hand, is excellent for corn and tobacco, although even for corn such temperatures as often prevail in Baltimore and St Louis are too high

741 A second factor in making the American Cyclonic area different from the European is its position on the east side of its continent instead of the west side This means that when westerly winds blow, as they do predominantly, the American area gets temperatures resembling those of the interior, whereas the European area gets those of the sea. This is the chief reason for the high summer temperatures in

the American Cyclonic Region. It should be noted, however, that along the Atlantic Coast a narrow strip from Boston as far south as Baltimore is much influenced by the ocean. Hence it is cooler in summer and warmer in winter than the regions farther inland. Some especially oceanic places such as Mount Desert, Newport, Nantucket, and Atlantic City, have climates which approach that of the Marine Cyclonic Regions.

742. In winter the continental effect is even greater than in summer for the tendency toward west winds is reinforced by the tendency for winds to blow outward from the interior of a cold continent. Hence in spite of a more southerly location the winter (January) temperature of the four American cities in Table 27 averages 6° cooler than that of the four European cities. This causes a drop in efficiency during the winter, followed by a correspondingly high peak in the deathrate. It appears to be the main reason why the American yields of wheat per acre, aside from those of the Puget Sound Region, are so much lower than those of the North Sea Region. In addition to this, although the rainfall is greater in the American Cyclonic Region than in the European, the relative humidity is lower. This is because our westerly winds blow from the dry interior, whereas those of Europe blow from the ocean. Hence Europeans are often greatly impressed by our bright blue skies in contrast with the haziness of their own. In similar fashion the North Sea region has more rainy days than the northeastern quarter of the United States, but not so much rain per day. This is partly because their rain is brought by westerly winds, while that of the eastern United States is brought by easterly winds. In winter a corresponding condition gives western Europe short cold waves due to easterly winds, whereas the northeastern United States gets more frequent and severe ones due to westerly winds. All these conditions are disadvantages to both people and crops in the United States.

743. THE RESTLESS ACTIVITY OF AMERICA. These disadvantages are partly offset by the fact that changes of weather in the Continental Cyclonic Region in America are more frequent and pronounced than in any of the Marine Cyclonic Regions. This is true also in the American Region of Cool Continental Agriculture in contrast to the corresponding area in Europe and Asia, and in the Mild East Coast Region of our South in contrast to the regions of that sort in other continents. Such variability of the weather arises from the fact that in North America the cyclonic disturbances not only swing far to the south, but also are more frequent and active than in any other region unless it be

around Antarctica This makes the American climate peculiarly bracing and stimulating Each storm with its changes of wind, sun, rainfall, atmospheric humidity, sunshine, and especially temperature brings with it a spur to activity It is generally recognized that in the United States we *do* more, but perhaps *think* less, than in Europe Our children are more lively and boisterous and much harder to restrain Our young people demand and receive greater freedom and are "on the go" far more than young people elsewhere Our older people throw themselves into both work and play with a vim that is unique As people grow old in this part of the world they want to keep on working instead of retiring comfortably as in Europe In Turkey, India, and China people often give up their work twenty years younger than we do

744 How much of all this is due to climate and how much to other conditions is a matter of great dispute There is good reason to believe that on the whole the people who migrate to new regions differ in temperament from those of the same social class who stay behind Other things being equal, the migrants tend to be alert, active, adaptable, and adventurous in proportion to the danger, risk, hardship, and initiative involved in migration On the other hand, those who stay behind generally include the more thoughtful and successful, as well as the more conservative and unadaptable, types. In addition to this, there can be no doubt that new circumstances, the wealth and opportunities of a new land, and the necessity for building up a new country act as powerful stimulants to activity and invention The automobile, the movie, and other modern innovations stimulate incessant activity, especially among young people. Thus biological inheritance and the economic and social conditions of a new and rich country clearly cooperate with the climate in producing the peculiar alertness, activity, impulsiveness, and lack of poise which are characteristic of America and especially of the Continental Cyclonic Region in which we find a large fraction of the people of the United States

745. In the Continental Cyclonic Region, then, we find a very active people who are always seeking for something new They display a passion for bigness, speed, and energy As a result they have fully utilized the coal, iron, cotton, corn, and other resources which lie within their borders or close by, and have built up a manufacturing industry which finds no rival except in the Marine Cyclonic Region of Europe They have also carried crime, moral reforms, religious activity, and universal education to levels almost unknown elsewhere The same intensity which causes business to boom also leads to intense and sudden depressions The case is like that of automobiles the faster a

car goes, the worse the accidents. In spite of depressions, the wealth per capita in the United States, especially in the northeast quarter and on the Pacific Coast (B557), is much greater than in the European Cyclonic Regions or any other part of the world. This is inevitable where people of the finest racial stocks find themselves in a highly stimulating climate with all the stored-up culture of the Old World behind them, and with an abundance of fertile land, vast forests, and untold mineral wealth before them. The astounding feature is not that the Continental Cyclonic Region of North America has attained such power and wealth, but that a more or less accidental chain of circumstances has caused so many and such great advantages to focus in one region. The way in which the Marine Cyclonic and the Continental Cyclonic Regions stand out upon map after map in this book (A431, A537, A540, A555, A556, A562, etc.), as well as upon many showing other features of human activity and progress, is one of the dominant features of economic geography.

746. TYPES OF CITIES. In the Continental Cyclonic Region of America, just as in the Marine Cyclonic Regions, large cities are numerous in proportion to the population, and many owe their growth primarily to manufacturing. In the following table the cities of more than 250,000 inhabitants are divided according to their relation to important navigable waterways. After each city is given the percentage of its men engaged in what may properly be called manufacturing, that is, in industrial pursuits other than the building industries.

TABLE 28

CHIEF CITIES OF AMERICAN CYCLONIC REGION

Seaports		Lake Ports		River Ports		Not Ports	
Baltimore	34	Buffalo	40	Cincinnati	37	Akron	57
Boston	26	Chicago	36	Kansas City	25	Columbus	30
Jersey City	33	Cleveland	45	Louisville	34	Indianapolis	35
New York	32	Detroit	54	Pittsburgh	37		
Philadelphia	38	Milwaukee	51	St. Louis	37		
Providence	45	Rochester	46	Washington	15		
Newark	40	Toledo	43				
		Toronto	—				

Among the 24 cities in this list all but 3 are ports of some kind, and in all except Washington at least a quarter of the men are employed in manufacturing. But note that although some of the coast cities, such as Providence, are so highly industrialized that nearly half their men are at work in factories, the industrialization of the lake cities

such as Milwaukee and Detroit has gone still further Even in Chicago, where commerce and transportation play great roles, 96 per cent of the men are in factories Only in the Marine Cyclonic Region of Europe do we find any other group of cities where manufacturing is so dominant

Chapter XXXI

OTHER CYCLONIC REGIONS OF HIGH PRODUCTIVITY

747 Natural Regions of Eastern Europe We have seen abundant evidence that proximity to the sea is a great help in economic development A comparison of the three sections into which the cyclonic portions of Europe are divided in Plate I makes this clear Practically all of Europe except the northern forests and the Mediterranean regions feels the influence of cyclonic disturbances at all seasons, and this influence extends in a thin wedge far into central Siberia Nevertheless, to the south and east of the Alps the cyclonic control becomes weakened and is combined with Mediterranean and especially with continental influences Hence that part of Europe is called the European Continental Cyclonic Region (A738) Farther north in Poland, Russia, and western Siberia the cyclonic aspect of the climate is even more modified by continental influences which manifest themselves in relatively warm summers with a fair number of storms, and very cold winters during which cyclonic changes become rare Hence this northern section is simply called a Cool Continental Region

748 The European Continental Cyclonic Region The part of Europe forming northern Italy (the Po Basin), old Austria, Hungary, Rumania, and most of Yugoslavia and Bulgaria differs from the Marine Cyclonic countries farther west in several significant respects These differences help to explain why the crops are less reliable and give lower yields than in the Marine Cyclonic Region, while the people display less efficiency The winter temperatures average near the freezing point or lower, and the summer temperatures between 70° and 80° in the hottest month In these respects the climate does not differ much from that of the northeastern quarter of the United States from New England and New Jersey to Missouri and Iowa The rainfall, however, is not so favorable as in the American area, as appears in Plate I. The western part of the European region, that is, the Po Basin in Italy, comes nearest to the American standard, but lacks the American regularity from year to year, and has pronounced maxima in the spring and autumn As one goes from west to east the European Continental Cyclonic Region becomes drier. Milan has 40 inches of rain, Belgrade 24, Bucharest 23, and Odessa 16. Toward

the east, to be sure, the decline in rainfall occurs mainly in winter and the maximum rainfall comes in summer, but this does not overcome the great handicap of the irregularity of the rain The average departure of the annual rainfall from the normal increases from 10 per cent in the west to as much as 25 per cent in the east. Such conditions mean that the danger of poor crops increases rather rapidly toward the east. The harm done by the eastward deterioration of the climate is evident in the fact that although the yield of corn averages only 22 bushels per acre in Italy, or half as much as in Connecticut, it falls even lower farther east The maximum wheat production per acre is not half so much as in Denmark (B330), and it, too, diminishes eastward

749 *The Po Basin* A brief description of the four sections of this European Continental Cyclonic Region will show how the conditions of life change from east to west The four are separated by mountains which increase the diversity. Rather mild winters and warm summers, young alluvial soils, a fair rainfall during the entire year, and water for irrigation from the snowy Alps enable the Italians to make the Po Basin the granary of Italy and the best section of southern Europe Intensive land utilization with fairly good yields per acre is the result Wheat and corn cover large areas, rice is raised in easily irrigated sections, and dairy cattle are kept in large numbers, especially on the marshlands bordering the Po River. Rows of mulberry trees along the boundaries of the fields are used for silkworms, and vineyards are plentiful on the slopes In this essentially agricultural area manufacturing flourishes far more than in any other part of Italy It is based on waterpower, and supplies manufactured goods to the dense population of a vigorously growing nation We have already seen how strongly this part of Italy is contrasted with the part south of Rome In many ways northern Italy is one of the world's highly productive regions and plays an important role in the political history of Europe.

750 *The Danube Basin.* The mountain-girt Danube Basin from Vienna southeastward through Hungary and Yugoslavia to the part of Rumania west of the Carpathians and their Iron Gate is much more continental than the Po Basin Its winters are colder and drier, and its summer rains more variable Nevertheless the conditions for raising grain are good, compared with most parts of the world Dark soils formed under grass prevail, the summer temperatures are favorable, and rain, as a rule, is fairly abundant When one travels from Vienna to Bucharest and farther south or east in July, the flat plain seems like one huge field of wheat and sturdy corn, and the many villages

with their whitewashed houses suggest general prosperity Nevertheless, there is gradually a pronounced change for the worse Although Austria, western Hungary, and western Yugoslavia have the aspects of western Europe in many respects, the poorer eastern parts begin to suggest the Orient Since the factors which foster the intensive development of manufacturing are missing, except at Vienna, and prosperity depends mainly on the crops, the variability in the yield from year to year becomes a disturbing element. The frequent recurrence of poor crops makes it hard for the small farmer to get a decent living In 1920, for example, the average Yugoslavian farmer got, on an average, only 53 bushels of wheat from a field that yielded 100 in 1933 In the Netherlands, on the other hand, in the worst year during this same period since 1920, the farmers got 71 bushels from a field which yielded 100 in the best years. Moreover, during this whole period the Yugoslavian farmers averaged only 1 bushel from an area which gave 2 2 in the Netherlands

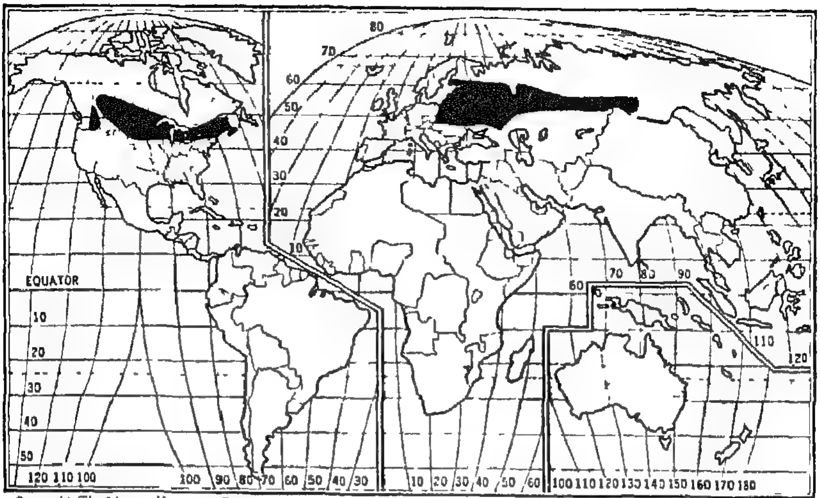
751 In the past such conditions have increased a tendency whereby the land in this part of the European Cyclonic Continental Region has fallen into the hands of large owners who can stand bad times better than small owners In the reorganization of society after the first World War the government divided the large holdings into small units which are now owned by the farmers themselves This is not in accord with the climatic conditions In order to support such a policy of dividing the land the government has been obliged to help the farmers with loans in bad times Such conditions promote political instability, and this is increased by the constant friction of one country with another over tariffs Nevertheless, the Danube Basin is looked upon by Germany as an important source of cereals and other food to be taken in exchange for manufactured goods

752 *The Rumanian Section.* The third section of the European Cyclonic Continental Region embraces much of Rumania, especially Walachia, or the plain of the lower Danube Here the disadvantages just discussed are even more pronounced than in Hungary and Yugoslavia Fields of wheat and corn abound in the lowlands, but the chance of crop failure is an ever-present menace In 1928 the Rumanian farmers got only 43 bushels of corn from an area that gave 100 in 1926. In Connecticut, on the contrary, the worst year between 1910 and 1936 gave 72 bushels for every 100 in the best year Unfortunately, regions where the crops are unreliable tend to be places where people depend to an unusual degree upon agriculture In Yugoslavia about 67 and in Rumania 71 men out of every 100 are engaged in agriculture, whereas in the Netherlands the corresponding number is 26,

and in Connecticut? Where the crops are so variable, where so large a part of the people are farmers, and where the population is dense, the majority of the people are almost sure to stand on a low cultural level. Under our present type of social organization, such a condition is almost sure to be accompanied by a great contrast between rich landowners and poor peasants, and between luxurious cities like Bucharest and peasant villages where the people live in whitewashed little adobe houses with floors of dried mud, and the whole family crowds into two or three rooms.

753. *Southern Russia.* The last of the four sections of the European Cyclonic Continental Region lies in southern Russia. The Ukraine, which is located on its northern border and should in some respects be counted as part of the Cool Continental Region farther north, is considered the most fertile part of Russia. It has long been regarded as the granary greatly to be prized by Germany. Nevertheless, it shares the handicaps of Rumania, and its people are poor. As one proceeds eastward into drier areas, corn is replaced by wheat and barley. Then, in spite of fertile soils, which are chocolate brown in the south and darker in the north, the type of cultivation approaches more and more to dry farming. Finally, near the border of the continental grasslands in southeastern Russia (Plate I), livestock replace crops just as on the great plains of America. The density of population is low, and the peasants are poor. The Soviet policy of replacing the former large land holdings by great government farms, especially in the section near the Caspian Sea, is in harmony with the character of the climate, for the small cultivator is unable to solve the problem of climatic instability.

754. *Cities.* Large cities are not numerous in the European Continental Cyclonic Region. Vienna, however, which formerly had more than two million people, lies close to the border between this region and the Marine Cyclonic Region. Budapest, a little farther east, has a million inhabitants. The cities, like the farming, show a rapid change of quality from west to east. Vienna is strongly west-European, with important manufactures of many complex types. Budapest, although magnificent, begins to show a hint of the east, especially in the simple quality of its factories which turn out a far greater value of flour than of machines. The factories make plain cotton cloth, leather, and beer rather than complex goods such as automobiles. In the Po Basin, the cities of Turin and Milan are like Vienna in being manufacturing centers of the active west-European type, but Venice and Trieste begin to show the eastern simplicity in their manufactures. Farther east there are only a few great cities—Belgrade, Bucharest, Odessa, and



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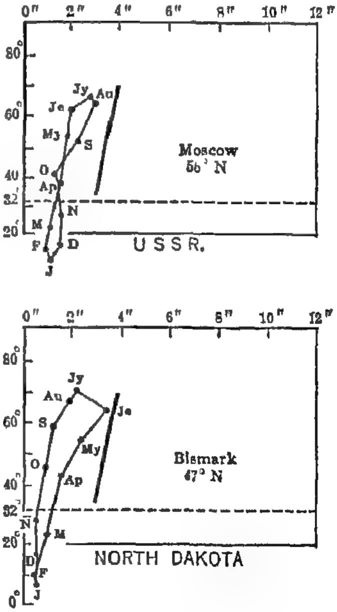
Goode's Semi-homolosine Equal Area Projection

A755—Location of Cool Continental Regions

Rostov. All of them are primarily commercial centers and seats of government, although vigorous efforts are being made to establish factories modeled on those of America.

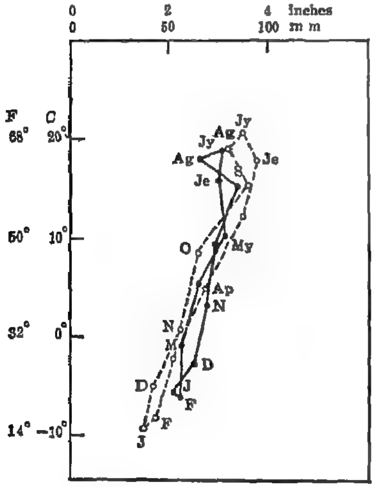
755. COOL CONTINENTAL REGION. *Climate.* Regions of this type (A and B755) differ from the preceding type (A738) because of a distinctly cooler summer, but principally because of a much colder winter. They include a considerable area in both of the northern land masses. The North American Cool Continental Region (Plate I) lies north of the Corn Belt and the American Continental Cyclonic Region. It stretches from northern New England and the Maritime Provinces of Canada to the plains of Alberta and Saskatchewan. The Eurasian Cool Continental Region lies east of the Marine Cyclonic Region and north of the Continental Cyclonic Region that we have just been studying. It tapers away eastward into a



B755—Climographs of Cool Continental Regions

narrow strip which extends far into Siberia, and ends only at Lake Baikal where it is replaced by high uplands and mountains The American and Eurasian parts have enough in common so that they are here grouped together Yet they differ in somewhat the same way as the two Continental Cyclonic Regions and might reasonably be treated separately.

756 Both regions consist largely of vast plains, and only in limited areas such as the Adirondacks, the White Mountains, and the Urals is the relief important Monotonous lowland plains, where glacial moraines often form the only break in the flatness, are typical, although in the eastern part of North America a rolling dissected upland takes the place of the lowlands In both the American and Eurasian regions, relatively warm but short summers and cold, long winters are among the dominant factors in man's economic life In both the rainfall decreases from the coast inland, while the extremes of temperature, especially the low temperatures of winter, increase greatly (B755).



A757—Climate of Opposite Sides of Lake Michigan Dotted line, Green Bay, Wisc., 44° 31′ N Solid line, Ludington, Mich., 43° 57′ N

757 *The American Portion* The differences between the two regions can be best described by giving a more detailed picture of each The climate of the northern fringe of the United States and of the well-inhabited part of Canada as far as the Rocky Mountains is cool and continental, but is greatly modified by cyclonic storms and large lakes The cyclonic influence means that even in winter the cold periods are interrupted by southerly winds, bringing warm air from the Atlantic and the Gulf of Mexico The hot spells of summer are broken by cooler days more often than in the continental interior of Eurasia. The cyclonic influence also means that the rainfall in the American region is usually sufficient for crops almost to the Rocky Mountains, although decreasing toward the west. The rain is rather evenly distributed throughout the year in the east, while in the west a summer maximum is noticeable, although the winters have a fair amount of snow The Great Lakes reduce the severity of the climate, especially where the prevailing westerly winds bring the winter warmth

and summer coolness of the water to the eastern shores. A757 illustrates the fact that places on the east shore of Lake Michigan are 4° or 5° warmer in winter and 2° or 3° cooler in summer than those in the same latitude on the west shore. The rainfall is also more uniform at all seasons on the east shore. These differences, especially the fact that winds off the lake reduce the danger from frost in the spring, explain why fruit is grown much more abundantly on the east side of the lake than on the west.

758 The natural vegetation of the American section of the Cool Continental Region follows the climate. In the east the evenly distributed rainfall permits forests to cover land that is not cleared for crops, farther west, prairies prevail (A61). The soils vary similarly, being light colored and leached in the forested east, and dark and fertile in the grassy west (Plate II). The farmer in North Dakota faces long, cold winters with blizzards and interruptions of communication because of snow. January averages 3° at Devil's Lake. Frosts may occur in June and August, and the average frostless season is only 125 days. Nevertheless, the early summer rains, the long summer days, and a relatively dry late summer permit the growth of grains, especially wheat, but not of many other crops. Hence one-crop agriculture, with its attendant difficulties, is dominant. The yield of crops in both the Canadian and American parts of the Cool Continental Region varies greatly from year to year. Sometimes the rainfall is too scanty or comes too late, frosts in the late spring or early autumn may prevent the grain from ripening properly. Harm is done not only by the variability of the weather from year to year, but also by locusts which sometimes eat up every green crop over many square miles, and by sudden hailstorms which in a single hour may destroy crops that are almost ready for harvest. This means that the average yield per acre is relatively low, hence very large farms are the rule in the western parts of the Cool Continental Region. In North Dakota the average size is nearly 500 acres, of which close to 300 are planted each year. In Maine, on the contrary, where the rainfall is more reliable, the average size is 120 acres, of which only 35 are planted.

759 Because of the variability of the crops, periods of prosperity and stringency alternate rapidly in the western part of the Cool Continental Region, that is in the Dakotas and the Prairie Provinces of Canada. This, as we have seen, is also true along a north-and-south strip extending all the way from Manitoba to Texas. In A130 it is noticeable that the risk of frosts added to that of droughts makes North Dakota the American state with the most variable crops. The adjacent Prairie Provinces of Canada share this difficulty. Moreover, the prob-

lem of getting a living is further complicated by the fact that this region is far from large markets The race and training of these western farmers, as well as the energizing quality of their climate, do not permit them to submit to misfortune in the apathetic fashion of the Russian peasants Hence, as we have seen, they turn to political agitations, such as those connected with the Grange in the 1870's, then the Farmers' Alliance, the Populist Party, the Free Silver Movement, and in later days the Non-Partisan League and the Farm Bloc In times of stress it becomes popular to denounce old methods, organize new political leagues, advocate state operation of grain elevators and banks, and vote for the cancellation of debts and reimbursement of losers in closed banks by the state When good crops come again, and prosperity returns, people become more conservative

760 In the eastern part of the Cool Continental Region in America, the more stable and less extreme climate and the modifications due to the Great Lakes and the ocean make life less strenuous than in the west Although the growing season is short the crops are by no means so variable as in the west (A130). The more abundant and regular rainfall enables potatoes to thrive wonderfully, Michigan and Wisconsin raise sugar beets, and in large sections many other vegetables and fruits grow excellently The eastern shore of Lake Michigan shows a long zone of peaches, cherries, and other fruits Apples of fine flavor are raised in huge quantities in northern New York and the Annapolis Valley of Nova Scotia Nevertheless, such conditions are typical only in the warmer southern portion of the Cool Continental Region, where there is also good rainfall In these same more-favored parts the cool, moist climate makes grass grow excellently, and promotes a large production of milk (A464), butter, and cheese Hence from Wisconsin and Michigan eastward through Ontario, Quebec, northern New England, and the Maritime Provinces dairying is the dominant method of land utilization Cattle, especially black and white Holsteins, are a typical part of the landscape Frosts do less harm to the dairy industry than to crop-raising, for the forage to which the dairy farmers devote their main efforts can be used even though it does not ripen its seeds before the arrival of frost. Seed for the next year can be bought from warmer regions Then, too, the proximity of this region to the great manufacturing area of the Continental Cyclonic Region and to the waterways of the Great Lakes, the St Lawrence, and the ocean makes it relatively easy to find a market not only for potatoes and apples but also for milk and its products Wisconsin cheese and Minnesota butter are widely familiar Ocean liners carry dairy products from Montreal down the St Lawrence to Europe.

761. The forests which cover practically all except the cleared farm land in the eastern part of the Cool Continental Region help the farmers in two ways. They provide abundant firewood, and furnish a winter occupation. Similar regions in Russia, being remote from good waterways and market, cannot so easily profit by shipping out lumber, pulpwood, and pulp. The great manufacturing district just south of the American region also provides a ready outlet for surplus labor, and many people take advantage of this in winter, returning to the farms in summer. The forests, together with the rolling terrain, the mountains, and the abundant glacial lakes and winding rivers, also help to make this eastern part of the Cool Continental Region attractive for summer tourists, and thus still further stabilize the income of the farmers. Nevertheless, many farms have been abandoned in northern New England and New York, but only where poor soil and rugged relief make it unprofitable to raise vegetables, apples, or dairy cattle. And finally the glaciated mountain topography with its lakes, rapids, and waterfalls combines with the regularity of the rainfall to make the development of waterpower easy and profitable.

762. When conditions such as these cooperate with a stimulating although somewhat rigorous climate, a high degree of progress and some important cities may be expected. The number of large cities is not great, to be sure, although it is fairly large in proportion to the population. Montreal is the only one with more than a million inhabitants, St. Paul and Minneapolis form a large metropolitan district, and Winnipeg is growing rapidly. These cities are engaged in active manufacturing, much of which is such work as flour-milling or the preparation of other local products for market.

763. *The Eurasian Portion.* Very different is the development in the Eurasian Cold Continental Region in spite of a similar range of temperature and the same general flatness. The difference lies partly in a weaker cyclonic influence in Eurasia, greater dryness in the south, and greater isolation. The lack of stimulating changes of weather is especially evident in winter. Then the huge size of Asia leads to such low temperature and high atmospheric pressure that outblowing winds almost exclude cyclonic storms from the interior. Hence in the Soviet Union from the borders of the Baltic States and Poland *eastward* the winters are colder, drier, and less broken by warm periods than the corresponding portions of the American Cool Continental Region from southeastern Canada *westward*. In summer, on the other hand, a fair number of cyclonic disturbances penetrate the continent, and the precipitation is largely concentrated within a few warm months. It is worth noting that the Baltic States, which Russia lost

in the World War, are the part of this natural region most closely resembling the more-favored parts of the American region. This, as well as their location on a part of the Baltic Sea, more open and less frozen than the gulf that leads to Leningrad, was one reason why the U.S.S.R. was so eager to recover them in 1939. Note also that, whereas in the well-inhabited parts of the American region the main contrast is between the east and west, in the similar parts of Eurasia there is also a considerable contrast between north and south. In the northwest of the Eurasian area, near the Baltic Sea, we find the best seasonal distribution of rainfall, the finest forests, the easiest access to the world's main markets, the most-leached soils, the best of grassy meadows, and the greatest development of dairying. Toward the southeast, although the summers are warmer, the rainfall is more concentrated in a few months, thus increasing the danger of crop failures, grass takes the place of forests, the "chernozem" soil is dark and fertile provided there is moisture enough, and cereal crops largely replace cattle. The colder northern section with its less fertile soil is used for rye, potatoes, and flax, but in the southeast the fertile black soil supports a wheat belt which runs far into Siberia. There, even more than in the corresponding part of America, the lack of snow in winter and especially the uncertainty due to both drought and frost cause the yield of the crops to vary greatly. So serious is this that even within the present century this section of the Soviet Union has several times suffered badly from shortage of food and even starvation.

764 The character of the Russian peasants seems to reflect these environmental conditions. One of the most noteworthy features of the Soviet Union is that at least a hundred million of its people live in almost the same fashion. This is possible because here, as nowhere else, climate and relief provide comparative uniformity over an enormous area. One of the outstanding characteristics of the peasants is their passive submission to authority. Formerly they lived under a rigid village system, or under landlords who had almost complete power, now they live under the almost equally rigid control of the Soviet government. It concentrates some of them as laborers on state farms, it lets others live in their old villages as members of great cooperative farms or "collectives" which are dominated by a handful of leaders; and it demands obedience from all.

765 The fact that these hundred million peasants submit so easily may be partly racial and partly due to a long series of historical events, but the geographic environment seems also to have been important. From the geographic point of view one of the potent features of peasant life in Russia is the long period of idleness in winter. Near Moscow, for example, the ground usually freezes in mid-

October and soon the farmers have practically *no* work. Despite the cold winter, only a little firewood has to be cut because the houses are very small, everyone crowds into one room, and the animals, which are often kept within the same walls as the people, help to provide heat. The care of the animals takes little of the men's time, for the women do most of it. Moreover, the average number of animals for each Russian peasant is only 1 or 2 horses, 3 cattle, a pig, and perhaps 10 hens. Contrast this with the 3 horses or mules, 9 cows and calves, 20 pigs, and 80 hens on an average farm in Indiana. In the past the Russians have not known how to read, their houses have been badly lighted, and there has been little in the way of handicrafts. Inasmuch as all the Russian peasants live in villages, the men do not have to clear the snow from the roads to town or go to town to do errands and get the mail. Nor have they had much of the stimulus to both health and activity that comes from frequent changes of weather. Moreover, the high latitude causes the winter days to be very short, and poverty makes it difficult for the peasants to provide themselves with light. What wonder, then, that in winter they often spend the long 16 hours of darkness in bed, and the rest of the time do little except huddle over the fire, eat, drink and talk. Not till April does the frost leave the ground at Moscow, and then for some weeks mud prevents plowing and other outside work. Thus for practically 6 months each year many of the Russian peasants have lived in enforced idleness.

766 In May, when the spring work really begins, two great difficulties confront the Russians. One is that the men are soft from lack of exercise. The other is that the growing season is short. At Moscow only about 110 days have an average temperature above 50° F, and even in the far south of Odessa, the number is only 180. Hence the work of plowing and planting must be condensed into a short time. Even a little delay means that frost will nip the crops in the early fall before they are ripe. Accordingly the men have to work extremely hard no matter how weary they become. Fortunately the days are long, but this makes the work all the more exhausting. One result is that the Russian peasants alternate between extreme idleness during a large part of the year and cruelly heavy work at planting time, and again to a less degree at harvest time. Such a life is a potent instrument in deadening initiative and promoting inefficiency. It appears to be one of the reasons why it has been easy for a few forceful people to dominate the vast peasant population. Another grave difficulty arises from the fact that the shortness of the planting season made it inevitable in the old days that each man could cultivate only a small area. Even now the average amount of harvested land

amounts to only about 13 acres for each man engaged in agriculture. The corresponding figures elsewhere are Germany 14, Mississippi 16, the United States as a whole 34, and Iowa 70 The productivity per acre in Russia averages only 11 to 14 units against 18 in Iowa, 32 in Germany, and 49 in Belgium It is evident, therefore, that the hundred million peasants, more or less, in the Cool Continental part of Russia can produce only a small surplus available for the purchase of manufactured goods or for foreign trade Calculate for yourself what a farm family's income would be in Russia compared with Germany or Iowa if an index of productivity of 13 means a net revenue of about $20 per acre according to our prices Remember, too, that additional income is derived from farm animals, but that this is scanty in Russia. The rapid increase of manufacturing and education in the Soviet Union, and the more frequent migration back and forth between country and city are helping to do away with idleness in winter and to arouse the ambition and energy of the Russians Except by cultivating fields formerly left fallow relatively little can be done to enlarge the cultivated area without using land of poor quality Therefore for a long time to come there will probably be an important difference in the prosperity of the American and Eurasian portions of the Cold Continental Region

767 In the past the cities have shown this difference very clearly Warsaw, Moscow, and Leningrad are all huge cities with over a million people, while Lodz, Breslau, Konigsberg, Kiev, Kharkov, and Riga are important Until after the World War none of them except Lodz and perhaps Warsaw was a manufacturing city to any such degree as Montreal and Minneapolis For the most part they were centers of trade and of the smaller and simpler kinds of manufacturing. Lodz, to be sure, was one of the most thoroughgoing manufacturing cities in the world, but it manufactured little except cotton cloth Since the war the Russian cities have made strenuous efforts to reach the industrial level of the European and American Cyclonic Regions They have succeeded in building huge manufacturing plants and in employing a large industrial population This population is engaged in a great variety of industries, especially those that provide materials for war Nevertheless, only an inadequate supply of such things as shoes, woolen clothes, furniture, radios, motor cars, and the many little conveniences of life is available The amount of manufacturing will have to be increased severalfold before the cities of the European Cool Continental Region are as important in proportion to the whole population as the great cities of the American Cool Continental Region

PART X

THE MINERAL FACTOR

CHAPTER XXXII

THE GEOGRAPHY OF MINERALS

768 RELATIVE IMPORTANCE OF MINERALS The importance of minerals in comparison with vegetable and animal products is much greater than one would suppose from the value of the production each year According to the statisticians of the League of Nations the annual production of minerals of all kinds in the world as a whole has a value less than that of milk or meat alone Such a comparison, however, overlooks the fact that many minerals last a long time, and can be used again and again. This is true of practically all metals and stony products, but not of fuels and fertilizers Food is rarely kept more than a year, and when once it is consumed, that is the end of its usefulness Textiles and goods made of leather and rubber may last several years, but in general their life is quite limited Wood is the only major organic product which lasts many years, but its life is short in comparison with that of stone or iron

769 CLASSES OF MINERAL PRODUCTS AND THEIR USES I. *Metals* The class of minerals that we usually think of first is metals Practically all metals are durable, but their degree of durability varies greatly The precious metals last longest because they are not put into such active use as most of the others, and do not easily rust or corrode Some gold or silver is lost each year by abrasion and in other ways, but the world's supply is constantly increasing The baser, but in the long run more useful, metals are not quite so durable as the precious metals, but aluminum lasts almost indefinitely As time goes on the world's supply is sure to become greater and greater Copper, lead, and zinc are more subject to oxidation and to destruction by acids than the precious metals and aluminum Hence supplies of these do not accumulate so steadily The metal in use today has been mined mainly within the last generation or two, during which time

the major part of all the world's mining has been done. Iron, the most widely used of all metals, is also the one that disappears fastest, mainly because it rusts so easily. The average life of iron is measured in decades instead of centuries. Nevertheless, the iron in use at any one time represents the accumulation of many years. The baser metals find their chief use in (1) machines and tools, (2) means of transportation, (3) buildings and their fixtures, including the pipes and wires that are necessary inside and outside of every modern house, (4) other minor uses such as lead for shot, and both lead and zinc for paint. Fortunately iron is one of the most common elements in the earth's crust, and usable deposits are found in most countries. Aluminum, which stands next in general usefulness, is even more widespread. It is estimated to form 81,000 parts out of every million of the earth's crust, while iron forms 50,000. Only oxygen (466,000) and silicon (277,000) are more abundant. Copper (100 parts in a million), zinc (40), and lead (16) are far less abundant.

770 II *Stony and Earthy Products* The stony and earthy mineral products are as durable as the metals, perhaps more so, and are very widely distributed. They include not only stone, but also clay, sand, gravel, and cement. Common clay is probably the most important of these, for it is the world's chief building material. Except in a few places like Japan, city buildings are constructed of brick and tile much more commonly than of stone, wood, or any other material. Among the world's rural population probably more than half live in houses made largely of clay or its products. In advanced regions like France and Germany, the clay is burned into bricks. In much larger areas it is mere adobe, or dried mud. Except in wealthy regions, such as California, the dry summers permit the great majority of houses in the Mediterranean type of climate to be made of adobe, with roofs that are tile, or else flat and made of dry mud. In large parts of Spain, Italy, Greece, Turkey, Chile, and other countries one sees few other types of houses. In the Wet and Dry Low Latitudes, the Chinese Monsoon Region, and large sections of the Mild East Coast Region, houses of adobe, often thatched with grass, are a dominant type. In warm regions this type often degenerates into wattle where houses made of brush are plastered with mud on the outside. Neither adobe nor wattle houses are very durable, especially where the rain falls heavily during the rainy season. Nevertheless, a well-made adobe house sometimes lasts a generation. Stone, of course, is also used in large amounts for buildings, especially in mountain regions and in wealthy cities. The other main use of stony products is in the form of cement, concrete, and macadam for founda-

tions, walls, bridges, and above all for roads In almost any well-populated region outside the wet tropics stony and earthy products are used in far greater volume than either metals or wood Roads require a vast amount of material, and houses of brick, tile, stone, or adobe are widely prevalent

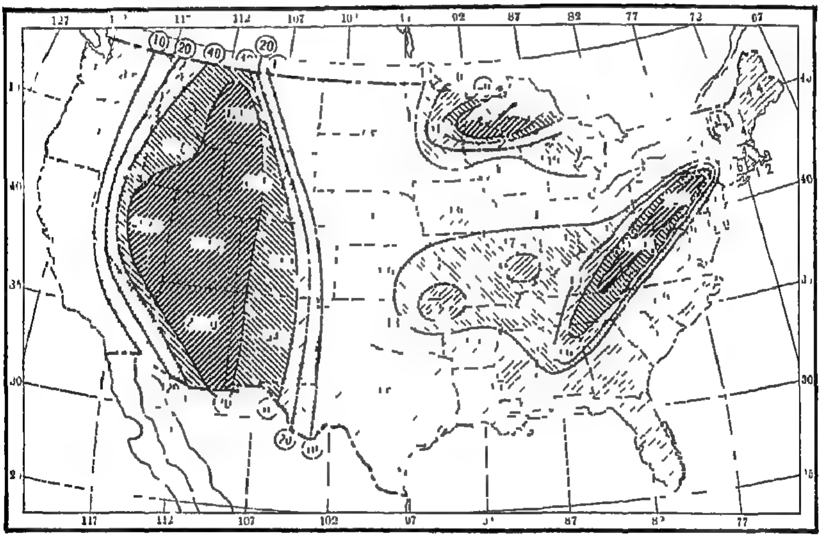
771 III *Sources of Power* The third great type of mineral products resembles food in that when once the product has been used its value is destroyed It includes two main sources of power, namely, fuels and water The importance of coal and petroleum is so well known that we need not dwell on it The same is true of water No matter whether water is employed for drinking, washing, or power, it is a mineral just as truly as liquid oil or solid coal After it has been used nature ultimately carries it back to the hills so that it can again be used On the other hand, when the fuels have gone up in smoke, their usefulness is ended Millions of years must elapse before nature can produce new supplies. Moreover, the distribution of mineral fuels in the earth's crust is very uneven, so that some countries have much and others have little or none Thus the wise utilization and conservation of fuels form one of man's greatest problems

772 IV *Fertilizers* The last major type of minerals includes many sorts of rock used as fertilizers We have already seen how fast the need for these expands, and how small a part of any fertilizer actually becomes a part of the crops When a mineral fertilizer is applied to the land, its useful life is likely to be short, for most of it goes off to the ocean with the rainwater Moreover, the geographical distribution of fertilizers is even more uneven than that of fuels Phosphates, as we have seen, are widespread, but nitrates and potash occur in usable amounts in only a few places

773 PRINCIPLES OF MINERAL DISTRIBUTION I *Influence of Plains* The principles which govern the worldwide distribution of mineral resources are far less definite than those which govern the distribution of plants and animals The geologist cannot say that a certain region is fit for copper-mining in any such precise way as the agricultural expert can say that climate, soil, and relief make a region good for peanuts Nevertheless, certain broad principles apply to all mineral resources, and certain others apply to one or another of the four types described in preceding paragraphs The first general principle is that all kinds of *mineral resources are relatively scarce and hard to find in plains, especially alluvial plains* Petroleum, to be sure, can sometimes be detected by surface indications, even when it lies beneath a level plain, but in such locations other kinds of mineral wealth usually give little indication of their presence The effect of this is evident

in A773, showing the per capita value of all mine and quarry products (omitting petroleum) in the United States. The Atlantic Coastal Plain, the Great Plains of the interior, and the plain south of the Great Lakes all show low values. The lowest of all, less than a dollar per capita, is found in Louisiana and Mississippi, states of low relief with large areas covered by the alluvial deposits of the Mississippi River

774 II *Influence of Old Age and Soil Minerals of all kinds, including petroleum, are also hard to find in regions where the bedrock is deeply covered with residual soil,* that is, in regions where the



A773—Value of Mine and Quarry Products in Dollars per Capita, 1929
(Not including petroleum)

level of the land in respect to the sea has remained almost unchanged so long that the slopes have become well graded and deeply shrouded with soil This is one reason why the Atlantic Coastal Plain from North Carolina to Texas shows such low values in A773 Closely allied to this is another principle, namely, that *minerals of all kinds are hard to find where the climate is such that rocks decay rapidly and vegetation grows rankly* This is one reason why the mineral wealth of tropical regions still remains largely unknown and is likely to continue so for a long time

775 III *Influence of Rugged Relief* The next general principle is that *minerals are especially abundant and easy to find in regions of*

rugged relief In A773 the rugged region of the Appalachians is conspicuous because coal causes the value of the products of mines and quarries to be high from Pennsylvania to Alabama A second small, rugged area, the Green Mountains of Vermont, is important as a source of marble. In the Lake Superior Region of Minnesota and Michigan old mountains have been worn down to mere stumps rising only a few hundred feet above the surrounding country Nevertheless, so far as the structure of the rocks is concerned, they are real mountains, and their presence accounts for vast supplies of iron ore and copper Even in Illinois it is the more rugged southern portion which supplies coal. Finally, in the West the rugged Rocky Mountain area is the part of the United States where mining assumes the greatest importance in proportion to other occupations

776 In addition to the value per capita, the total value of the mineral output and the percentage of workers who get a living from mining should be considered as measures of the importance of that occupation. In A773 three Rocky Mountain States show higher mineral values per capita than West Virginia, while two others surpass Pennsylvania Nevertheless, the total value of mine and quarry products in the six Rocky Mountain States is less than 40 per cent as great as in Pennsylvania and West Virginia, and only 50 per cent more than in Minnesota and Michigan In considerable parts of each of these ten states nine tenths of the men are employed in the mines Taking the entire states, however, the following percentages of the men who are at work are engaged in extracting minerals (including petroleum) 5 to 14 per cent in the Rocky Mountain States, 8 in Kentucky, 10 in Pennsylvania, and nearly 23 in West Virginia Oklahoma also, because of its numerous oil fields, has 8 per cent of its men engaged in extracting minerals, while Kansas, Texas, southern California, and western Louisiana have high percentages for the same reason

777 In Europe the relation between mining and mountains is much the same as in the United States The greatest mining area, corresponding to our Appalachian coalfields, comprises the coalfields of Wales, England, Belgium, western Germany, and northern France Less important is the eastward continuation of this region, first in Silesia where Germany, Poland, and Czechoslovakia formerly came together and then in the Donets Region of southern Russia These regions range from rough mountains in Wales to hilly uplands in England, Belgium, and Germany, and still gentler hills in Russia None of them is flat Metallic ores show the same tendency toward abundance in rugged regions The rolling plateau of northern

Sweden, the uplands of central England and eastern France, the Erz Gebirge (Ore Mountains) on the west of Bohemia, and the Mediterranean ranges of Spain, Italy, and the Balkans are all rugged regions which contain metallic ores, sometimes in great variety. Famous mining regions are also located in the mountains or plateaus of Alaska, Mexico, Colombia, Peru, Bolivia, Chile, South Africa, and Australia. Valuable ores are known to exist in the rugged parts of the Caucasus, Turkey, Persia, northeastern Siberia, central Asia, and China. In contrast to these are the lowlands of all the continents where even stone is often scarce, and metallic ores and coal cannot usually be easily discovered or profitably worked even if they exist.

778. Reasons Why Mountains Favor Mining. Although the tendency for rugged relief and mining to go together is not universal, it is strong enough to require an explanation. In the first place, the geologic structure of mountains favors the formation of ores. Mountains are regions where the earth's rocky crust has been bent, folded, cracked, broken, and shoved upward or downward along what are called fault lines. This produces cracks and fissures along which water can move and deposit ores. Here, too, molten rock is pushed upward, thus forming dykes, sheets, or irregular masses. Where these come in contact with cooler rocks there is opportunity for ores to be formed. Hence the upper layer of the earth's crust contains metallic ore more abundantly in mountainous regions than in parts where the earth's crust has suffered little disturbance.

779. Mountains also have an advantage because ores can easily be discovered among them. On steep slopes bare rock is often exposed. Mountains, too, are full of valleys which expose rock of different geologic periods for a vertical distance of hundreds or thousands of feet, while similar rocks beneath alluvial plains are out of economic reach. To a certain degree coal, the most important non-metallic mineral, is also a product of more or less rugged relief. In the Cumberland Plateau, for example, the many river gorges expose undisturbed horizontal coal layers and make mining easy, whereas similar valuable coal layers which may perhaps underlie the deep alluvial deposits of the Mississippi River are out of reach. In recent years, to be sure, new methods of measuring the density of the rocks below the surface have given some insight into the geologic structure beneath alluvial plains. Salt domes, for instance, where salt, oil, or sulphur might be obtained by boring, have been successfully located. Nevertheless, a flat state like Mississippi, with its gently sloping strata, deep soil, vast river deposits, moist climate, and abundant vegetation, is at a great disadvantage compared with New Mexico, where mountains, deep erosion,

scanty soil, and a dry climate make it easy to examine the rocks at many geological levels.

780 The mere presence of mountains does not insure a supply of minerals Although the Sierra Nevada, for example, is famous as the place where the Forty-niners found gold a century ago, it does not produce much else in the way of valuable minerals Neither do the Appalachians aside from coal, although in early days iron, gold, silver, lead, and copper were all mined in small quantities in the Middle Atlantic and New England states The Cascades, Alps, and Himalayas are examples of young mountains, like the Sierra Nevada, which contain few metals or other valuable minerals On the other hand, the Urals and some of the mountains of eastern Siberia are examples of old mountain ranges which are full of many kinds of metallic ores

781 MINERALS AND CIVILIZATION *Production* In proportion to their area and population highly progressive countries produce far more than their share of minerals In a few cases, such as the coal and iron of the United States, the potash of Germany, and the nickel of Canada, this is because they possess supplies of outstanding size or quality In most cases the reason is that they have exploited their minerals to an unusual degree When compared with China, even the new expanded Germany is a small place with limited mineral resources Nevertheless, Germany supplies at least 1 per cent of the total production of 29 of the 52 minerals listed in *The Mineral Industry*,* whereas China produces that amount of only 11 In the same way little Belgium produces at least 1 per cent of 9 minerals, but the corresponding number for Brazil, which is 5 times as populous and 280 times as large, is only 4 Moreover, the Belgian minerals include such important products as coal, lead, phosphates, and nitrates, whereas the manganese, nickel, arsenic, and zirconium of Brazil are of relatively minor importance Another way of putting the matter is seen in the first column of Table 29 Out of every $1,000 worth of minerals produced each year in the world as a whole, the United States is responsible for approximately $290, Germany for $91, the Soviet Republic $92, and Canada $54 A big country, however, does not necessarily produce much even if it is very populous The figure for India is only $22, China $10, and Brazil 25 cents, too small to appear in the table Contrast this last figure with $7 for Belgium. The failure of countries such as India, China, Brazil, and many smaller ones to utilize their minerals is due to many diverse factors Mineral development in Brazil, for example, in spite of fine iron ore

* A statistical summary published by the Imperial Institute of the British government.

and other promising minerals, is handicapped by vast plains, a tropical climate, lack of good coal, and tropical inertia, poverty, and indifference

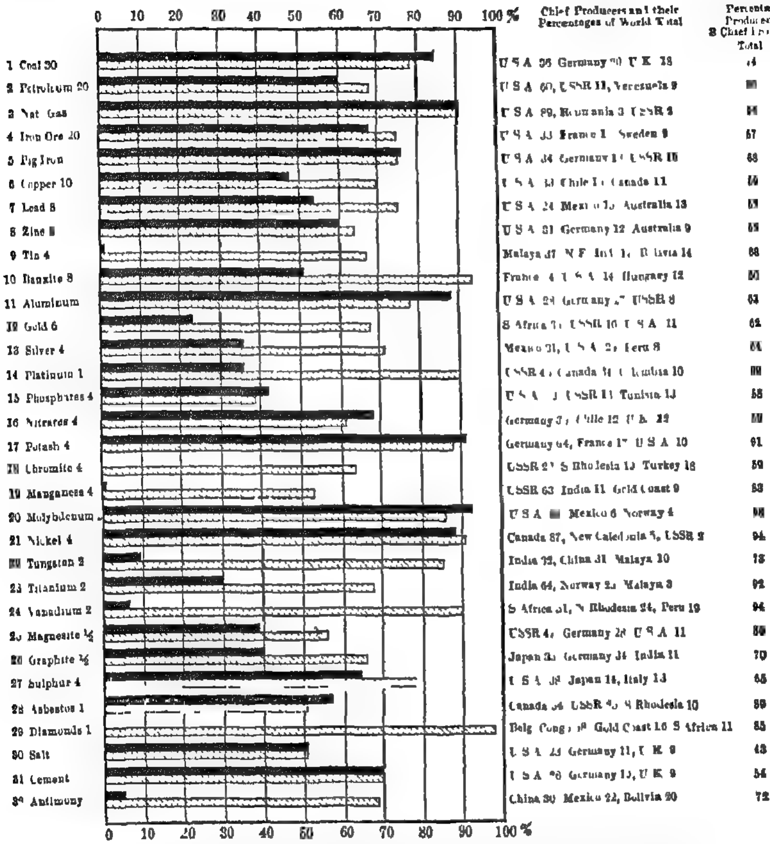
TABLE 29

RANK OF COUNTRIES IN MINERAL PRODUCTION

Country	A Value of minerals per $1,000 of world production in 1936, by Sampson	B Number of minerals in which each country ranks first, second, or third out of 32 in A782	C Index number of production per capita
U. S. A.	290	20	19
Germany	94	10	7
U S S R	92	12	4
Canada	54	4	41
France	47	3	9
United Kingdom	45	4	8
Japan	29	2	3
South Africa	26	3	22
Mexico	25	4	11
India	22	4	1
Australia	19	2	23
Italy	15	1	3
Chile	14	1	25
Sweden	12	1	17
Malay States	12	3	21
Venezuela	11	1	27
China	10	1	0 2
Netherland E Indies	10	1	1
Yugoslavia	10	0	5
N Rhodesia	9	1	52
Belgian Congo	9	1	8
Spain	8	0	3
Peru	8	0	9
Belgium	7	0	7
Norway	7	1	21
Hungary	6	0	6
S Rhodesia	6	2	38
Rumania	5	1	2
British Guiana	5	0	125
Bolivia	5	1	14

782 The contrasted productivity of progressive countries and the rest of the world is illustrated in A782 The solid black bars represent the percentage of each mineral produced in the manufacturing

regions of western Europe and North America The countries included are the British Isles, France, Germany, Scandinavia, the Netherlands, Belgium, Switzerland, Italy, the United States, and southern Canada These countries comprise 10 per cent of the world's area and 20 per cent of the population, but they produce 56 per cent



U S A Canada, Germany (Austria, Czechoslovakia, W Poland), Scandinavia, France, Italy, Luxembourg, Netherlands, British Isles, Sweden and Switzerland = 20 % of World Population (418,000,000)

A782—Production and Approximate Consumption of Chief Minerals in Manufacturing Countries, Expressed as Percentages of World Total in 1936

of the minerals If they produced merely in proportion to their area, the black bars of A782 would end at the vertical line indicating 10 per cent. If they produced in proportion to population the bars would end at the line for 20 As a matter of fact, only the bars for chromite, manganese, tungsten, vanadium, diamonds, and antimony fail to reach the 20 per cent level, while coal, petroleum, natural gas, iron,

zinc, aluminum, nitrates, potash, and various minor products all rise above 60 per cent

783 On the right of A782 we also see that the production of most minerals is largely concentrated in a few countries The numbers indicate the percentage of production in each of the three leading countries. The United States, for example, produces 36 per cent of the coal, Germany 20 per cent, and the United Kingdom of Great Britain 18 per cent This makes a total of 74 per cent for only three countries The concentration of petroleum is still greater—60 per cent in the United States, 11 in Russia, and 9 in Venezuela In the whole list of A782 a single country produces more than 50 per cent of the total output of no less than 10 out of 30 minerals * More than a quarter of every other mineral except lead, bauxite, salt, and chromite comes from the greatest producer Equally if not more significant is the fact that except for salt, which is found in almost every country and is very easily procured, the three chief producers furnish more than half of the world's entire supply In 6 cases—natural gas, potash, molybdenum, nickel, vanadium, and titanium—they supply more than 90 per cent, and in 6 others—petroleum, manganese, magnesite, asbestos, platinum, and diamonds—more than 80 per cent

784. One reason for this concentration is that progressive countries work their resources very fully A second is that in all countries it is expensive to mine low-grade ores, or those that occur in small amounts Hence people concentrate upon a few of the best deposits And finally, minerals are distributed so sporadically that one country may have far more than it needs of some product, while all other countries have little or none, so far as is yet known This is true of molybdenum in the United States, nickel in Canada, and diamonds in the Belgian Congo and the Gold Coast Potash, also, is largely concentrated in deposits that are divided between Germany and France, South Africa and its neighbor, Northern Rhodesia, have most of the vanadium, Russia and Germany produce three fourths of the magnesite The Soviet Republic in company with one other country produces about three fourths of several minerals, namely, magnesite with Germany, manganese with India, and asbestos and platinum with Canada In A782 the United States stands among the three chief producers in 19 cases, the Soviet Republic in 12, and Germany in 10 No other countries approach these three, but Great Britain, Mexico, and India each stand among the first three producers in 4 minerals. These facts lead to three important conclusions (1) *the distribution of mineral*

* A782 gives 32 names, but iron and aluminum (bauxite) each appear in two forms.

deposits which can be economically and profitably developed is very limited, (2) the progressive manufacturing countries not only have more than their share of such deposits, but also far excel other countries in developing their deposits even when these are meager, and (3) *a few big countries, notably the United States and Russia, with Germany in its enlarged form as a third, are far better off in minerals than the rest of the world*

785 The peculiarly fortunate position of the United States is evident in Table 30 where all the minerals given in *The Mineral Industry* are listed according to the general type to which they belong For 27 out of the 53 minerals there named, the production of the United States amounts to at least 20 per cent of the world total This means that we produce or could easily produce sufficient to supply our needs The United States also produces 10 to 20 per cent of the world's supply of aluminum, gold, china clay, and arsenic, so that we are not in serious need of these On the other hand, our production of 11 minerals amounts to only 1 to 10 per cent of the world total, which is much less than we need in order to maintain our industries and agriculture at their normal level Moreover, 10 others are not produced here at all, or in amounts too small to be significant The minerals in which we are deficient include such important kinds as tin for plating iron; chromium, manganese, nickel, and tungsten for making special grades of steel, and nitrates and potash to maintain the fertility of our fields We also lack other less vital, but highly useful, minerals, such as antimony, arsenic, diamonds, and graphite. Thus even the country which is by far the best supplied with minerals lacks many which are becoming more and more essential as our mechanical civilization grows more complex

786 *Consumption.* In the progressive manufacturing countries the consumption of minerals is even greater than the production. In A782 the lower, lightly shaded bars indicate the approximate percentage of the various minerals used in the manufacturing countries of North America and western Europe. Aside from phosphates, for which the statistics are doubtful, at least 50 per cent of every mineral is consumed in those countries (see names in Paragraph 782) For such highly valuable commodities as coal, iron, lead, aluminum, and potash, the consumption rises to at least 75 per cent In fact, on the basis of value the western manufacturing nations, with their 20 per cent of the world's population, probably use three fourths of the world's total production of minerals This is true even though the rest of the world includes such manufacturing countries as Japan and Russia, as well as immensely populous countries such as China and

TABLE 30

TYPES OF MINERALS

A. *Fuels*
1 Coal***
2. Petroleum***
3. Natural gas***

B. *Structural metals*
4 Aluminum**
5. Copper***
6. Iron***
7 Lead***
8. Tin
9. Zinc***

C *Ferroalloy metals*
10. Cadmium***
11 Chromium
12 Manganese*
13 Molybdenum***
14 Nickel
15 Titanium
16. Tungsten*
17 Vanadium*

D *Precious metals*
18 Gold**
19 Platinum*
20. Silver**

E *Other metals* (used for coloring matter, chemical reactions, etc)
21 Barium***
22 Columbium*
23 Mercury***
24 Monazite group
25. Strontium
26 Tantalum*
27. Tellurium***
28 Uranium group***
29. Zirconium

F *Earthy substances*
30 Abrasives***
31 China clay**
32 Cement***
33 Diatomaceous earth***
34 Felspar***
35 Fluorspar***
36 Fuller's earth***
37 Gypsum***

G. *Fertilizers*
38 Nitrates*
39 Phosphates***
40 Potash*

H *Other nonmetals*
41 Antimony*
42 Arsenic**
43 Asbestos*
44 Bismuth
45 Borates***
46 Cobalt
47. Diamonds
48 Graphite
49 Mica***
50 Salt***
51 Selenium***
52 Sulphur***
53. Talc*

* U S A normally produces at least 1 per cent
** U S A normally produces at least 10 per cent
*** U S A normally produces at least 20 per cent

India Only in a few cases, such as aluminum, iron, and potash, do the western manufacturing countries produce more than they consume. When this occurs it means that the manufacturing countries happen to have unusually good deposits which they can work cheaply because

of their coal and their mechanical equipment. Among rarer minerals, such as chromite, manganese, and diamonds, most or all of the production is in other countries. The people of manufacturing countries often own and run the mines in these other countries and consume two thirds to four fifths of the product. The amount produced and the amount used in any one country are the same only for such products as natural gas, which cannot be easily transported, or salt and cement which can be procured almost everywhere, and are expensive to transport because of their low price per pound.

787. Political Influence of Minerals. The facts now before us explain why minerals, even more than tropical colonies, create much trouble politically. Inasmuch as progressive manufacturing countries cannot produce certain essential minerals and tropical commodities, they make great efforts to procure them elsewhere. In Germany, for example, the nature of the geology and climate forbid the drilling of oil wells and the raising of rubber. Under such conditions, as we saw in Chapter XVII, the Germans are impelled to search for these products in other regions which they can control, and from which they cannot be cut off even in time of war. If trade were free, or at least if equal opportunities for trade were somehow guaranteed to all countries so that no nation feared what might happen in war, the political control of mines and plantations would be of small importance.

788. *Germany's Predicament.* Before 1939, Germany produced a good amount of coal and could increase production to satisfy increased needs. Even by using scrap iron and low-grade ores, however, she was able to supply only one third the iron needed in peacetime. For the other two thirds she had to depend on France, Sweden, or more remote sources. Hence when war cut off the supply from France and the west, Germany at once had an intense interest in Sweden's far northern iron mines, and was very anxious to prevent Sweden from going to war to help Finland. In this age of electricity copper follows iron in importance, especially in time of war. From the Mansfeld district pre-war Germany was supplying about 14 per cent of her needs. By using poorer ores which are expensive to smelt, she might have raised this to 25. For the other 75 per cent she had to rely on neighbors such as Norway, Finland, Yugoslavia and Russia. These countries produce 9 per cent of the world's copper in comparison with Germany's 2 per cent, our 33 per cent, and 22 per cent within the limits of the British Empire. In preparation for war Germany insisted that people replace copper by aluminum and other lighter metals, but this had no appreciable effect on the need for copper from other countries. Again,

pre-war Germany had no commercial ores of manganese, but was able to meet 40 per cent of its peacetime needs from manganiferous iron ores. The rest came from Russia, which produces 63 per cent of its peacetime needs, and is able to raise this to 100 per cent by going to added expense and using low-grade ores. In lead Germany is not so well off, producing only 35 per cent of its needs, with the possibility of increasing this to about 60. Low-grade ores might permit the country to supply 10 per cent of its needs for mercury, although in peacetime the German ores are not worth working. Similar conditions prevail in respect to nickel, which, like manganese and other ferro-alloys, is essential for the production of certain types of steel. Moreover, Germany has to import all, or nearly all, of its aluminum ore (bauxite), antimony, chromite, magnesite, mica, phosphates, sulphur, tin, and tungsten. Most serious of all is Germany's lack of petroleum. Although gasoline can be made from coal, Germany can scarcely supply half her needs from this source, and such gasoline costs four times as much as the kind derived from petroleum.

789. *Deficiency of Minerals Elsewhere.* Italy is far worse off than Germany for minerals. Its own mines supply only about 8 per cent of the coal needed in peacetime, 20 per cent of the iron, 7 per cent of the oil, and 0.5 per cent of the copper. Other minerals are scarce. Japan is better off, partly because it controls Manchukuo and northern China. The way in which countries reach out after minerals is illustrated by Japan's ownership of iron mines in the Philippines, Malay States, British Columbia, and Australia. Even before the European war, however, political difficulties had arisen in respect to the Australian mines. Japan's coal is poor in coking types and anthracite, but better supplies are available in Manchukuo and China. This helps to explain Japan's insistence on conquering those countries. Japan, like many other countries, has trouble about petroleum. It supplies 7 per cent of its own needs, and gets some from shale in Manchukuo and from wells in Russian Sakhalin and Dutch Borneo. These sources are not dependable in war.

790. Even the most fortunate countries have their troubles in respect to minerals. France has superb supplies of iron ore, bauxite, and potash, and her North African possessions furnish abundant phosphates. Coal, copper, lead, and zinc, however, are deficient, and there is no petroleum. The Soviet Republic stands next to the United States in mineral wealth, and produces at least a little of a great number of minerals. Nevertheless, the situation is difficult because of the remoteness of a large part of the deposits, and the fact that many of the ores occur in small bodies or are of low grade. Hence

the cost of production is high One of the points which the man in the street often fails to realize is the rapidity with which the cost of production rises when poor ores are used A pound of copper from 10 per cent ore in a small and remote deposit may easily cost 10 times as much as a pound from a 40 per cent ore occurring in large masses in an accessible location

791 *America's Deficiencies* Even in the United States, which is far better off than any other country, we could not build a modern automobile if our imports of metals were cut off Our chief deficiencies are in the ferroalloy group, especially chromite, manganese, nickel, tin, and tungsten. No one has yet found a satisfactory way of making steel without manganese, although only 15 pounds is needed for every ton of metal. We also have a deficiency in potash, and in such minerals as antimony, mercury, graphite, and certain kinds of mica From the standpoint of industrial progress few things would be more helpful than a political system which enabled every country at all times to buy minerals freely in the places where they can be produced most cheaply Such a system would cause the closing of many small mines and the increased development of the main sources of supply

792 MAJOR METALLIC MINERALS *Iron* Although iron ore is found all over the world, more than 60 per cent of the mining is concentrated in two spots namely, the Upper Lake region in the United States around Duluth, and the Minette region in French Lorraine and Luxemburg Of these two, the American region is still the leader From the lake harbors of Minnesota, upper Michigan, and Wisconsin some 75 million tons of ore are shipped in good years to the blast furnaces in Illinois, Indiana, Ohio, New York, and especially Pennsylvania The Minette region, which before the World War of 1914 18 belonged mostly to Germany, but now belongs to France, smelts two thirds of its ore locally, importing coal for this purpose Other regions which combine ore-mining and iron plants include Great Britain, where the Cleveland fields are the most important, Germany where ore is mined in several sections, and Russia where iron mines and the coal of the Donets Basin are relatively near each other Alabama is still another of this group, it is particularly fortunate in having iron, coal, and limestone—three requisites of steel-making—almost within a stone's throw of one another Another group of regions exports iron ore These include Sweden, Newfoundland, and Chile A typical example of the way in which coal and iron from different countries come together is found in the great iron works at Sparrows Point, Maryland, where cheap ocean transportation makes it possible to use ore imported from Algeria, Australia, Chile, Cuba, France, Morocco, Newfoundland, Norway, Spain, and Sweden Among these sources Cuba is the most important Aside from times of world depression the production of iron ore has long increased steadily, the increase in some exporting countries such as Sweden and northern Africa being especially noteworthy Of much greater importance is the increasing production of Russia Such places as Magnitogorsk in the Urals, and the iron works of western Siberia, illustrate the Russian activity.

793 *Copper* The United States mines about half of the world's copper and imports another 7 per cent in the form of copper concentrates and refined copper Arizona, Utah, and Montana, together with Michigan and Nevada, are the main domestic ore producers, the region from Connecticut to Baltimore is the great consumer and importer In Chile, the second largest producer, the copper mines are controlled by American capital and function almost as part of the American copper industry Copper production is unimportant in Europe except in Spain, and in Asia except in Japan In central Africa, however, the so-called Katanga region in the southern Congo and northern Rhodesia is rich in high-yielding copper ore, and production has increased rapidly Elizabethville, the main mining center, is a prosperous town with the largest white population between North Africa and the Transvaal The new railroad westward to Lobito in Angola provides a considerably shorter outlet than the old route eastward to Mozambique The chief difficulties in Katanga are the small supply and poor quality of native labor

794 *Lead* A little more than one third of the world's lead is mined in the United States Missouri still produces the most, but Idaho, Utah, Colorado, and Oklahoma are also of importance. The large output of Mexico and of a single great mine in British Columbia also finds its main market in the United States, partly in the form of ore and partly as unrefined lead In Europe, Spain is the main producer The great Baldwin silver-lead mine in Burma is located in the densely wooded Shan Plateau, a center of action in an almost uninhabited region Most of the Australian production also comes from a largely uninhabited area, that is, from the silver-lead-zinc mines of Broken Hill in the desert and a modern plant at Mount Ida in Queensland

795 *Zinc* The general distribution of zinc ore production is similar to that of lead, except that Europe produces relatively more It furnishes one fourth of the output, with Silesia as the main zinc mining district The share of the United States in world production is 40 per cent The leading centers are Oklahoma, Missouri, New Jersey, and some of the Rocky Mountain States The amount of foreign ore smelted in the United States is relatively unimportant British Columbia and Mexico export most of their zinc in the form of ore or unrefined metal Western Europe, especially Belgium, is an interesting example of an ore importer Although without zinc mines, Belgium is the world's second largest zinc producer, its production being based entirely on imported ore The zinc of Burma and Australia comes from the same mines as the lead, for the two kinds of ore are often found together

796 *Tin* The area around Singapore, including the southern part of the Malay Peninsula and the islands of Banka and Billiton in the Netherlands Indies, produces about 55 per cent of the worlds tin If Siam and southern Burma are also included, the production rises above 60 per cent Only three tin regions of importance lie outside this section of southeastern Asia Nigeria, Yunan in southwestern China, and Bolivia, where mines at a great elevation on the central plateau of the Andes produce nearly a quarter of the world total The ores of the Malay region are smelted locally, but those of the other regions are carried to Europe, especially Great Britain That country formerly had the world's chief tin mines, and Cornwall still produces 2 per cent of the worlds supply

797. *Gold and Silver* Gold, the most-sought-after mineral, is found in small quantities in most parts of the world, but more than half is mined at one spot, the Witwatersrand at Johannesburg in the Transvaal Elsewhere the output of gold is not of much importance in world affairs, although Mexico, California, Alaska, and Australia are famous as places where gold once attracted thousands of

adventurers Ontario with its gold mines in the Laurentian Upland, has been the world's second largest producer Recently, however, Soviet Russia, with the help of great dredges in northeastern Siberia, has taken second place The rate at which gold is produced is particularly important because gold is the standard of value for money The production of silver is still increasing in spite of low prices Except for the Inca region in the Peruvian Andes, and the mines already mentioned in Burma and Broken Hill, the production of silver is almost entirely concentrated in North America Mexico produces about 40 per cent of the total output, the United States nearly a quarter, Utah, Montana, and Arizona being the leading states British Columbia and Ontario enable Canada to supply nearly a tenth of the world's production

798 *Bauxite* The production of aluminum has recently increased with such speed that this commodity deserves a place among the major metals France, with a fourth of the world's output, is the main producer of bauxite The United States comes second with about one seventh, nearly all of which comes from Arkansas British and Dutch Guiana (Surinam) in South America account together for nearly another fifth, and the rest comes mainly from Hungary Italy, and Yugoslavia Bauxite ores are transported a great deal Since electric power is the main factor in recovering aluminum from the ore, Germany and the United States with their abundant coal are the great producers of aluminum while Canada, Norway, and Switzerland with their abundant water power produce a good deal although they have little bauxite

799 Economic Aspects of Mining Communities Mining towns as a rule suffer certain distinct handicaps because of their main occupation In this respect they are like lumbering and manufacturing towns, although the handicaps are different In the first place, many mining towns are short lived As soon as the ore is exhausted people go away Hence both the mine owners and the miners hesitate to put much money into homes or public improvements like churches and schools, or even into business. As a result buildings of all sorts are likely to be as cheap as possible Moreover, women and children are comparatively scarce, because many miners do not take their families with them Of course this is not true of the more permanent mining communities, such as many coal towns, or those of the iron mines around Lake Superior A good example of the rise and fall of mining centers is seen in such places as Virginia City in Nevada where a city grew up in the last century, but faded away so that by 1915 the traveler saw little except deserted streets, empty houses, hotels with every window broken, and great unsightly dumps composed of rock brought up from the mines In the present century there has been a mild revival, so that now the town is much less dead than formerly Another example was seen by the author in Australia The city of Broken Hill, with 20,000 inhabitants, broad streets, stores, theaters, and comfortable if not beautiful hotels, does not seem at all appropriate to a treeless desert where even sheep can scarcely get a living,

and there is no irrigation worth mentioning. But already the city is on the wane, and in due time is likely to become like Cobar, 300 miles to the east That place was once a thriving little city famous for its copper mines Exhaustion of the ore brought the complete downfall of the city Today one sees merely grass-grown, dusty streets, cracked houses with broken windows, partly demolished mining plants, and a handful of people who still hope that something will bring a revival

800 Many of the difficulties of mining towns arise from the irregularity of the work Most metals are used mainly in new construction Newly produced iron is consumed chiefly in new railroads, new ships, new skyscrapers, new machines, new automobiles, and all sorts of other new articles most of which will be used for a long time As soon as hard times arrive and business becomes depressed, people stop new construction They buy almost as much food and gasoline as ever, they cut down only a little on clothing and coal, but they do not construct new railways and apartment houses, or install new machinery, or buy new automobiles at the same rate as formerly Thus the demand for iron drops off far more than the demand for flour Copper suffers similarly because most of it is used in installing new lines for telephones and power, and in equipping new buildings and machines All of these activities promptly fall to low proportions in hard times Lead and zinc, too, are used mainly in constructing buildings, or in painting them, so that they also suffer The result is that in a civilization like ours, which has not yet learned how to prevent the alternation of periods of inflation and depression, the demand for the metals varies enormously From the boom year of 1929 to the depressed year of 1932 the production of iron ore in the United States fell from 73,000,000 tons to 9,600,000, that of copper from 999,000 tons to 344,000, lead, 775,000 to 287,000, and zinc, 625,000 to 207,000 This means a diminution of about 87 per cent in the use of iron, and 63 to 67 per cent in the use of the other main metals. Bituminous coal did not decline so much, about 43 per cent, because trains must run and houses must be lighted and heated, even in a depression Since houses must be heated every winter, however, the production of anthracite coal fell off only 27 per cent from 1929 to 1932, and much of this was because of exceptionally warm winters The use of cotton also fell off about 40 per cent because people wore their clothes longer than before On the other hand, people eat about as much in bad times as in good, so that the consumption of wheat changed scarcely at all The alternation between good times and bad causes even the most stable mining towns to fluctuate violently from intense

activity to painful stagnation and unemployment Thousands of miners are thrown out of work, and thousands of others have only a few hours' work each week. Moreover, many of the mines close entirely

801 In all kinds of mining there is a growing tendency to work the biggest mines as fully and steadily as possible They can often make a profit even when production and prices are low The smaller mines cannot do this Accordingly they run when prices are high and the demand great But as soon as unfavorable conditions ensue they close down and employ only a few caretakers to watch the property and keep the pumps running so that the mines will not be filled with water This, of course, creates great hardship, for the miners have to move away They are likely to flock to the bigger mining centers, crowding them with men who want jobs, and thereby lowering wages All this makes mining a difficult occupation It leads many competent and reliable men to choose something else, it increases the tendency toward strikes, radicalism, and violence, and it discourages people from living in mining towns unless their work compels them to stay there In recent years the organization of large companies has led to some improvement in these respects. Better communications, attractive houses, and a deeper insight into the need for social aids such as sports and libraries have brought a higher level of life Nevertheless the temporary character of mining is still a tremendous obstacle.

Chapter XXXIII

FUELS AND POWER

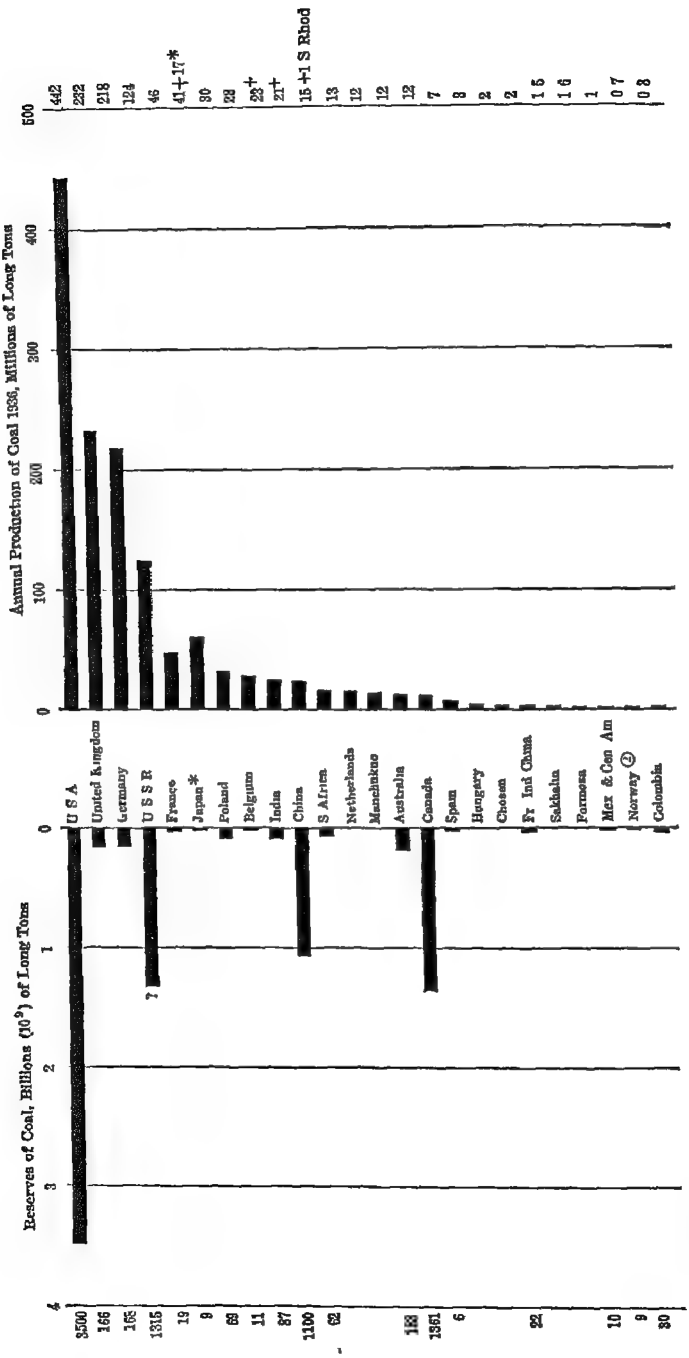
802. **Sources of Power.** The relative importance of the world's three main sources of power is rapidly changing. In 1900 coal supplied most of the power, there was a little waterpower, and petroleum was negligible. In 1913, although the amount of coal consumed each year had greatly increased, the other two sources of power had so increased in importance that the figures were coal 89, oil 7, and water 4. In 1925, still with a great increase in coal, the percentages had become 76, 16, and 8. Between 1925 and 1939, however, the use of coal increased only a little, but that of the two other sources still kept on growing. Therefore the percentages had become approximately coal 60, oil 26, and water 14. A similar change is still going on. In due time, presumably, the percentage of the world's power derived from oil will decrease, because the supply of that commodity is limited. The use of waterpower will doubtless go on increasing until all available supplies are utilized. Meanwhile, however, new sources of power, such as the heat of the sun and of the earth's interior, or alcohol made from rapidly growing plants, especially those of tropical countries, are likely to come into prominence. Thus the geographical distribution of sources of power has been rapidly changing during the past century, and will probably keep on changing. If all the available waterpower were in use, it would just about supply the world's present requirements. Since 37 per cent of the waterpower available at all seasons without storage is in Africa and 22 per cent in Asia, it will probably be a long time before it is all utilized. Therefore, although the importance of coal may diminish, it will doubtless long be the main source of power.

803. **Coal.** The world's annual production of coal has in late years risen to about 1,400 million metric tons. This includes the three main kinds of coal, semibituminous, bituminous, and anthracite, as well as the impure sandy or clayey coal known as lignite, or brown coal. Lignite is here reckoned in terms of its actual content of coal. Peat might also be added, but the total production is small, and has merely a local value, although in countries such as Ireland and Holland it is

still of importance as a household fuel. Coal, of course, is used not only as fuel, but also for a great many other purposes, such as raw material for dyes, explosives, antiseptics, fertilizers, gasoline, and even perfumes The total for such purposes, however, is almost negligible compared with that for fuel Much coal is also turned into coke by heating it to drive off the gaseous portions The coke and much of the gas are burned as fuel, but part of the gas becomes solid or liquid when cool and is the source of paraffin, dyes, and a certain amount of gasoline

804 *Uneven Distribution of Coal* An inspection of A804 shows great unevenness in the geographical distribution both of the annual production of coal and of the reserves underground On the right of the diagram four countries—the United States, Great Britain (United Kingdom), Germany, and the Soviet Republic—are seen to produce more than 80 per cent of all the coal mined each year The bars on the left show that these four countries possess more than 60 per cent of all the unmined coal still left in the ground The United States alone, with well over 40 per cent of the known reserves, mines about 30 per cent of the total annual production Great Britain and Germany, however, have smaller reserves than either China, Canada, or Australia The Soviet Republic, China, and Canada together have reserves slightly larger than those of the United States These four countries together own almost 90 per cent of all the known coal deposits

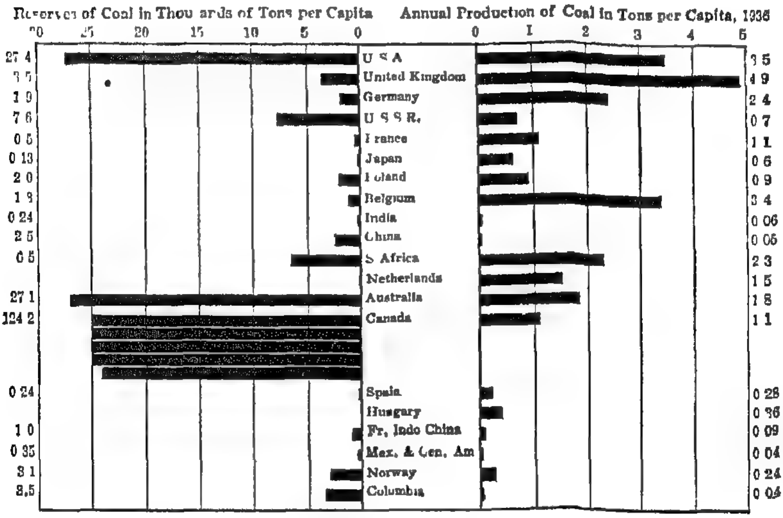
805 The facts illustrated in A804 become more interesting when the population of the various countries is taken into account In A805 the countries are arranged in the same order as in A804, but the left-hand side shows the reserves in thousands of tons per capita and the right side the annual production in tons per capita The left side shows that in proportion to its population Canada has vastly more coal than any other country The United States and Australia stand second and third, and are far ahead of any other countries Since South Africa and the United Kingdom come next, aside from the U S S R, it is evident that the English-speaking parts of the world have an overwhelming advantage over the rest of the world In addition to the Soviet Republic the non-English-speaking parts of the world with good reserves of coal include Germany, Poland, Belgium, China, Norway, and Colombia, but the Colombian coal is of poor quality That of Norway is found in the cold island of Spitzbergen The poverty of Japan, India, Spain, Mexico (with Central America) is noteworthy, and practically all parts of the world not included in A805 are equally poor so far as coal is concerned



A804—Reserves and Annual Production of Coal

* In addition Sakhalin produced 1 5 million tons, Manchukuo 11 8, and Formosa 1 6
② Spitsbergen

806 On the right of A805 it is evident that the amount mined has relatively little to do with the reserves underground The United States, to be sure, stands high in both respects, and the United Kingdom and South Africa come next Such countries as Germany, France, Japan, Belgium, and the Netherlands are removing their supplies at a rate which spells scarcity before many generations have passed That is what normally happens to a progressive country with moderate or small supplies of a mineral It develops its resources rapidly, and then has to look around for more in some other part of the world On the other hand, China and Colombia are examples of countries with mod-



A805—Reserves and Annual Production of Coal per Capita

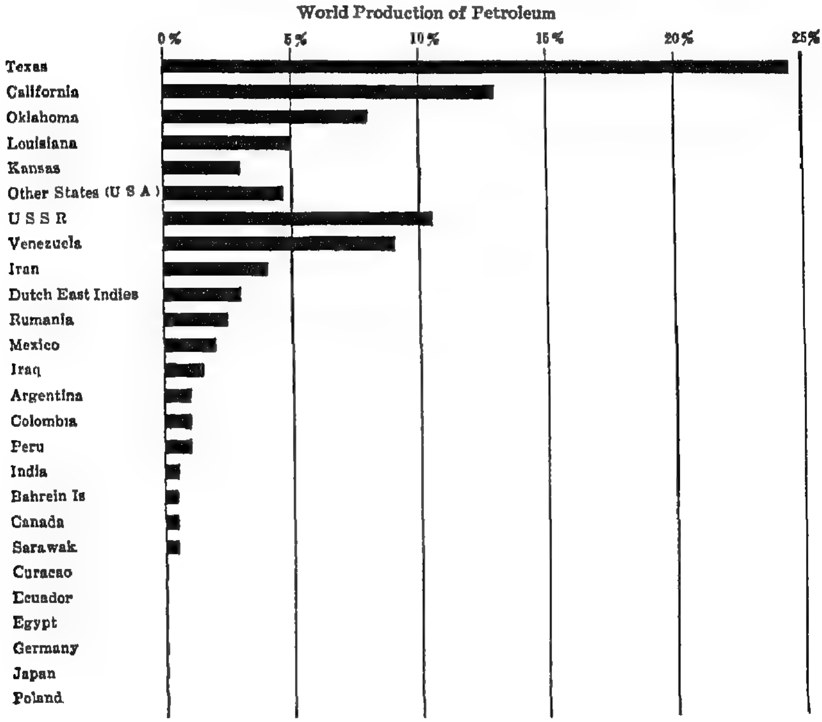
erately large supplies of coal which they are mining so slowly that the mines will be productive for a long time if the present rate continues India is badly off, for its bars in A805 are small on both sides This means not only that it has little coal in comparison with its population, but also that the supply will not last long, even though the amount mined per person is small The existence of such intense geographical differences in so vital a commodity as coal bears in itself the seeds of bitter national rivalries and jealousies These have already done much damage and will do far more unless all countries are enabled to get what coal they need, and at the same time the rate of production is kept low enough to preserve at least part of the coal for the distant future

807 The 40 per cent, more or less, of the world's coal produced in the United States is largely concentrated in the central eastern states, especially Pennsylvania, West Virginia, Kentucky, and Illinois While Pennsylvania and West Virginia now rank nearly the same as producers of bituminous coal, Pennsylvania ranks first in total coal production because of its monopoly of anthracite Canadian mines in Nova Scotia and New Brunswick attempt to compete with American coal in Montreal, which is one of the chief Canadian centers of consumption, but they are too remote to compete in Toronto, the other main Canadian center, and in the other cities of Ontario Europe produces one half of the world's coal, with Great Britain first and Germany second Together these two countries produce about as much as the United States This leaves only one fifth of the coal for all the rest of the world put together

808. PETROLEUM The production of oil has been increasing rapidly for many years In each decade from 1900 to 1930 it doubled During the next decade it did not increase so fast, but the growth in volume was huge, and the rate of increase was well over half as great as formerly Although oil wells have become exhausted, and the yield of oil fields has changed greatly, no country which has once been an important oil producer has ceased to be of significance Moreover, among the oil-producing countries Mexico and Poland are the only ones where the production has reached a maximum and then seriously declined In Mexico a production of 193 million barrels in 1921 fell to 33 in 1932, but has since increased Poland produced 13 million in 1910, but only 4 in more recent years On the other hand, in 1920 Venezuela and Colombia produced no petroleum, but in 1938 their production was one tenth of that in the whole world. The production of Argentina, Peru, Trinidad, Rumania, and Iran, which amounted to only 4 per cent in 1920, had also risen to nearly 10 per cent in 1938 In that same period the production of the United States and the Netherlands East Indies increased threefold or more, and that of Russia also rose to three times its maximum before the World War How long any such increase can continue is doubtful, although the widespread fears of exhaustion of petroleum which were entertained from 1910 to 1920 have not been realized

809 Petroleum is by no means the only source of gasoline Gasoline is produced from oil shales, from coal, including low-grade material of little value otherwise, and from natural gas In the United States about one tenth of the gasoline now comes from natural gas Germany is producing a good deal from coal Scotland, Estonia, and Australia, all of which lack petroleum, are producing some from the

oily forms of shale. Such conditions by no means indicate that the danger of exhausting the available supplies of petroleum and of other sources of oil and gasoline has passed. Far from it. There is grave danger that future generations will say very harsh things of us because we have exhausted nature's choice resources so rapidly and above all so wastefully. Nevertheless, at the present time the economic problem is not so much to find new sources of gasoline and lubricating oil as

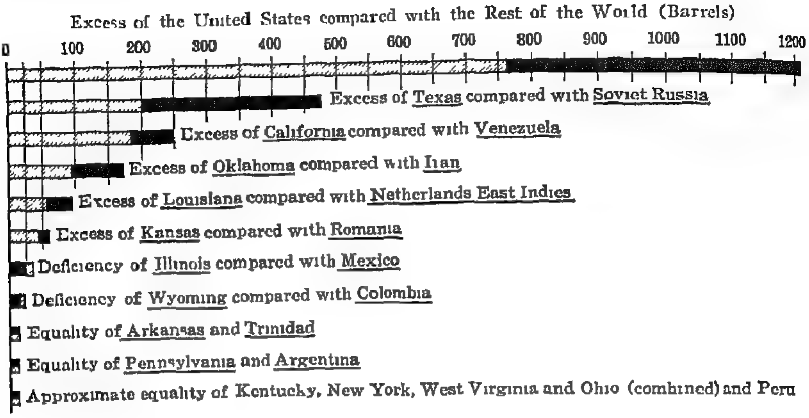


A810—World Production of Petroleum, 1938

to use those that we now have wisely and to prevent the waste which still goes on apace.

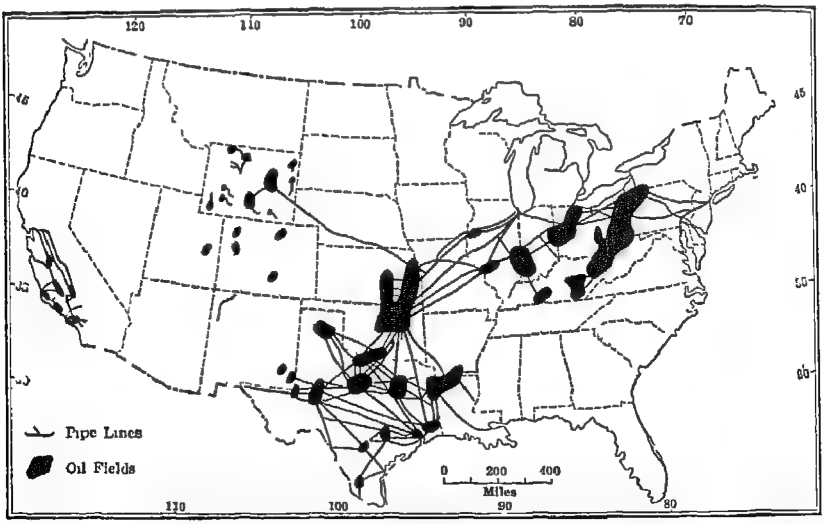
810. *Distribution of Oil Production.* In spite of the rapid growth of other oil fields, the United States still continues to furnish almost two thirds of the world's total petroleum (A810). California, which for some years was the leading producer in the United States, and in the world, is now overshadowed by Texas, which alone produces a quarter of the world's petroleum. California and Oklahoma each produce nearly one eighth, while Russia and Venezuela do nearly as well. Louisiana and Kansas stand next among the states, having a

production which rivals that of Iran among foreign countries Thus five states and three foreign countries produce three fourths of the world's oil.



B810—Comparative Production of Petroleum in the United States and Elsewhere

811 In most parts of the world pipelines play an important part in marketing oil In the United States a complex system of pipes,



A811—Oil Fields and Pipe Lines in the United States

with an aggregate length of 250,000 miles, brings oil thousands of miles from centers of production to centers of consumption and refining, or to harbors. In A810 note how pipe lines from the Texas-

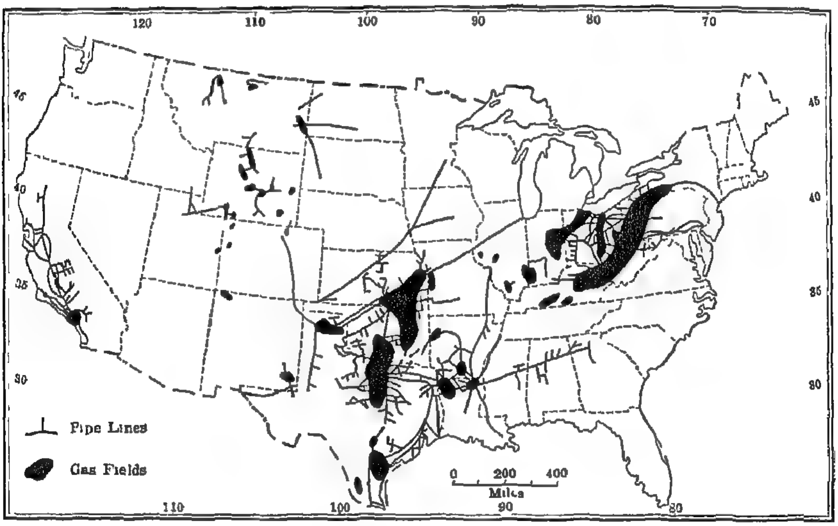
Oklahoma-Louisiana fields head toward the Kansas field There they are joined by a line from Wyoming Then at least five lines head northeastward toward Chicago and other parts of the great northeastern manufacturing region Still other pipelines head toward the Gulf of Mexico, where the oil can be pumped into ships for carriage to foreign countries There, too, part of the oil is treated in huge refineries, as is done also near Jersey City, where the world's largest refineries are located (A811)

812 In Russia the two main oil fields of Baku and Grozny, on opposite sides of the Caucasus Mountains, are each connected with the Black Sea by a pipeline The oil fields of Iraq, near the lower Euphrates River, send their oil by pipelines to the Mediterranean Sea in Palestine and Syria A number of other oil fields lie so close to the ocean that they have little need of pipelines In Mexico near Tampico and in Venezuela around Lake Maracaibo much of the oil is pumped from the wells directly into tankers lying a little distance offshore At Maracaibo many wells actually stand in the water, so that ships can lie alongside them The oil of Peru and of Borneo and Sumatra in the Dutch East Indies is also produced close to the sea, and of course the same is true on the island of Trinidad That of Rumania comes from inland fields Part is carried up the Danube by boat, but a great deal goes by pipeline to the Black Sea Difficulties of transportation from a backward interior are one of the main reasons why certain other promising oil fields such as those which Bolivia and Paraguay have quarreled over in the Gran Chaco still remain largely undeveloped Political factors, however, as we have seen, also enter into the matter In Mexico the government's seizure of the oil wells, and the bargains whereby it tried to sell the oil to Germany instead of the United States and Great Britain, have been productive of much friction

813 Natural Gas In practically all oil fields vast quantities of natural gas are associated with petroleum For a long time the gas was wasted, and this is still largely true except in the United States The waste thus involved may be judged from the fact that one tenth of our gasoline now comes from natural gas A good deal is also used for other purposes, and some is still wasted Gas, like oil, is now transported long distances in pipelines, but unlike oil the gas is piped directly to the consumer The distribution of the main pipe lines for natural gas is much more local than that of pipelines for petroleum, as is clear from the many short lines in A813 The total length of the gas lines is something like 70,000 miles For two generations large sections near the gas fields in western Pennsylvania have relied on

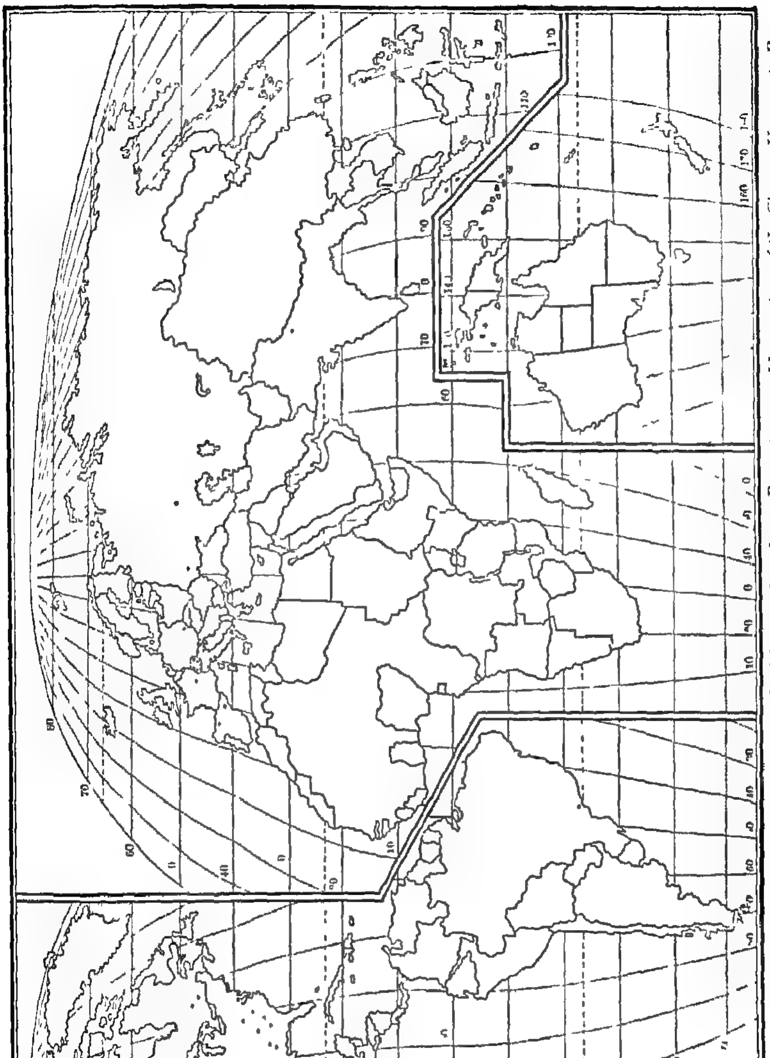
natural gas for heating and cooking, and in earlier days for light. Today natural gas is used also as a source of "dry ice," and of helium for airships In the United States it is worth about half a billion dollars each year In Europe and the eastern United States a vast amount of artificial gas is made by heating coal, leaving the residue as coke

814 WATERPOWER. A comparison between the distribution of developed and potential waterpower shows that there is little relation between the two The developed power (A814) is chiefly located in western Europe and in the eastern United States with adjacent parts of Canada Two other important centers are the Pacific Coast of



A813—Gas Fields and Gas Lines in the United States

North America, and Japan Mexico, Brazil, southern Russia, northwestern India, and Australia with New Zealand are other regions where fairly large waterpower developments are located On the other hand, B814 shows that potential power, that is, water which might be utilized regardless of whether it is or not, has a far wider distribution Each of the regions mentioned above must, of course, have considerable potential power, or it could not have so much that is developed Nevertheless, the greatest supply of potential power is found in tropical Africa, especially in the Congo Basin, but also along the Niger, Zambesi, and other rivers The rivers of Africa rise and become of great size on a high plateau When they pour down from this they form the cataracts of the Nile, the rapids of the Congo (Stanley Falls,

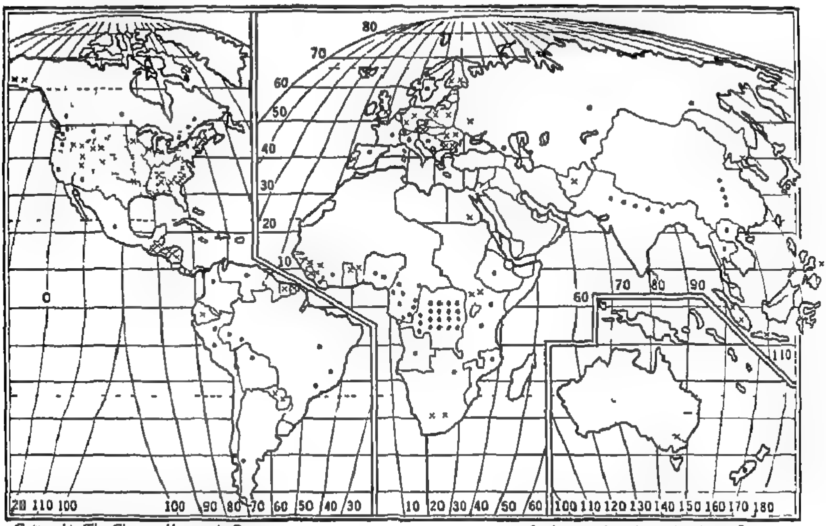


Goode's Semi-homolosine Projection, used by courtesy of the Chicago University Press

A814—World Map of Developed Water Power, 1937-39

Each dot indicates one half of one per cent, but fractions between one quarter and one-half have been indicated by dots

for example), and the Victoria Falls of the Zambesi Such sites and many others offer wonderful opportunities for the development of power. Equatorial Africa, in the Congo Basin, but also along the Niger, Zambesi, and other rivers, including Madagascar and Ethiopia, might supply well over a third of the world's waterpower, but actually it supplies practically none, and is blank in A814 It has the advantage of heavy rainfall most of the year, as well as of the great rivers on the plateau It has the disadvantage of an unstimulating climate and a backward people, and hence of remoteness from the active parts of the world



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B814—World Map of Potential Water Power Available 90 Per Cent of the Time

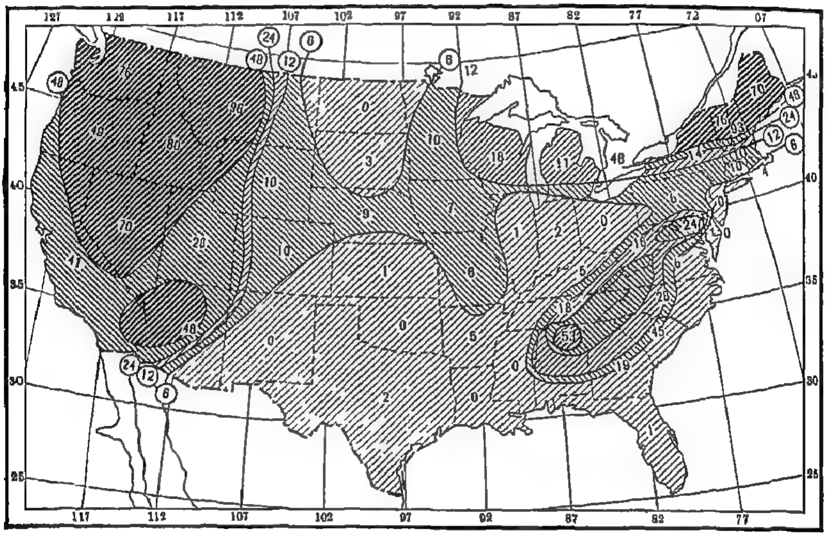
815 Other great centers of potential waterpower are located (1) on the humid east side of the tropical Andes from Venezuela to Bolivia where the tradewinds help the normal equatorial rains, and on the west side in southern Chile where west winds from the Pacific drench the mountain slopes in rain, (2) on the Atlantic border of the South American plateaus in Guiana and especially in southeastern Brazil, (3) on the south slope of the Himalayas, (4) on the western border of China proper where the great Hwangho, Yangtse, and other rivers break away from the mountains, and (5) among the mountains of Indo-China, Borneo, Sumatra, New Guinea, and other East Indies All these regions together have a larger potential supply of waterpower than Europe, the United States, and Canada. Some

people think that, when coal and petroleum become scarce and expensive, this will cause them to become great centers of manufacturing. Others believe that the climate will prevent this. They expect that the people of the more energizing climates will find ways to carry power from these undeveloped regions to those which are now the main centers of industry.

816. The 1 per cent of the world's waterpower in Brazil is especially interesting because it represents a unique method of development. In southern Brazil, from Rio de Janeiro to Santos and onward, the Atlantic Coast is bordered by a mere strip of coastal plain only a few miles wide. Back of this there is a steep rise of 2,000 feet or more to a plateau which then slopes away to the west and southwest. Many streams rise on the plateau within 10 or 20 miles of the ocean, but flow away from the ocean and do not get back to it until they have flowed 1,500 miles or more through the Parana River to the estuary of La Plata near Buenos Aires. Near São Paulo, for example, which is the most progressive city of Brazil and the main seat of manufacturing, American engineers have dammed many of these streams so that lakes lie close to the edge of the plateau. Then a short tunnel takes the water off to the front of the escarpment, and lets it drop almost straight down for nearly 2,000 feet. The presence of the plateau is a good thing for Brazil in other ways too. It makes São Paulo cool enough to be comfortable and healthful, and gives Rio de Janeiro some very beautiful and relatively cool suburbs within easy reach.

817. *Waterpower in the United States.* The location of waterpower in the United States is evident in A817, which shows the horsepower available at hydroelectric plants per 1,000 persons in each state. The Appalachian Mountains are largely responsible for the important development which begins in Maryland and extends through West Virginia and the mountainous parts of Kentucky, Virginia, Tennessee, and North Carolina. South Carolina, although not mountainous, gets the benefit of mountain rivers which cross the so-called "Fall Line" within its territory. Alabama has much waterpower because the Tennessee River flows into its northern part. The Fall Line is the place where the streams leave the hard ancient rocks of the piedmont section and flow onto the softer rocks of the coastal plain. During the time since the plain and the piedmont were last uplifted the rivers have been able to cut deeply into the plain, but not into the harder piedmont. Hence the edge of the piedmont is marked by a series of rapids which sometimes extend as much as 25 miles along the larger rivers. The line connecting these rapids is called the Fall Line. It begins at Philadelphia where the lowest rapids lie close to sealevel

The same condition prevails at Baltimore. Farther south the Fall Line passes through Washington, Richmond, Raleigh, Columbia, Augusta, and Macon (Georgia), where it is nearly 300 feet above the sea. Thus geological structure as well as altitude and general slope have a good deal to do with making it easy to develop waterpower. The Fall Line determines the location of the main southern manufacturing belt from Virginia to Alabama. On the west of the Appalachians there is no definite line of rapids, but such rivers as the Tennessee have sufficient slope so that waterpower can be developed by means of dams, at numerous points along their courses.



A817—Developed Water Power per 100 Persons in the United States

818. The Appalachians have a great advantage for waterpower in that they have more rain than any other part of the United States except the northern Pacific Coast. Moreover, rain falls at all seasons, so that normally the power plants do not have to shut down for lack of water. Nevertheless, there may be considerable variation between times of summer drought and those when rain is falling and winter snows are melting. For example, the Potomac River carries 250 times as much water in extreme flood as in drought. Such great variations are rare, but smaller ones make trouble, and deforestation has increased their frequency. They enforce one of several difficult possibilities: (1) the power plants may be so built that they use only the minimum amount of water; (2) they may install auxiliary steam power; (3) they may curtail operations at low water, thus shutting

down factories and inconveniencing other users of power, or (4) they may construct reservoirs All these methods are employed, but they are expensive The use of reservoirs is especially discouraging because rapid erosion, due largely to unwise utilization of the land, causes many of them to be filled rapidly with silt

819 A development of waterpower even greater than in the Appalachians is found in the glaciated northeastern section of the United States and the adjacent parts of Canada Maine has 710 horsepower for every 1,000 people (nearly 1 horsepower per person), and Vermont has 513 Even in such densely populated sections as southern New England hydroelectric plants furnish about 100 horsepower for every 1,000 people The reasons for this high development include not only relief and rainfall, which are always the main factors, but also a human factor in the form of long experience and full utilization of every opportunity Further advantages are found in the fact that seasonal variations in runoff are less than farther south, and the whole region has been glaciated The ancient icesheet made the topography so irregular that lakes and ponds abound These natural reservoirs tend to keep the flow of water even throughout the year They also lessen the danger of floods and remove the difficulty which arises in many regions because so much mud is deposited behind the dams The icesheets also blocked the courses of most of the rivers, obliging them to seek new routes where they frequently encounter rock or glacial deposits that give rise to rapids and falls which make ideal sites for power plants. The New England States have developed practically all the waterpower sites where a flow worth using is available as much as nine tenths of the year. A similarly high development is found in Italy and Switzerland, which are like New England in having neither coal nor oil, and in Germany.

820 The Niagara River, with its huge supply of power for both the United States and Canada, was displaced by glacial blocking of its former outlet. That is why the water from all the Great Lakes drops more than 160 feet in a single great fall The volume of water at Niagara is more uniform than at practically any other major waterpower site except along the same waterway at Beauharnois and the Lachine Rapids of the St Lawrence River above Montreal The Great Lakes hold so much water that seasonal variations in the flow are almost negligible, except when due to wind or ice Such conditions account for the fact that New York State with 1,900,000 horsepower has almost as much waterpower as California (2,500,000) The only other state which ranks higher is Washington with its huge new developments on the Columbia River. In A817 New York, with 145

horsepower per 1,000 people, ranks with West Virginia (148), but if New York City and the Hudson Valley are omitted because they get little benefit from waterpower, New York ranks close to Oregon (360)

821 All three of the Pacific states stand high in respect to waterpower Even Norway will not rival Washington when the Columbia River project is completed, as appears in Table 31 Newfoundland, Canada, and Switzerland are the only other foreign countries which surpass Oregon in their developed supply of waterpower The abundance of waterpower on the Pacific Coast is interesting when one recalls that, although the rainfall on the west slope of the Coast Range in Oregon and Washington is the highest in the United States, it is highly seasonal with a great excess in winter This unfavorable feature, however, is largely overcome by the fact that much of the precipitation is snow which lasts far into the summer, thus keeping a good volume of water in the rivers most of the year A dense cover of forests, relatively porous rocks, and many glacial lakes also help in this respect Another interesting point both on the Pacific Coast and in all the region from the Great Plains westward is the close connection of waterpower with irrigation In progressive countries the two tend more and more to go together wherever there are mountains and seasonal rainfall The use of water for cities also plays its part in the development of power plants. The union of at least two of these three uses is illustrated by the huge Boulder Dam on the Colorado River as well as by the Columbia dams Other illustrations are the superb systems whereby Los Angeles and San Francisco bring water from the high Sierras through huge tunnels and across mountain ranges When water can be used for power, irrigation, and the support of cities, it pays to spend a great deal of money on dams, reservoirs, and aqueducts

822. *Waterpower Outside the United States.* Table 31 sums up the use of waterpower outside the United States The regions numbered 1 to 7 are especially favored by rugged relief The parts of each where waterpower is developed are also helped by glaciation Norway is highly favored because its many small rivers plunge swiftly off a high plateau to a seacoast which is open to navigation in winter, even though located far to the north Deep sheltered fiords where ships are always safe lie at the very foot of waterfalls so high that they furnish abundant power at surprisingly low cost In Alaska somewhat similar conditions prevail, for though that territory lies far north, its southern coast, where the waterpower is developed, is no colder than New York in winter Newfoundland, Canada, and Sweden are quite cold in winter, but get their power from rivers which never

freeze sufficiently to prevent the use of the water. Canada, of course, gets a considerable fraction of its waterpower from the Niagara River, just as does New York. Switzerland and New Zealand owe their great supply of waterpower to high mountains, covered many months with snow and well supplied with rapids and waterfalls. In places such as Alaska, Newfoundland, and New Zealand the total amount of developed waterpower is small, but the amount per person is large, and

TABLE 31

DEVELOPED WATERPOWER
(Horsepower per 1,000 Persons, 1937)

I. Advanced countries with favorable relief and rainfall		
1	Norway	970
2	Newfoundland	942
3	Canada	740
4	Switzerland	672
5	Alaska	582
6	Sweden	300
7	New Zealand	255
8	Old Austria	148
9	Italy	142
10	U.S.A.	136
11	France	125
12	Finland	123
II. Countries with less favorable relief		
13	Old Germany	33
14	Australia	21
15	Old Czechoslovakia	20
16	Yugoslavia	17
17	European Russia	11
18	Great Britain	9
19	Argentina	6
20	Belgium	4
21	Asiatic Russia	1
III. Handicapped by seasonal rains or tropical inertia		
22	Japan	75
23	Panama	72
24	Hawaii	67
25	Spain	56
26	Costa Rica	54
27	Chile	42
28	Mexico	24
29	Brazil	17
30	French Morocco	12
31	Guatemala	10
32	Chosen	10
33	Peru	8
34	Bolivia	4
35	Venezuela	4
36	Honduras	4
37	Colombia	3
38	India	2
39	Nicaragua	1
40.	China	0

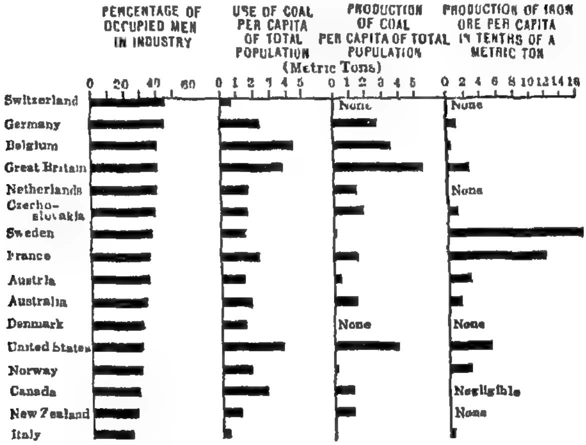
that is the essential point so far as prosperity and progress are concerned.

823. Three of the next five regions in Table 31, namely old Austria, Italy, and France, get their water mainly from the Alps, but, inasmuch as a large part of their people live well beyond the Alpine regions, they do not have nearly so much per capita as does Switzerland. Finland is much like Canada, with a great abundance of glacial lakes and rivers, which provide a peculiarly favorable environment for waterpower. Most of the countries of Group II in Table 31

(Nos. 13-21) have done well in developing what waterpower they have, but cannot do much because of the gentle relief of the earth's surface. Old Germany, to be sure, includes a small section of the Alps and many lower mountains such as the Black Forest, but it has an enormous population in the lowlands farther north. Belgium, on the other hand, except for one small section, is so flat that only very industrious people would think it worth while to build any dams at all. Asiatic Russia has many rivers which might furnish power, especially in its eastern Pacific section, but these are too remote to be used at present. Moreover, the extremely low winter temperatures make it vary hard to keep power plants in operation. Finally in Group III of Table 31 we have a large number of countries which are handicapped by seasonal rains, as in Japan and Chile, or by these plus the general conditions of inertia and poverty which hinder progress in oriental and tropical countries. When rain falls heavily at one season and in small quantities at another, and there are no snowy mountains to serve as reservoirs, the flow of water is naturally very uneven. Waterpower development in China and India will always be difficult on this account. In China there is practically none as yet, and in India, despite the steep, snow-capped Himalayas, only 1 horsepower for every 1,000 people. Japan, on the other hand, has developed her waterpower very fully in spite of seasonal variations in rainfall. Her difficulty in this respect, however, is much less than that of China, India, and many of the Andean countries of South America. Panama has much waterpower in proportion to its population because of the dams built in connection with the Panama Canal. Countries such as Peru and Colombia have high and sometimes snowy mountains, as well as waterfalls, but the rainfall is seasonal, and the people are poor and generally unprogressive.

824. ELECTRICITY. Coal and waterpower are the two important sources of electricity, their relative shares depending on the local natural resources. In the United Kingdom and Holland, for example, electricity is derived mainly from steam engines which burn coal; in Germany the ratio is 5 horsepower from coal to 1 from water; in the United States it is 1.5 to 1; in France the two sources are about equal; in Japan only one sixth of the electricity is produced by coal; Switzerland, Sweden, Norway, Spain, Italy, Canada, and Mexico get their electric power almost wholly from water. Of the world total the United States produces about 33 per cent, Germany 11, Great Britain 7, and France 5. The highest production of electricity per capita is in Canada, twice as high as the United States, with Switzerland second. Canada and Switzerland export electricity to neighboring countries.

825 SOURCES OF POWER AND THE DISTRIBUTION OF MANUFACTURING There is widespread misapprehension as to the part played by coal and waterpower in determining the location of manufacturing Many people suppose that the presence of coal or waterpower, or at least of these together, is the main reason why manufacturing has developed so remarkably in the northeastern United States and the North Sea region They seem to imply that, if Italy, Rumania, Egypt, Peru, or Borneo had been equally well supplied, they too would have become great centers of manufacturing This is by no means true Coal —and the same may be said of waterpower or iron, or of all three together—is a *great help* in manufacturing, but it is not a *cause* of manufacturing It does not determine in which of the natural regions



A826—Industrial Development Compared with Use of Coal and Production of Coal and Iron

manufacturing shall be most active Nevertheless, *coal greatly increases the efficiency of regions which would otherwise stand high in manufacturing It also helps to determine what kinds of industries are dominant in special sections of those regions*

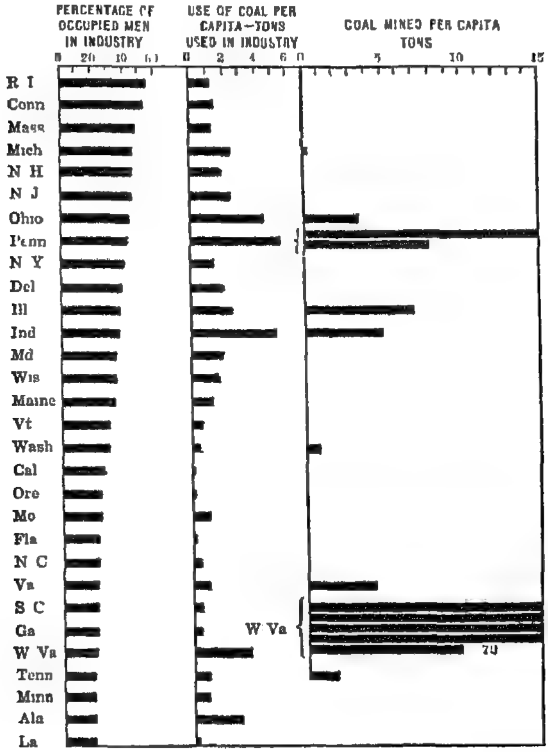
826 We shall discuss this matter more fully in the chapter on manufacturing, but the preceding general principle will be considered a little further here The general relation between coal and manufacturing is illustrated in A826 The left-hand bars show the percentage of the occupied men engaged in industry, the middle bars show the *use* of coal per capita when the entire population is taken into account, those that come next indicate the corresponding *production* of coal, and the fourth set shows the production of iron ore. If coal were a primary cause of industrialization the bars in the first

three columns ought to vary in essentially the same way Yet among the sixteen countries where industry is most highly developed, Switzerland, which has no coal, has the largest percentage of its men engaged in industry It imports a small amount of coal, it also uses water-power But its chief method of making up for lack of coal is to manufacture goods that need only a little power Similar reasoning applies to iron A826 shows no tendency for a large production of iron ore to be associated with a high percentage of men engaged in industry. The fact is that, if a nation is progressive, it will develop its iron ores, as well as its coal, if it has any Even if it lacks one or both, it will, nevertheless, devise some method of making progress industrially.

827 Some figures as to the amount of power required for various products will make the matter clear In the United States the cost of fuel and power for every $1,000 worth of products amounts to only about $5 for jewelry and shoes, $6 for automobiles, $8 for watches, clocks, and printing, $10 for knit goods and flour, $12 to $13 for electrical machinery, furniture, silk goods, and rubber goods, and $16 to $21 for machine tools, agricultural implements, cutlery, foundry products, woolen goods, and the more expensive kinds of cotton goods On the other hand, it rises to $40 for cheap cotton goods, $65 for paper and the products of steel works and rolling mills, $75 to $80 for chemicals and pottery, over $100 for glass-making, $160 for brick, $200 for cement, and $280 for blast furnaces One fifth to one fourth of the expense of making cement or pig iron goes for fuel. The Swiss, being wise people, originally developed their industries by making watches, clocks, jewelry, silk goods, knit goods, and other products that require relatively little coal Then, when electrical development made it possible for them to bring cheap power from the mountains to the cities, they took up such work as the preparation of aluminum and the fixation of nitrogen Sweden, Denmark, Austria, Norway, and Italy, all of which have very limited supplies of coal, also specialize in industries that can get along without much coal for power, although their choice is somewhat different from that of Switzerland On the other hand, Germany, Great Britain, and Belgium, with plenty of coal, go in for the heavy industries in which iron plays the chief part and coal is highly necessary If they have iron ore, as in Great Britain, they use it, if they are not well supplied, as in Belgium, they import it

828 A828 shows a similar condition among the 30 states of the United States where industry is most highly developed States that mine a great deal of coal generally consume a great deal also, but they

are far from being the states where manufacturing is most active. Virginia, West Virginia, Tennessee, and Alabama fall among the lowest 8 states on the left of A828, but are among the highest on the right. On the other hand, among the 30 states that are most highly industrialized 21 mine no appreciable amount of coal per capita. Some of these can indeed get coal easily from neighboring states, but others such as California and Florida are so remote that coal is expen-



A828—Relation of Industrial Development in the United States to Use and Production of Coal

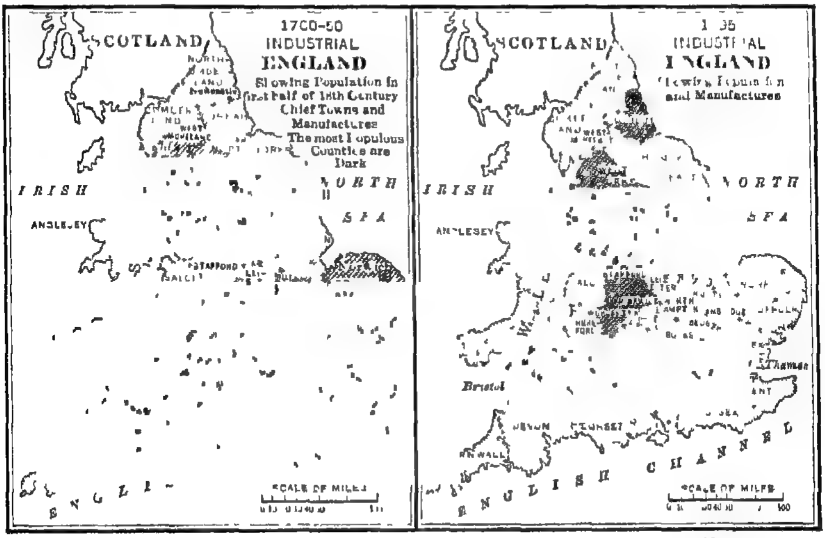
sive. If iron ore were added to A828, the only states with a large production per capita would be Minnesota, Michigan, Alabama, and Wyoming. A similar condition exists in Canada. Most of that country's coal is mined in Nova Scotia or Alberta, but neither of those provinces is highly developed industrially. On the other hand, Ontario, which includes the city of Toronto, and Quebec with Montreal, mine no coal at all, but lead in industry. Ontario brings coal from the United States, and Montreal from Nova Scotia, but both

cities are put to considerable expense in this respect Yet Ontario and Quebec are far more active industrially than Nova Scotia or Alberta and their near neighbors Newfoundland mines a huge amount of iron ore per capita and might get coal cheaply by water from Nova Scotia, but its industrial development is almost limited to paper pulp

829 Historically the conditions are similar Since the dawn of civilization *the regions of greatest industrial activity have always been those where civilization in general has stood highest* All industries, of course, were formerly primitive, but even then the work of artisans reached its highest level and greatest activity in the most highly civilized regions such as Egypt, Babylonia, and later Greece and southern Italy In the Middle Ages, northern Italy and Spain took the lead Then, as the world's culture and power migrated farther north with man's increasing control over nature, the center moved to France, Germany, Great Britain, the Low Countries, and Scandinavia Before people learned how to use coal for smelting iron the forests of certain parts of Great Britain became exhausted because cut so freely for this purpose Laws were passed to prevent further cutting Before the invention of the steam engine, Paris was famous as a center of artistic manufactures, the Swiss were the best clock-makers, the Flemings of Belgium were noted for their linen fabrics, and British cutlery was sold all over the world Previous to the nineteenth century the making of woolen goods for clothing was the most important of all types of manufacturing, or at least it employed the greatest number of people It had reached a higher stage of development in Great Britain than anywhere else before the invention of the steam engine and before iron was used to any appreciable extent for machinery This made it possible for England to take the lead in cotton-manufacturing as soon as the cotton gin and power loom were invented The essential point is that before either coal, steam, or iron machinery was used the highest development of industry was found in the same general section of Europe as at present, that is, in the Marine Cyclonic Region where the climate is especially healthful and stimulating The fact that wonderful supplies of coal and iron happen to be present in this same region greatly intensified this development after the steam engine was invented, but did not materially alter its general location

830 Nevertheless, coal did cause a shift in the local distribution of manufacturing as shown by the shading in A and B830 In 1750 industrial occupations were most common in the three parts of England which are heavily shaded in A830 (1) the rich agricultural counties of Norfolk, Suffolk, Essex, and Kent, (2) the southwestern

counties north and south of Bristol. (3) Lancashire and other regions surrounding Liverpool. By 1900, as is evident in B830, manufacturing had become relatively insignificant in the first and second of these sections. On the other hand, the third area had expanded greatly so that it now extends from west Yorkshire in the north past Liverpool, Manchester, and Sheffield to Worcestershire, with Birmingham as an especially important new center. In addition two new industrial sections have grown up, one in the far north in Durham and the other in the south in Wales. Each of the three new areas, namely, Durham, the Birmingham area, and South Wales, is located close to coalmines.



(*From Industry in England by H. de Gibbins.*)

A830—Industry in England, 1700-1750 B830—Industry in England, 1895

Thus coal and iron have caused a local concentration of industry. Moreover, they have enabled manufacturing to make remarkably rapid progress. It must be remembered that most of England is now highly industrialized compared with the world as a whole, or with the England of 1750. Moreover, although London is not shown as a modern industrial region in B830, the goods manufactured by the 10,000,000 people within a radius of 20 miles of that place are worth about one fifth as much as all the manufactures in England, and more than those of the whole of Spain and old Poland with their much larger populations. Thus this London area without coal has increased its manufactures almost as much as the regions with coal. Moreover, Switzerland, Massachusetts, New York, and other places

that are not on coalfields have increased their manufacturing during the last century as much as places such as Birmingham, Pennsylvania, or the Essen District of Germany which are located directly on coalfields.

831 *Waterpower and Manufacturing* Waterpower, like coal and iron, greatly helps alert, active people who have already reached a high stage of progress, but it never stimulates backward people to any appreciable extent Hence the general distribution of manufacturing throughout the world has little to do with the potential waterpower Belgian Congo, the Asiatic part of the Soviet Republic, French Congo, and Brazil, with a total of 42 per cent of all the world's potential waterpower, have practically no hydroelectric development Even in India, which has long been managed by Great Britain, only 1¼ per cent of the waterpower has been developed Among the 25 countries with the greatest potential waterpower only 5—the United States, Canada, Norway, Japan, and France—are among the 16 countries that are most highly industrialized Such considerations by no means detract from the importance of waterpower In geographic regions which are otherwise fitted for manufacturing, its presence is an immense help New England illustrates the point. Manufacturing began there without any help from waterpower Almost as soon as Europeans settled in the United States they began to set up little industries of the primitive type in their own homes Thus when power looms were invented New England was ready to seize upon them and set up factories run by waterpower This is what almost always happens *The regions which are most advanced are the ones that seize upon new inventions and thereby are enabled to utilize resources which were formerly wasted The use of these resources in turn stimulates further development*

PART XI

MANUFACTURING

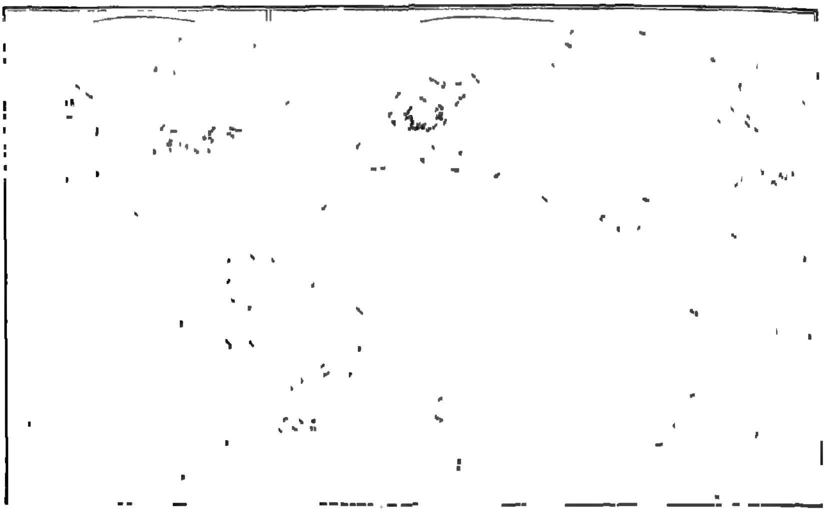
Chapter XXXIV

FOUR GREAT TYPES OF INDUSTRY

832 The Measurement of Industrial Activity Although agriculture is the greatest occupation of the world as a whole, manufacturing is the most important industry in highly developed countries of the cyclonic type Such countries are the ones of highest productivity not only because they have the most profitable agriculture, but because they also produce manufactured goods most abundantly It is much more difficult to measure industry than agriculture In measuring agriculture the main features are indicated by the yield of a relatively small number of products such as wheat, corn, oats, cotton, flax, and rubber For most of these, fairly accurate statistics are available for all countries except China, Iran, Turkey, Ethiopia, and a few minor areas Moreover, although the quality varies from country to country, the difference between the best and the worst is not excessive Only an expert can tell, for example, whether cocoa comes from Ecuador or west Africa With manufactures, on the other hand, the number of articles to be considered is enormous, running into tens of thousands Then, too, such industrial pursuits as house-building, painting, printing, smelting, and all sorts of work done by carpenters, masons, plumbers, electricians, blacksmiths, engravers, and so forth, are usually classified with manufacturing Moreover the same materials are used over and over in different kinds of manufacturing Iron ore is smelted in blast furnaces, then rolled into sheets or drawn into bars in rolling mills, next it may be made into special shapes in other steel mills, after that another manufacturing plant may convert it into some kind of machine Thus the same iron may be reckoned as part of the output of four different factories Again each special kind of manufactured article usually varies more than the products of agriculture. The different kinds of watches, typewriters, pens, books,

desks, and electric-light fixtures vary much more than the different kinds of rice, barley, jute, oranges, or grapes.

833. In addition to all this, statistics of manufacturing are not so accurate and complete as those of agriculture, mining, or even lumbering. Many countries publish no data at all as to the kind and value of their manufactures. Among the countries that publish such data there is so much diversity of method that it is very difficult to make accurate comparisons. Within the limits of the United States, however, we can make accurate comparisons by means of several exact



A833—World Map of Percentage of Men Employed in Industrial Pursuits

criteria such as the total value of manufactured goods, the value added to the raw materials by manufacturing, the percentage of goods belonging to different classes, and the number of people employed in making them. For the world as a whole, the most satisfactory basis yet available for comparing industrial activity is the one used in A833, namely, the number of people engaged in industry. For this there are fairly good statistics in most countries, although for some, such as China, Afghanistan, Iran, and many tropical regions, it has been necessary to use estimates in preparing A833.*

* In their ordinary form statistics as to persons engaged in industry are not always a safe guide. For example, according to the ordinary tables Mexico appears to be more industrial than Bulgaria, having 11.4 per cent of its occupied population engaged in industrial pursuits as against 7.9 in Bulgaria. This, however, is misleading, because these percentages are based on totals which include women. In

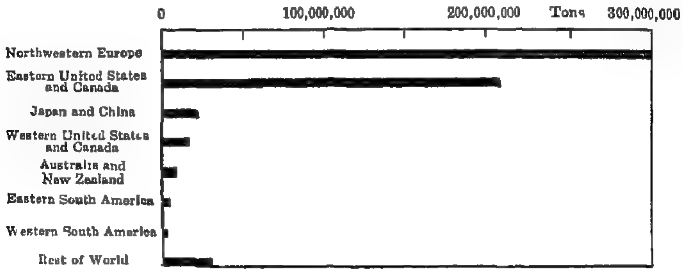
834. WORLDWIDE DISTRIBUTION OF INDUSTRY. The central features of A833 are two areas where more than 40 per cent of the men are engaged in industrial pursuits. One of these surrounds the North Sea and includes much of western Europe. Switzerland and old Germany, with 45 per cent of their men in industry, are the most highly industrialized of all nations. Scotland with 43 per cent, and England, Belgium, and the Netherlands with 40 follow closely. The United States as a whole falls far behind, having 31 per cent of its men in industry. Sweden, France, Australia, and Denmark all surpass it. Nevertheless, individual states such as Rhode Island with 56 per cent, Connecticut 53, and Massachusetts 49, are even more highly industrialized than either Switzerland or Germany. These southern New England states form the eastern end of a second main industrial area in which more than 40 per cent of the men are engaged in industry. This extends westward to Chicago and includes Toronto in Canada. In size it almost equals the North Sea area of high industrialization, although its population is far less. From both of these areas the percentage of men employed in industry decreases gradually outward, but the southward decline in the United States is broken not only by the cotton factories of the South, but also by the fact that even Florida and Louisiana, which have few cotton factories, have more than 20 per cent of their men engaged in industry.

835. A833 shows also several minor areas where the percentage of men engaged in industry rises to 20 or in most places 30 per cent. Notice how symmetrically the industrial areas are located. They usually lie on the east and west sides of the continents in middle latitudes. Western Europe on a large scale balances Japan on a small scale. The Japanese area is beginning to spread into China, but even in Japan the highly industrialized section comprises only a narrow coastal belt from Tokyo and Yokohama through Nagoya to Osaka and Kobe. In North America the area from New England to the Missis-

Bulgaria practically all the women on the farms are recorded as agricultural workers, whereas in Mexico only a few are thus recorded. Yet actually the Mexican women help a great deal in the fields. Even if they did not, we should get a better comparison by taking only the men, for the best available measure of the degree to which a country depends on industrial pursuits is the percentage of families which depend on such pursuits for a living. Reckoned in this way, Bulgaria's percentage is 12.3, while that of Mexico is 9.1. Even this gives only an approximation to the truth, for in Bulgaria many men who are reckoned as farmers spend much of their spare time in making knives, cloth, wooden articles, and other small manufactured goods at home, while in Mexico this is comparatively rare. Nevertheless, the percentage of the occupied men who are engaged in industrial pursuits is the best available measure of the degree to which a country depends upon industry.

sippi is balanced by the Pacific area from British Columbia to California. As times goes on, the Pacific area will doubtless become more and more like the two main manufacturing regions. The state of Washington already has 30 per cent of its men engaged in industry, and the coastal parts of Oregon and California rise to that level. Utah is a minor outlier of the Pacific industrial area.

836. In South America the region around Buenos Aires and Montevideo stands opposite the one around Santiago and Valparaiso in Chile. Argentina, Uruguay, and Chile are still mainly agricultural, but the more they prosper, the greater becomes the industrial population in the great cities. Nearer the equator the growth of industry on the relatively cool plateau at São Paulo joins with a number of factories at Rio de Janeiro in producing an outlier of the eastern South American industrial area. Even in Australasia the states of



A837—Approximate Use of Power in Manufacturing. Power derived from water or oil is expressed in terms of coal.

Victoria and New South Wales, together with New Zealand, balance West Australia, with the Adelaide section of South Australia tucked in between. Australia is even more industrialized than our Pacific Coast. In Victoria no less than 35 per cent of the men are engaged in industry, chiefly around Melbourne, while in New South Wales (around Sydney), South Australia (Adelaide), and New Zealand (Auckland and Wellington) the percentage rises above 30. Even Brisbane in Queensland is moderately industrialized. In Africa the continent ends in such a low latitude that the moderately industrialized area in the Union of South Africa is the only one that has developed. In many ways the Australian and South American industrial areas resemble one another, just as do those of the eastern United States and western Europe. A little manufacturing is carried on here and there in other regions such as India, but it is of relatively slight importance.

837. The overwhelming bulk of the world's manufacturing is carried on in the heavily shaded areas of A833, especially the two largest

One way of judging the amount of manufacturing is by means of the power employed. This is mainly derived from coal, but water power and petroleum must also be considered. These can be expressed in terms of the amount of coal needed to produce the same amount of power. On this basis the approximate industrial activity in the various areas where at least 20 per cent of the men are engaged in industry is shown in A837. The importance of the North Sea region and the United States is so overwhelming as to need no comment. When we analyze industries according to their complexity and the degree of skill required in them, the outstanding importance of these advanced areas becomes even greater.

838. FOUR TYPES OF MANUFACTURING. I. *The Primitive Type.* Industrial activities fall into four great types—primitive, simple, community, and complex. Primitive industries are those which are carried on by hand, generally in people's homes and without the use of power. Here are some modern illustrations: the making of bows and arrows by Indians, the daily grinding of grain in hand mills by hundreds of millions of people in China, India, Iran, and Turkey, the making of toys from wood in Swiss homes, the making of lace by women as they watch the flocks and herds in central France, the weaving of linen at home in Poland, the making of Turkish rugs, the canning of peaches or blueberries in our own kitchens, dressmaking, sewing, and knitting, the making of paper dolls, kites, or whistles by children, ordinary cooking, which is, of course, a kind of manufacturing. Such industries in one form or another are found everywhere from the lowest savages upward. As people rise in the scale of civilization the relative importance of primitive industries steadily diminishes. Among savages they supply the only kinds of homemade manufactured goods, among highly advanced urban people who live in apartment houses and take their meals in restaurants they have dwindled to mere pastimes. Nevertheless, in our day they are still highly important in the form of cooking, sewing, and washing, but their products rarely go outside the home.

839. II. *The Simple Type.* Simple industries resemble many primitive industries, but are carried on in factories and by means of power instead of in homes without power. They are designed to reduce the weight or bulk of raw materials, or to make it possible to preserve or transport them easily. Their general purpose is to prepare products for use in other industries, which in turn put them into shape for final consumption. Old-fashioned flour-grinding, for example, not only takes out the oily germ, or little plant, which tends to become rancid, but also removes the bran. Thus the keeping qualities

are improved, the weight to be transported is reduced, and the product is made ready for the next manufacturing process, namely baking. Other examples of simple industries are the sawing of wood into lumber, or its conversion into paper pulp, the smelting of ores, the grinding and heating of lime and cement, the canning of fruits, vegetables, and fish in factories, the work of slaughtering and meat-packing, the tanning of hides, the ginning of cotton and the extraction of cottonseed oil As a rule these occupations are carried on close to the places where the original products are produced The main exceptions are ores that are sometimes smelted at the mines and sometimes carried to supplies of coal, as in the case of iron, petroleum which goes to refineries in pipes or ships, and materials such as hides or wool that can be kept a long time in their original state and lose little weight in the process of manufacture Another noteworthy feature of simple industries is that as a rule few or no other raw materials are needed in order to prepare them Thus a flour mill needs no raw material except wheat, a cotton gin needs only cotton, and a sawmill only logs Even when other raw materials are required, they are usually either flavorings or preservatives, such as the salt used in meat-packing, chemicals such as are employed in tanning, or fluxes which do not form part of the finished product, like the limestone used in smelting iron

840. III *The Community Type* Community industries are those which are needed locally in order to keep the community going Every civilized community wants its own newspapers and local printing plants It also wants bakeries, and plants to supply it with ice, ice cream, gas, electricity, and telephone service It needs iron foundries, railroad repair shops, and garages within easy reach Then, too, it must have carpenters, plumbers, masons, electricians, and all the other kinds of workers who build new structures and repair old ones Not all of these are required in every small town, but no civilized community can get along unless practically all are within easy reach Nobody in Panama wants to depend on New York for daily newspapers, bread, ice cream, railroad repairs, or the setting of new panes of glass Even though a community makes nothing at all in the way of goods to be sent away for sale, it must have a considerable number of people engaged in community industries.

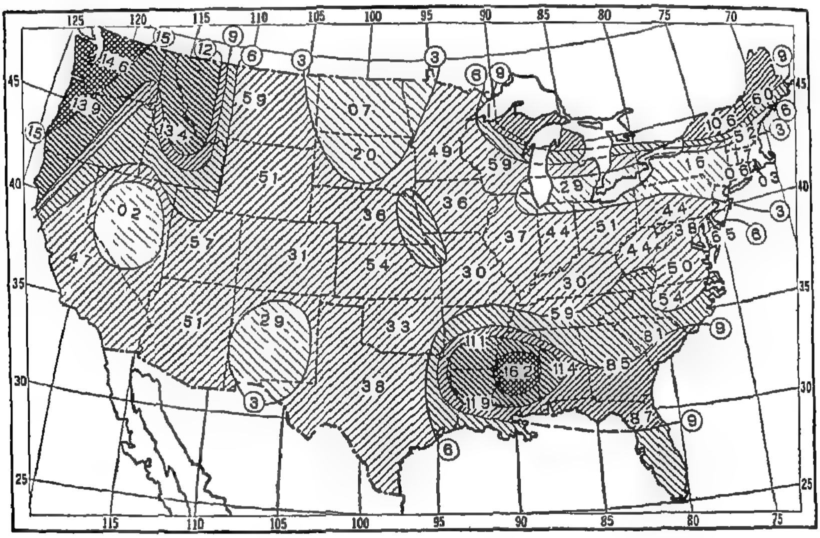
841 IV *The Complex Type* The remaining industries belong to the complex type They are complex in their methods both of manufacture and of distribution Here are their outstanding qualities (1) in general they employ complex machinery, (2) they usually combine two or more kinds of raw material, for even a steel machine may have brass cups, wooden handles, or glass dials, (3) the purpose

in manufacturing is not to preserve the materials or make them easy for transportation, but to fit them for some special use, most of them produce goods for the ultimate consumer, that is, the person who finally uses them, and in so doing ultimately destroys them or throws them away. Such goods are not designed for purely local use, but for sale in many places. After leather has gone through the simple manufacturing process of tanning, some of it may be sold to cobblers who will use it in the community industry of resoling the shoes of the people who live nearby. A larger part will go to complex industries to be made into belts, suitcases, straps, and especially boots and shoes. There it is combined with metal, cloth, wood, oil, rubber, and other raw materials to produce finished products ready for the ultimate consumer. Some of these products may, of course, be sold locally, but the greater part are sold in widely scattered markets. Iron, after the simple process of smelting in a blast furnace, usually goes through an intermediate or semi-simple process of being rolled into sheets, bars, plates, and rods in steel works and rolling mills. These semi-final products may go to railroad repair shops, blacksmith shops, and garages to be used in community industries. Or they may go to factories where machinery, screws, automobiles, locomotives, or stoves are made. There they enter into the complex industries and are prepared for the factory, house, or transportation system where the final product is ultimately consumed in the sense of being worn out.

842. Distribution of the Four Types of Manufacturing. I. *Distribution of Primitive Industries.* Each of the four types of industries has its own distinctive geographical distribution. A map of primitive industries is almost the opposite of A833, which shows the distribution of the other three types combined. In central Asia, for example, there is little manufacturing of the three higher types. In the villages and among the nomads, however, many people still make their own clothing, bedding, and even tents out of the wool of their sheep, goats, camels, or yaks. Some of them make their own saddles out of wood and leather which they themselves have tanned, while their saddlebags and containers for milk are made of the skins of their own animals. Even in the towns of western and central Asia a multitude of people spin wool in their spare moments, make rugs, felts, and even cloth at home, slaughter animals for meat, and build their own houses out of dried mud with a little rough wood. The relative importance of primitive industries as a factor in human life is at a maximum among primitive people like the Australian aborigines who sometimes make arrowheads out of the glass of old bottles instead of the flint that was formerly employed. It falls to a minimum among

the dwellers in the great cities of the United States and western Europe Nevertheless, primitive and barbarous people have such poor tools, and usually work so slowly and spasmodically, that the actual production by means of primitive industries is greatest in fairly active but rather poor civilized countries such as Bulgaria

843 II *Distribution of Simple Industries* In studying the remaining types of manufacturing, we can illustrate the main principles by means of maps based on the exact statistics of the United States, even though no accurate maps of the world, or even of Europe, are now possible. A843 shows that among the nonagricultural workers



A843—Percentage of Nonagricultural Workers in the United States Engaged in Simple Industries.

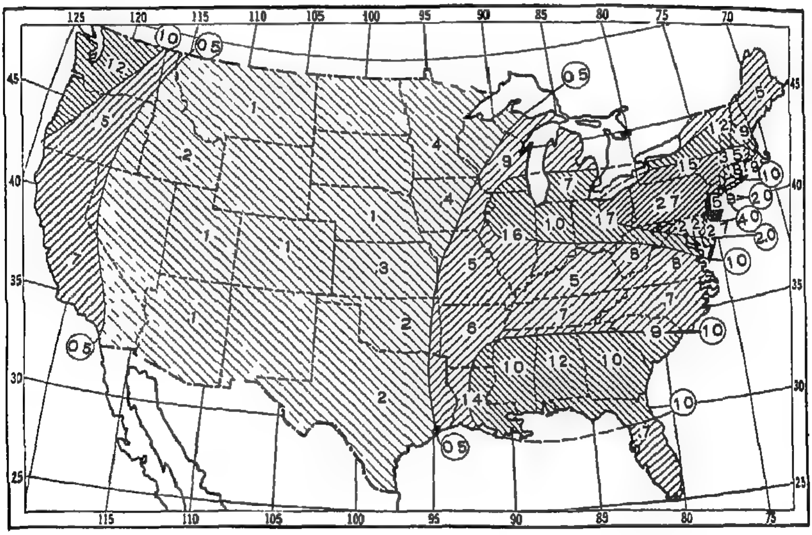
of southern New England only a small percentage are engaged in simple manufacturing A few sawmills, some brickyards, and some works where marble and granite are shaped into blocks, or where concrete, cement, and paving materials are prepared, almost complete the list New York, too, stands low in spite of some flour-milling, canning, brick-making, slaughtering, tanning, and work in stone, cement, lumber, and pulpwood Pennsylvania rises somewhat higher on account of its blast furnaces, cement works, brickyards, coke ovens, slaughtering, tanning, and stonework Ohio, Indiana, Illinois, Missouri, and Iowa rank about like Pennsylvania with 3 to 5 out of every 100 nonagricultural workers engaged in simple industries As one goes westward, however, agricultural products such as butter, cheese, corn

syrup, canned vegetables, slaughter-house products, and flour gradually replace lumber and products of stone, clay, and iron. Nevertheless, in a belt from southern New England to Chicago the simple industries are relatively unimportant. Farther north in northern New England, Michigan, and Wisconsin, the presence of lumber, together with canning and the making of butter and cheese, raise the percentage engaged in such industries. Over the line in Canada there is a still greater increase. To the south there is also a rapid increase until 8.7 per cent is reached in Florida and 16.2 in Mississippi. This is due partly to canning, to the making of cottonseed oil and cake, and to the preparation of peanuts, pecans, and sugar. In some states, especially Louisiana, Oklahoma, and Texas, it is also due partly to petroleum refining. But the greatest factor almost everywhere is lumbering and the preparation of turpentine and rosin.

844. In the western half of the United States, aside from the Dakotas, there is also an increase in the percentage of men engaged in simple manufacturing. In the Rocky Mountain and Plateau states this is due largely to the smelting of ores. On the Pacific Coast the canning industry accounts in part for a still greater rise, but petroleum-refining is important in California. Lumbering plays a great part everywhere, and causes Washington, with 14.6 per cent, to surpass all other states except Mississippi in the proportion of its nonagricultural population engaged in simple industries.

845. This picture of the distribution of simple industries will give a wrong impression unless we correct it by considering the amount of simple manufacturing per square mile as well as in proportion to the population. The fact that Massachusetts has the lightest shading in A843 does not mean that the state fails to exploit its resources, but rather that it has a dense population. A845 shows that in proportion to its *area* it ranks second only to New Jersey. Stone-working, brick-making, planing-mills, and the like employ a good many men in proportion to the area. New Jersey far outranks all other states because it brings from elsewhere such quantities of crude petroleum, hides, and ore, but even if it were limited to simple industries based on its own products, its cement, bricks, and other commodities would still put it close to the top. The neighboring states and those directly west as far as Illinois, by reason of their coal, clay, and agricultural products, have at least one man in simple industries for every square mile of area. Elsewhere the percentages decline in fairly close harmony with the density of the population, except in the lumbering areas of the Southeast and Northwest. From such facts we infer that all parts of the country are fairly active in exploiting their natural

resources and in basing simple industries upon them Wherever there are many people nearly all the available resources are being drawn upon so far as they are needed In regions such as the Dakotas there are few people and only scanty or remote supplies of good stone, cement, clay, lumber, and fish for canning Naturally the simple industries are not well developed In southern New England and New York the resources are not large and many of them have been more or less exhausted, but the population is so dense that they are very fully exploited Nevertheless, this exploitation needs only a small percentage of the workers In the South and West the resources



A845—Men Engaged in Simple Industries per Square Mile in the United States

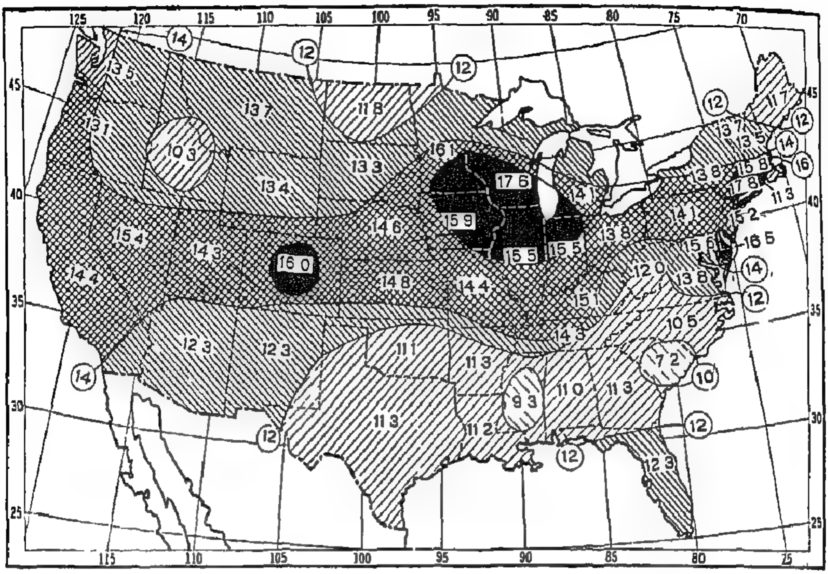
of lumber, agriculture, petroleum, and other products are so abundant and the people in some places so few that a large percentage of the workers is needed in simple industries, even though the natural resources are not so completely utilized as in the East

846 In the world as a whole the same general situation prevails Because of their coal, iron, fish, and intensive agriculture, fairly high percentages of the workers in England, Belgium, and Germany are engaged in simple manufacturing The same is true in Norway and Sweden because of their lumbering, fishing, and dairying On the other hand, France, Holland, Scotland, northern Italy, and especially Switzerland, although ranking high as industrial nations, have only a small percentage of their men in simple industries, they devote most

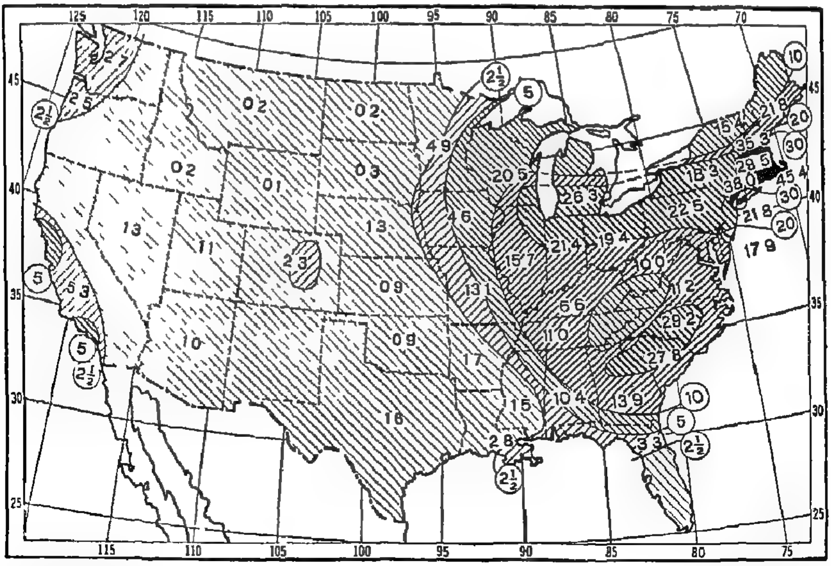
of their attention to other kinds From all this it is evident that, although the location of natural resources has a decided effect upon manufacturing, it is only a minor factor in determining where the chief manufacturing regions shall be located.

847 As one goes away from the main centers of manufacturing, the relative importance of simple manufacturing increases, even though the actual amount per square mile may decrease. In India almost the only large-scale complex manufacturing is the weaving of coarse cotton cloth, and that is one of the simplest kinds belonging to this group A similar condition prevails in China, where the cotton industry has established itself around Shanghai Up the Yangtse River at Hankow there are iron works which are often heralded as the beginning of a great industrial development This may be so, but thus far these factories have been mainly blast furnaces with relatively few rolling mills and practically nothing in the really complex iron industries Even in Japan, aside from the great cotton and silk mills, which after all are relatively simple, most of the manufacturing consists of simple operations such as preparing raw silk, cotton yarn, bean oil, and crude copper In tropical countries like Brazil, Venezuela, Java, and most parts of Africa the greater part of the small amount of modern manufacturing now carried on takes the simplest forms extracting oil from palm nuts or coconuts, making raw sugar, shelling peanuts, and curing coffee, cacao, and rubber In reading accounts of the industries of a country it is necessary to have a true idea of how largely they are of these simple kinds It may safely be said that outside the two heavier types of shading in the map of climatic efficiency (A537) simple industries completely outshadow those of the complex type

848 III *Distribution of Community Industries* A848, showing the percentage of the nonagricultural workers engaged in community industries, is quite different from A843, showing simple industries Here we find a considerable degree of uniformity all over the United States There is, to be sure, a relatively high strip across the country from southern New England and New Jersey to California, but the highest percentage, 17 8 in Connecticut, is only 2½ times as great as the smallest, 7 2 in South Carolina Moreover, all except 4 states have percentages of 10 to 16 This is quite different from the map of simple industries, where the largest percentage (Mississippi) is 81 times the smallest (Nevada) It shows that in a civilized country like the United States all parts employ nearly the same proportion of their nonagricultural workers to keep up the ordinary activities of bakeries, printing plants, planing-mills, repair shops, garages, and the like



A848—Percentage of Nonagricultural Workers in the United States Engaged in Community Industries.



A849—Percentage of Nonagricultural Workers in the United States Engaged in Complex Industries

849 IV *Distribution of Complex Industries.* In A849 the general distribution of complex industries presents certain features like the maps of health (B510), yield of milk per cow (A413), yield of eggs per hen (A431), and number of persons per automobile (A560) The contrasts, however, between the East and the West, and even between the North and the South, are greatly accentuated This means that an astonishingly large part of the United States takes practically no share in making the complex kinds of goods that are the most distinctive element in modern manufacturing The Pacific Coast does a little such manufacturing, the mountain states and those of the plains as far east as Iowa do only a negligible amount, even when we make allowance for their scanty population. The South likewise does relatively little except for cotton-manufacturing It is interesting, however, to note that among all the types of complex manufacturing the spinning of thread and the weaving of cloth are almost the simplest When the spindles and looms have once been set up, they run a long time with practically no alteration and with no care aside from ordinary upkeep The more complex processes which are needed to make the cloth ready for the final consumer are not carried on in the South to anything like so great a degree as in the North. The cloth must be dyed, bleached, cut into shape, and sewed to make clothing, bedding, awnings, and a host of other products For these purposes much of the southern cloth is carried north Thus the cotton industry of the South, like the steel works and rolling mills of both North and South, is a semi-simple industry This merely accentuates a fact which would stand out far more clearly if A849 were based on all workers, and not merely on non-agricultural workers That fact is that the complex industries are mainly developed in the northeastern United States There the most highly industrialized state, Rhode Island, has 45% of its non-agricultural men engaged in such industries, and Connecticut has 38%

850 Outside of the United States a similar condition prevails. Practically all the complex manufactured goods are made within the limits of the area where at least 40 per cent of the men are engaged in industrial pursuits as shown in A833. Even in the southern half of France, and in Italy outside the Po Valley, the manufacturing is mainly of the simple and community types Australia does a little complex manufacturing, and there are the beginnings of this kind in the five great South American cities of Santiago, Buenos Aires, Montevideo, São Paulo, and Rio de Janeiro All this, however, is on so small a scale that these countries supply only a minor fraction of their own needs for machinery, tools, motor vehicles, clocks, chemicals,

and even shoes and clothing Moreover, many of the factories for this sort of work are branches of those within the two main areas of complex manufacturing, or else are run by people from those areas Thus it appears that *complex manufacturing, more than almost any other occupation, is largely limited to a few highly progressive areas, where temperature, humidity, and changes of weather due to cyclonic disturbances give the greatest efficiency, best health, and greatest surplus energy* Later we shall see how strictly certain types of manufacturing are limited to these areas We shall also see that, *as a type of manufacturing becomes simplified into a mere matter of routine, it tends to spread from the areas of highest efficiency into those which are not quite so stimulating* This is notably true of the cotton industry, not only in the southern United States, but also in Japan, and even in India, Mexico, and Brazil

851 ECONOMIC FACTORS IN THE GEOGRAPHY OF MANUFACTURING. Manufacturing industries thrive and grow great only if they have skilled and energetic managers, an abundant and capable labor supply, sufficient capital, good markets, and ready access to raw materials All five of these factors tend to go together, although the labor supply and raw materials behave somewhat differently from the others Skill and energy among both managers and hand workers are the greatest though not the only factors in providing capital on the one hand and markets on the other Where these qualities are present, a country shows a strong tendency to produce abundant goods of high quality and great variety. It also tends to acquire wealth which can be used as capital for further industries Moreover, it has money to spend so that it provides a good market This combination of good workers, good savers, and good spenders is most strongly developed in the cyclonic regions, including those which have cyclonic disturbances during the winter Where people are good at working, saving, and spending, the population is almost certain to become fairly dense in proportion to the agricultural resources Hence a good labor supply is available Where such a labor supply is present, together with favorable conditions of working, saving, and spending, raw materials are almost invariably available They may not be produced just there, but transportation facilities are sure to be developed so that it is comparatively easy to bring them from a distance Nevertheless, *an unused labor supply, or raw materials in a region with little manufacturing, often attract industries and lead to considerable shifting of their geographical location* Moreover, *within the general areas which are favorable for manufacturing there are great local differences in the degree to which places are right for factories*

852 The United States illustrates these principles In general the entire Continental Cyclonic portion (Plate I) is one of the best places for manufacturing Nevertheless, a location near a harbor on the eastern coast is unusually desirable, and was especially so before railroads and motor vehicles became common A century ago a location there put a factory close to the best available American market It made it easy to bring raw materials from overseas, and to replenish the labor supply by means of immigrants from Europe It also made it easy to get capital Trade and industry had already made the North Atlantic cities the wealthiest part of the country. Hence local capital was available to start factories Thus many economic and geographic factors combined to give manufacturing an early and prosperous start in the northeastern United States

853 As time went on, these early geographic advantages became relatively less important The population expanded westward into regions which from the standpoint of agriculture and of the two most important minerals—coal and iron—have greater advantages than the eastern coast These new regions could not get labor and raw materials from across the ocean so easily as could the coastal cities, nor were they so well located to serve the largest market Nevertheless, the energy of their people and their favorable geographic conditions soon caused them to have great buying power and to provide a large and profitable market of their own Such advantages, as well as the mere fact that people moved west, enabled the more favored inland sections along the Great Lakes and the upper Mississippi to build up industries many of which surpass those of the East Some of these industries, such as flour-milling and the making of harvesting machinery, use local products or supply local needs Others such as the motor industry find their raw materials and their markets all over the country and all over the world

854 Stage of Civilization and Type of Manufacturing It is not to be expected that a new country will have either so much or the same kind of manufacturing as an old country We have already seen that simple industries tend to be dominant as long as a region has a relatively sparse population and large resources These conditions long prevailed in California, Oregon, Washington, and British Columbia In due time, however, this region, especially its cooler parts, will probably resemble New England, Switzerland, and central Sweden in being highly industrialized, and in having a large part of its population engaged in the lighter types of complex industry New Zealand and southeastern Australia show strong tendencies in this same direction, as do Buenos Aires, Montevideo, and São Paulo to

a much milder extent. Eastern Canada is farther along, but by no means fully developed South central Chile also gives similar promise, but as a home of the white race it is still too new to have developed greatly. Industrially, India, China, Russia, and Japan are also relatively new In view of the natural regions in which they lie and of the achievements of their people thus far, it is scarcely to be expected that even Japan and Russia will rival the regions where manufacturing is already highly developed Nevertheless, we may confidently expect a considerable growth in manufacturing, if only to supply their own needs As various types of manufacturing become highly standardized and therefore easy, we may expect them to spread to regions that are less advanced industrially.

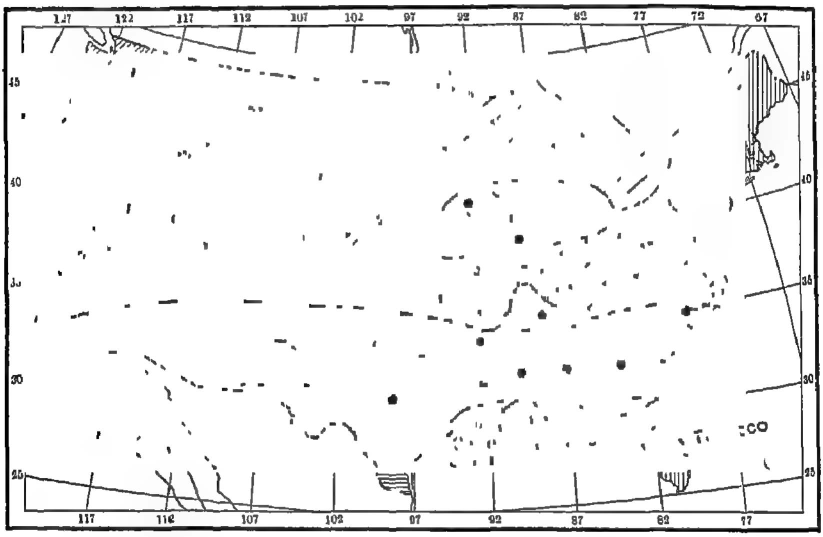
Chapter XXXV

FACTORS IN INDUSTRIAL GEOGRAPHY

855 Factors in the Distribution of Industries The geographical location of an industry almost invariably depends upon the combined effect of several diverse factors Some factors determine the general region, and others the exact spot within the region. In a broad way the conditions that determine the general location belong to three groups A geographical group includes (1) raw materials derived from field, forest, mine, or ocean, (2) fuels and other sources of power, (3) climate, and (4) the degree to which waterways and the relief of the earth's surface facilitate transportation A second group consists of economic factors such as (5) markets, (6) supplies of labor and capital, (7) competition with other industries, and (8) the advantages of an early start, acquired skill, and prestige Finally a group of human characteristics includes (9) the ability, energy, and activity of the people, and (10) their stage of cultural development The last of these sums up a large part of the influence of most of the others Thus manufacturing, like progress in general, reaches its highest development in the cyclonic types of climate, where human energy is especially high. Nevertheless, even in northwestern Europe and the northern United States, which are especially desirable climatically, there are great differences from place to place because of other factors In the following pages we shall discuss some of these factors and describe examples that illustrate them The reader, however, must constantly remember that the distribution of an industry never depends wholly on a single factor. In fact it usually depends upon the combined effect of so many that a really complete description is almost impossible Therefore, even though an industry is taken in the following pages as an illustration of the effect of only one factor, the reader will do well to ask himself what other factors are also at work

856 Raw Materials Among the many factors which determine where an industry shall be located the distribution of raw materials is one of the most obvious This is especially true in the simple industries A856, for example, shows the distribution of five simple types of manufacturing whose location depends largely upon that of their

raw materials * Cane sugar is the most sharply localized of the five. It is manufactured only in southern Louisiana where a district west of New Orleans employs an average of about 4,200 wage-earners when all 12 months are taken together The cane-sugar industry is localized here because this is the only part of the United States where the combined conditions of temperature, rainfall, and relief make it worth while to raise sugar cane Inasmuch as the raw material (the cane as it comes from the fields) weighs far more than the sugar made from it, and has only slight value after the sap has been squeezed out, sound business policy requires that it be transported as little as pos-



A856—Wage Earners Engaged in Manufacturing Agricultural Products, 1937
Where the number is less than 1000 the following symbols are used
+ 700-999, ⊢ 500 699, L 300 499, I 100-299

sible Hence mills for the manufacture of raw sugar are almost invariably located close to the cane fields This kind of simple industry is by far the largest type of manufacturing in Cuba and the other West Indies, and occupies a relatively large place in Java, India, Brazil, and the lowlands of Peru

857 North of the cane-sugar industry in A856, factories for converting cottonseed into oil, cake, and meal are located in a belt extending across the United States This industry is developed only in

* The circles in this map and in others like it are all on the same scale They are proportional to the number of wage earners when all 12 months of the year are averaged together

cotton-raising states, especially from Georgia to Texas. After an interruption in the western two thirds of Texas it revives in warm irrigated tracts farther west, especially those watered by the Rio Grande in New Mexico and Texas, and the Colorado River in Arizona and the Imperial Valley of southern California. North of the western part of the belt where cottonseed industries prevail lies a large area in which beet sugar is manufactured. The location of this depends on the presence of a relatively cool but nevertheless fairly sunny climate where sugar beets grow well with the help of the natural rainfall or of irrigation. California and Colorado are the leaders, but Michigan also stands high.

858. COMPETITION. The most centrally located industry of A856, namely, the manufacture of corn products (syrup, sugar, oil, and starch) is greatly limited by competition with other industries and raw materials. About half of the corn-products industry is concentrated in Illinois, and the rest in the neighboring parts of the Corn Belt. The location of the industry is obviously determined by that of the raw materials. One wonders, however, why corn products are not manufactured more widely in the twenty or more other states that raise a good deal of corn. One would think that the East and South might profitably engage in this industry. These sections provide a market comparable to that of the Corn Belt, and the raw material can easily be raised in abundance. In order to understand the matter, it is necessary to take account not only of markets, but of other factors as well. One such factor is competition with other industries and products. We have already seen that in the East corn is largely driven off the farms by competition with dairy products and high-priced crops such as vegetables. The low prices which factories can pay for the cornstalks and grain do not make it worth while for the farmers to plant cornfields. Therefore no raw material is available, even though a large supply might easily be raised. In the South, similar competition arises from tobacco and cotton. In the past it has paid so well to raise these crops that in many sections the farmers have not even raised enough corn to feed their own animals. Naturally, then, there has been no cheap surplus to attract capital and stimulate the manufacture of corn products.

859. One of the interesting points about A856 is that the five types of manufacturing there shown overlap very little, even though all are based on agricultural products. In most states there is only one of these industries, and in only two states, Louisiana and Ohio, is there a significant development of three. Even in these two states the raw materials come in general from different sections. In Louisi-

ana, sugar cane is raised south of the cotton districts, while the tobacco manufactured there is exceptional because it comes mainly from Cuba. In Ohio, corn is especially important in the western part of the state, sugar beets in the north, and tobacco in the south. Apparently the farmers in each section of the country tend to raise a surplus of some particular crop which grows unusually well, or is otherwise especially profitable. In the hands of farsighted, energetic people with sufficient capital, such a surplus often leads to a manufacturing industry, provided there is a proper supply of labor and a reasonably good market. Invariably, several factors must be considered before the location of an industry can be well understood. The idea to which we are thus led may be expressed in the form of the following general principle: *Raw materials often determine the location of manufacturing industries, provided other conditions are favorable, but their presence by no means insures the development of such industries.*

860. MARKETS. The distribution of the tobacco industry, as shown in A856, illustrates the way in which the influence of raw materials upon the location of manufacturing may be modified by other factors, especially the location of markets. The influence of the raw material is evident in the fact that North Carolina not only produces more tobacco than any other state, but also employs more people (29,700 wage-earners) in manufacturing that product. Kentucky and Virginia, which come next as raisers of tobacco, also manufacture a good deal, but are much exceeded in this respect by Pennsylvania, which raises far less of the raw product. Pennsylvania, in fact, produces only one tenth as much tobacco as North Carolina, but employs almost as many people in its tobacco factories. Among the other large producers of the raw material Tennessee has only a moderate manufacturing industry, while South Carolina and Georgia have not enough to appear on the map. Other states such as New York and especially New Jersey, which are negligible as producers of tobacco, stand fairly high in its manufacture. The same is true to a less degree of Florida and Louisiana. Other factors besides the location of the raw material are obviously at work.

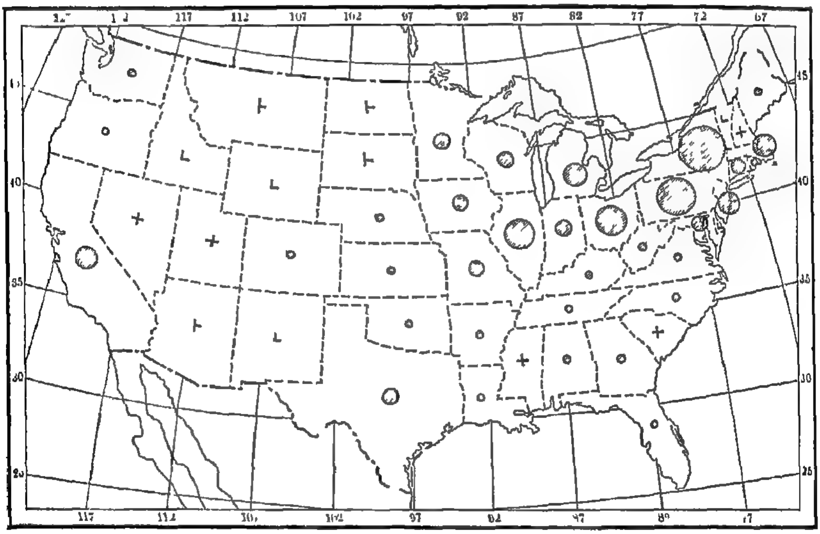
861. The discrepancy between the location of the raw tobacco and that of the simple industries to which it gives rise is due partly to the fact that the largest market for tobacco lies farther north than the places where most of the raw material is grown. The northern manufacturing section from New Jersey and New England to the Mississippi River and beyond is so populous and prosperous that it uses far more tobacco than any other part of the country. The tobacco manufactured and used in the Middle Atlantic and North Central

states comes largely, although not wholly, from Virginia, Kentucky, the Carolinas, and Tennessee. Thus in supplying the great northern market raw tobacco moves northward so that the place of manufacture is nearer the market than is the place where the raw material was raised The tobacco which is manufactured in Florida and Louisiana shows a similar tendency to move toward the market It comes mainly from Cuba and is largely used for cigars The importation of Cuban tobacco calls attention to the fact that the location of industries is modified by the quality as well as the quantity of the raw material Cuban tobacco has a flavor which makes it especially desirable as the filler, or main part of cigars, whereas the tobacco of the Connecticut Valley has smooth, unblemished leaves which make it especially good for the outer wrapper The northward movement of tobacco also calls attention to the fact that the largest markets tend to be the places where labor is most skilled, capital most abundant, and the ability to organize business most highly developed In a general way skill and high buying power go together, for skillful people tend to produce a surplus that can be exchanged for manufactures Thus in moving from Cuba to the United States, or from the south toward the north within the United States, tobacco tends to go not only toward the market, the also toward regions of greater skill in manufacturing and of greater buying power per person Such considerations introduce us to a general principle *Raw materials for manufacturing generally move from regions of lower skill, less wealth, and poorer markets to those of higher skill, greater wealth, and markets with higher buying power*

862 The importance of the market in determining where industries are located is even greater in the bakery industry (A862) than in tobacco-manufacturing In this case the raw materials play only a minor part, and the main factor is the market Bread is the most common of all foods Part of it is made in homes, but in the United States there is a rapidly growing tendency to buy bread from bakeries In cities this is now almost universal, and the same thing is fast becoming true in the rural districts of the North and West It is not surprising, then, that A862 shows a distribution of bakeries very different from that of the cottonseed, sugar, or tobacco industries No state is without bakers, and in none do the bakeries employ less than 300 wage-earners On the other hand, even in New York State, with its 12 million people, the wage-earners in bakeries show no such concentration, relatively speaking, as the makers of corn products in Illinois Nor is there any hint that bakeries are more abundant in wheat-raising states, such as the Dakotas and Kansas, where the raw material is abun-

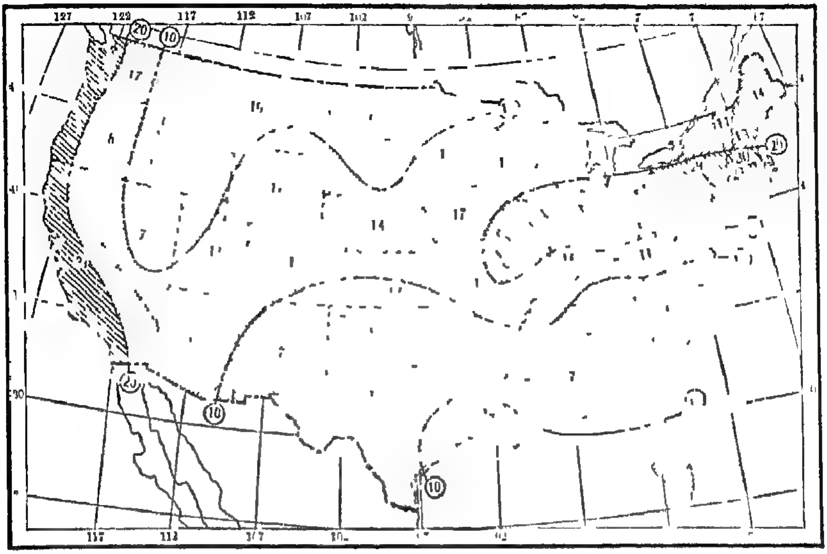
dant, than in those such as Massachusetts or Florida which raise little wheat. Transportation, too, seems to have only a slight effect upon the nationwide distribution of bakeries One of the main reasons for this is that people want their bread fresh Another is that bread-making is a simple kind of manufacturing which does not require much capital or any kind of skill that is not easily acquired Hence bread is baked close to where it is eaten

863 From what has just been said one might suppose that the amount of bread baked in any given region depends directly upon the number of people, but this is not so We see in A863 that among every 10,000 inhabitants only 4 are wage-earners in bakeries in Missis-

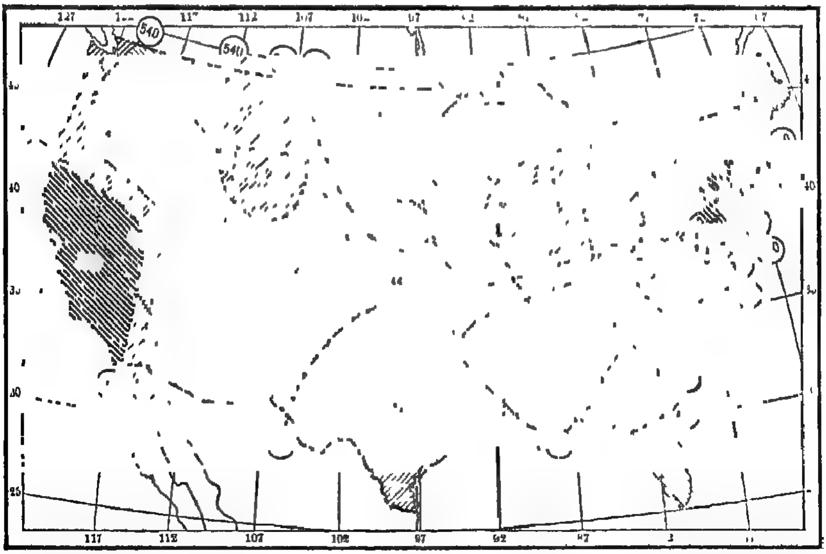


A862—Wage Earners Employed in Bakeries, 1937, 245,810 workers
(For meaning of symbols see A856)

sippi, whereas the number is 30 in Massachusetts One cause of this difference is that a large part of the population in Mississippi is rural, whereas Massachusetts is highly urban Rural people do not find it convenient to buy bread every day or two, so they bake it at home In addition to this the people in Mississippi, on the average, are by no means so well off as in Massachusetts Their average income per capita (B863) is only about one third as great Therefore people cannot afford to buy wheat bread so commonly Cheaper substitutes, such as corn pone, partly take its place in Mississippi Moreover, even in the cities a certain degree of conservatism leads people to bake bread at home more often in Mississippi than in Massachusetts A

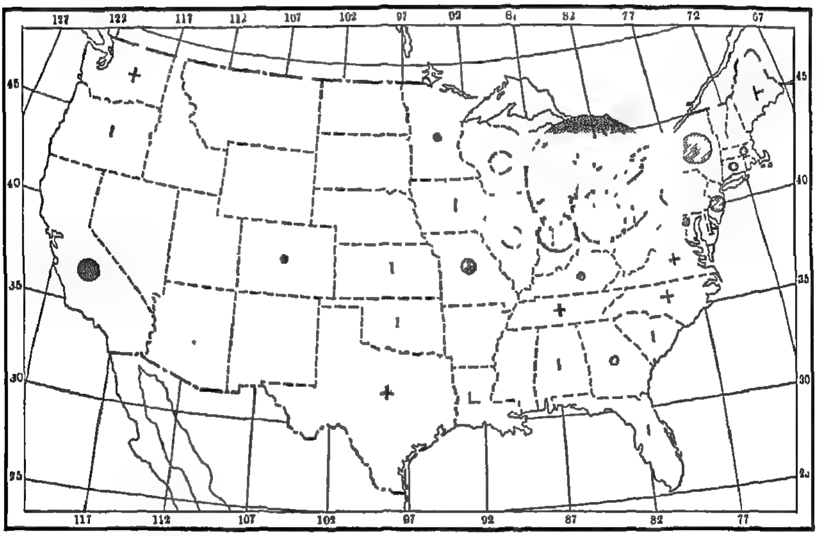


A863—Wage Earners Working in Bakeries per Ten Thousand Population 1937



B863—Income per Capita, 1937 Compare this map illustrating a relatively prosperous year, with B557, representing mainly a period of depression The greatest difference is in states such as West Virginia, New Mexico, Maryland, and Indiana, and the least in New York where income is especially high, and in the northern great plains

final contrast is that people outside the state—chiefly in New Orleans—bake some bread for Mississippi, whereas Massachusetts has many bakeries which make crackers that are shipped out to other states If all the people everywhere depended on bakery bread, if all ate the same amount of bread, and if no bread were shipped to a distance, about 25 wage-earners out of every 10,000 population would be needed in bakeries The fact that in A863 there are such wide departures from this number indicates that the demand for bakery products varies greatly from one part of the country to another Nevertheless, the local demand for bread, that is, the local market, is by far the greatest



A864—Wage Earners Engaged in Making Motor Vehicles and their Parts, 1937, 684,279 workers

Except for the huge circle for Michigan, the symbols in this diagram, as in others, are placed at the centers of the states

factor in determining where this industry is carried on The market is large in industrial, urban, and wealthy states, and small in agricultural states and those with little wealth.

864 Transportation The importance of transportation as a factor in the location of industries is so great that we have repeatedly referred to it in previous paragraphs, but here we must consider it more specifically Its effect is well seen in the manufacture of motor vehicles Between 1900 and 1910, when automobiles were first being made in the United States, the industry was mainly located in New England. A museum of old cars is interesting not only because of the

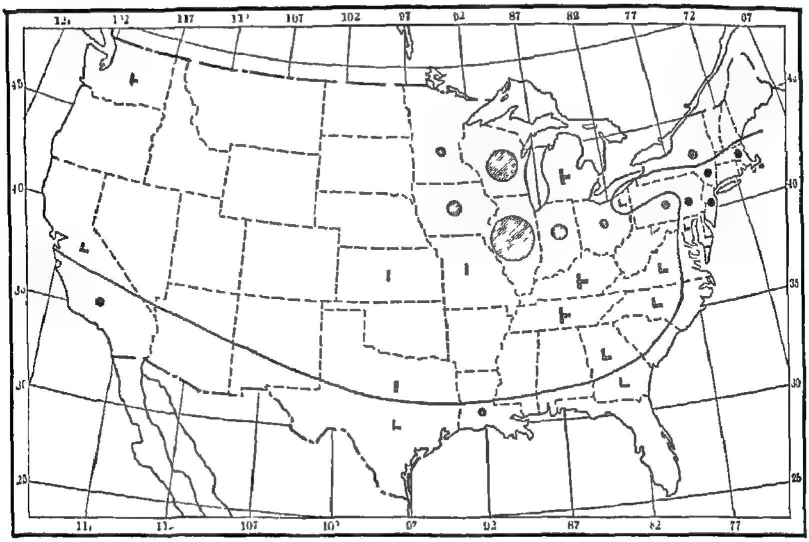
variety of models—some like old-fashioned buggies, others long, narrow, and only wide enough for one passenger—but also because of the great number of models made in small cities of southern New England. It was natural that the work should begin there, because that was where skilled manufacturing of machinery was most highly developed. New England was the center not only for textile machinery, but also for bicycles, which have a close connection with automobiles In the end, however, the automobile industry moved to Detroit and the general region at the western end of Lake Erie, as is evident in A864 The main reason was that Detroit is extremely well located for the transportation both of the finished product and of raw materials and fuel. Automobiles in their finished form are expensive to transport by rail If they travel under their own power each requires a driver and they do not seem new when they reach the customer. Even when carried on special trucks, transportation is still a large item The fact that the raw material consists largely of iron, and that the consumption of fuel in manufacturing is considerable, also makes transportation important.

865 Detroit lies near the center of the automobile market A circle slightly more than 600 miles in diameter with its center at Detroit includes Boston, New York, Philadelphia, Baltimore, Washington, Atlanta, Memphis, Kansas City, Omaha, and Montreal, as well as many less distant cities. This area now owns about 70 per cent of all the cars in the United States and half of those in the entire world. When automobiles were first used, it contained an even larger portion of the available buyers Nowhere else in the whole world does a similar circle include one fourth as many automobile users Cheap transportation by water on the Great Lakes and the St Lawrence River makes this central location still more important Cars can be carried by water to within 300 miles of practically the whole area Thus from the standpoint of transportation to the main market for motor vehicles Detroit's location is unrivaled From the standpoint of raw materials and fuel the location is almost equally good Detroit can bring iron ore at small expense by water from the Duluth region Coal can come cheaply from mines in Pennsylvania or Ohio. It is not strange, then, that in A864 Michigan shows a huge circle representing about 300,000 wage-earners, while four neighboring states which touch the Great Lakes have large circles representing another 100,000

866 A remnant of the condition which made southern New England the first home of automobile manufacturing is seen in the circles east of Ohio which represent about 45,000 wage-earners Many of these make some of the more delicate parts of automobiles, such as electrical fixtures In other words, they make light parts which require

much skill and little raw material or fuel Such articles cost little for transportation in proportion to their value In California a somewhat different situation prevails There, too, A864 shows a considerable number of wage-earners employed in making automobiles They are at work largely in assembly plants The parts of an automobile can be shipped more cheaply than the whole machine in its final form Therefore for a market so distant as the Pacific Coast it pays the Michigan factories to build assembly plants where the parts are put together. Some work of this sort is also done in the East and in Texas

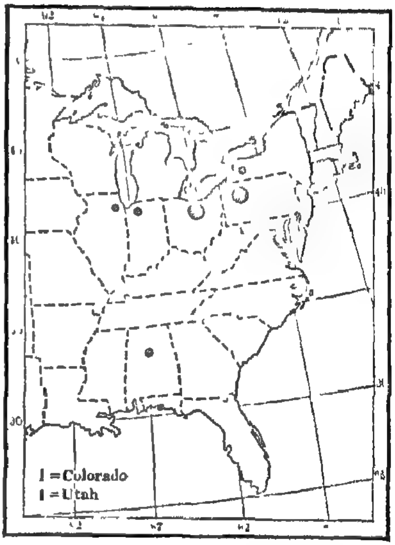
867 Agricultural machinery (A867) is another kind of manufac-



A867—Wage Earners Employed in Making Agricultural Machinery and Sugar Refining, 1937 Symbols above the curved line refer to agricultural machinery, 70,512 workers Symbols below the line refer to sugar refining, 14,024 workers

turing in which transportation is especially important The location of the market is even more significant than that of the raw materials, but both play a part Agricultural machinery is bulkier than automobiles, and hence more expensive to ship Its market lies farther west than that for automobiles This is not only because large sections of the Middle Atlantic and New England states are too rugged for farming, but also because those states generally have small farms which require little machinery The South, too, with its small farms and hand-picked crops like cotton, tobacco, and corn, does not buy much agricultural machinery Hence the market lies mainly in the plains of the North Central states from Ohio to Kansas, Nebraska, and

the Dakotas, with a southward extension to Oklahoma and Texas. Chicago's position at the southern end of Lake Michigan causes railroads from all directions except north and northeast to converge upon it. Thus it lies in an admirable position for shipping agricultural machinery to the entire prairie region from Ohio westward. It is also excellently located for bringing coal, iron, and wood by water from Pennsylvania, the eastern end of Lake Superior, or the forests of northern Michigan. Southern Wisconsin shares these advantages, and is especially well located in respect to the northern wheat region of Minnesota and the Dakotas, where much large machinery is used. Hence Milwaukee and Racine are centers of agricultural machinery. The part played by transportation is also illustrated by the sugar refineries shown outside the curved line in A867. Raw sugar from Cuba, Puerto Rico, and Hawaii is brought to convenient seaports. There the greater part of it is refined before being shipped by rail to the interior.



A868 — Wage Earners Engaged in Manufacturing Blast Furnaces, 1937, 23,075 workers

868. Fuel and Power. The presence of some source of power is important for the majority of manufacturing industries, but is especially important in determining the location of those which require much heat. In Chapter XXXIII we saw, for example, that in order to make $1,000 worth of shoes only $5 worth of fuel is needed, whereas for glass the amount is $100. Naturally the industries which require much fuel tend to be located on coal fields. Blast furnaces (A868) illustrate this.* The separation of iron from its ore in such furnaces requires coal and limestone as well as ore. The coal supplies heat, the limestone acts as a flux which, when melted, unites with the non-metallic parts of the ore and separates them from the iron. Only in rare sites, such as the Birmingham district in Alabama, are all three found close together. If the three are located in separate places, and if

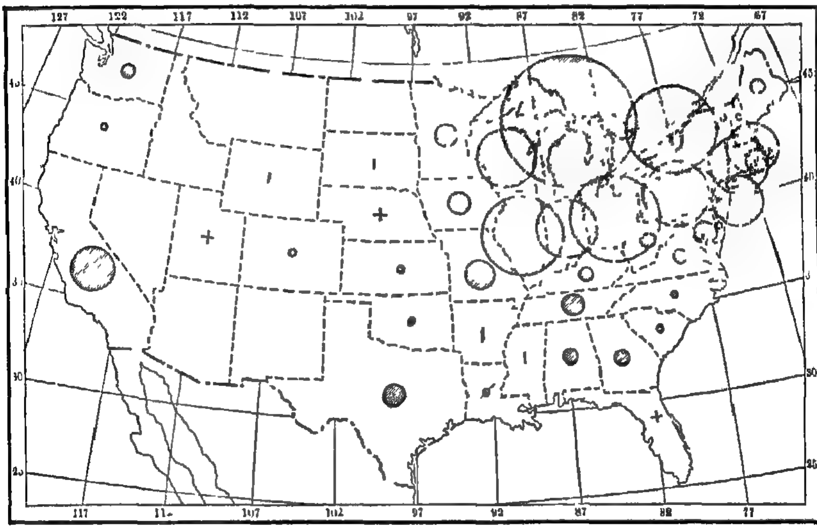
* Only the eastern United States is shown in this map, because the western half of the country has no blast furnaces aside from a few in Colorado and Utah.

there is no question of markets, the cost of transportation is normally lowest if the blast furnaces are placed near the coal rather than near the ore or the limestone. This is because the weight of the coal exceeds that of either of the other materials. For geological reasons limestone is more likely to be found near coal than near iron ore In the United States the best deposits of iron ore happen to be located close to the western end of Lake Superior The best coal, on the other hand, together with abundant limestone, is located within easy access of the southern shore of Lake Michigan and especially Lake Erie This happens to be the part of North America where other conditions discussed in this book have led to a relatively dense population which has great skill and great buying power

869 Because of this combination of favorable circumstances the states which touch Lake Erie or the southern part of Lake Michigan have about 87 per cent of all the blast furnaces in the United States Most of these are located near the lakes, or at least within a hundred miles of them Another 8 per cent of the blast furnace work is done in Alabama, leaving only 5 per cent for the rest of the country Even this 5 per cent is located mainly in states such as Tennessee and Colorado, which produce coal, or else in northern Minnesota, where ore vessels which would otherwise return empty bring some coal to Duluth and Superior The amount thus brought, however, is small because the western end of Lake Superior lies in a region of relatively sparse population where the main occupation is agriculture and there is little market for iron.

870 *America's Coal Belt.* The contrast between the large amount of iron ore that moves eastward and southward on the Great Lakes, and the small amount of coal moving in the opposite direction, emphasizes the fact that the market for iron and the facilities for transportation on the Great Lakes, as well as the fuel, are important reasons why blast furnaces are located as they are The greatest market for iron is found in factories where iron and steel are used for all sorts of machinery, including vehicles for transportation and machines for textile manufacturing, printing, household uses, public utilities, agriculture, and office work A870 shows that this work is highly developed in the states which have the most blast furnaces It is by no means confined to them, however, for it spreads eastward to southern New England and New Jersey, and westward to St Paul, Des Moines, and St Louis In the far west, too, the great cities are beginning to supply the needs of the Pacific Coast for machinery, but have to get their iron from the East Thus before the iron of the blast furnaces is put to its final use in making machinery it is distributed far more

widely than the blast furnaces. Nevertheless, the region within a hundred miles or so of the southern shore of the Great Lakes from Buffalo and Pittsburgh through Cleveland and Detroit to Chicago and Milwaukee outranks all others in making machinery almost as much as in blast furnaces It employs 80 per cent of the 23,000 wage-earners engaged in operating blast furnaces (A868) and more than 60 per cent of the 1,400,000 engaged in making machinery (A870) Thus there can be no doubt that the availability of coal, together with the location of the Great Lakes and of iron ores on their shores has a great tendency



A870—Wage Earners Engaged in the Manufacture of all Sorts of Machinery, 1937, 1,112,200 workers

The huge circle covering the western Great Lakes represents Michigan It is placed north of its real position to avoid overlapping, as is the circle for New York

to concentrate the manufacture of machinery in a rather narrow strip extending from Buffalo and Pittsburgh to Chicago and Milwaukee

871 Although coal, iron ore, and limestone can be brought together with even more ease in Alabama, at Birmingham, than in the coalfields south of the Great Lakes the effect of this upon the location of industries is by no means proportionally great Alabama and Tennessee together employ 9 per cent of the workers in blast furnaces, but all seven of the states from Tennessee and North Carolina southward employ only 2 per cent of the workers engaged in making machinery. The Pacific Coast, on the contrary, with no blast furnaces, makes 3 per cent of the machinery. These facts indicate that other factors in addi-

tion to coal and iron play a part in the development of the industries that manufacture machinery These other factors include climate, agricultural prosperity, and related conditions which cause the continental cyclonic region of the United States and the immediate coast of the Pacific Ocean to be unusually active

872 The American Coal Belt is such a remarkably fortunate part of the world that we may well examine its advantages still further In addition to having coal and limestone in one of the climates that is best for both man and agriculture, it has some of the world's best soil and finest plains not far away, while the abundant high-grade iron ore of the Lake Superior region lies not too far away in a similar stimulating climate It is especially fortunate in having all these advantages located close to a vast waterway with great irregular arms which penetrate far into the heart of the land in several directions The fact that Lake Superior projects westward, Lake Michigan southward, Lake Huron southeastward, and Lakes Erie and Ontario northeastward gives the lakes far more value than they would have if they were all strung out in a single line By sheer geographical accident it happens that the world's greatest coalbeds lie near the center of a remarkable region which includes the best part of Canada as well as the main area in the United States Thus the influence of the coal in causing iron to be brought from a distance is greatly reinforced by the fact that, when the iron has been smelted in the blast furnaces, it is located in almost the best possible position for supplying a huge market From this springs the important result that the highly varied and complex heavy industries based on iron and steel are mainly located in the same region as the blast furnaces, or in a belt surrounding them

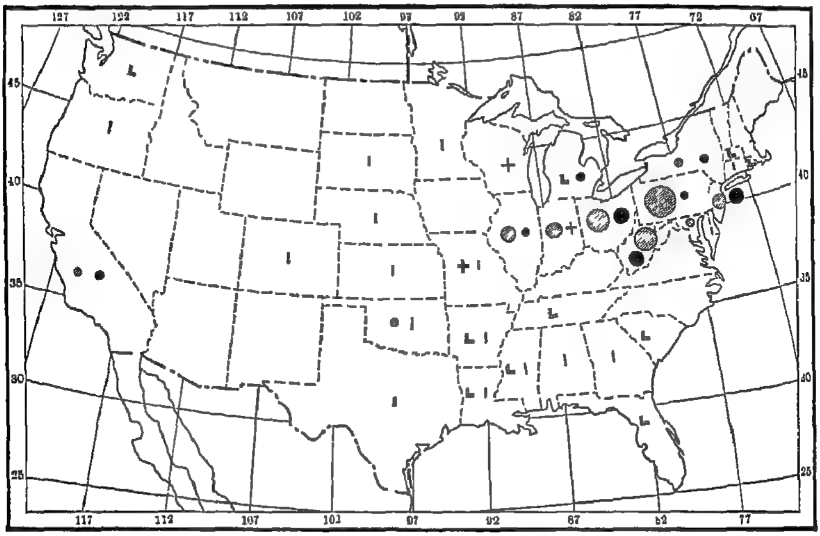
873 *Western Europe's Coal Belt* The blast furnaces of Europe, like those of America, illustrate not only the power of fuel to determine the location of an industry, but also the way in which other factors reinforce the influence of the fuel The North Sea region from Great Britain to the Netherlands, Belgium, northern France, and northwestern Germany is the only region which rivals the coalfields of the United States in its combination of advantages Coal, limestone, and iron ore are found more closely together there than in the Lake Region of the United States This condition is true in Great Britain, although now the supply of ore is running low The situation prevails to an even more marked degree on the continent where the wonderful iron ore of Lorraine is located scarcely 200 miles from the coal of northwestern Germany The transportation of these ores is helped by the Rhine River and the canals which connect that river with others such as the Meuse, Moselle, and Scheldt The work of

bringing iron ore from more distant mines is made easy by arms of the sea such as the English Channel, the Irish Sea, the North Sea, and the Baltic, which penetrate far into the land Thus ore from the coast of northern Spain is brought cheaply to the coal of Great Britain, while that of northern Sweden comes to Germany The German coal district is helped by numerous rivers and canals in the plain of northern Germany and the Netherlands Agriculture, as we have seen in earlier chapters, is more reliable and profitable in the coal belt of western Europe than in almost any other part of the world Belgium and the region near Essen have unusually good soil for their type of climate The climate in turn is so excellent for both man and agriculture that it makes up for deficiencies of the soil in other sections near the coalfields Thus in the Birmingham and Glasgow sections of Great Britain, in eastern Belgium, and above all in the adjacent section of Germany, where Essen, Cologne, and many other great cities are located, blast furnaces send out their smoke amid a combination of advantages unrivaled except in America Around them, and spreading out somewhat more widely, just as in America, lies the main area where machines and other iron and steel goods are manufactured

874 How much of this concentration of industries should be ascribed to the coal and how much to the other advantages it is impossible to say It is clear that, even if there were no coal, both the American and European areas would be the seat of active manufacturing The character and energy of the people, and the prosperity, capital, and labor supply associated with their high type of agriculture make that certain It is equally clear, however, that, if there were no coal, there would be few if any blast furnaces, and the great iron and steel industries would have only a minor development Other parts of the Old World make it clear that coal alone, or even coal and iron without the advantages of climate, agriculture, waterways, and human progress, would not lead to the development of this great type of industry Northern China has coal and iron well located in respect to each other, but blast furnaces and an iron industry have only begun to be developed under the leadership of people from western Europe and America In the Soviet Republic a fairly good combination of coal and iron north of the Black Sea has had considerable effect in stimulating the iron industries On the other hand, the great coalbeds of Kuznetsk in central Siberia have done only a little in this respect because they are wrongly situated Unfortunately the coal is located a thousand miles east of the nearest available iron ore, that of Magnitogorsk in the Ural Mountains The main market for iron is, of course, in the cities of European Russia, such as Moscow Hence if the Rus-

sians carry the ore eastward to the coal, they must bring the iron back again to the west. Moreover, they have to do this on railroads instead of on convenient and inexpensive waterways. Hence the effect of fuel in attracting iron ore and leading to a great iron and steel industry and to the making of machinery is far less here than in the United States and western Europe.

875 *Fuel, Pottery and Glass.* The location of other industries, such as the making of pottery and glass, is greatly influenced by the presence of coal, but in these also it is necessary to consider other factors as well as fuel. A875 shows the location of pottery-making in the



A875—Wage Earners Engaged in Making Glass and Pottery, 1937
Glass, light shading, 67,183 workers
Pottery, heavy shading, 33,060 workers

United States by means of solid black circles and of glass-making by lighter, but larger circles. In the main the distribution of these two industries is alike and is similar to that of blast furnaces, but there are interesting differences. For instance, the influence of the Great Lakes largely disappears. Deposits of pottery clay and of sand for glass are so widespread that the transportation of the raw material is relatively unimportant compared with the presence of fuel. Within the regions where fuel is easily accessible, however, the local distribution of factories for pottery and glass depends largely on that of deposits of clay and sand, together with facilities for transportation. Accordingly the presence of coal in states such as West Virginia and

Maryland enables those regions to use their clay and sand, and thus to stand much higher in pottery-making or glass-making than in blast furnaces. West Virginia ranks next after Pennsylvania and Ohio as a glass-blowing state and second only to Ohio in making pottery. In the same way the presence of another kind of fuel, namely, petroleum, enables southern California to become more and more important in both glass-blowing and pottery-making, while Oklahoma is making good progress as a producer of glass.

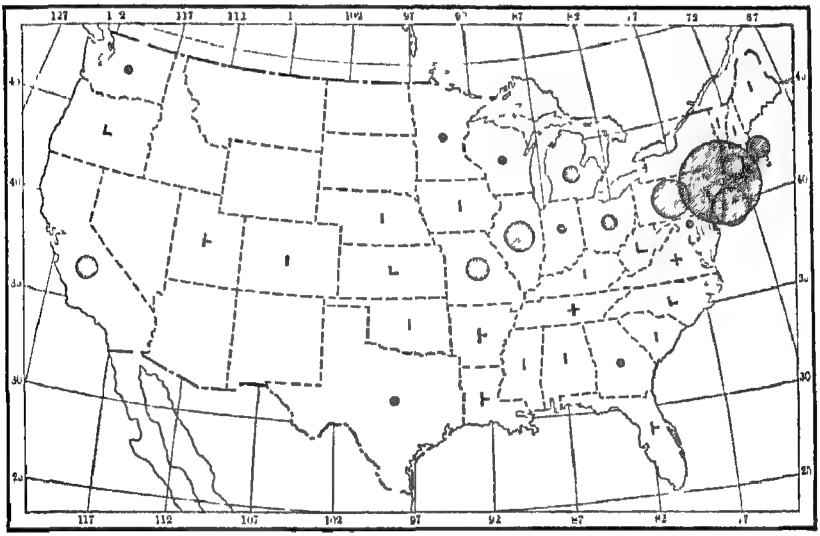
876. EARLY START AND PERMANENT INVESTMENT. The fact that New Jersey stands next after Ohio and West Virginia as an employer of wage-earners in making pottery, and is exceeded only by those two states together with Ohio and Indiana in glass-making, shows that the influence of fuel and other factors may be counteracted by the advantages of an early start and high skill. A century or more ago New Jersey was the chief American center of both these industries. In those days wood was still the chief fuel, and it grew in New Jersey as well as anywhere. Good deposits of both sand and clay helped the industries, and the workers acquired unusual skill. As time wore on, several factors combined to make these advantages of less importance. In the first place, the central and then the western parts of the country became settled, so that New Jersey was no longer near the center of the market. Then, too, the amount of pottery and glass required by the growing population became so great that coal had to be substituted for wood as fuel. At first coal was brought to New Jersey from the anthracite field farther north and finally from the bituminous fields farther west. Then both the glass-making and the pottery-making industries expanded westward, first to the coalfields and now to the petroleum fields. In recent years, automatic processes of blowing glass have introduced another new factor, and relatively unskilled men can do work which formerly required long training and no small degree of native ability. The old industry did not die because of these changes. It was too well established, it had too large an investment, and too many skilled workers. It suffered, to be sure, but the main effect was that it changed its form, and specialized more than before in highly skilled work. Some of the skilled workers moved to the new factories, or were taken there by the management to teach new operatives. Others remained in the old factories and devoted themselves to making expensive glass and pottery which require skillful handwork and sell for a high price.

877. This kind of change in the nature of an industry is common. It does not necessarily mean that an industry actually declines in its old home, provided the people there are skillful enough. What it does

mean is that, as the population grows, as new resources are developed, and as simpler methods of production are invented, the industry increases in volume and expands geographically. As a rule the parts of an industry that require least skill are the first to move into new territory This is natural for three main reasons First, the workers in the new place are generally less skilled than in the old home of the industry Second, as a rule, the places where industry is newly developing do not contain a large proportion of persons sufficiently prosperous to buy the more expensive kinds of goods The new local market is more attractive to cheap goods than to expensive ones Third, the simpler parts of an industry generally require less capital than the more complex parts It is harder to get capital in a new region than in an old one. The net result is that, although the early centers of an industry often decline in *relative* importance when new centers grow up, they may still do as much business as before, or even more The most significant change is that the kind of manufacturing that they carry on tends to become more highly skilled. A change of this sort has taken place in many other industries In respect to woven cotton goods, for example, England has lost an important part of its market because of new factories in India, Japan, China, and elsewhere New England has suffered a similar loss because of new factories in the South. Nevertheless, in the more expensive kinds of cotton textiles both England and New England are still leaders Their skill and their heavy investments, as well as the good market right around them, enable them to maintain their hold on the most difficult parts of the industry.

878 Skill and Prestige The manufacture of women's clothing is almost as closely concentrated in a single area as is that of motor vehicles, but the reasons are different Half of all the women's clothing in the United States is made in New York State, mainly in New York City (A878) Another quarter is made not far from New York in southern New England, New Jersey, and Pennsylvania Half of the rest is made in Illinois (Chicago) and California, but these centers of manufacturing are relatively small compared with New York But why does New York have such supremacy in this particular kind of manufacturing? The answer is certainly not found in the raw materials, for neither cotton, wool, silk, nor linen is raised near New York It is not found in fuels or power, for New York City has no water-power, and must bring steam coal from western Pennsylvania or West Virginia The local market, to be sure, is large, but not large enough to warrant putting half the industry in one city Nor can transportation be a primary factor, for a city such as Indianapolis lies much

nearer the center of the market than does New York. The main factors which give New York its supremacy in manufacturing women's clothing are an early start, a high degree of skill in operating the business, and the prestige attaching to the country's greatest city. Paris has a similar and more nearly world wide prestige in women's clothing. A Paris hat is prized by civilized people all over the world. Switzerland has a similar prestige in watches and clocks. In all these examples the prestige rests on a solid foundation of skilled work. Both the managers and the workers in New York, Paris, and Switzerland know



A878—Wage Earners Engaged in Making Women's Clothing, 1937, 299,726 workers

The clothing factories in practically all states are mainly located in the largest cities, such as Cleveland, Chicago, and St Louis

how to do good work, and how to keep the world aware of what they are doing.

879. CENTRIFUGAL AND CENTRIPETAL TENDENCIES IN MANUFACTURING. The distribution of manufacturing, as it exists at any given time, represents the interaction of two opposed forces. Viewed from the standpoint of the world's main manufacturing centers, one of these is a centripetal, or inward tendency, while the other is centrifugal, or outward. We have seen repeatedly that the two main centers of manufacturing are western Europe and the northeastern quarter of the United States with the adjacent part of Canada. We have also seen that there are minor centers such as the Po Valley in Italy, the Pacific Coast of North America, Japan, southeastern Australia together with

New Zealand, and South America near Buenos Aires and Santiago These regions differ considerably in industrialization, but each owes much of its progress not only to human causes, but also to a combination of at least two of the following geographical conditions. (1) a climate that is stimulating and healthful for man, (2) soil, relief, and climate that foster a productive and reliable agriculture, (3) mineral resources, especially coal and iron, but also water power, petroleum, and other types, and (4) a location such that transportation by land or more especially by water is easy Where all these are combined, as in the northeastern United States and western Europe, they possess a wonderful drawing power for industries, for raw materials, for capital and labor, for able people, and for ideas A steady flow of all these diverse attributes of civilization takes place toward these centers, particularly toward their main cities

880 Because of such conditions the great industrial regions are the places where the majority of new inventions are made and new industries developed Automobiles, radios, airplanes, rayon, and surgical instruments are a few of the many lines of invention that illustrate the matter Airplanes, for example, are needed everywhere, but are manufactured almost wholly in a few highly active industrial centers, such as Hartford and Los Angeles Although their use is less restricted than their manufacture, it is largely concentrated in the great industrial regions The United States has about 10,000 commercial airplanes, or close to half of all those in the whole world Most of these fly between the cities of the northeastern quarter Almost every new invention is utilized first and most fully in the advanced parts of the world A man who wants capital, engineering ability, and skilled labor in order to develop a new idea heads for New York, Chicago, San Francisco, or some corresponding European city far more often than for Bombay, Ningpo, Cairo, Rio de Janeiro, or Havana Thus a strong centripetal pull tends to bring a great variety of industries and activities together in the relatively small parts of the earth which are highly favorable to industry Those regions tend to become more and more industrialized, urbanized, and wealthy The difference between them and remote parts of the earth, such as Chinese or Peruvian villages, increases from generation to generation Such villages know little or nothing of modern inventions, and are much the same now as they were a hundred years ago

881 The centripetal tendency of industry is always in conflict with a centrifugal tendency In other words, many circumstances constantly cause industries to expand in regions outside the main industrial centers The opening of coal mines in China, the discovery of oil in

Venezuela, the planting of new rubber plantations in Brazil demonstrate the desire to utilize undeveloped fuels, raw materials, or agricultural possibilities which has caused industries to be established far from the great centers. The Chinese coalmines have made it possible to establish iron works at Hankow and elsewhere The oil at Lake Maracaibo has led to the building of great refining works on the Dutch islands of Curaçao and Oruba The rubber plantations necessitate not only the preparation of the latex, but also local iron foundries, carpenter shops, electric light plants, and other industries on a small scale Similar conditions, together with the desire to be self-supporting and to locate industries where they will be safe from the attacks of enemies, have led to the building of iron works in Siberia Another reason for establishing industries outside the main manufacturing areas is the presence of a market, or of an abundance of cheap labor Both these considerations have had much to do with the remarkable spread of the cotton industry not only to our South but to Japan, India, China, southern Brazil, and elsewhere

882 The centrifugal tendency is evident chiefly in simple industries and in those such as waterworks, electric light plants, and printing offices which are of the community type, and are needed locally wherever there are civilized people It is noteworthy, however, that as an industry becomes more mature its machinery often becomes more perfect so that unskilled people can use it, as is evident in our motor vehicles and clocks Almost anyone can keep a clock going, and almost anyone can run the machines employed in flour-milling, glassmaking, and the spinning and weaving of cotton cloth When this happens, an industry usually shows a strong tendency to spread out from the old centers and to develop in places which furnish raw materials, fuel, cheap labor, or a local market The net result is that, although vast sections of the earth still remain without manufacturing industries and will doubtless continue thus for a long time, industrial development becomes more and more widespread Nevertheless, new inventions and discoveries are made so fast that the contrast between the centers of industry and the rest of the world still continues to increase, as is evident in the use of motor vehicles and radios During the last 30 years, for example, the equipment of homes has changed far more in American cities than in farms in Hungary, but farm equipment in turn has changed in Hungary far more than in Venezuela

CHAPTER XXXVI

FOOD AND CLOTHING INDUSTRIES

883. GREAT DIVISIONS OF INDUSTRY. In the United States, as in every other country, a profitable method of studying the manufacturing industries is to divide them into classes. In a previous chapter we divided such industries into three types: primitive, simple, and complex. We also found that some are community industries in the sense that they are ordinarily developed locally as part of the orderly functioning of all civilized communities. Such a division of industries is not sufficient for the more detailed study contained in the present chapter. Here we plan to inquire what kinds of industries are found in each main section of the country, how far the industries are developed above or below the level where they supply local requirements, and why there are such great differences from section to section. For these purposes we shall find it convenient to divide all industries into six groups according to the needs which they satisfy: (1) food, (2) clothing; (3) shelter, (4) equipment for work, (5) facilities for transportation, communication, and trade, and (6) appliances for satisfying higher needs. Each of these great groups contains industries of the primitive, simple, and complex types. In each, also, certain industries belong to the community type because they are customarily carried on more or less fully by each local community. In a general way the industries increase in complexity as we go from food and clothing to shelter, and thence to equipment for work, facilities for transportation and trade, and appliances for satisfying the higher needs.

884. FOOD INDUSTRIES. The food-manufacturing industries are the most basic because they are devoted directly to the problem of maintaining life. They represent such essential operations as grinding flour, baking bread, churning butter, slaughtering animals, refrigerating meat, and canning all kinds of food. Among the six groups, however, they rank near the bottom in the number of wage-earners whom they employ, as appears from Table 32. Farmers are not included in Table 32, for we are here dealing only with manufacturing processes

which are carried on after the basic commodities have left the hands of the primary producers.*

TABLE 32

PERCENTAGE OF INDUSTRIAL WAGE-EARNERS IN MAIN DIVISIONS OF INDUSTRY, 1937

Food industries	11 5
Clothing industries	25 5
Shelter industries	19
Equipment for work	22
Facilities for transportation, communication, and trade	13
Equipment for higher needs	9

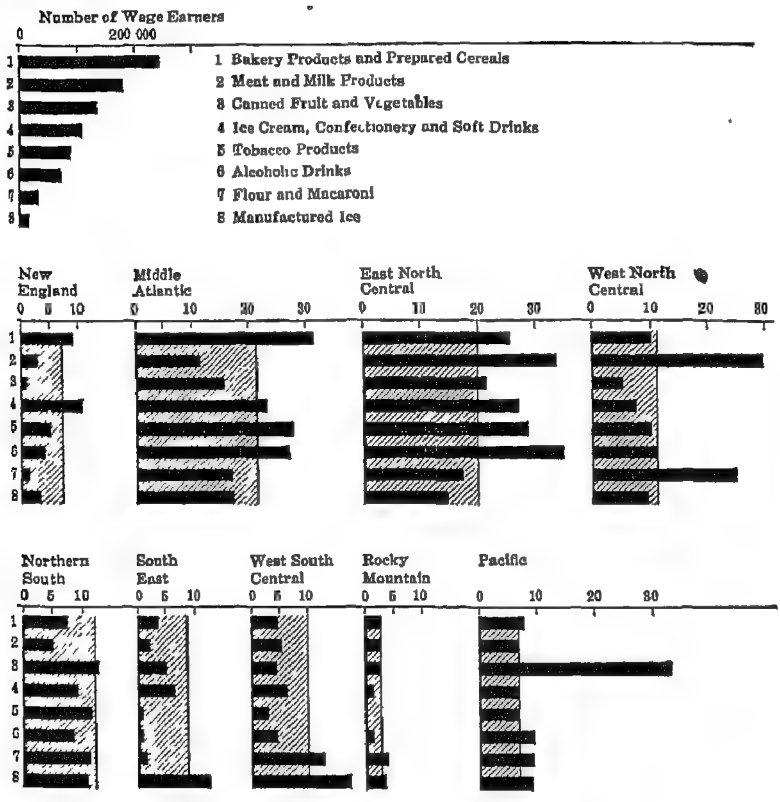
885 The percentage assigned to the food industries in Table 32 does not show their full importance because it omits a vast amount of primitive manufacturing, such as threshing grain, cutting silage, making butter on the farm, preparing jelly, canned fruit, or grape juice at home, and, above all, cooking If we include cooking, these other food-preparing occupations take part of the time of perhaps 50 million people in the United States alone Nevertheless, in comparison with other groups the food industries are less important in the United States than in more backward countries In Afghanistan, or Nigeria, for example, the women spend hours each day grinding grain by hand One of the outstanding facts about the food industries is that, for the most part, they are either primitive, like slaughtering animals in China, or simple, like making maple syrup or cigars Many, however, are community industries such as the work of bakeries and icecream factories. Only a few, such as the preparation of flavoring extracts, margarine, or fancy sauces, are complex

886 *A Diagram of Food Industries.* In studying this group of industries we want to know how much of each kind of food is pro-

* Among the many ways of measuring the relative importance of industries, one of the best is the average number of wage earners during the year as a whole The value of the products of an industry is often used for this purpose, but it includes values derived from many sources in addition to the industry under discussion These other sources include (1) the original raw materials while still in a state of nature, (2) the work of primary production by farmers, miners, lumbermen, and others, (3) transportation, (4) preliminary manufacturing processes, such as smelting ores, spinning yarn, producing steel sheets, or preparing chemical reagents, and (5) profits on sales between the preceding stages of production

Value added by manufacture is another measure of the importance of an industry It is good except that rates of wages and so forth are often much higher in one region than another, and therefore the value added by manufacture does not show how many people get a living from the industry The average number of wage earners gives an accurate picture of the direct importance of the industry in providing people with a living

duced in each section of the country, and how much of this is a surplus that can be sent elsewhere for sale The available statistics do not make it possible to determine this exactly, but A886 gives the essential facts so far as they are revealed by the census The bars at the top show the relative number of wage-earners engaged in producing each of eight main kinds of food products. The other bars show what per-



A886—Food Manufacturing Industries in the United States *

* Sugar, Flavoring Extracts, Corn Products, Cottonseed Products, Canned Fish, and other Minor Items are Omitted

centage of each kind of wage-earners is found in each of nine sections of the United States With one exception, these sections are the ordinary census divisions The industries of the northern portion of the South Atlantic states (Delaware, Maryland, Virginia, West Virginia, and North Carolina) resemble those of Kentucky and Tennessee much more than they resemble those of the southern portion Therefore the northern portion has been grouped with Kentucky and Tennessee to

form the *Northern South,* while the southern portion (South Carolina, Georgia, and Florida) has been placed with Alabama and Mississippi to form the *Southeast* In each of the nine regional diagrams the shaded portion indicates the percentage of the country's population contained in that particular section If a given geographical section has the same percentage of the country's population and of the wage-earners in a given kind of industry, the length of the bar in A886 is the same as the width of the shaded area

887 *An Eastern Deficit* With this explanation in mind, let us examine the New England section of A886 New England contains 6 6 per cent of the population of the United States Its 21,200 wage-earners in bakeries (very few happen to be engaged in the preparation of cereal foods) form 8 7 per cent of the wage-earners of that sort in the whole country Therefore the top bar for New England projects beyond the shaded area indicating population This may mean one of two things Either the average New Englander uses more bakery products than the average person in the country as a whole, or else New England ships bakery products to other parts of the country As a matter of fact, both are true A863 shows that on an average 30 persons out of every 10,000 men, women, and children in Massachusetts are wage-earners in bakeries We have already seen that this is partly because the great majority of New Englanders use baker's bread and partly because a good many bakeries, especially in Massachusetts, make crackers which are shipped outside of New England

888 The next bar in A886 shows that New England has only about half of its fair share of wage-earners engaged in making milk products, or in the slaughtering and meat-packing industry. We have already seen that this is largely because the industrial cities make it pay the farmers to sell fresh milk rather than beef Going on downward in the diagram, we find that New England cans only about one tenth as much fruit and vegetables as it would need if its people's average use of canned goods were the same as that of the entire United States. As a matter of fact, it is much more than the average, because New England is relatively prosperous and only a small part of its people live on farms where they can raise their own food On the other hand, the fourth bar indicates that New Englanders manufacture more than their share of icecream and confectionery and do fairly well in making soft drinks. This is mainly because they can afford these luxuries On the other hand, they are so far from the main supplies of raw tobacco and wheat that they do not supply their own needs in the manufacture of tobacco or flour. Nor do they make alcoholic drinks to any great extent. The fact that New England does only half of its fair share

in the manufacture of ice (the lowest bar) is explained partly by climate and partly by prosperity. New England, especially the northern part, is cold enough so that natural ice is cut from ponds and lakes in considerable quantities, and stored in great icehouses. On the other hand, really hot weather does not last very long, so that the need for ice is by no means so great as farther south. And finally the prosperity of New England and the fact that it has a high percentage of city people has led to the widespread introduction of mechanical refrigerators, so that ice plants are not necessary on a large scale.

889. *A Central Surplus.* Having seen what the bars of A886 mean for New England, we can quickly interpret those for the other states. The Middle Atlantic states are like New England in manufacturing more than their share of bakery products and of icecream. They manufacture less than their share of meat and milk products, canned goods, flour, and ice, but in all these they do better than New England in proportion to their population. They do noticeably more than their share in respect to tobacco products, and somewhat more in alcoholic drinks. Thus, on the whole, the Middle Atlantic states approach self-sufficiency much more closely than New England. Farther west the East North Central states from Ohio to Illinois and Wisconsin manufacture more than their share of every main group of food product except flour and ice, and even in these they come close to the average. Beyond the Mississippi the West North Central states from Minnesota and Missouri to the Dakotas and Kansas make up for eastern deficiencies by having more than twice the average proportion of people at work on meat products and flour. They do not do much canning, and they fall a little short on icecream, presumably because so many people live on farms or in small villages. In other respects they manufacture about the amount of food that they would need if they were like the average of the whole country. The North Central states, both east and west of the Mississippi, not only raise a large surplus of food, but also do considerably more than their share in manufacturing it and getting it ready for sale elsewhere.

890. *A Minimum of Food Manufacture.* The Central South from Maryland to Tennessee forms a transitional area so far as the manufacture of food products is concerned. Bakery products, meat and milk products, icecream and confectionery, and alcoholic drinks are not manufactured on a large scale. On the other hand, canned goods, tobacco products, flour, and ice are made in about average amounts. Little can be sold to other regions, however, except as the people of this section consume less than the average. In the Southeast, that is, in the states from South Carolina to Mississippi, the manufacture of

food products falls to a low ebb Much ice, to be sure, is manufactured, not only because the long summers create a great demand, but also because relatively few people can afford mechanical refrigerators at home Thus the local market for ice is excellent Otherwise only canned goods and icecream are manufactured to even half the average extent, but ice, of course, is not a product that is shipped more than a few miles A similar but less extreme condition prevails in the West South Central states beyond the Mississippi River from Louisiana and Arkansas to Oklahoma and Texas Flour, to be sure, becomes a product of more than local importance, but on the whole the entire South manufactures far less food than it would consume if its people used such products as freely as the rest of the country

891 *Western Self-Sufficiency* The Rocky Mountain states introduce us to still another type of food production Of course, the total production is small because of the scanty population In proportion to the numbers, however, the people in the states from New Mexico and Arizona to Montana and Idaho hold their own in manufacturing all the main groups of essential food products and in ice-making Only in luxuries such as icecream and confectionery, tobacco, and alcoholic drinks, do they fall behind Finally, the Pacific Coast is by far the most perfectly balanced of the nine sections of the United States In eight out of nine groups of products it manufactures just a little more than its fair share The one exception is canned fruits and vegetables, of which California produces one third of the country's supply Taking A886 as a whole, it is clear that New England at one end of the Atlantic Coast, and the southeastern states at the other, are the least effective as manufacturers of food products The East North Central states, on the other hand, with their large areas of Corn Belt country and northern Dairy Region, stand far at the head as manufacturers of a surplus for sale elsewhere The Pacific Coast from California to Washington stands between these two extremes as a section that is just about self-sustaining, except in its one big export of canned goods In other ways also, although by no means all, the Pacific Coast acts more or less as a self-contained unit, supplying its own needs.

892 CLOTHING INDUSTRIES The industries that supply clothing include two main branches, based respectively on textile fibers and leather The textile fibers in turn are divided into three kinds, namely, vegetable fibers of which cotton is the chief, animal fibers including wool and silk, and chemical fibers such as rayon, nylon, and the recently invented thread made of glass Although both vegetable and animal fibers are utilized for other purposes, such as housefurnishings, containers for merchandise, and equipment of vehicles for transpor-

tation, by far their main use is for clothing. In the same way leather has a wide variety of applications, including upholstery for furniture and automobiles, belts in factories, and harness for horses, but much the greater part is used for boots and shoes. The main branches of the clothing industries and the number of wage-earners employed in them are shown in Table 33.

TABLE 33

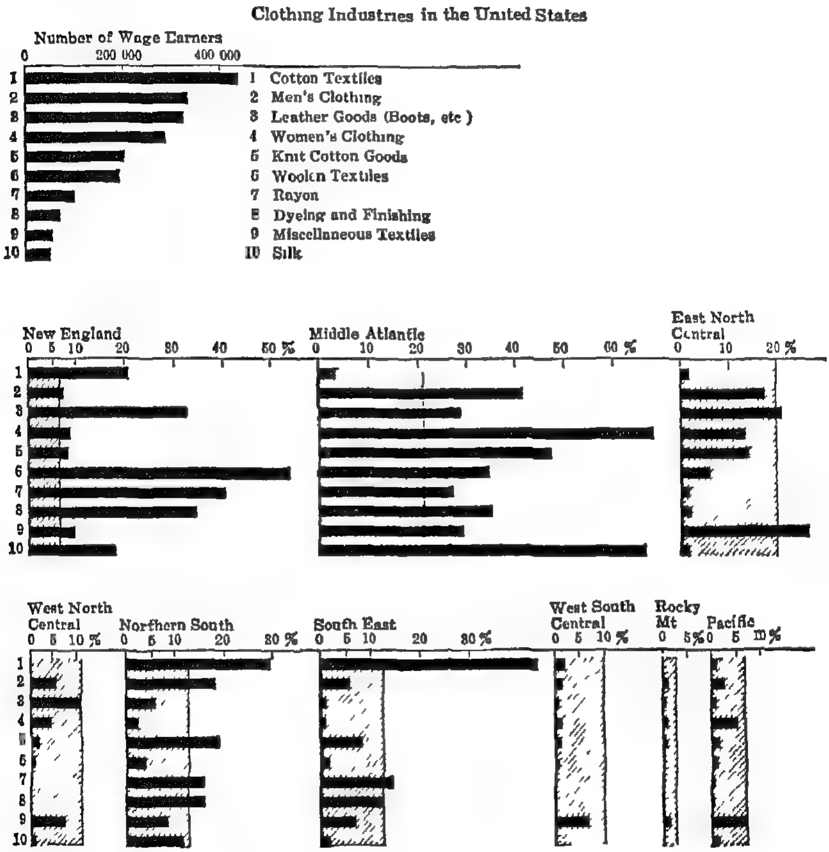
WAGE-EARNERS ENGAGED IN MANUFACTURING CLOTHING, 1937

Cotton: making of thread and yarn, weaving, knitting, dyeing	731,000
Leather: tanning, dyeing, cutting, sewing—boots, shoes, gloves	339,000
Wool: carding, spinning, weaving, knitting, dyeing	173,000
Rayon: chemical processes, thread-making, weaving, knitting, dyeing	136,000
Silk: weaving, knitting, dyeing (also some spinning)	62,000
Rubber (usually combined with some other material)	30,000
Miscellaneous: buttons, pins, needles, feathers, etc.	24,000
Finished clothing of all kinds	684,000
Total	2,179,000

893. The clothing industries range all the way from the most primitive, such as the curing of sheepskins in the encampments of nomadic Mongols, to the most complex, like the preparation of fabrics where many kinds of thread are interwoven into elaborate patterns. The main processes of spinning thread, weaving cloth, and making clothes are comparatively simple. All of them are carried on at home by hundreds of millions of people, especially in Asia and Europe. The last part, that is, the final preparation of clothing, still prevails to a large extent as a home industry, even where an exchange economy has reached its highest level. Only in a few families does the purchase of clothing go so far that there is no form of dressmaking or even of mending clothes in the home. Many home occupations, in addition to cooking, are genuine processes of manufacture, even though we do not commonly call them so.

894. *The Northeastern Clothing Section.* The industries that supply clothing are much more concentrated than those that manufacture food products. New England and the Middle Atlantic states, with 28 per cent of the country's population, have 55 per cent of the wage-earners engaged in the various industries that help to clothe us. The South, to use that term in its old sense of the states south of Pennsylvania and Ohio and east of the Mississippi, has another 21 per cent of the population and 27 per cent of the wage-earners in the clothing industries. All the area west of the Mississippi River, with about 20 per cent of the population, has only 7 per cent of this kind of

wage-earners. In A894 all the bars for New England extend beyond the limits that would be expected on the basis of population alone. Even in the cotton-textile industry, in which New England has lost its former leading position, the wage-earners are still three times as numerous as would be expected if all parts of the country shared equally in the work In woolen textiles, there are nine times as many



A894—Wage Earners in Clothing Industries, 1937

as would be expected, while rayon, leather goods (mainly boots and shoes), and the work of dyeing and finishing cloth have at least five times the number that would be expected In the silk industry, too, which is a specialty of the Middle Atlantic states of Pennsylvania and New Jersey, New England has more than twice the expected number of workers

895. In a general way the parts of the clothing industry in which New England is weak are those in which the Middle Atlantic states are strong. This is especially true of men's and above all women's clothing, and of cotton goods that are knit instead of being woven. New York City is so great a center for women's clothing, and so much is made in the surrounding area, that the Middle Atlantic states as a whole employed 68 per cent of the workers. Silk is almost equally a specialty of eastern Pennsylvania and northern New Jersey. The Middle Atlantic states employ 66 per cent of the workers. New England and the Middle Atlantic states together, although comprising only 28 per cent of the country's population, contain the following percentages of the wage-earners engaged in the clothing industries: Cotton textiles 24, men's clothing 49, knit goods 56, dyeing and finishing 56, leather goods 61, rayon 70, women's clothing 77, silk goods 84, and woolen goods 88.

896. *The American Cotton Industry.* The only clothing industry in which sections of the country other than New England and the Middle Atlantic states are conspicuous is the so-called cotton industry. This includes the spinning of cotton yarn and thread, and the weaving of cotton cloth, but not the almost equally great knit-goods industry. The old South, that is, the northern South and the Southeast of A894, have 73 per cent of the wage-earners in this industry, which is more than three times as much as would be expected on the basis of their population. The 21.4 per cent of the country's population in the old South fully hold their own in several other clothing industries. They have 22 per cent of the workers in dyeing and finishing, 24 per cent in men's clothing, 27 per cent in the knit-goods industries, and 29 in rayon. Even the South, however, does not supply its own needs in all lines of clothing, especially boots and shoes, women's clothing, and woolen goods. So far as a supply of goods for sale in other sections is concerned, the rest of the country is almost negligible. Chicago, Milwaukee, and especially St. Louis, to be sure, have flourishing shoe factories. The East North Central states also do well in miscellaneous textiles such as embroideries, but the upper part of A894 shows that the total number of workers in this branch of the clothing industries is small.

897. *Early Localization of Cotton Industry.* The history of the cotton industry provides an excellent illustration of the centripetal and centrifugal tendencies discussed at the end of the last chapter. During colonial days the northern colonies gradually became the main industrial part of America. The industries, to be sure, were very small and simple, but, in spite of British laws to the contrary, the early

colonials made iron tools, guns, ammunition, cloth, carts, harness, furniture, and various other manufactured goods

898 The industrial revolution during the latter part of the eighteenth century acted in the same way as inventions in general It had its first and greatest effect upon the main centers of industry Old England and New England were the places where the immediate effect was greatest The British, to be sure, did their best to prevent new inventions, such as the spinning jenny and power loom, from being used in other countries, but emigrants soon carried them to America Inasmuch as woolen, linen, and cotton textiles already had a good start in New England, and the people there were looking for new means of making a living, a little coastal area from Portsmouth to New London soon became second only to Lancashire as a textile center The New Englanders who organized the new factories form an interesting contrast to the leaders of the textile industry in Philadelphia The Philadelphians were mainly master weavers rather than merchants They were interested in good workmanship, and were not in the habit of trying new ventures The skill of their hands prevented them from seeing the advantages of machines Some of this skill was carried to New England by weavers who were hired to serve as teachers in the new factories The contrast between New England and Philadelphia was much like that between Old England and France. In Philadelphia and France the cotton industry is still organized with special reference to small shops which make especially fine goods and carry on the most difficult parts of the industry, in the two Englands the big factory surrounded by the houses of hundreds of factory workers is the rule

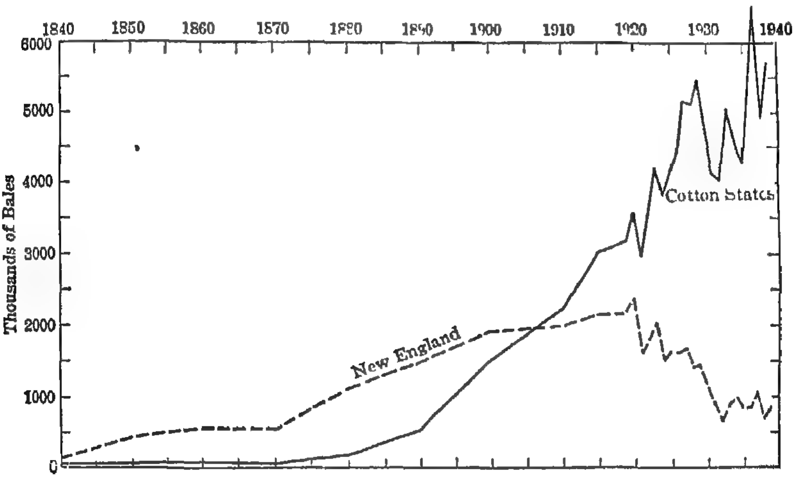
899 PAST AND PRESENT ADVANTAGES OF NEW ENGLAND (1) *Waterpower* Although New England does not owe its textile industry to waterpower, as is sometimes said, such power was certainly a help in giving the industry an early start After deciding to replace their already active hand industries by factories in which power looms and spinning jennies were used, the New Englanders looked around for sources of power. Some tried horses, but sites where water wheels were already employed for grinding grain proved better Waterpower is more widely distributed, nearer the sea, and more regular in volume at all seasons in New England than farther south Glacial lakes, ponds, and swamps, as well as a cover of forests, make the New England water unusually free from mud and hence excellent for bleaching Moreover, at places such as Fall River the waterpower is so near the sea that the cost of land transportation for raw cotton and for the finished goods is negligible. This advantage still continues. It is far less

important than formerly, however, because transportation by land has improved and become cheaper to a far greater degree than by water.

900. (2) *Climate* The climate of southern New England was a great help to the cotton industry in early times, but is not so great a help now This was especially true at oceanic places like New Bedford and Fall River, which were long the greatest cotton-manufacturing cities of America There, as in the Manchester climate of England, humid winds from the sea keep the cotton yarn moist enough so that it does not break easily. Such a climate, being well-nigh the most healthful and stimulating in all North America, is one reason why the New England textile workers were unusually efficient It is also one reason why Newport continues to be a famous and wealthy summer resort. In the South, on the other hand, the hot dry days which frequently occur in summer lead to much breakage of the yarn, thereby increasing the cost of manufacture and almost prohibiting the making of fine goods On the other hand, hot humid days which are also frequent in summer, have a bad effect on the workers, who become weary and careless. When artificial humidity was introduced, it overcame the effect of dryness but made matters even worse for the workers Nevertheless, when air conditioning reached such a development that a factory could be cooled as well as warmed and at the some time given the right amount of humidity, the climatic advantage of new England largely disappeared

901 (3) *Labor and Capital.* From the standpoint of both labor and capital New England had a greater advantage over the South in the past than at present We have seen that the combination of poor farms and energetic farmers was a factor in leading the early New Englanders to carry on hand industries The same conditions made them, or their daughters, ready to work in factories. After the cotton industry had become well established, immigrant labor was easily available in southern New England The South, on the contrary, had to rely on inefficient slave labor until after the Civil War. Now, however, when it is no longer considered degrading to work in a mill, a large supply of efficient white labor actually gives the South an advantage because wages are lower there than in the North Financial resources were probably more important than labor in developing the cotton industry in New England rather than in the South, where the raw material is grown Personal wealth in early America was mainly held by two groups, the planters and the merchants. The southern planter held riches in the form of land and slaves, but rarely had ready money beyond the limits of what came each year from the crops. In other words, he had little readily avail-

able capital The northern merchant made his money by taking risks, putting a large sum of capital into a single venture, and waiting for profits His capital was liquid at frequent intervals after a ship returned with a profitable cargo Thus it was not at all revolutionary for the merchant to buy stock in a corporation, or even to start a factory, whereas it required a complete change of habits for the planter to do so Moreover, the War of 1812 made foreign commerce unprofitable in New England at the very time when money was needed for cotton factories, which were proving highly profitable Hence, in the early nineteenth century, almost every merchant in Boston owned shares in cotton mills This advantage, like various others, still persists, but has lost much of its importance People are much

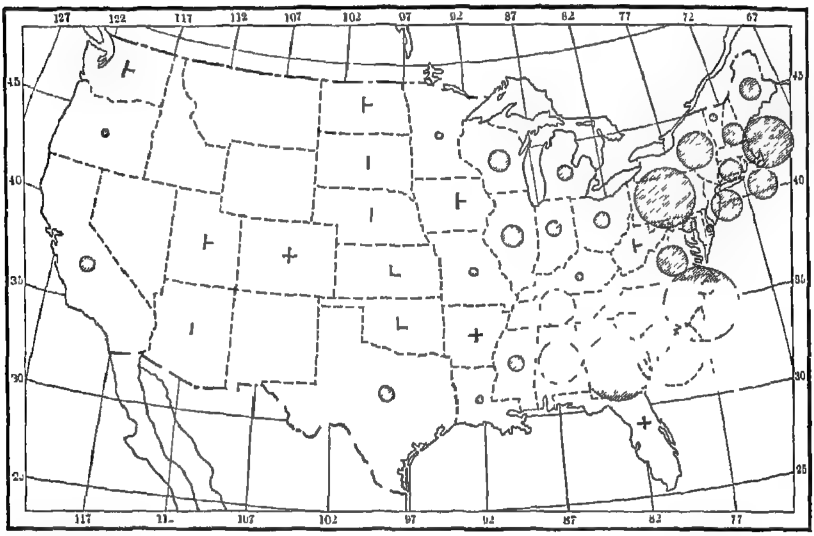


A902—Consumption of Cotton in Factories of New England and the South

more ready to invest capital in distant parts of the country than they were a century or more ago Many large cotton mills in the South are owned by northern companies which still have mills in the North

902 The Shift to the South. The conditions described in the last paragraph have combined with improvements in machinery to shift the center of cotton spinning and weaving from New England to the Cotton Belt A902 shows how this has taken place from one census to another in the textile industry The decrease in the amount of cotton spun in New England, and the amazing increase in the South, show that the simpler part of the clothing industries now belongs mainly to the South, especially the Carolinas. The somewhat more difficult work of weaving cotton cloth has not shifted quite so much, but the South now makes far more than any other section In knit

goods, which require more skill than woven goods, Pennsylvania is still decidedly the leader, and the region within a radius of 200 miles of New York, that is, the three southern New England states and much of the three Middle Atlantic states, together with the industrial parts of Delaware and Maryland, still employs 40 per cent of the workers in comparison with 11 per cent in the remaining South Atlantic states and 12 per cent in the East South Central states Pennsylvania alone has 27 per cent of all the workers in knit goods, while New York, Massachusetts, and New Jersey have another 20 per cent On the

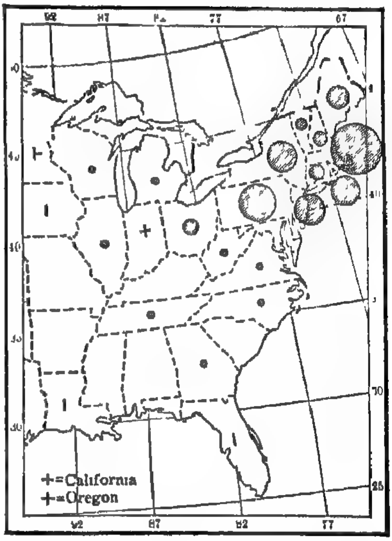


A903—Wage Earners Engaged in Making Cotton Textiles This includes woven, knit and miscellaneous goods, 1937 Total number of wage earners, 694,500, of whom 30% were in southern New England and the Middle Atlantic states, 41% in the South Atlantic states and 56% in all cotton raising states excluding Virginia and Missouri

other hand, North Carolina alone has 24 per cent of cotton spinners and weavers, while South Carolina and Georgia come next, with 35 per cent between them In each industry three states do more than half the work Such figures illustrate the way in which industries tend to concentrate in certain areas which have special advantages in respect to raw materials, transportation, markets, high skill, or an early start Woolen goods have not shifted their location to any such extent as cotton goods

903 North and South in Present Clothing Industries We shall conclude our study of the textile group of industries with some maps showing the location of five great branches A903 illustrates the entire

cotton industry including spinning, weaving, and knitting.* North Carolina, South Carolina, and Georgia are easily the leaders by reason of the yarn that they spin and weave into cloth Knit goods put Pennsylvania next, and then comes Massachusetts which still weaves the finer grades of cloth The greatest concentration of cotton industries is found not far from the Fall Line from Winston-Salem southwestward across North Carolina, South Carolina, and Georgia to Alabama There the centrifugal influence of the raw material, cheap labor, waterpower, and a local market gives these four states 46 per cent of the whole industry On the other hand, the centripetal effect of an early start, high skill, and an even larger market enables the smaller area within 200 miles of New York to retain 30 per cent of it

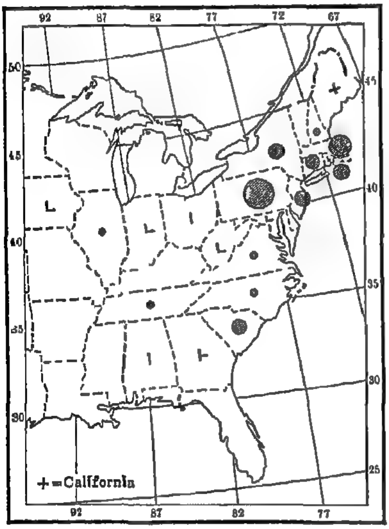


A904—Wage Earners in the Woolen Goods Industry, 1937 78% in southern New England and Middle Atlantic states, and 1% in Southern Atlantic states

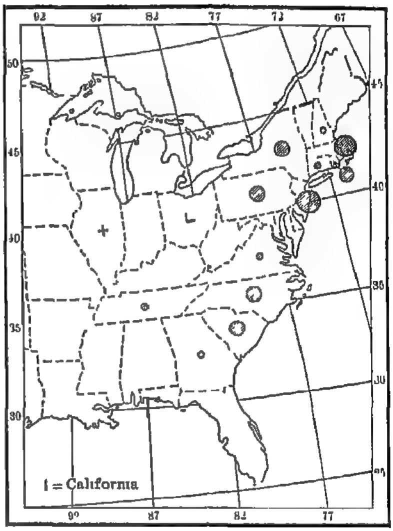
904 A map of woolen manufacturing (A904) presents quite a different picture The four southern states which excel in cotton manufacturing employ only 3 per cent of the workers in wool, while the area within a radius of 200 miles of New York employs 78 per cent, with the greatest concentration in eastern Massachusetts and Rhode Island The market for woolen cloth lies farther north than for cotton because of the temperature Raw wool comes from distant regions such as the Rocky Mountain states and Australia, where there are not enough people to make it worth while to establish big industries. Moreover, wool is so expensive that the cost of transportation is relatively less than for cotton One reason why the industry is slow in expanding westward is that the states bordering the Great Lakes have their hands full with the iron and steel industries Hence New England and the Middle Atlantic states are left with 80 per cent of the workers in wool and a still larger percentage of the value of the goods produced

* Knit goods consist of such articles as stockings, shirts, skirts, and gloves Nearly two thirds of the material, when reckoned by value, is cotton, one sixth wool, and one sixth silk

905 FACTORS THAT LOCATE NEW INDUSTRIES Silk and rayon afford an interesting example of what often happens when new industries are established Silk, to be sure, has been used from time immemorial, but only in the last generation or two has it been used in America on any large scale Rayon, of course, is a chemical product of the present century The experience, skilled labor, machinery, and factory technique needed in converting silk and rayon into cloth are practically the same as in the cotton and woolen industries The North Central states have been so busy with their iron goods and the



A905 — Wage Earners Engaged in Manufacturing Silk and Rayon, 1937, 82,238 workers 78% in southern New England and Middle Atlantic states, and 17% in South Atlantic states

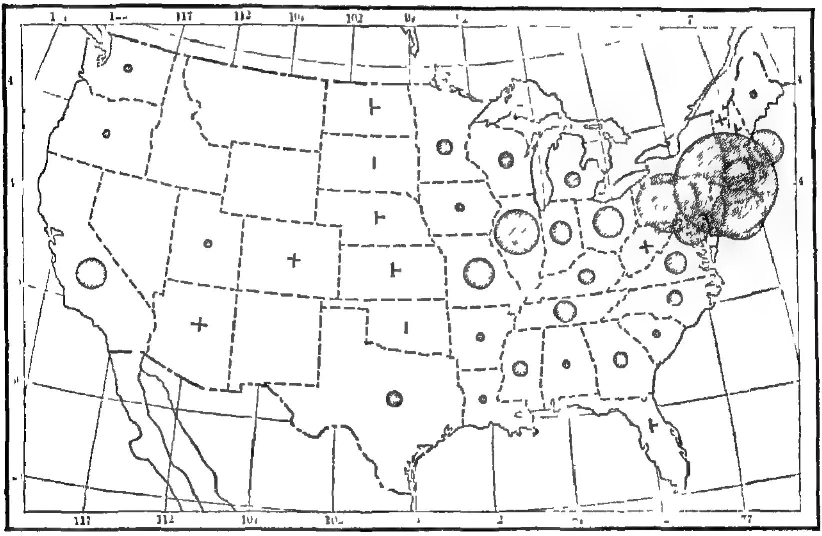


B905—Wage Earners in the Dyeing and Finishing Industry, 1937, 74,983 workers

Great Plains and the West Coast with problems of recent settlement and agriculture that they have as yet taken little interest in textile manufacturing Therefore the North Atlantic and South Atlantic states have been the competitors for new business in textiles It appears from A905, however, that the area centering in New York has absorbed 78 per cent of the new business (rayon, etc) and the entire South only 19 per cent This is another example of the centripetal power of a region where skill, good markets, and the long establishment of industry tend to hold old industries and draw new ones. The same thing is illustrated by the work of dyeing, bleaching, and

finishing cloth (B905), which requires more skill than almost any other phase of the textile industry. Some of this is done wherever much cloth is made. It is of interest, however, that here, too, the centripetal force is strong. As a result of the large migration of the cotton industry to the south that section employs 27 per cent of the workers engaged in dyeing, bleaching, and finishing cloth, but the area within 200 miles of New York employs 68 per cent.

906. THE GARMENT WORKERS. The last step in the preparation of clothing, that is, the fashioning of cloth into garments, might almost be called a community industry. In a simple way it is carried

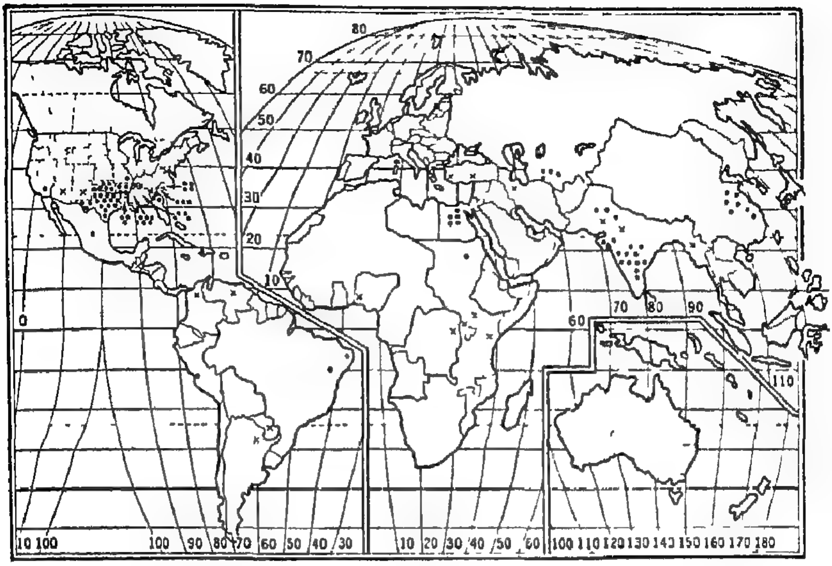


A906—Wage Earners Engaged in Making Clothing. This includes 693,100 wage earners of whom 61% are in southern New England and the Middle Atlantic states and 9% in all southern states, 1937.

on in practically all parts of the world. As a well-developed manufacturing industry, however, it is developed mainly in a few of the most intensive industrial areas (A906). The New York sector from Massachusetts to Maryland employs 66 per cent of the workers and makes most of the more expensive garments. We have already seen that this owes much to the widespread idea that New York is the center of taste and fashion, but we should also note that it is an outstanding example of the centripetal or centralizing tendency. This tendency is best able to express itself in the kind of manufactures which are light in proportion to their weight, which owe their value largely to human skill rather than raw materials, which require a

variety of goods to produce the finished article, and in which established reputation has a great influence upon sales Nevertheless the pull of a great western market is strong enough so that Illinois, Missouri, and Indiana employ 13 per cent of the workers and the Pacific Coast nearly another 4 per cent On the other hand, the South, in spite of its great production of cloth, makes only a little clothing From Virginia southward the South Atlantic states employ merely 4 per cent of the garment workers Most of these are engaged in making workshirts, overalls, and other inexpensive rough garments. The secret of this distribution of the garment industry lies largely in buying power and in the persistence of a home industry. The South cannot yet afford to buy so much of the more expensive kinds of readymade clothing as the North Moreover, the people who can afford such clothing feel especially well dressed if their garments bear a New York label.

907 World Distribution of the Cotton Industry In the world as a whole the relation between the production and manufacture of cotton is much the same as in the United States A907 shows that most of the world's cotton is raised in latitudes lower than 35°, while B907 indicates that half is manufactured in regions north of 40°. The greatest nonproducing manufacturers are (1) the North Sea countries and central Europe, including Great Britain, Germany, France, and northern Italy, (2) the coastal part of the northeastern United States, (3) Japan, and (4) the Russian regions near Kiev and Moscow The United States and Soviet Russia are so large that they use their own cotton grown in areas much warmer than the relatively cool manufacturing districts Western Europe gets its cotton mainly from the United States, Egypt, and India, Japan draws on India and the United States The main areas which produce cotton and also manufacture it are our southern states, China, India, Brazil, and Mexico Yet even in the southern states the mills take a good deal of cotton that has moved northward, coming from Texas and Mississippi to the Carolinas, while in Brazil, although a little cotton grows near São Paulo, the main supply which is there manufactured comes from near Recife, 1,400 miles to the northeast The southern states and China, being in the Mild East Coast Region, show a distinct resemblance so far as cotton is concerned In China, however, the land is so fully needed for food that some cotton is actually imported India, on the contrary, sends the major part of its cotton to Japan, England, and other European countries Its cotton for home consumption goes mainly to the seacoast near Bombay, where the climate, though constantly warm, is at least moist instead of very dry as is the



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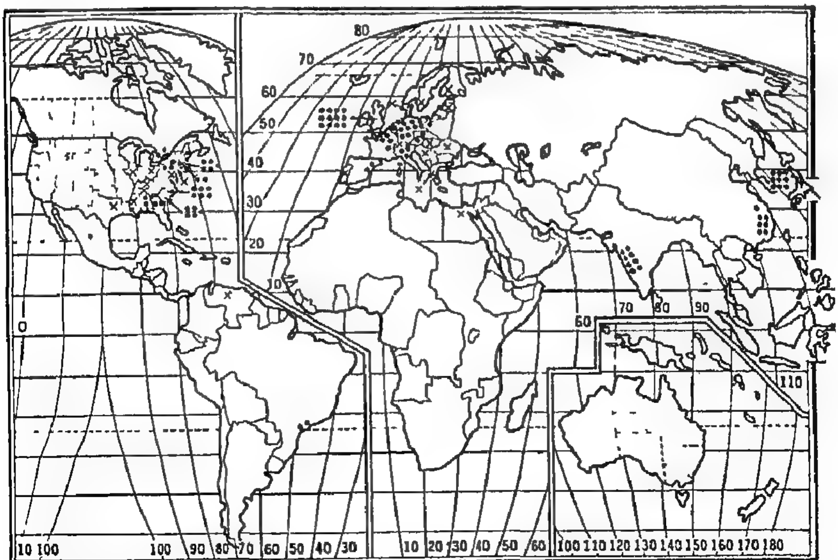
A907—World Map of Cotton Production



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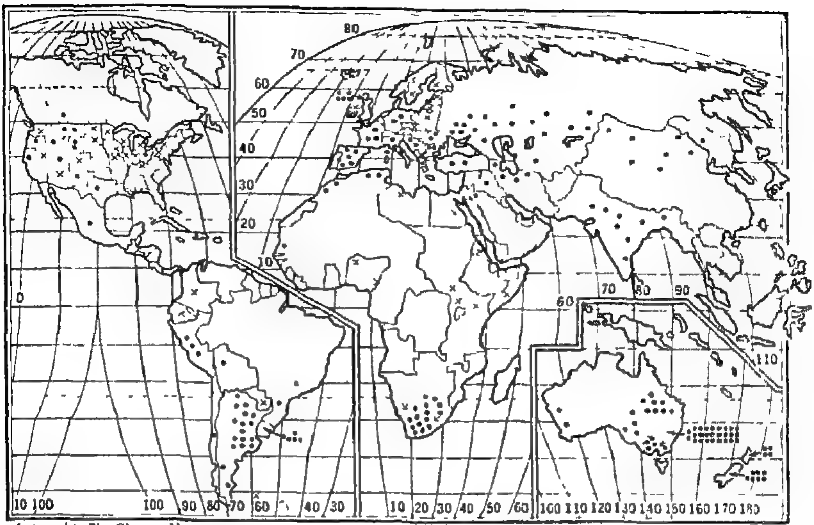
B907—World Map of Cotton Manufacuring

case farther north Egypt is the only important producer which is not also an important manufacturer The labor supply there is abundant, and its quality is probably at least comparable to that of India Coal might be imported cheaply from England by sea The extreme dryness of the climate, however, puts a severe handicap upon cotton manufacturing

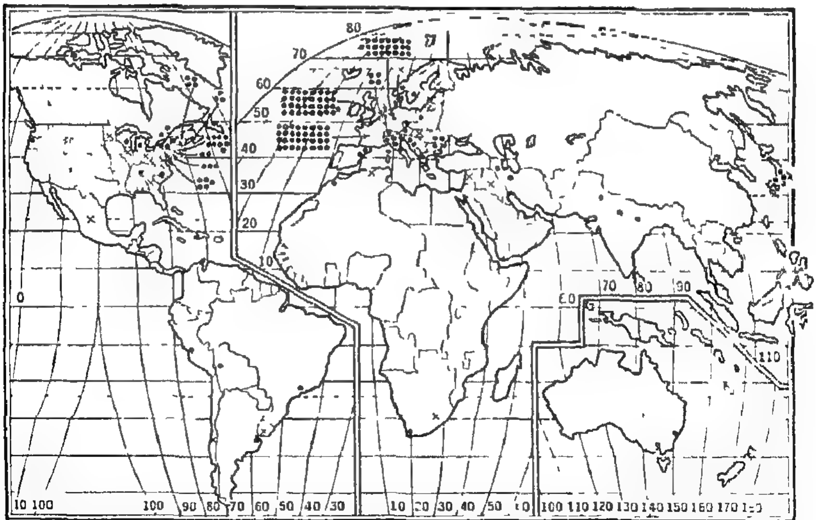
908 *The Equatorward Migration of Cotton Manufacturing* Most of the cotton manufacturing within 40° of the equator represents a development since 1880 Before that date our South, Japan, China, Brazil, and Mexico bought practically all their cotton cloth from higher latitudes The movement of the center of the American cotton industry from New England to the Carolinas is part of a widespread general tendency which is apparent all over the world The main reasons for it are abundant and cheap supplies of labor in lower latitudes, low costs of living there, local supplies of raw material, large local markets, low taxes or other political help, and the fact that much of the cotton industry, as we have seen, is nearly automatic. This last reason is commonly neglected, but without it the others would lose much of their force

909 THE WOOLEN INDUSTRY. Many of the main facts about the woolen industry have already been stated, but a comparison of A and B909 with the corresponding cotton maps (A and B907) brings out two curious contrasts between wool and cotton Wool (A909), unlike cotton (A907), is produced abundantly in all continents and all the main regions of the world except where Tropical Rainforests, Wet Tropical Agriculture, Cool Coniferous Forests, Tundras, and iceshects prevail Even in equatorial latitudes and deserts sheep are found if the altitude is great enough, while llamas, camels, and goats supply substitutes for wool Another contrast is that in the case of cotton, but not of wool, the primitive industry has practically come to an end. The majority of the dots and crosses in the Asiatic and tropical parts of the woolen manufacturing map (B909) and many of those in Mediterranean lands, represent wool which is used locally in primitive household industries for clothing, rugs, bags, felts, and thick tent-cloth The habit of making these things was established long before factories came into existence The fibers of wool are so long and so covered with tiny barbs that primitive people can spin them into thread far more easily than cotton. Another contrast is that, while part of the cotton industry has recently expanded into areas where the raw material is raised, the western United States, Argentina, South Africa, Australia, and New Zealand, in spite of an enormous number of sheep, show only a slight development of manufacturing



 Goode's Semi-homolosine Equal Area Projection

A909—World Map of Wool Production



 Goode's Semi-homolosine Equal Area Projection.

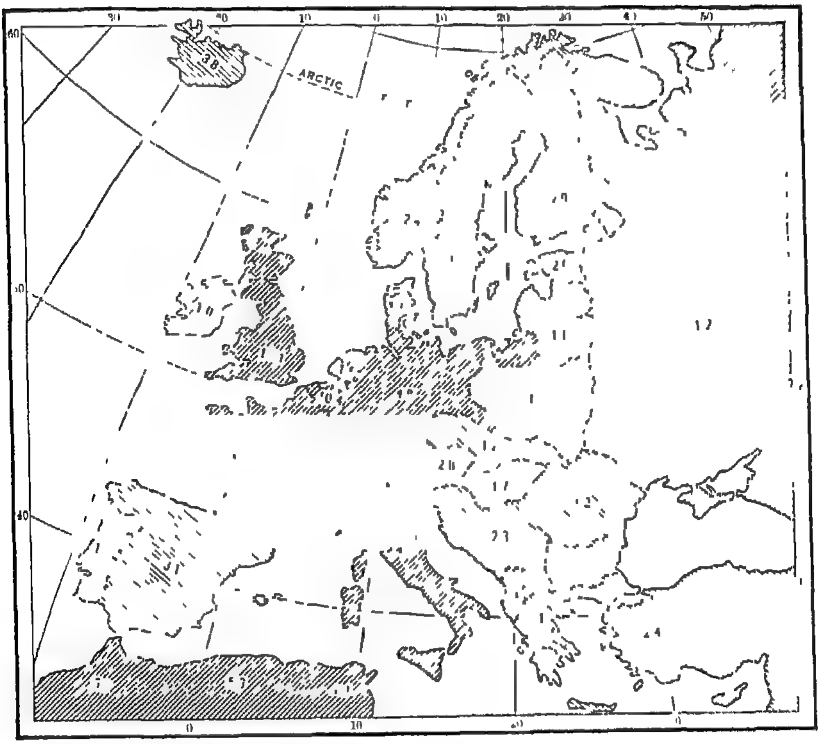
B909—World Consumption of Wool.

England, France, the Low Countries, Germany, and the northeastern United States continue to be great centers of the woolen industry, even though some of them have few sheep

910 One reason why the woolen industry has not changed its location so much as the cotton industry is its greater complexity. Although wool can be made into cloth more easily than cotton when primitive methods are employed, the opposite is true in modern factories Woolen mills require more skillful labor than cotton mills. The difference in skill is evident in the fact that in 1919, when both industries were working under pressure, the average wage-earner in Massachusetts received $895 in the cotton industry and $1,045 in the woolen industry Before the depression of 1929-32, but after the New England cotton industry had suffered its post-war setback, a similar ratio prevailed, $970 per year for cotton workers and $1,220 for woolen workers Thus wool provides another example of the principle that, *the greater the skill required in an industry, the more likely that industry is to be located in one of the great centers of industry, and the less likely it is to change its location* Another factor in keeping the woolen industry in the northeastern United States and western Europe is the price of wool compared with the cost of transportation A pound of washed wool costs two or three times as much as a pound of cotton The more expensive product can, of course, stand transportation much better than the less expensive Still a third factor is the supply of labor The five main regions where wool is raised abundantly but not manufactured to any great extent are all newly settled sections with only a sparse population This is equally true in the western United States and in four southern areas—South America, South Africa, Australia, and New Zealand In such places the labor supply is not large, new enterprises connected with the development of the country demand much labor, and wages are high Therefore, although the labor is skillful, there is little to tempt the woolen manufacturer to use it Finally, the regions where most of the world's wool is raised do not afford a large market In our western states the population is small All four of the wool-raising areas of the southern hemisphere together have fewer people than a single province of China or India The buying power of these sheep-raisers, to be sure, is relatively high, but in general their climates are sufficiently warm so that they need less woolen clothing than the people of the northern United States and Europe

911 The distribution of the use of wool in Europe (A911) illustrates some of the points that have just been made The figures there indicate the number of pounds of wool used by the factories each

year for each inhabitant Great Britain with 13 pounds per capita and Belgium and France with 10 pounds manufacture far more than their people need Hence they have a large amount for export Germany, Switzerland, Italy, and French North Africa, with 4 or 5 pounds per capita, also have somewhat more than they need Another large section, embracing the Scandinavian, Baltic, and Balkan regions, together with Spain, either raises or imports 2 to 4 pounds per capita, which is enough for local needs but not for export The



A911—Annual Consumption of Wool in Pounds per Capita in Europe, 1933 35

Soviet Republic, Poland and several small countries, on the other hand, have less than 2 pounds per capita, which is not enough to provide proper clothing in a cold climate The supremacy of the North Sea region as a woolen manufacturing region is obvious in A911, but is not shared by all the North Sea countries The Netherlands and Denmark manufacture so little wool that they have to import cloth in order to be properly clothed Another important point brought out by A911 is that most countries evidently manufacture a

large share of the woolen clothes that they use This does not necessarily mean factories, however, for much of the woolen cloth in countries such as the Balkan States, Spain, North Africa, and Turkey is made by hand at home, and not in factories. A similar study of linen would show that a surprising amount of linen cloth is still made by hand in Poland, the Baltic States, and Russia

912 THE SILK INDUSTRY In the silk industry some of the principles which we have just discussed are illustrated still further Production is mainly limited to Japan and China, with minor areas in north Italy, other Mediterranean lands, and southern Russia In practically all these places the primitive type of silk manufacturing has attained a fairly high development, thus illustrating how the presence of a resource stimulates people to use it In our day the primitive industry still continues on a small scale in Iran and on a larger scale in China and Japan On the other hand, the silk industry in its complex form has developed mainly in France, Japan, Germany, and above all the United States, with a little in Italy, Switzerland, and England In Japan, France, and Italy this began as a response to a native product In France the response has gone so far that large imports are needed, and the mills around Lyon use the most modern methods The outstanding fact, however, is that the United States consumes nearly three fifths of the world's silk The single city of Paterson, New Jersey, manufactures close to one twelfth of the entire world production, while nearly half of the world's supply is utilized within a hundred miles of New York or Philadelphia In few other lines of manufacturing is there so remarkable a concentration The reason for the choice of Paterson rather than some neighboring city is largely accident The city has waterpower, and someone happened to use it for silk mills rather than some other kind But the concentration near New York and Philadelphia is by no means accidental It is the ultimate expression of (1) the energy and skill of the northeastern United States, (2) the great natural resources of the United States as a whole and the concentration of wealth in the northeast, and (3) the fact that New York and to a less extent Philadelphia, by virtue of having the best locations on the Atlantic Coast in the Continental Cyclonic Region, have become two of the country's main foci of population, wealth, and industry Raw silk is worth about thirty times as much per pound as raw cotton Therefore the cost of transportation is of little importance in the final cost of the goods The cost of the power used in silk manufacturing is also of minor importance Hence the final product is manufactured where people of high skill and great energy, such as the old Philadelphia weavers, are available, and close

to the main center for the manufacture of dresses and other garments to be bought by women who are able to afford such expensive materials.

913 LEATHER AND SHOES Leather shoes are a necessity to us, but a luxury to fully half the world's people In the cyclonic regions and in the cool temperate interior, practically everyone wears them, although they are still displaced to a small extent by wooden shoes in parts of the Netherlands and elsewhere In Mediterranean regions almost everyone owns leather shoes Nevertheless, in countries like Turkey they are such a luxury that people often walk barefooted, but carry their shoes and put them on for looks when they reach their destination In the rest of the world, substitutes for leather are used in the more progressive countries, but bare feet become more and more predominant as one goes toward the equator Even in our own South barefooted Negroes are numerous In Japan, cloth shoes and wooden sandals are common, and in China they are worn by the more prosperous people In both countries, however, the vast majority go barefooted while at work in the house or the field This is still more true in India, the East Indies, and the tropical parts of Africa and America Such a condition is bad for health Shoes are least common in the very regions where the hookworm and other parasites are most abundant, and where bruises and cuts on people's feet are most likely to form persistent sores If everyone wore shoes, the hookworm disease would be greatly reduced, and the efficiency of the vast area within 35° of the equator would be much increased

914 The unfavorable distribution of the wearing of shoes arises partly from the temperature, for in warm weather shoes are often uncomfortable It also arises partly from the scarcity of good leather outside the Cool Continental, Cyclonic, Mediterranean, and Desert climates, and partly from the distribution of energy and skill which is so important in all phases of industry In A463 India appears to have an enormous number of cattle, and so it has—far more than any other country in the world, and almost as many as the United States per capita Four animals for every ten people in India, however, provide far less leather for each person than we get from five animals for every ten people Our animals are twice as heavy as those of India, and of course have larger hides Half of them are killed when two or three years old, and few live beyond ten years In India, on the contrary, the cattle are used mainly for plowing and hauling, and are almost never killed for meat Hence the animals that die each year form a much smaller percentage of the total than with us Inasmuch as the animals that die are old, their hides are usually poor because full of holes where the animals have been

galled by the yoke, wounded in fights, or stung by insects which have produced sores Moreover, the hides are not well collected or tanned. The poor peasants usually skin the animals where they die, but many hides are not collected at all and others are cut in skinning Furthermore, the tanning is poorly done, and sometimes the hides are used with only meager preparation This illustrates the way in which the quality as well as the quantity of goods tends to be poor in hot countries, and improves from equatorial to cyclonic regions Even though its hides are poor and relatively scanty, India has to export a great many of them because the people are too poor to afford such an expensive luxury as shoes Against this, however, must be set the fact that in breeding cattle for tender beef and abundant milk we have also bred thin hides Then too we kill great numbers of animals before their hides have attained full strength In spite of all this the hides of our own cattle and the great number that we import provide us with stout shoes, whereas most of India goes barefooted and suffers from hookworm disease even though cattle are abundant.

CHAPTER XXXVII

THE SHELTER INDUSTRIES

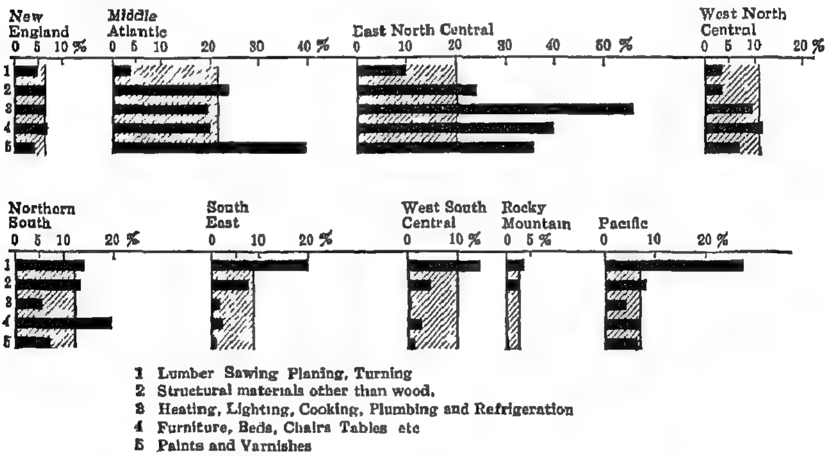
915 NATURE OF INDUSTRIES The amount of work devoted to the manufacture of materials with which to build and furnish houses, offices, factories, and other forms of shelter is amazing Some idea of it may be obtained from Table 31 This does not include the carpenters, masons, plumbers, painters, and others who construct a building and prepare it for occupancy, nor the lumbermen who cut the logs from which the lumber is prepared These workers are primary producers, not persons engaged in manufacturing Only the manufacturing part of the lumber industry is included in the table —the part carried on in sawmills, planing-mills, and wood-turning establishments The wage-earners engaged in making the materials commonly used in erecting and equipping our buildings outnumber those engaged in the manufacture of foodstuffs (1,700,000 against 1,100,000), and are three fourths as numerous as those engaged in making cloth and clothing Their number is large because among highly civilized nations the users of a building rarely take any direct part in erecting it, or in making its furnishings Nevertheless, they equip their buildings most lavishly Primitive people, on the other hand, and even the rural population of fairly advanced countries, such as Chile, Russia, and Rumania, do a surprisingly large part of the work of making and equipping their own houses A Brazilian house of plastered mud with a thatched roof of leaves consists of structural materials which require practically no manufacturing No paint or equipment for heating, lighting, and plumbing is used The cooking arrangements consist of an outdoor oven of mud, and a few pots and pans Tables and even bedsteads are scarce and often home-made. The dishes for an entire family can be put on a single tray Contrast all this with our own elaborate arrangements It seems clear that, as we go from food to clothing and then to shelter, we begin with industries which in large measure still remain relatively simple, even among the most advanced people In the clothing industries, however, and still more in the shelter industries we come to groups which remain simple among backward people, but become highly complex in the most progressive countries.

TABLE 34

WAGE-EARNERS ENGAGED IN MANUFACTURING THE MATERIALS AND EQUIPMENT NEEDED FOR HOUSES AND OTHER FORMS OF SHELTER, 1937

Lumber sawing, planing, turning, etc	417,000
Other structural materials brick, iron, glass, stone, etc	298,000
Heating, cooking, lighting, refrigerating, and plumbing apparatus	321,000
Furniture chairs, tables, beds, etc	252,000
Interior furnishings carpets, rugs, pictures, hardware, dishes	333,000
Paint, varnish, etc	40,000

916. *Distribution of Shelter Industries in the United States* The degree to which the different sections of the United States supply their own needs for structural materials is illustrated by the two upper bars in the various parts of A916 In the United States, unlike most countries, lumber is still the main material for houses New England, the Middle Atlantic states, and the West North Central states produce far less lumber than they need and do much less than their fair share



A916—Wage Earners in Shelter Industries

of sawing, planing, and turning the lumber to prepare it for use All other sections, especially the Southeast and Northwest, do more than their share With other structural materials such as brick, stone, cement, and concrete it is quite different. All parts of the country except the West North Central and West South Central states manufacture just about as much as they need This is natural, for clay and stone are widely distributed and are costly to transport They are relatively scarce, however, in the great plains of the West Central states beyond the Mississippi

917 The equipment of a house or other building falls into three main parts. (1) apparatus for plumbing, heating, lighting, cooking, cleaning, and refrigerating, (2) furniture, and (3) furnishings such as carpets, rugs, dishes, clocks, pictures, and ornaments A916 shows that household apparatus is manufactured in proportion to the number of people in New England and the Middle Atlantic states, and in vast quantities in the East North Central states where the iron of which it is largely constructed is most easily available The rest of the country relies heavily on the states from Ohio to Illinois and even as far as Iowa for means of making houses comfortable and for equipment for such work as cooking and ironing. The bars for furniture have a length proportional to the population in four sections of A916, but the East North Central states and the Northern South make more than their share, while the Southeast, West South Central, and Rocky Mountain states show a deficiency The wide distribution of forests and the bulkiness of furniture encourage the manufacture of enough furniture for local needs The presence of hardwood forests south of the Great Lakes and in the southern Appalachians helps to make Michigan, Indiana, Illinois, and North Carolina conspicuous as furniture-makers Inasmuch as wood as well as plaster can be beautified and preserved by paint or varnish, these chemical products belong to the shelter industries The Pacific Coast, as often happens, takes care of its own needs in this respect, as appears in the lower bars of A916 The Middle Atlantic and East North Central states make far more than they need—three quarters of the whole—and the rest of the country relies largely on them. New Jersey is the greatest state for paints and varnishes, with Illinois, New York, and Ohio next.

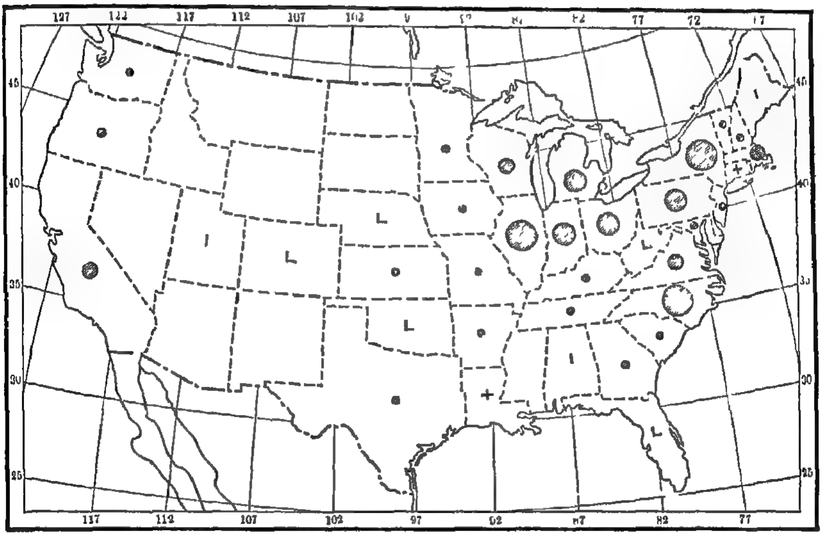
918. Forest Products Wood, like many other products, furnishes the material for a simple industry, a community industry, and one main complex industry, as well as several minor ones such as the making of matches and lasts The simple industry is that which is carried on in sawmills It begins with skidding logs onto rollers which carry them to circular saws There slabs are sliced off to get rid of the bark, and the logs are sawed into rough beams and boards, or sometimes into shingles, laths, staves, mine props, ties, or other forms The distribution of this industry (A164) is practically identical with that of the cutting of lumber in the forests A large share of the mills get their power by burning the waste wood cut from the logs, but in recent years hydroelectric power has increased in importance Some sawmills have steam-heated drying kilns, but others let the lumber dry in the open air. The size of the mills varies

geographically Large mills with modern equipment are most common on the Pacific Coast The great size of the logs there requires mechanical handling The fact that large tracts are owned by single interests also favors large mills In that region three great companies hold 11 per cent of all privately owned timber in the United States Accordingly, about a third of the mills on the Pacific Coast are really large In the South, where neither the trees nor the lumber tracts are generally so huge, about a quarter of the mills are large In the Appalachian and Great Lakes regions the preponderance of small holdings reduces the large mills to about a tenth In New England the only really large tracts under a single ownership are held by a few paper companies which control about half the pulp wood of New England Accordingly, few sawmills there are large, and small mills that are moved every year or two are common Sawmills are one of the big industries in the United States, employing 324,000 wage-earners in 1937

919. The community industry based on lumber is carried on in planing-mills, and by contractors and carpenters The census reports 4,500 planing-mills compared with nearly 14,000 sawmills, but wood is also planed in thousands of sawmills, and probably tens of thousands of carpenters' shops Such mills and shops take rough lumber as it comes from the sawmills, and plane, bevel, saw, and chisel it to the shape and size desired for building operations Naturally the distribution of planing-mills is very different from that of sawmills Cities such as New York, Detroit, Chicago, and Los Angeles are conspicuous as the sites of planing mills because many buildings are erected there Leaving out the three Pacific states, which employ 32 per cent of the sawmill workers and 17 per cent of those in planing mills, the seven other states with the most sawmills are all in the south—Louisiana, Arkansas, Alabama, Mississippi, Florida, North Carolina, and Texas These employ 54 per cent of the sawmill wage-earners, but only 25 per cent of the planing-mill workers On the other hand, the ten states from southern New England and New Jersey westward to Iowa do only 3 per cent of the sawmill work, but bring so much rough lumber from outside that they do 33 per cent of the planing-mill work

920 FURNITURE. The furniture industry (A920) affords a clear example of the influence of transportation on the distribution of industries whose products are heavy or bulky New York City and Philadelphia are good places for the furniture industry because lumber reaches them by water, while the finished furniture finds a market close by among a vast number of people who are active in building

and furnishing new structures. Skill, however, is as important as transportation, and the furniture industry requires high skill as appears from the fact that the furniture worker of Massachusetts earned about $1,400 a year at a time when only $970 was earned in the cotton industry. Certain towns, with numerous skilled men in large, well-equipped furniture factories, are able to reach a widely scattered market by transporting their products partly in the "knockdown" condition. Thus Grand Rapids, Michigan, Sheboygan, Wisconsin, Gardner, Massachusetts, and several towns in Indiana, Tennessee, and North Carolina, have been able to hold and expand their markets



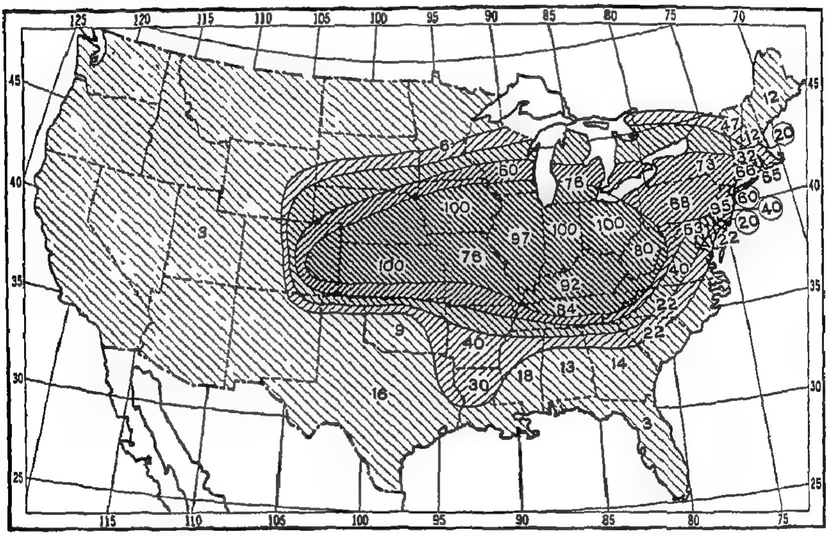
A920—Wage Earners Engaged in Making Furniture. Total 170,072 in 1937

at the expense of local factories elsewhere. In Massachusetts a gigantic chair at the railroad station proclaims that Gardner is "Chair Town," just as its neighbor, Winchendon, with a huge wooden horse at the station, is "Toy Town." Each excels in a branch of the wood industry.

921. The quality of the wood joins with transportation and human skill in determining the location of the furniture industry. The high development of North Carolina, and the conspicuous position of Chicago, Detroit, and Grand Rapids in the furniture industry, arise partly from the nature of the forests. Most furniture is made of hardwood. Because of their irregular grain, freedom from pitch, and capacity for taking a high polish, oak, curly maple, chestnut, and

walnut are not only more durable but much more beautiful than pine or spruce. A921 shows that in the part of the East North Central states where the furniture industry is best developed more than 80 per cent of the timber is hardwood In the southern Appalachians a similar condition prevails Thus the furniture factories in the Carolinas and Virginia, just as in Indiana, Illinois, Michigan, and Wisconsin, were originally located close to their raw materials, as well as in rapidly growing regions where there is a large market for the product

922 THE STONE AND EARTH INDUSTRIES Although millions of tropical and oriental houses are built without stone, brick, cement, or



A921—Percentage of Hardwood in Lumber Cut in the United States

glass, civilized people use at least two of these in practically every building Almost any house that can be called good has a brick or stone chimney, and glass windows, and many have cement cellars In cities the walls are commonly made of brick, stone, or stucco. In advanced countries with humid climates even the rural houses are often made of stone or brick with tiled roofs, as in much of Europe In less advanced and drier regions such as Egypt, India, and China, the great majority of rural houses and many in the cities are made of adobe, or dry mud They have either sloping, thatched roofs, or flat roofs made also of dry mud Thus clay, brick, stone, cement, and glass are of first-rate importance in the world as a whole In the United States they are usually worth over a billion dollars per year—

more than all the flour, and as much as the bread, cake, and crackers made in bakeries

923 *Cement Mills.* It is especially desirable that cement plants be widely distributed, but the nature of the industry limits the number of plants So far as these raw materials are concerned, Portland cement, the kind now mainly used, might be manufactured in practically all states The demand for cement is also universal wherever civilization is high In the United States it follows the general lines of our maps of wages, consumption of baker's bread, health, and climatic energy But cement is cheap and heavy, so that transportation by rail for 300 miles doubles its cost Therefore, unless it travels by water, it cannot be used in large quantities far from the mills The making of cement requires large, heavy machinery, and hence abundant capital and a large market for each mill Moreover, the preparation of each ton of cement demands half a ton of coal, or a corresponding amount of natural gas, or oil Cement requires greater heat than steel-making

924 Because of all this the United States has only about 150 cement plants, which are grouped around the great cities in fairly close proportion to their population The greatest single producing area is the Lehigh Valley in northeastern Pennsylvania and northern New Jersey There the following conditions give great advantages dense and prosperous population, unusually good limestone and cement rock in the same pits, cheap bituminous coal, the price of which is kept down by competition with the powdery refuse of the anthracite mines, good railroad facilities The coal of Birmingham and the calcareous slag produced by limestone in blast furnaces make Alabama the largest southern producer of cement Abundant oil for fuel, great wealth which permits the building of many good roads, and dryness which gives concrete a distinct advantage over wood for houses, cause California to produce more than twice as much cement as would be expected on the basis of population From the standpoint of competition, cement is peculiar. All the manufacturers have agreed to produce cement of only one quality, making it according to a strict formula Thus competition does not depend upon quality, but upon price The cost of production and hence the price depend mainly on geographical conditions, the chief of which are the location of markets, fuel, and stone

925 *Clay-Using Industries* Clay is the raw material for three main products—pottery, bricks, and tile. Pottery is perhaps more interesting than bricks, but in the United States bricks are four times as valuable. The pottery industry, as we have seen, is concentrated in

two places because of the conflicting influences of long-established skill and fuel One is in New Jersey (especially Trenton), where the population is dense and foreign trade is easy The other and greater concentration is in Ohio (East Liverpool) near coalfields where fuel costs scarcely half as much as in New Jersey. Cincinnati is famous for its Rockwood pottery, one of the few artistic products in which America rivals the best work of Europe Trenton also makes china which is entitled to high praise for its beauty England, France, and other European countries have long been famous for such products as Wedgwood china and French porcelain, but the Chinese and Japanese are in many respects the world's most famous potters

926 Tile and brick are two of the forms in which clay is a structural material In the United States, however, the main use of tiles is for drainage. Ohio is the main area of production In Europe, Asia, and Latin America, on the other hand, hundreds of square miles of roofs are red with tiles Bricks are used in practically all countries. They are like cement in three main ways (1) their raw material is widely distributed, so that the industry is possible almost everywhere, (2) they require much fuel, since they have to be fired from six days to two weeks, (3) they are so heavy and cheap that transportation is expensive compared with their value The chief demand for brick is in regions where wood is scarce, or where its use in buildings brings danger of fire Sun-dried bricks, or adobe, as we have seen, give rise to an important though simple industry in practically all dry regions, burned bricks give rise to a more complicated but still simple industry where cities are large and population is dense, and also where coal is abundant but lumber expensive Wooden houses in cities have brought disaster again and again. The fact that London was a city of wood in 1666 permitted a great fire to destroy much of it It was reconstructed of brick. Chicago had large tracts of wooden houses until 1871 when a disastrous fire destroyed 17,450 buildings and rendered 100,000 people homeless Today Cook County, which includes Chicago, makes more brick than any other county in the United States The region surrounding New York is a second great brick-making area Bargeloads of bricks come to the city down the Hudson, from the flats of northern New Jersey, and even from Connecticut Philadelphia, Pittsburgh, Cleveland, and Cincinnati are built mainly of brick Pennsylvania and Ohio, with their abundant coal, large supplies of natural gas, and relative scarcity of lumber, make more than a third of the country's bricks One reason why Philadelphia is the "City of Homes" is probably the cheapness of bricks This has fostered the construction of mile after mile of two-story brick houses in solid rows like those which dominate most

of the industrial districts of England In Pennsylvania and Ohio many villages and farmhouses are built of brick. A similar condition prevails in California and the dry Southwest, and in Belgium, Holland, and parts of Germany, where fuel is abundant In Pennsylvania brick roads were formerly common

927 In one respect brick-making is quite different from the manufacture of cement It requires only simple machinery, and hence can be carried on profitably in small units Accordingly the average brick plant in the United States employs about 50 wage-earners, while the average cement plant employs 225 The small investment in machinery and buildings makes it feasible to run many brick plants only in summer when the clay can easily be dug and less fuel is needed The cement plants have to run more steadily in order to earn the interest on their large investment

928 *Building-Stone Industries* Stone and brick fill almost the same needs Stone, however, is generally more expensive than brick, except for rough construction like cellar walls Accordingly it is used abundantly where lack of fuel or proper clay makes bricks costly, or where great wealth permits stone to be used in spite of its cost New England, having no coal and being a rocky place where the icesheet scraped away most of the clay as well as the other soil, produces only 2 per cent of the country's bricks On the other hand, it produces close to one fifth of the marble, granite, slate, and other building stones Michigan, Wisconsin, and Minnesota, although extremely active states with 18 per cent of the country's population, lack fuel Hence they make only about 2 per cent of the bricks, but quarry and dress 8 per cent of the stone In Georgia and Indiana a different situation appears. Both states make a fair amount of brick, but have so much good stone that they rank among the large producers Indiana, in fact, comes next to New York and Vermont It has a kind of limestone so soft that it can easily be sawed, although later it hardens on exposure to the air At the mills near the quarries this limestone is cut into blocks exactly the size and shape needed in the buildings for which they are designed The blocks are numbered according to a plan so that they can easily be fitted together hundreds of miles away

Chapter XXXVIII

HEAVY AND LIGHT METALLIC INDUSTRIES

929 Equipment for Work. Machines and chemicals may be regarded as tools with which man converts raw materials into commodities ready for use Table 35 states the number of wage-earners engaged in manufacturing the main kinds of tools, machines, and other equipment which we employ in making food, clothing, household goods, transportation facilities, means of communication, scientific equipment, and luxuries *

TABLE 35

Wage-Earners Engaged in Manufacturing Tools, Machines and Chemicals, 1937

Iron and steel pigs, sheets, bars, wire, bolts, etc	603,000
Nonferrous metals copper, lead, zinc, etc , pigs, bars, sheets	117,000
Machines of all sorts, except for transportation, household use, etc	846,000
Tools for hand use and machine tools	118,000
Chemicals used mainly in industry, but including agricultural (25,000) and household use (20,000) .	176,000

930 The most significant feature of Table 35 is that the making of machinery, even when vehicles for transportation are omitted, takes the full time of 846,000 wage-earners—an even larger number than are engaged in the manufacture of cotton goods (731,000) If we include those who make motor vehicles, locomotives, washing machines, clocks, watches, refrigerators, radios, sewing machines, typewriters, and adding machines, the total for machines becomes about 1,635,000 wage-earners This is by no means the total number of people who help in the work of making machines in the United States It is merely the average number of wage-earners during the year as a whole. In

* The entire work of manufacturing iron, nonferrous metals, and chemicals, after the ores or other minerals have been dug from the earth, is placed in this division of industry, because that is where the major part is used A portion, to be sure, goes into the construction and equipment of buildings, or is utilized in other ways aside from machines and manufacturing processes Inasmuch as the amount thus used is unknown, no attempt is made to separate it from the much larger part employed as a means of producing tools, machinery, chemicals, and other equipment for manufacturing

some months, when business is urgent, the number is far larger than this, in other months, far less. In addition to the wage-earners more than 200,000 salaried workers help directly in running the factories that make machines. Other hundreds of thousands work sometimes in making machines, but unfortunately cannot find work all the year. Then, too, some hundreds of thousands of people in mines, quarries, forests, and fields are engaged in producing ores, fuels, and raw materials which will be used for the machines which serve as tools. Perhaps the best way to get an idea of the importance of machines is to note that about one fifth of all persons engaged in industrial pursuits are making them. The other four fifths are using the machines to manufacture food products, clothes, materials for building and equipping buildings, equipment for carrying on transportation and trade, and luxuries, or special kinds of goods to help in such matters as preserving health. The making of machines with which to produce other goods is one of the most characteristic features of highly advanced countries. Even this, however, is largely limited to a few specially favored regions.

931. Facilities for Transportation, Communication, and Trade. Rapid and cheap transportation, easy means of communication, and hence active trade, are among the most important results of an abundance of machines. In order that trade may be carried on swiftly and easily we need not only many kinds of vehicles for transportation, but also office facilities, means of communication such as the post office, telephone, and radio, and many sorts of packing equipment such as bales, boxes, and crates. To a large extent the manufacture of the articles used in the work of transportation, communication, and trade is limited to the most advanced parts of the world, and to the sections where manufacturing is most highly developed. Packing materials (wooden boxes and barrels, paper cartons, steel drums, etc.), to be sure, and such products as cement or railroad ties, are produced in backward as well as advanced regions, but the machines which are the central feature of modern transportation and communication are made only where the level of achievement is high. Table 36 shows that it takes a great many wage-earners to make the equipment needed for transportation, communication, and trade.

932. Higher Needs. By higher needs we mean health, recreation, art, music, science, and other pursuits which go beyond the mere needs for food, clothing, and shelter. The most important means of satisfying these needs is the printed page. Therefore paper and printing machinery form by far the greater part of the goods which must be

TABLE 36

WAGE-EARNERS ENGAGED IN MAKING EQUIPMENT FOR TRANSPORTATION, COMMUNICATION AND TRADE, 1937

Materials for road construction	34,000
Railroad equipment	49,000
Ships and their equipment	62,000
Airplanes	24,000
Lubricants and fuels (manufacturing processes only)	85,000
Motor vehicles	570,000
Office equipment typewriters, adding machines, cash registers, etc	77,000
Packing and shipping supplies boxes, bags, twine, etc	175,000
Miscellaneous	31,000
Total	1,107,000

manufactured for this purpose Nevertheless, Table 37 shows that there are other important aspects, which center mainly around esthetic enjoyment, health, recreation, and science Job printing and the printing of newspapers are universal occupations among civilized people The making of books, on the other hand, as well as the preparation of equipment for science, medicine, sports, and recreation, and of articles pertaining to music, art, and drama, is largely localized in the same regions where machines are made

TABLE 37

WAGE-EARNERS ENGAGED IN MAKING EQUIPMENT FOR MAN'S HIGHER NEEDS, 1937

Paper making, printing, and publishing	507,000
Equipment for music, art, and drama	64,000
Scientific equipment	22,000
Medical equipment, including drugs, etc	64,000
Equipment for recreation	39,000
Personal and other adornment, perfumes, etc	65,000
Total	761,000

933 PRESENT DISTRIBUTION OF IRON INDUSTRIES Iron plays so dominant a part in the manufacture of machinery, transportation equipment, and appliances for the satisfaction of higher needs that we may well consider the entire group of industries that depend mainly on iron Most of the iron ore of the United States now comes from the Lake Superior region, but the Appalachians still furnish about one tenth of the supply The Birmingham region in Alabama produces most of this, but five other states have a share The only other sources of iron ore worth mentioning in this country are the Rocky Mountain region, Utah, and Missouri A map of blast fur-

naces shows that the western ores come to the coal of Colorado to be smelted, the Lake Superior ores meet eastern coal near the south shores of the Great Lakes, and the Appalachian ores find coal for smelting near their source We have already seen that the most notable feature of the blast-furnace map is the concentration of the non-smelting industry along the southern shores of Lakes Michigan and Erie from Chicago and Gary through Toledo, Cleveland, and Erie to Buffalo, with a southward projection to Pittsburgh and Wheeling Our next objective is to follow the iron into other industries and see how far their distribution depends upon that of the blast furnaces and how far upon other factors.

934 A List of Iron Industries. The nature of the industries that depend primarily upon iron, and the degree to which they are concentrated around the southern shores of the Great Lakes, are illustrated in Table 38 There the metallic industries listed by the United States census are arranged in order according to column A which shows the percentage of their wage-earners in Pennsylvania and the East North Central states (Ohio, Indiana, Illinois, Michigan and Wisconsin) These states form the Heavy Industry Section of the United States The percentages belonging to this section should be compared with those of the Light Industry Section (New York, New Jersey, and the six New England states), as given in column B, and of the rest of the country (column C) The number of wage-earners in each industry is shown in thousands (column D) Columns E to H name the four leading states in each industry and state the number of wage-earners in thousands Table 38 would be improved if the census figures permitted us to add the Buffalo district of New York and the Wheeling district of West Virginia to the Heavy Industry Section and put eastern Pennsylvania and the southern parts of Ohio and Indiana in the Light Industries Section Although Columbus, Dayton, Cincinnati, and Indianapolis carry on industries that depend on iron, they tend to do the lighter kinds of work, and thus belong to the same general type as Rochester, Syracuse, and Hartford

935 The Heavy Industry Section Even as Table 38 now stands it emphasizes the fact that industries which require much iron, or that use iron in forms that require relatively little work in comparison with the amount of raw material, are concentrated on the south side of the Great Lakes Motor vehicles (No 1 in the table) use not far from one eighth of the iron produced in the United States Table 38 shows that, although the Heavy Industry Section contains only 28 per cent of the population of the country, it includes 86 per cent of the 471,000 wage-earners in the motor-vehicle industry. On the other hand, the

TABLE 38

METAL INDUSTRIES IN THE UNITED STATES

	A	B	C	D	Leading states and thousands of wage-earners							
	East North Central and Pennsylvania	New England, New York, New Jersey	Rest of country	Thousands of wage-earners	E First		F Second		G Third		H Fourth	
1 Motor vehicles	86	7	7	474	Mich.	298	Ohio	42	Ind	33	N Y.	21
2. Agricultural machinery	82	3	15	78	Ill	39	Wisc	17	Iowa	*6	Ind	4
3 Cranes	81	5	14	19	Ohio	5	Wisc.	4	Ill.	2	Mich	2
4 Refrigerators	79	9	12	50	Ohio	14	Mich	10	Ind	8	Pa	5
5 Steel works	78	7	15	479	Pa	166	Ohio	96	Ind	43	Ill	41
6 Blast furnaces	76	8	16	23	Pa	8	Ohio	6	Ill	2	Ala	*2
7. Bolts, forgings, etc	75	19	6	35	Ohio	9	Pa	5	Ill	5	Mass	2
8 Iron pipe	74	6	20	14	Ohio	9	Pa	7	Mich.	5	Ill	4
9 Foundry products	72	15	13	120	Mich	37	Ohio	16	Ill	11	Pa.	9
10 Plumbing supplies	71	15	14	19	Wisc.	3	Pa	3	Ohio	3	Ill	2
11. Engines, steam turbines	66	27	7	22	Wisc	6	Pa	6	Ohio	5	N Y.	3
12 Screws	65	33	12	21	Ohio	6	Ill	3	Conn	3	Mich.	3
13 Steel barrels, etc	63	12	25	6	Ill	2	Ohio	1	Calif	*1		
14 Stamped and pressed metal	63	22	15	61	Ohio	15	Mich	7	N Y	6	Ill	6
15 Machine tools	62	35	3	81	Ohio	20	Mich	14	Conn	8	Ill	8
16 Heating and cooking apparatus	61	17	22	89	Ill.	19	Ohio	13	Pa	10	Mich	7
17 Railroad cars, etc	60	9	31	41	Pa	9	Ill	9	Ind	5	Mo	*3
18 Scales	59	32	9	3	Ill	1	Ohio	1	Wisc	1		
19. Pumps	55	27	18	28	Ill	5	Ohio	3	N J	3	N Y	3
20 Boilers	55	9	36	24	Ohio	5	Pa	5	Ill	2	Calif	*2

21 Nails and springs	54	33	13	6	Pa	1	Mass	1	Mich	1	Conn	1
22 Metal doors, screens	54	29	19	8	N Y	2	Ind	1	Mich	1	Ill	1
23 Electrical machinery	54	38	8	258	Ill	35	Ohio	34	Pa	34	N Y.	28
24 Aluminum products	52	18	20	24	Pa.	6	Ohio	4	N J	2	Mich	2
25 Electroplating	51	41	8	18	Pa.	4	Conn	3	N Y	2	Ill.	2
26 Structural metal	49	16	35	39	Pa	9	N Y.	4	Calif	*3	Ill.	3
27 Miscellaneous machinery.	48	28	24	147	Ohio	19	N Y.	18	Ill	17	Pa	14
28 Hardware and cutlery	47	49	4	99	Conn	25	Mich	18	N Y	11	Ill	10
29 Sheet metal	46	19	35	58	Ohio	3	Ill	2	Calif	*2	Pa	2
30. Printing machinery	45	51	4	14	N Y	4	Ill	3	N J	1	Ohio	1
31 Nonferrous metals .	42	47	13	88	Conn	22	N Y.	11	Ohio	10	Mich	10
32 Radios	41	55	4	48	N J.	14	Ill	9	Pa	5	R. I.	5
33 Clocks and watches	40	59	1	26	Conn	9	Ill	7	Mass	4	N Y	2
34 Tin cans	40	22	38	33	Ill	8	Md	*4	N Y	4	Calif.	*3
35. Sewing machines .	38	62	0	3	Conn	2	Ill.	1				
36 Washing machines	35	15	50	10	Iowa	3	Ill.	2	Ohio	1	N Y.	1
37 Wire	32	34	34	58	Mich	14	Ill.	7	Pa.	6	Mass	6
38 Office, machines, typewriters, cash registers	32	66	2	45	Conn	15	N Y.	13	Ohio	7	Mich	4
39 Ship building	31	36	33	77	Mich	17	N J	11	N Y.	9	Va	*7
40 Scientific equipment, surgical, photo .	28	62	10	51	N Y	24	Pa	7	Ill	5	N J.	2
41. Textile machinery	23	72	5	25	Mass	11	Pa	5	Me.	3	R I	4
42 Silverware, jewelry	8	87	5	34	R I	9	N Y	8	Mass	7	Conn	3

* States not included in either Heavy or Light Industry Section.

Light Industry Section includes only 7 per cent of the motor-vehicle workers, although it has 20 per cent of the population. The entire remainder of the country with 52 per cent of the population has to be content with the remaining 7 per cent of the motor-vehicle workers. We have already discussed the reasons for all this, and for the concentration of the automobile industry at Detroit. The manufacture of agricultural machinery which comes next in Table 38 displays a similar condition. It differs from automobile manufacturing chiefly in two respects. First, Chicago, Milwaukee, and the small cities near them are the chief places concerned. Second, because of the location of the prairies the manufacture of agricultural machinery has expanded westward into states such as Iowa and is negligible in the Light Industry Section.

936. Run down the list of Table 38 and note the change in the type of product as one goes from industries that are largely concentrated in the Heavy Industry Section to those that are less prevalent there than in the region of Light Industry. Cranes (item 3) require large amounts of iron in the form of stout beams, heavy cogwheels, and boilers. Refrigerators (4) require relatively large sheets of metal and not much in the way of delicately adjusted machinery. Steel works (5), which include rolling mills, represent the first stage of manufacture after iron leaves the blast furnaces. In such mills pig iron, as molten iron as it comes from the furnaces, is purified and then combined perhaps with "scrap." Carbon is also added to produce ordinary steel, and minor metals, such as manganese, vanadium, or tungsten, to produce steel alloys with special qualities of hardness, toughness, or electrical conductivity. The steel may be sold in ingots, but more often it is at once rolled into sheets and bars, drawn into rods, or shaped in other ways so that it is nearly ready for use in other mills. In the new "strip" mills the metal does not have to be handled from the time the ore is melted until it comes out at the end as strips of steel. "Scrap" plays an important part in steel mills. The annual production of steel is generally about a quarter greater than that of pig iron, because scrap, in the form of old iron and steel, is melted up again. Each year, in this country, railroads turn in about 5,000,000 tons of scrap. About a million automobiles are junked, and many of them go to the steel plants. In Europe the production of steel generally exceeds that of pig iron by only about one eighth instead of one quarter, as with us. This difference affords a rough measure of the way in which our old machines are scrapped while still useful, whereas in Europe they are generally used until they wear out. Although blast furnaces and steel mills are often run separately, their general distribution is

almost identical Pennsylvania makes about 33 per cent of the pig iron and 36 per cent of the steel-mill products

937 From the steel mills and rolling-plants the steel or iron has to move only a little way to reach other factories which manufacture the products listed in the upper part of Table 38 Bolts, forgings, iron pipe, foundry products, and the tanks, hot-water boilers, metal bath-tubs, and so forth, which constitute plumbing supplies (7 to 10) are relatively simple products the manufacture of which requires considerable heat It is wise to manufacture such articles close to the blast furnaces (6), because coal as well as iron is conveniently near, and it costs little more to ship the finished product than to ship the raw material Going down still farther in Table 38 we find heavy machines such as steam engines (11), machine tools (15), pumps (19), and electrical machinery (23) which consist largely of iron and are often of large size We also find products such as screws (12), nails (21), and metal doors and screens (22) which consist of iron cut into comparatively small pieces, but not built into machines From 50 to 70 per cent of such products are manufactured in the Heavy Industry Section, but the Light Industry Section also does a good deal of this kind of work, and so do other sections of the country, as appears in columns B and C

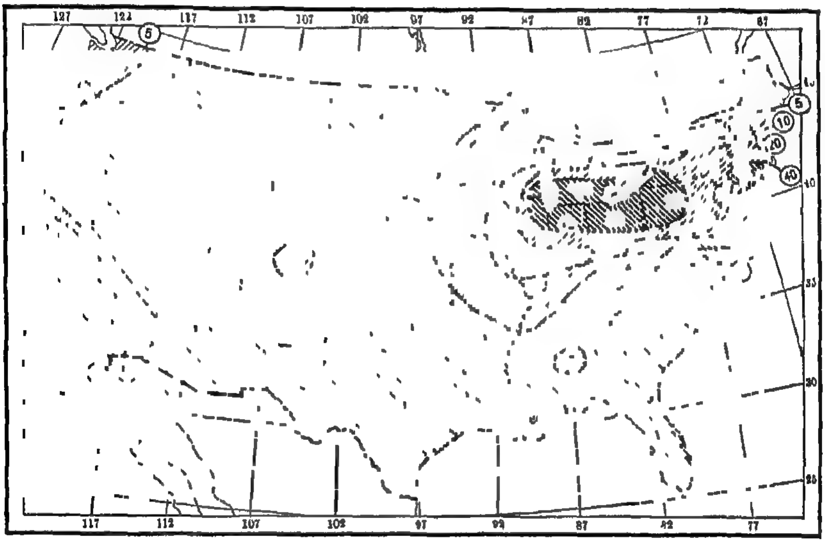
938 Next we come to products which are manufactured more extensively in the Light Industry Section than in the Heavy Industry Section They include such articles as hardware and cutlery (28), printing machinery (30), radios (32), clocks and watches (33), office machinery (typewriters, adding machines, cash registers, and so forth) (38), textile machinery (41), and scientific equipment (40), including such items as surgical and dental implements, scientific apparatus, and cameras Products made of nonferrous metals (31), including zinc, lead, and especially the mixture of copper and tin known as brass, also are produced most extensively in the Light Industry Region. In a general way, as we go from products which are manufactured mainly in the Heavy Industry Section to those manufactured mainly in the Light Industry Section, there is a gradual change in the relative values of raw material and labor A large percentage of the value of a huge crane, for example, lies in the metal of which it is composed The iron in a complete outfit for a dentist's office, on the other hand, may be worth only two or three dollars, although the dentist pays two or three thousand for it The actual amount of work required to manufacture a hundred pounds of iron in making a crane may be far less than that required to convert only a pound or two into the finest dental implements In the same way a microscope may cost as

much as an automobile, although the iron used in it weighs no more than the ornament on a radiator cap, and the glass lens, before it is ground and polished, is not worth a cent. The costliness of the microscope arises from the great amount of labor put into it.

939 Facts such as are illustrated in Table 38 lead to important principles (1) *The greater the bulk and weight of the raw material in proportion to the value of a manufactured article, the more likely the article is to be manufactured near the source of the raw material.* On the other hand, (2) *if the cost of transporting raw material is slight compared with the cost of labor, a product is likely to be manufactured where the accumulated experience of the past and the general conditions of life have concentrated great skill* A third principle is illustrated by the fact that 87 per cent of the manufacture of jewelry and silverware in the United States is concentrated in the Light Industry Section within 100 or 200 miles of New York (3) *If an article consists of expensive raw materials brought from a variety of sources, its manufacture tends to be concentrated in places where accumulated experience and skill are especially high* All these principles, of course, must be interpreted in relation to others such as those relating to fuel. Each principle modifies the application of many others.

940 METAL INDUSTRIES OUTSIDE THE MAIN INDUSTRIAL SECTIONS One of the interesting features of Table 38 is the types of manufacturing which show high percentages of wage-earners outside the regions where either Heavy or Light Industry is the dominant occupation For instance, in column C steel barrels (13), boilers (20), structural metal (26), sheet metal (29) and tin cans (34) all show percentages above 30, indicating that they are manufactured in unusually large amounts outside of these two sections. In each of these cases California is starred in one of the columns showing the leading states Such stars indicate states which attain a leading position and are not located in regions that belong distinctively to either the Heavy or Light Industry Sections Steel barrels and the other products mentioned above are relatively simple products, generally made from sheets of metal, and needed in many parts of the country. The other asterisks in Table 38 are for blast furnaces in Alabama (2), tin cans in Maryland (4), railroad cars in Missouri (3), and shipbuilding in Virginia (7) Since shipbuilding must be carried on where there is water, it needs no further comment, except that it is surprising that an inland state such as Michigan should build more ships than New York, New Jersey, or any other state. As for the other products, many of the railroad cars are the freight type, made of heavy iron plates. Such products as boilers, structural metal, and tin cans require only a simple type of manufac-

ture All of them are used in occupations which are widely distributed throughout the country Thus they illustrate the centrifugal tendency whereby the easier parts of a highly concentrated industry tend to spread out into other regions It is especially interesting that California is the chief state where iron goods are manufactured outside the main centers of the metal industries It is also interesting that the products in which that state has thus far come to the front are the simpler ones These facts indicate that here we have an example of an expanding industry, like the cotton industry in the South In California, however, the market and a new fuel in the form of oil,



A941—Wage Earners Engaged in Metal Industries per One Thousand Population, 1937.

rather than the raw material, have been the factors which have drawn the industry away from its old moorings In both California and Maryland the growth of the sheet-metal and tin-can industries is connected with the fact that those states rank high in the canning of fruits and vegetables The essential point, however, is that the iron industry, like the cotton industry, is spreading out from its old center into new territory, and that in doing so it is beginning with the simpler processes

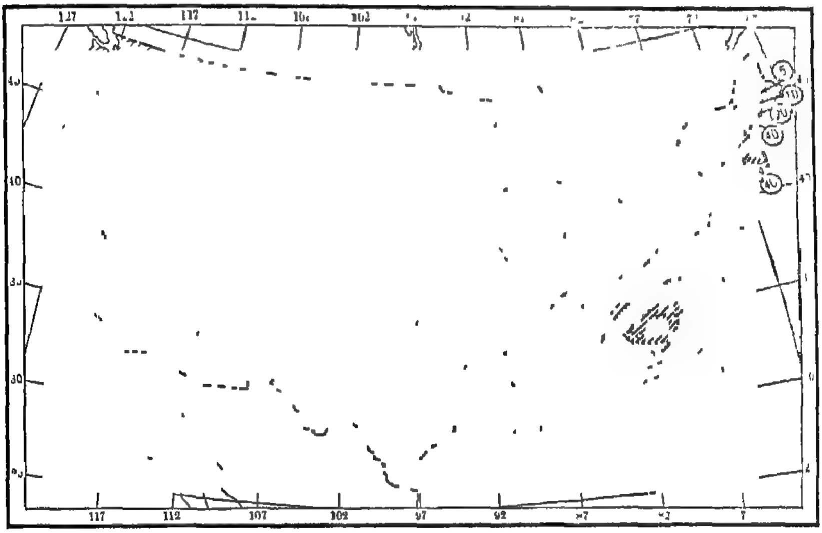
941. A Map of Metal Industries A good summing up of our study of metal industries is seen in A941 This map shows the number of wage-earners in all the metal industries per 1,000 of the population

Two major centers are evident—one for heavy metal industries and the other for light industries The center for heavy industries occupies an area within 100 miles or so of the southern shore of Lakes Erie and Michigan. It begins on the east with Buffalo, and includes Erie and the Pittsburgh region in Pennsylvania, Wheeling in W. Va, Cleveland, Youngstown, Canton, Akron, and Toledo in Ohio, Detroit, Flint, and Pontiac in Michigan, Fort Wayne, South Bend, and Gary in Indiana, Chicago in Illinois, and Racine and Milwaukee in Wisconsin, together with many smaller cities In this general region there are at least 40 wage-earners in the metal industries for every 1,000 inhabitants, which means that metal industries are by far the most important occupation of the cities. The center for light industries is in Connecticut and Rhode Island In Connecticut metal workers are relatively more numerous than in any other state except Michigan. Their work is based partly on iron, which they use for small goods such as hardware,,cutlery, guns, typewriters, and locks, but it is also based on extensive use of the nonferrous metals, especially copper and tin in the form of brass Hartford, New Haven, Waterbury, and Bridgeport are the chief cities in this center The Connecticut center is just as intensive as the Great Lakes center, but carries on kinds of work that require relatively small amounts of metal and fuel, but much skill. Surrounding these two centers there is a broad band where metal industries are well developed, but not dominant Richmond, Louisville, Memphis, St Louis, and St Paul lie near the outer limits of this area In these border regions, however, states such as Virginia, Tennessee, Kansas, and Minnesota have only about one tenth as great a percentage of workers in the metal industries as the central areas of heavy and light industries

942 The outlying minor centers of the metal industries seen in A941 afford interesting evidence as to the reasons for the development of manufacturing The center at Birmingham, Alabama, employs 18 per cent of the workers in blast furnaces, but has not developed any corresponding group of industries to use the iron thus prepared In fact, a large part of the iron is shipped elsewhere even before it is made into steel Hence most of the requirements of the South for metal goods are still supplied from the North The presence of an almost perfect combination of coal, iron, and limestone at Birmingham in the midst of a large market has not yet enabled people to overcome other handicaps and establish extensive industries In Colorado a somewhat similar situation is seen, but the market is smaller than in the South, and the metal industries though still simple are somewhat more fully developed The presence of mines not far away in the

Rocky Mountains has been a great help in providing a good market Finally in the California cities of San Francisco and Los Angeles, and to a smaller extent around Portland and Seattle, the presence of a good market and of active people is leading to a development which becomes more and more complex from year to year

943 CONTRAST BETWEEN METAL AND CLOTHING INDUSTRIES One of the outstanding facts about the development of manufacturing industries in the United States is the contrast between the clothing group, including both textiles and clothing, and the metallic group, including machinery and other kinds of equipment This is well brought



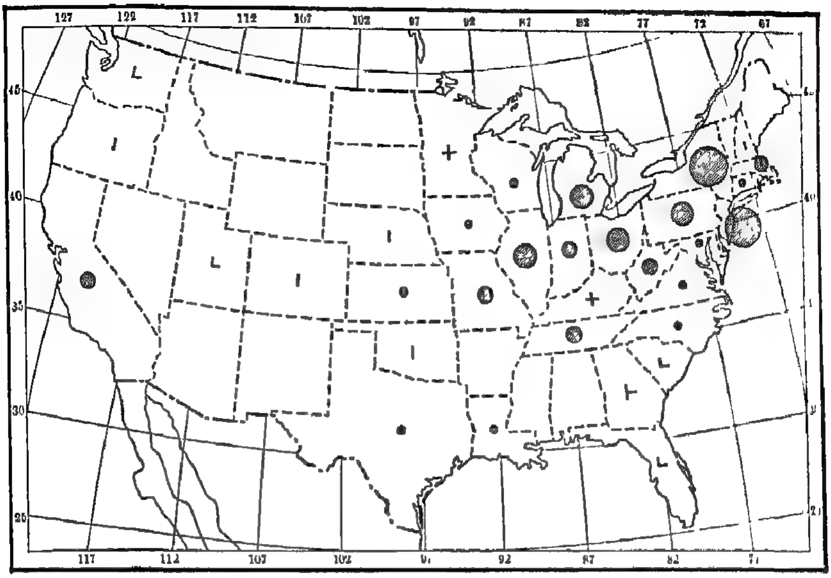
A943—Wage Earners Engaged in Making Textiles and Clothing per One Thousand Population, 1937

out by the difference between A941 and A943, each of which shows the number of wage-earners per 1,000 inhabitants In southern New England each map shows a center where more than 40 out of every 1,000 inhabitants are wage-earners in one of these two great types of industries. Each also shows another larger center some distance away, but these other centers are far apart, one in the southern Piedmont area and the other in the Great Lakes area The clothing map, however, gives distinct indication that its type of industry is spreading westward in the region of the heavy industries This is evident from the fact that in Indiana and Illinois 7 people out of every 1,000 are wage-earners in the textile industries This expansion has occurred

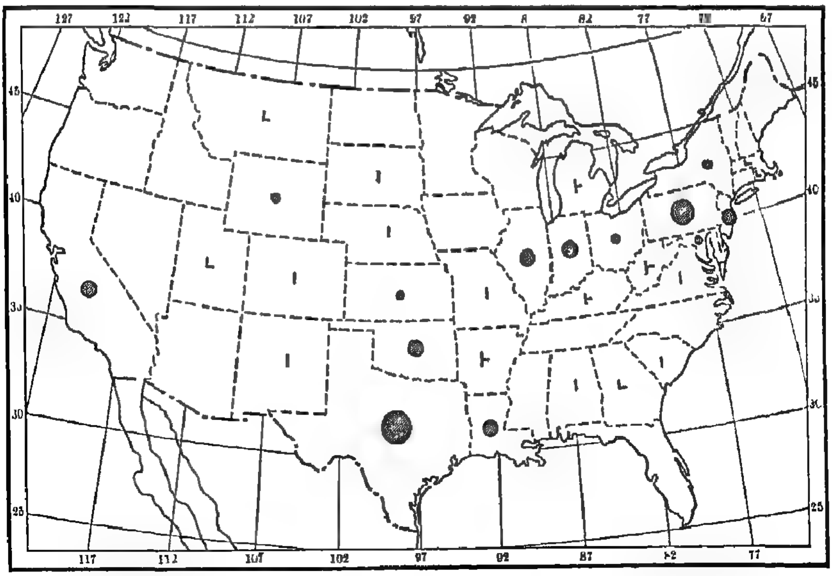
because of the presence of a good market, together with energetic people, even though there is not much advantage from the standpoint of raw material In the South the corresponding condition is that in Tennessee 5 persons out of every 1,000, and in Alabama 7, are wage-earners in the metal industries Here the presence of coal and iron ore is the main factor in locating the industries It is evident from all this that the geographical distribution of industries is governed primarily by the general conditions of climate and human progress, but that the distribution which would result from these factors alone is greatly modified by the location of raw materials, markets, and lines of transportation

944 CHEMICALS AND PETROLEUM-REFINING The distribution of the chemical industries and of petroleum-refining bear out what has just been said The commodities grouped as chemicals include not only ordinary chemicals such as the acids, ammonia, and other products used in the chemical phases of manufacturing, but also drugs, perfumes, tanning materials, and paints A944 shows that the greatest center of these industries, with close to 40 per cent of the workers, lies near New York and Philadelphia Another center, with over 25 per cent, coincides with the Heavy Industry Section south of the Great Lakes. Tennessee and Missouri, with their lead paints and other products, also do a good deal in the chemical industries California does still more, thus again sustaining the reputation of the West Coast as a region where many types of industry are fast developing to a stage where local needs are fully supplied The map of petroleum-refining (B944) is quite different from that of chemicals By far the greatest activity centers in Texas, spreading out into Louisiana, Oklahoma, and Kansas. The obvious reason for this location is the presence of vast supplies of oil in the ground The same thing is true of a second center in southern California Two other centers of oil refineries, one near Chicago and the other near New York and Philadelphia, have little to do with the location of the raw material They have grown up where great centers of industry and population create a large demand for petroleum products At these points previous developments have created so great a concentration of transportation facilities by water and by pipeline that oil can be brought cheaply from the wells. The map of pipelines (A811) illustrates the matter.

945 INDUSTRIES THAT SERVE HIGHER NEEDS *Paper* Among the products which serve man's higher needs, paper is preeminent because of its relation to printing. In the world as a whole this is among the most widely used products Yet in South and Central America the production of paper is estimated as only about two thirds of 1 per



A944—Wage Earners Engaged in Making Chemicals This includes ordinary chemicals, tanning materials, drugs, paints, perfumes, etc., 1937 Total 161,688



B944—Wage Earners Engaged in Petroleum Refining, 1937 83,182 workers

cent of that of the United States and Canada, and the consumption is not much greater Africa and Asia outside Japan produce even less than Latin America, and their consumption is almost negligible. Even Japan, which is by far the greatest nation of readers outside those of European stock, uses only about one twentieth as much paper as the United States

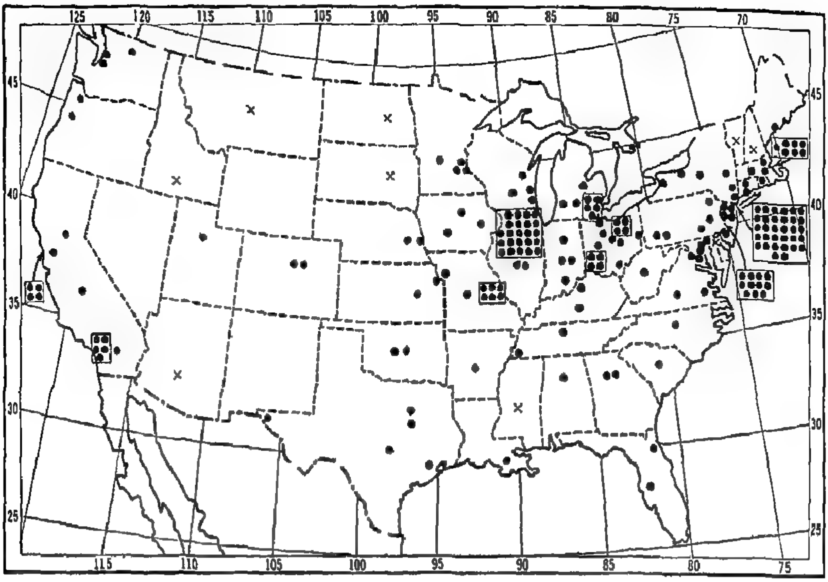
946 The first stage in the manufacture of paper is the making of pulp from wood or other cellulose material In the United States a vast amount is made from local wood in Maine, New Hampshire, and Wisconsin, while northern New York and Pennsylvania also manufacture a good deal from Canadian wood The importance of Canada in this respect is conspicuous Although Canada tries to have her pulpwood made into paper inside the country, much still comes to the United States The amount of wood pulp and paper made in Canada and imported to the United States exceeds our own supply Virginia and neighboring Appalachian regions, together with Louisiana, make pulp for the South, while the Pacific Coast with 6 per cent of the country's pulp production from its coniferous forests, is just about self-sufficing In a general way it is clear that the wood-pulp industry is primarily located near the forests which are best situated to supply the needs of the parts of the country that use most paper The main users are newspapers, although books, magazines, writing paper, cardboard, and cartons are important In the general regions where pulp wood is available, or to which it can be brought cheaply by water, sites with plenty of pure water and waterpower, such as Niagara, or the tumbling rivers in the glaciated regions of Canada, northern New England, and northern Wisconsin, are preferable

947. The great market afforded by the newspapers and other consumers of paper in the most active manufacturing areas draws the paper industry to a more southerly location than that of the pulp industry This is partly because the best grades of paper are still made of rags which once were almost the sole raw material Massachusetts, with its famous factories for high-grade paper in the Connecticut Valley around Holyoke, the "Paper City," and other states such as Connecticut, Ohio, Indiana, Illinois, and California, which make little or no pulp, are important as paper-makers On the other hand, in the southern states of Virginia, North Carolina, Tennessee, and Louisiana the production of paper is relatively less important than that of pulp Thus a comparison of the distribution of pulp and paper suggests that from the South as well as the North pulp moves toward

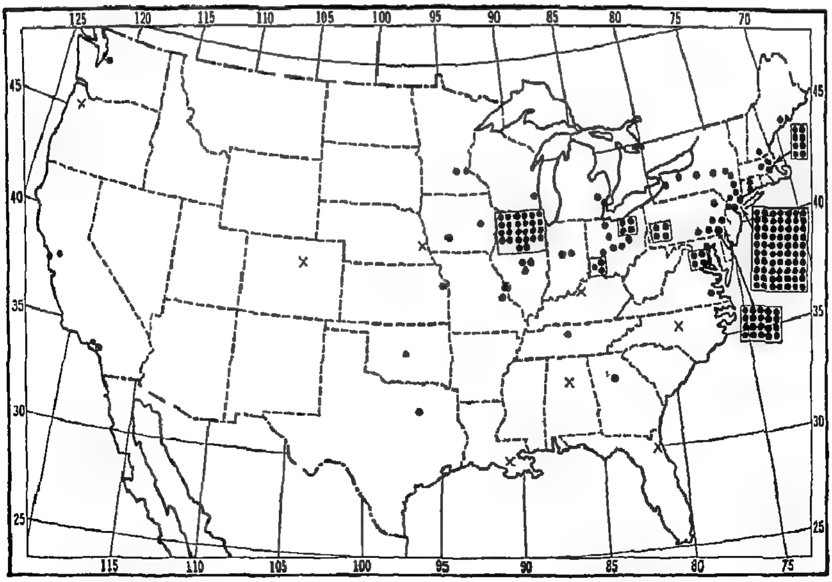
the manufacturing belt, for that is the area where the market for paper is largest

948 *Job Printing and Newspapers* The printing trade, which is by far the greatest user of paper, is divided into two parts One is the community industry which includes newspapers and the printing of local advertisements, programs, and notices The other is the complex industry of making books and magazines Of course the two industries are much alike and are sometimes combined, but they are different in that one supplies local needs, while the other serves larger areas, or even the whole country This difference is evident when A and B948 are compared In these maps a dot stands for one half of 1 per cent of the country's production, and crosses stand for smaller quantities which amount to at least one tenth of 1 per cent The dots for the larger cities are enclosed in rectangles In A948, showing newspaper and job printing, there are dots or crosses in all states except the four that are least populous This is natural, since no part of the country is without its local newspapers and local offices where minor printing jobs can be done. Nevertheless, it is also clear that the big cities, the Northeast in general, and the Pacific Coast all have more than their share of dots in proportion to their population Massachusetts claims 5 per cent of this kind of printing but only about 3 5 per cent of the population, New York City 16 per cent with less than 6 per cent of the people, Chicago 12 against less than 3, and the Pacific Coast 9 against 6. This indicates two main facts One is that the cities print newspapers for many people outside their limits, the other is that Chicago is particularly noteworthy in this respect Its metropolitan district has only 4½ million inhabitants compared with 12 million in New York's, and yet Chicago does three fourths as much job and newspaper printing as New York

949 *Magazines and Books.* With books and magazines (B948) the story is different. Metropolitan New York prints three times as many as Chicago—one third of all the books and magazines in the country The metropolitan districts of Boston, New York, Washington, Philadelphia, Pittsburgh, Cleveland, Cincinnati, and Chicago publish three quarters of all the books and magazines combined, and a still larger portion of the books. Printing requires such high skill that printers, on an average, get higher wages than the workers in any other main branch of manufacturing Such people help to bring prosperity to cities that do much printing Moreover, authors from all parts of the country tend to flock to New York and other big cities, thus adding still more to the prestige of the cities and helping

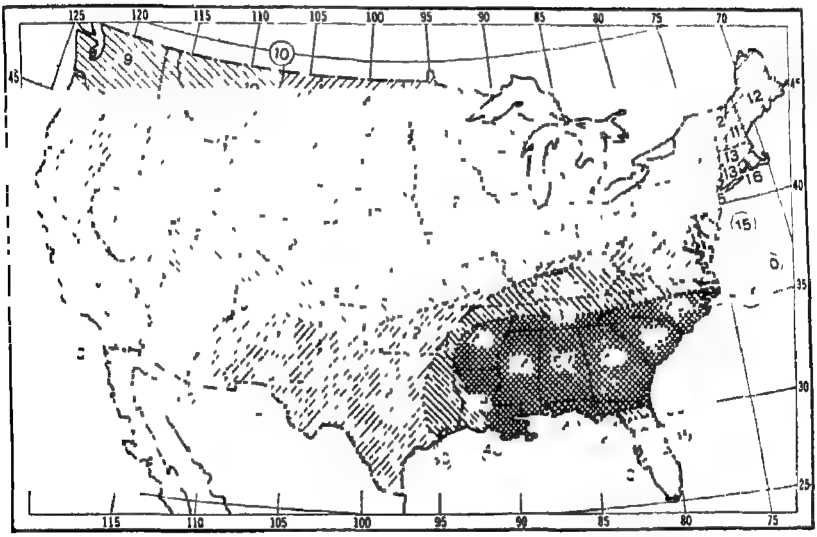


A948—Newspaper and Job Printing in the United States



B948—Periodical and Book Printing in the United States

still further to concentrate the printing industry there Such concentration is also helped by the fact that the cost of shipping books and magazines is small compared with their value The cost of fuel is a minor matter compared with that of the final product. In addition to all this the urge to write seems to be especially strong in the region from Boston to Chicago where the printing industry is most fully developed Thus, in the printing business the degree of human activity and progress is an especially strong factor in determining where the industry shall be located Such activity is also stimulated by education, by the spirit which pervades great cities, by the attraction



A950—Persons per Copy of Curtis Magazines

which such cities exert upon unusually able people, and by the facilities which they offer for selling and distributing their products

950 But how about the final use of the printed page? Where do people read the most books and magazines? This question cannot be answered exactly for most publications, but A950 shows how many persons there are in each state for each copy of certain widely read publications of the Curtis Publishing Company of Philadelphia, namely, the *Saturday Evening Post, Ladies' Home Journal,* and *Country Gentleman* A map of magazines and books in general would, presumably, present the same general features These have now become familiar through repetition in maps of many kinds ranging from yield of corn per acre, and egg and milk production, to wages, the

consumption of baker's bread, mortality, climatic energy, and manufacturing What this map and others show is that the manufacture of equipment for satisfying man's higher needs is one of the industries that is most strongly concentrated in the manufacturing section of the northern United States.

PART XII

CITIES AND COMMERCE

CHAPTER XXXIX

CITIES AND HINTERLANDS

951 NATURE OF CITIES Cities are primarily important as centers of trade and industry, but they are also important as centers of government, finance, residence, education, and so forth The commercial city is like a giant sitting at the gateway of his estate With one hand he sweeps up the products which the people of his hinterland prepare, with the other he reaches far out to other people, strangers perhaps, and offers his own people's products in exchange for something which he can hand back to his subjects Kansas City, for example, receives corn, wheat, cattle, and other farm products from the surrounding regions, especially those to the west of it It sells these commodities largely to the eastern states where the beef, pork, and flour of Kansas and Missouri supply food for cities of the industrial type. The industrial city resembles another kind of giant—one who works with his hands to make machinery, cloth, chemicals, furniture, and other goods in a great profusion of kinds He, too, engages in commerce, exchanging his manufactured goods for food and fuel, and for the raw materials that he must have if he would keep busy All cities are partly commercial and partly industrial, but the relative importance of these two types of activity varies greatly from city to city Omaha and Memphis, for example, are primarily commercial, whereas Waterbury and Rochester are primarily industrial.

952 HINTERLANDS OF CITIES The growth of a city depends largely on its *hinterland* The hinterland is the region for which a city, town, or village serves as a center For example, the hinterland of a certain small farming village includes an area extending outward in all directions for about 5 miles This is the approximate limit of the area within which people make use of the village's stores, movies, and railroad station Some of the more distant farmers use the village to satisfy only part of their needs, patronizing other villages for many

things. Thus they belong to two hinterlands which overlap each other Both of these villages and a number of others ship their milk, vegetables, fruit, and lumber to a small city 30 miles away When people want clothes, or other relatively expensive goods, they usually go to the city because its stores are better than those of the villages. If a village store has a hurry call for something which it does not carry, it sends to one of the larger stores in the city Thus the city's hinterland consists of the combined hinterlands of all the villages. The little city in turn ships the products of the farms, forests, mines, and factories of its hinterland to a larger city, perhaps a seaport Its merchants purchase their goods from large wholesale dealers in the big city Its manufacturers depend on the big city for raw materials, especially those that are imported Other small cities depend in the same way upon the large city Thus all their hinterlands are combined in that of the large city

953 Even if a city is almost ideal so far as its own climate, soil, relief, minerals, and waterways are concerned, it can never grow great if it has an inadequate hinterland St. George in the Bermudas, for example, is a delightful little town with 3,000 or 4,000 permanent inhabitants, but there is little prospect that it will ever become a large city Its hinterland consists of a few islands with a total area of only 20 square miles, and with few natural resources aside from attractiveness for tourists On the other hand, the growth of New York City is probably due less to the excellent quality of its immediate hinterland in New Jersey, Long Island, Connecticut, and southern New York, than to the wealth of the far larger hinterland to which it lays claim by reason of its unique location. The fact that the coastal route from New England to the South meets the best route from the Great Lakes and the Prairies to the Atlantic Coast at a superb harbor gives New York an enormous hinterland. The great productivity and activity of this larger hinterland are the major factors in making New York one of the world's two greatest centers of population, commerce, manufacturing, and finance In recent years it has surpassed London, the capital and commercial center of the British Empire.

954. Overlapping Hinterlands The way in which the hinterland of a great city overlaps that of other cities is well illustrated by New York That city is the immediate center for about 12 million people who live in its metropolitan district Farther away in western Connecticut, southern New York, and northern New Jersey people live in the hinterlands of their own local cities Nevertheless, if a merchant, a manufacturer, a banker, or even an ordinary consumer in Waterbury, Poughkeepsie, or Elizabeth, for example, wants something un-

usual, he generally turns to New York for it He feels that New York is his main center, and that he lives in its hinterland as well as in that of his own smaller city Regions a little farther from New York, or a little nearer some other great city, have a more divided allegiance The Springfield district in Massachusetts does its wholesale trading partly with Boston, partly with New York, and to a small extent with the metropolitan district of Albany, Schenectady, and Troy. The districts tributary to Trenton, New Jersey, and to the anthracite center formed by Scranton and Wilkesbarre, Pennsylvania, look to both New York and Philadelphia In the same way Utica, Syracuse, and Rochester, with their local hinterlands, are largely New York's customers for wholesale trading, but show considerable tendency to deal with Buffalo Thus outside its main *inner hinterland* there is a broad zone where New York shares a larger *outer hinterland* with other great cities One sign of the degree to which a place belongs to one hinterland or another is the newspapers that are read Philadelphia and Boston, for example, read New York papers much more than New York reads their papers

955 Because of conditions such as this, not only do Boston, Philadelphia and Buffalo form parts of New York's outer hinterland, but so do such cities as Cleveland, Detroit, Chicago, St Louis, St Paul, Omaha, and Denver toward the west, together with Atlanta, New Orleans, San Antonio, and Oklahoma City more to the south All these cities, as well as others surrounding them, are reached by salesmen and buyers from the great eastern metropolis So, too, are Canadian cities such as Montreal, Toronto, and Winnipeg. Even an easily crossed international boundary, however, like that between the United States and Canada, tends to limit the hinterlands of cities. Tariffs, custom houses, and other restrictions make international trade relatively difficult Nevertheless, the outer hinterland of New York extends fanwise over the whole of the eastern United States and southern Canada east of the Rocky Mountains The strength of New York's influence is seen not only in the salesmen and buyers who travel back and forth across the land, but also in the capital invested by New York in all sorts of business enterprises

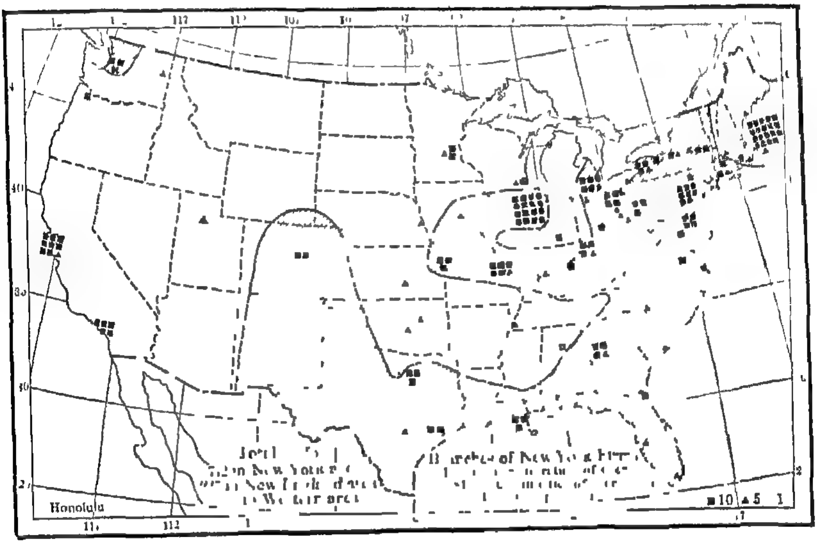
956 Let us see to what extent the Pacific as well as the rest of the United States belongs to New York's hinterland Branches of business houses in cities other than the one where the business has its headquarters are one of the many means of estimating this. According to Dun and Bradstreet's *Reference Book*, 469 business concerns that have their home offices east of the Rocky Mountains have branches in San Francisco. Although they form only 2½ per cent of the city's

18,000 business concerns, their importance far exceeds their number The 338 of them that have what Dun and Bradstreet call a "pecuniary strength" of over $500,000 comprise two thirds of San Francisco's concerns of that kind Naturally they do far more business than average firms One third (155) of the 469 branches of eastern firms in San Francisco have their headquarters in New York City Moreover, the New York firms include many of the largest, such as the United States Steel Company Chicago comes next with 81, then Boston 24, Philadelphia 18, and Milwaukee 14 Other important cities are Pittsburgh with 13 branches in San Francisco, Detroit 12, St Louis 12, Cincinnati 10, and St Paul-Minneapolis 10

957. The branch offices give a fairly good idea of how important the various eastern cities are in the business of the Pacific Coast Taking the list of branches at random we find firms that deal in drugs, asbestos, surgical dressings, air-conditioning equipment, steel, women's wear, woolen goods, rugs and yarns, carbon paper, typewriter ribbons, men's clothes, radio supplies, flour, drawing instruments, fiber boxes, meat, rubber goods, books, and a hundred other commodities and services The fact that the West Coast can pay for all sorts of luxuries as well as necessities makes it a profitable outer hinterland for the eastern cities, especially New York and Chicago In similar fashion, Los Angeles, San Francisco, Portland, Seattle, and other West Coast cities send out long fingers to claim the East as part of their outer hinterland The size of hinterlands varies according to the kind of work performed by a city As a wholesale seller of flour and lumber Seattle firms have branches in many eastern and southern cities The hinterland to which Los Angeles supplies these commodities is almost limited to southern California and Arizona On the other hand, as a source of movie films Los Angeles includes most of the world in its outer hinterland

958. MAPS OF HINTERLANDS Although it is impossible to set precise limits to a city's hinterland, there are various ways of measuring the relative amount of business done by any given city in different parts of the country One of the most feasible ways, as we have just seen, is by means of branch offices Eight maps, for example, which we shall now discuss, are based on all cities of more than 100,000 population in the United States and on a few smaller cities in states where there are no large ones The symbols (squares 10, triangles 5, and dots 1) show the number of branches maintained in each city by concerns whose main headquarters are in some particular city, such as New York. Only the larger concerns are included, those having a pecuniary strength above a million dollars Thus in A958 the 16

squares at the south end of Lake Michigan mean that in Chicago and its immediate neighborhood 160 great firms with headquarters in New York are doing an active business by means of their own branches. Hundreds of others, of course, are doing business there without establishing branches. Notice how widely the New York branch offices are distributed—7 far south in Miami, Florida, 14 in Dallas, Texas, 54 in Los Angeles, California, 6 in Spokane, Washington, one in Bangor, Maine, and a total of 1,353 in the entire United States. Some firms have only one or two branches, others have 10, 20, or more. In proportion to their size such places as Atlanta, St.



A958—Branches of New York Firms

Louis, Kansas City, San Francisco, and Seattle, as well as Chicago, are conspicuous as centers of the influence of New York.

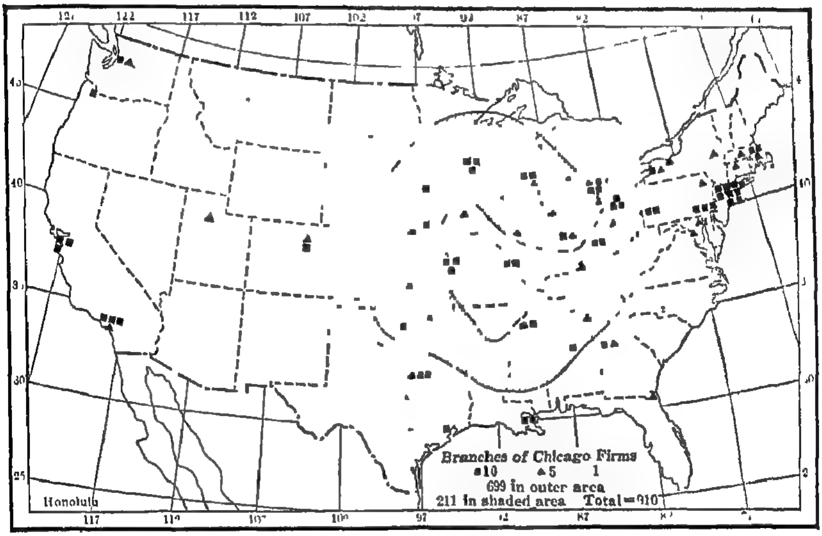
959 In spite of New York's great influence its branch businesses outnumber those of every other city in only part of the country. The shaded areas in A958 show the sections in which the branches of New York concerns outnumber those of any other city. Inasmuch as the New York firms average larger than the others, we may infer that within the shaded areas New York does more business than any other city except the one in whose inner hinterland we happen to be. Of course, each city has its own local hinterland where its influence is dominant, but we are talking now of the business that goes on between the cities of more than 100,000 population. In this larger sense

the whole of the eastern United States except New England belongs to New York's hinterland rather than to that of any other city such as Chicago, Philadelphia, Detroit, or Los Angeles. So do the South Atlantic states, and most of the South Central section from New Orleans across Texas to Denver. Moreover, Seattle, with 30 branches of New York concerns, seems to fall into the New York hinterland more fully than into that of San Francisco which has only 14 branches. This is not surprising when we recall that Seattle is a great port for the shipment of lumber and wheat, as well as for the importation of silk, a good proportion of which goes to the New York district. The 160 branches of New York firms in Chicago help to bring that city also within New York's hinterland. Another interesting feature of A958 is the way in which the New York hinterland pushes westward past Philadelphia and Pittsburgh to Columbus, Cincinnati, Louisville, Evansville, St. Louis, Kansas City, and Des Moines. Chicago is nearer to this latter group of cities than is New York. Nevertheless, the early start of New York, its financial power, and the fact that it is a great seaport and the center of a vast manufacturing industry draw these cities into its orbit to a surprising degree.

960. New York's preponderance in branch firms does not prevent Chicago from including these cities in its outer hinterland. In proportion to its population, Chicago's 23 branches in St. Louis and 20 in Cincinnati are about as numerous as New York's 55 and 35. Chicago's 31 in Kansas City and 6 in Des Moines are proportionally more numerous than New York's 33 and 9. Moreover, we have no data as to the exact amount of business done by each of these cities with either New York or Chicago, and Chicago may excel New York in places where New York has the most branch firms. Then, too, there are many other ways of measuring hinterlands, and some of these might give results somewhat different from A958. Nevertheless, it is clear that New York's great size, wealth, prestige, and foreign trade enable it to push its hinterland far to the west and south.

961. Now let us look at other hinterlands in addition to New York's. The maps which will now be considered show the branch firms of the six other American cities which are supreme in more than their local hinterlands when measured by the number of their branch businesses. A961 suggests that Chicago is the dominant outside factor in the business of a large group of cities in the central part of the United States. The slight extent of the shaded area east of Chicago seems to indicate that toward the east the efforts of Chicago firms to get business do not make that city dominant because it has to compete with Detroit, Pittsburgh, Philadelphia, and other cities, as well

as New York To the west and south, however, it has more opportunity, and is able to dominate a large area of rich agriculture The appearance of B961 suggests that Chicago, being blocked on the east by older cities, has pushed its dominance westward across the great plains as far as agriculture is profitable Judging by branch firms, however, it has not achieved the first position among the cities that do business with Denver The mining industries for which that city is the center find their best source of capital and their best markets in the industrial East, especially in New York Chicago has also pressed its business southward almost as far as New Orleans New York's push to the west, however, has been so strong that its zone of



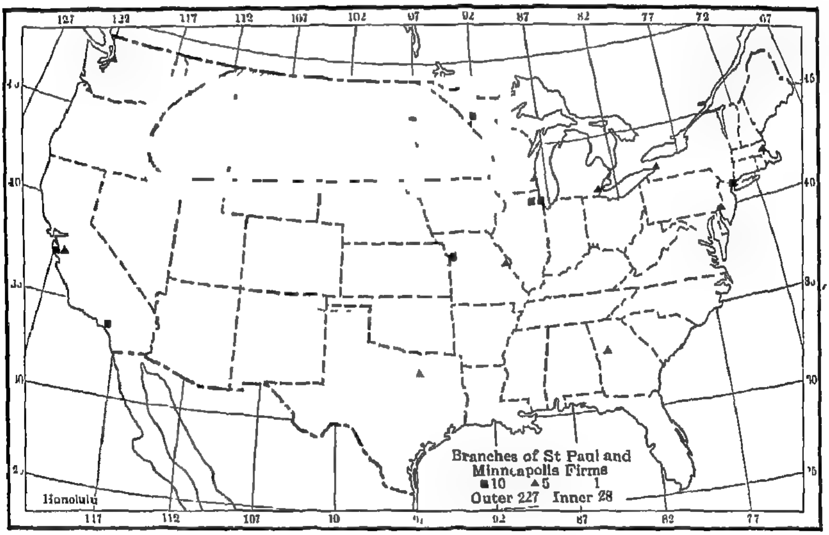
A961—Branches of Chicago Firms

supremacy, as measured by branches of million-dollar firms, extends right across the Chicago zone, cutting it almost in two along a line from Cincinnati to Kansas City.

962 The map showing branch firms with headquarters in Minneapolis and St Paul (A962) illustrates the same principles as the Chicago map The Twin Cities, lying as they do toward the western edge of the well-settled eastern section of the United States, and also being relatively young, have pushed their business mainly westward. Toward the east and south they are blocked by Chicago and New York, but toward the west they have had a free field. This illustrates a very important principle. *The size and location of the region where*

a city is economically dominant depend upon the location and activity of its competitors as well as upon the accessibility and resources of the areas to which the city has access. Wisconsin and Iowa are more accessible to Minneapolis than are Montana and Idaho Their developed resources and population are also larger Nevertheless, the Twin Cities influence the trade of Wisconsin and Iowa far less than they influence that of the states to the west of Minnesota Beyond Idaho, however, Minneapolis and St Paul yield first place to New York at Seattle and to San Francisco on the rest of the West Coast.

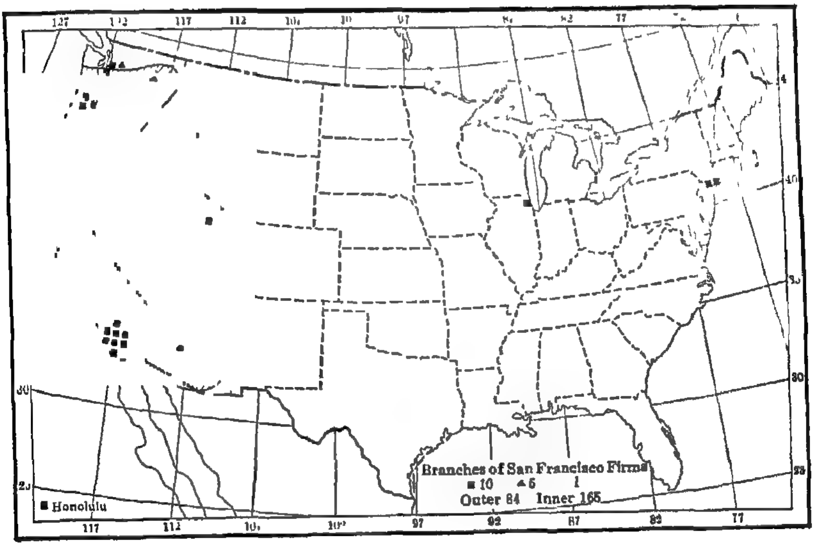
963 A963 shows that the region where branches of San Francisco concerns outrank those of any other city extends along the Pacific



A962—Branches of St Paul and Minneapolis Firms

Coast from Tacoma to San Diego Inland it includes Spokane in Washington, Salt Lake City in Utah, and Phoenix and Tucson in Arizona Its central location on the Pacific Coast, as well as its good harbor and early development, have enabled San Francisco to play a large part in the development of other western cities While the other cities were growing rapidly, the large firms of San Francisco were especially active in establishing branch offices in them, just as did New York and Chicago farther east San Francisco has become dominant as far east as the Rocky Mountains because the region between the Rockies and the Pacific did not develop until after San Francisco had made a good start, and also because that region is far removed

from the dominant eastern cities The main expansion of San Francisco's influence, however, has been northward and southward along the coast Although the Los Angeles metropolitan district is nearly twice as populous as that of San Francisco, it is less commercial. The immediate hinterland of Los Angeles, with its citrus fruits and petroleum, has become a great producer of primary commodities The city is also developing great manufacturing industries, which achieve a unique form in the movie industry at Hollywood In addition to this, metropolitan Los Angeles ranks especially high as a residential region, because its climate and scenery attract people who seek a pleasant place in which to live Such a combination of primary

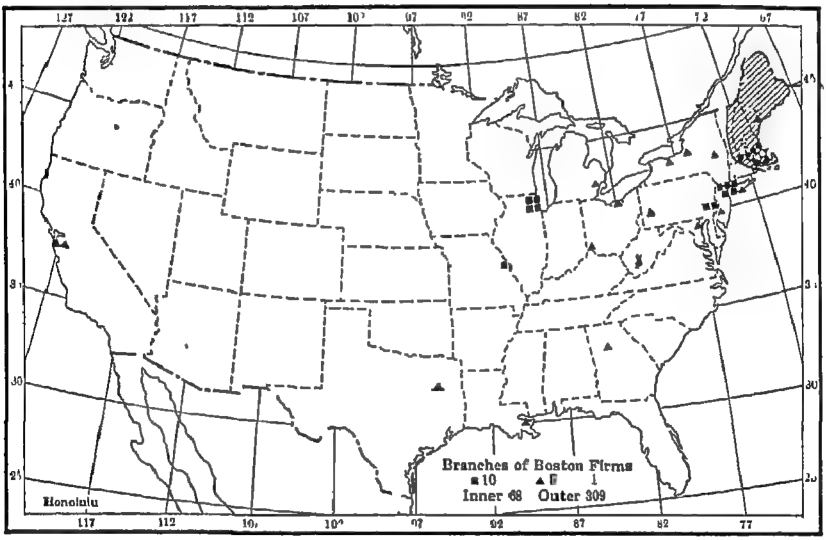


A963—San Francisco Branch Firms and Hinterland

production, manufacturing, and residential desirability is rare The contrast between San Francisco with its wide commercial dominance and Los Angeles with its intensely individualistic local hinterland and worldwide dominance in a single industry is a unique feature of the Pacific Coast

964. In spite of its size and age, Boston is dominant only in Massachusetts and New Hampshire (A964) Even in Maine, Vermont, and Rhode Island the great firms of New York have practically the same number of branches as those of the Boston district This is interesting because it illustrates still another principle· *The hinterland of a city tends to expand along the easiest lines of communication.* Maine

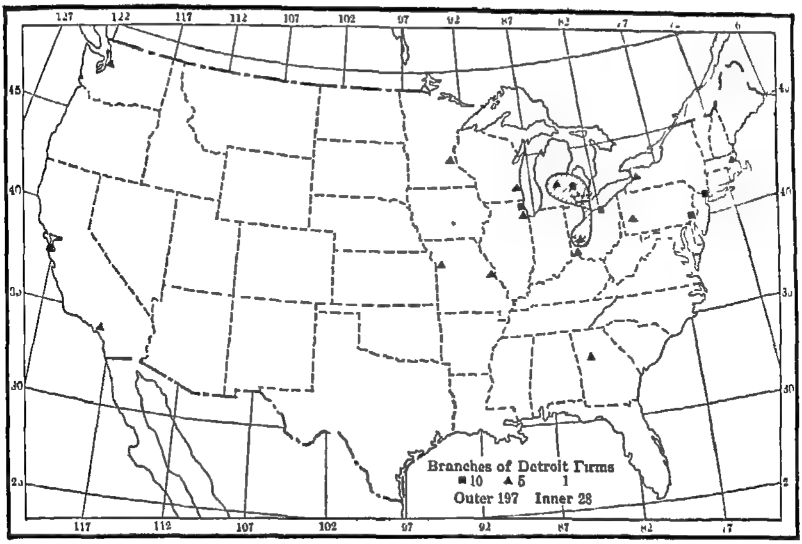
is reached by sea from New York so easily that Boston has only a slight advantage By the sea route Rhode Island is practically as near to New York as to Boston The cutting of the Cape Cod Canal, to be sure, has now given Boston the shorter sea route, but before the canal was built the influence of New York had become firmly established This influence is seen not only in such matters as branch firms and steamship lines, but in the fact that Rhode Island's summer resorts, especially Newport, have been frequented by New Yorkers far more than by Bostonians Then, too, the fact that Rhode Island is a cotton-mill state, makes it look southwestward rather than northward All



A964—Branches of Boston Firms

such factors play a part in determining what city will dominate a region An interesting illustration of the way in which the influence of a city declines as one goes away from it is seen along one of the two main railway lines from New York to Boston At Bridgeport, Connecticut, the first large city east of New York, the branches of million-dollar New York firms number 14, and there are none from Boston. At New Haven, the next large city, New York has 9 branch firms and Boston 2, at Hartford the numbers are 13 and 3 At Springfield, in the same state as Boston, New York's 8 branches are less numerous than Boston's 10 Finally, at Worcester, New York has only 3 and Boston 10—almost the reverse of the conditions at New Haven and Hartford.

965 The Detroit map (A965) shows an area of dominance smaller than that of Boston Even in Toledo the number of branch firms from Detroit is less than from either New York or Chicago Nevertheless, if the total volume of business could be measured, Toledo might prove to belong to the area dominated by Detroit On the basis of branches of large concerns, however, the large cities where Detroit is dominant are limited to Flint, Grand Rapids, and Dayton Whatever may be the exact area of Detroit's dominance, its size is clearly small One reason for this is that, although Detroit is one of the giants among cities, it has attained its present rank only within

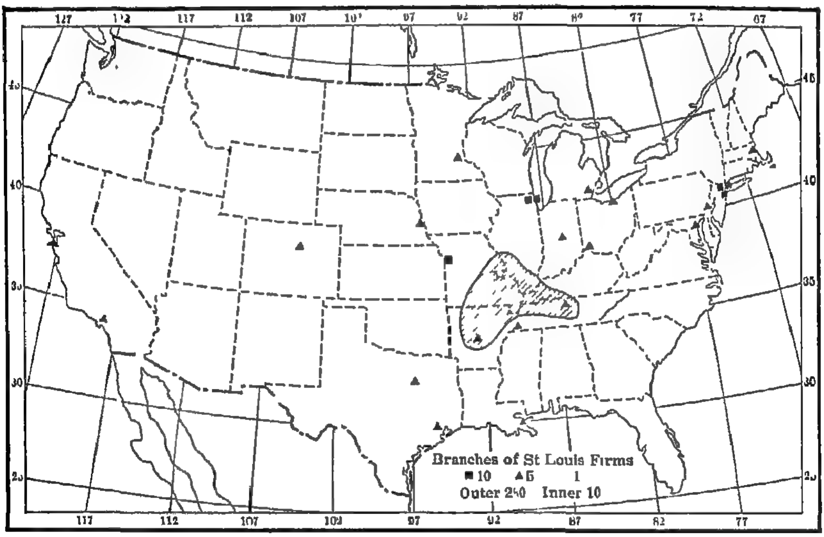


A965—Detroit Branch Firms and Hinterland

the twentieth century Moreover, Detroit is emphatically an industrial rather than a commercial city. Its relation to Chicago is something like that of Los Angeles to San Francisco Just as Los Angeles holds the preeminence of the whole world in manufacturing movies, so Detroit holds similar preeminence in motor vehicles Each of these cities has in one way a wider dominance than either New York or London Branches and agents of financial institutions of New York and London are found in all large cities, except in countries such as Russia which are almost closed to outside business London and New York, however, are rivals who have to divide the dominance of the financial world, whereas Detroit and Los Angeles, in their respec-

tive fields, stand so far above all competitors that it is difficult to say what other city comes second

966. The St. Louis map (A966) brings us again to a city which is strongly commercial, thus falling in the same class as New York, Chicago, Boston, and San Francisco The shape of the area dominated by St Louis is especially interesting Crowded by Chicago on the north and New York on the east, as well as by New Orleans and the Texas cities of Dallas, Fort Worth, and Houston on the south, St Louis has still been able to become dominant toward the southeast as far as Nashville and toward the southwest across Arkansas, where it is especially influential at Little Rock Here, as before, the num-

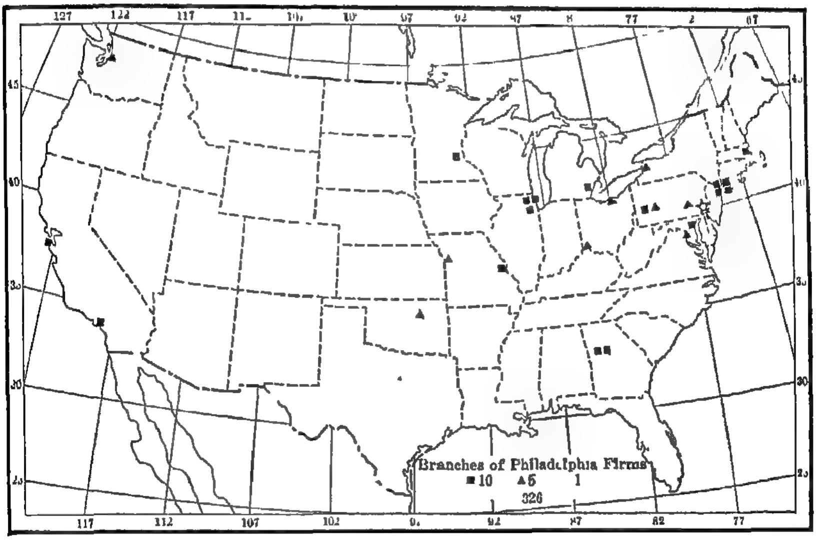


A966—St Louis Branch Firms and Hinterland

ber of branch firms is only one of many ways of measuring a city's dominance, and the outlines of the map would presumably be different if other methods were used Nevertheless, it is certain that the area in which St Louis is dominant is strictly limited by the competition of other cities, especially the two huge giants which dominate most of the United States east of the Rockies

967 A967, unlike the other maps in this series, has no shaded area It shows that although Philadelphia's large firms have more branches (326) than have those of any other cities except New York (1,353), Chicago (910), and Boston (377), the area where Philadelphia is the dominant city does not include any other city with more than 100,000

population aside from its suburb, Camden, just across the Delaware River At Trenton, a little farther up the river, there are only 3 branches of large Philadelphia firms, whereas there are 9 from New York Wilmington farther down the river has 2 from Philadelphia and 4 from New York, at Scranton and Wilkesbarre, farther north, the numbers are 1 and 10 This means that Philadelphia differs from New York as Detroit differs from Chicago Philadelphia devotes itself mainly to manufacturing, with finance and commerce as secondary occupations It lets New York spread its commercial influence far and wide, while it literally sticks to its knitting, for the Philadelphia region is preeminent in the manufacture of knit goods such as stock-



A967—Philadelphia Branch Firms and Hinterland.

ings Cleveland, Buffalo, and Pittsburgh are other great industrial cities of the Philadelphia type so far as hinterlands and dominance of outlying regions are concerned

968 In this whole discussion it is essential to remember that there are three distinct phases in the relationship of cities to other regions. First, every city, town, or village, no matter what its size, has an inner hinterland in which it is supreme. In other words, each such center is surrounded by an area within which the people look to the center as the place where they carry on the greater part of their buying, selling, and other activities Second, every center which can be called a city is surrounded by an outer hinterland within which it is domi-

nant over a certain number of smaller centers some of which may themselves be cities. In other words, the smaller centers look to it for certain goods and services which their own local businesses do not supply. The shaded parts of the maps that we have just been studying are areas of this sort in which large cities are themselves dominated by still larger cities. Finally, in civilized countries each center, no matter what its size, reaches out beyond its own hinterland, and beyond the area dominated by its main city to spots here and there all over the world. Such relationships are illustrated by the familiar story of the breakfast table with sugar from Cuba, coffee from Brazil, grapefruit from Florida, butter from Minnesota, chocolate from west Africa, and bread made of wheat from Kansas and baked with coal from Pennsylvania. They are also illustrated by the fact that woolen goods made in some New England village, rakes made in Chicago, rice raised in Louisiana, adding machines made in Ohio, or cotton raised in Texas may find a final sale in some remote part of any one of six continents.

969. PRIMATE CITIES. Our study of inner hinterlands, outer hinterlands, and regions of dominance naturally leads to a consideration of what Mark Jefferson has called primate cities. This means cities which are outstandingly dominant within their own countries. The last column in Table 39 names the three largest cities of each country and gives their population in thousands. Column C indicates the sizes of the second and third cities as percentages of the largest city. In 24 out of 42 countries the second city is less than half as large as the main city. Where there is so great a difference the larger city may properly be called a primate. In only 4 countries—South Africa, Australia, Spain, and Italy—does the second city attain to 80 per cent of the size of the first. The fact that there is usually so great a difference between the largest city and the next suggests that a primate city grows at the expense of the others. This seems to be actually the fact. When natural geographical conditions or other advantages, such as being the seat of government, cause one city to be larger and more famous than any other in its country, it acquires special advantages. Generally the city with the greatest natural advantages is also the capital. The United States is one of the few countries where this is not true. Even if the largest city is not the capital, its size generally enables it to do more than other cities in improving its railways, waterways, and airways, and making itself a center of transportation and industry. In France the railways focus on Paris, in Germany on Berlin, and in England on London. When the Russians chose Moscow instead of Leningrad as their new capital, they started new efforts,

TABLE 39

PRIMATE CITIES COMPARED WITH THE SECOND AND THIRD CITIES OF THEIR COUNTRIES

A	B	C	D
1. Denmark	1935	11–9	Copenhagen 843, Aarhus 91, Odense 76
2 Hungary	1936	13–12	Budapest 1052, Szeged 140, Debrecen 125
3 United Kingdom	1931	14–13	London 8204, Liverpool 1178, Glasgow 1089
4 Mexico	1930	18–13	Mexico 1029, Guadalajara 184, Monterrey 137
5 Rumania	1937	18–17	Bucharest 643, Chişinău 114, Cernauţi 110
6 Peru	1930	20–13	Lima 370, Callao 75, Arequipa 46
7 Argentine Republic	1937	22–13	Buenos Aires 2,290, Rosario 510, Córdoba 302
8 Turkey	1935	23–16	Istanbul 741, Izmir 171, Ankara 123
9 Cuba	1935	25–24	Habana 550, Holguin 135, Camaguey 133
10 Bolivia	1936	26–22	La Paz 200, Cochabamba 52, Oruro 45
11 Finland	1936	26–25	Helsinki 284, Viipuri 73, Turku 71
12 Chile	1930	30–11	Santiago 696, Valparaiso 193, Concepción 78
13 Belgium	1936	30–18	Brussels 905, Antwerp 273, Ghent 164
14 Philippines	1936	31–12	Manila 355, Cebu 110, Iloilo 46
15 France	1936	32–20	Paris 2830, Marseille 914, Lyon 571
16 Bulgaria	1934	35–24	Sofia 287, Plovdiv 100, Varna 70
17 Norway	1930	39–21	Oslo 253, Bergen 98, Trondheim 54
18 Greece	1928	40–10	Athens-Piraeus 592, Thessaloniki 237, Patras 61
19 Portugal	1930	40–4	Lisbon 594, Oporto 232, Coimbra 27
20 United States	1930	43–27	New York 7,781, Chicago 3,374, Philadelphia 2,083
21 Colombia	1937	43–36	Bogotá 420, Barranquilla 180, Medellín 150
22 China	1936	44–37	Shanghai 3490, Peiping 1556, Tientsin 1292
23 Germany	(1939)	44–32	Berlin 4,242, Vienna 1,874, Hamburg-Altona 1,372
24 Sweden	1937	48–26	Stockholm 544, Goteborg 263, Malmo 144
25 Japan	1935	51–16	Tokyo 5,876, Osaka 2,990, Nagoya 1,083
26 Egypt	1937	52–9	Cairo 1,307, Alexandria 682, Port Said 127
27. Poland (Old Boundaries)	1937	53–26	Warsaw 1,233, Lódź 653, Lwów 317
28 Venezuela	1926	55–28	Caracas 135, Maracaibo 71, Valencia 37
29 Switzerland	1936	59–50	Zurich 250, Basle 148, Geneva 124
30 Iran (Persia)	1935	61–39	Tehran 360, Tabriz 219, Meshed 139
31. Brazil	1936	66–28	Rio de Janeiro 1,711, São Paulo 1120, Pernambuco 473
32. Yugoslavia	1937	70–36	Belgrade 267, Zagreb 186, Subotica 100
33 New Zealand	1937	70–62	Auckland 214, Wellington 151, Christchurch 133
34 Afghanistan	1931	75–38	Kabul 80, Kandahar 60, Herat 30
35 Netherlands	1937	76–62	Amsterdam 783, Rotterdam 599, The Hague 487
36 Soviet Union	1933	76–19	Moscow 3,663, Leningrad 2,776, Baku 709
37 Canada	1931	77–30	Montreal 819, Toronto 631, Vancouver 246
38. India	1931	79–44	Calcutta 1,485, Bombay 1,161, Madras 647
39 Union S Africa	1931	80–29	Johannesburg 330, Capetown 265, Durban 95
40. Australia	1936	80–25	Sydney 1,267, Melbourne 1,016, Adelaide 317
41. Spain	1934	91–31	Barcelona 1,148, Madrid 1,048, Valencia 352
42 Italy	1936	96–75	Rome 1 156 Milan 1 116, Naples 866

such as a Volga canal, to increase the degree to which the transportation system centers on the new capital

970 Good transportation and the market afforded both by the city's large size, as well as the presence of the government, make the primate city a favorable location for trade, industry, education, and other activities Hence the city gets the reputation of being the place to which the most able people are ambitious to go The seeker for work thinks that, because the city is so big, there must be plenty of jobs The ambitious beginner hopes that the size of the city will increase his chances to become a leader in business The woman who wants an unusually attractive dress believes that she has a better chance of finding it in the primate city than anywhere else Artists, authors, scientists, and musicians, as well as people in the so-called practical occupations, all hear about the big city and about the famous people who live there The majority of tourists head for the capital of a country. The big city's name is far better known than that of the second city Paris, Bucharest, and Tokyo are much more familiar than Marseilles, Chişinău, and Osaka Most people know about Copenhagen, Athens, and Lisbon, but relatively few can correctly locate Aarhus, Saloniki, and Oporto

971 The relative difference in size between the primate city and the other cities is likely to be greater in a small country than in a large one Among the first 19 countries in Table 38 (those where the second city has no more than 40 per cent as many people as the primate city) the United Kingdom and France are the only ones with more than 20 million inhabitants. Mexico, Peru, Argentina, and Bolivia, to be sure, have areas of about half a million square miles, but only Mexico has as many as 10 million people The two populous countries where the primate city is especially dominant, that is, the United Kingdom and France, are not divided internally by strong regional contrasts Different sections of their people are not greatly separated by mountains or deserts, or by contrasts in language and customs On the other hand, three of the four countries where there are two large cities of nearly equal size are separated by discordant cultural or racial elements. North and south Italy, and the Dutch and British parts of South Africa, differ much in progressiveness and type of activity, and are diverse in race and language There is a similar difference between the northern Catalan-speaking part of Spain centering around Barcelona and the Castilian-speaking southern part that rallies around Madrid Another reason why London and Paris are so much larger than the other cities of the United Kingdom and France is that they are to a considerable extent primate cities of the whole

world New York is the only other city of which this is equally true. People from foreign countries flock to these cities as to no others The fact that the primate cities of the world have such a reputation does much to give them an increased amount of commerce, industry, and other activities

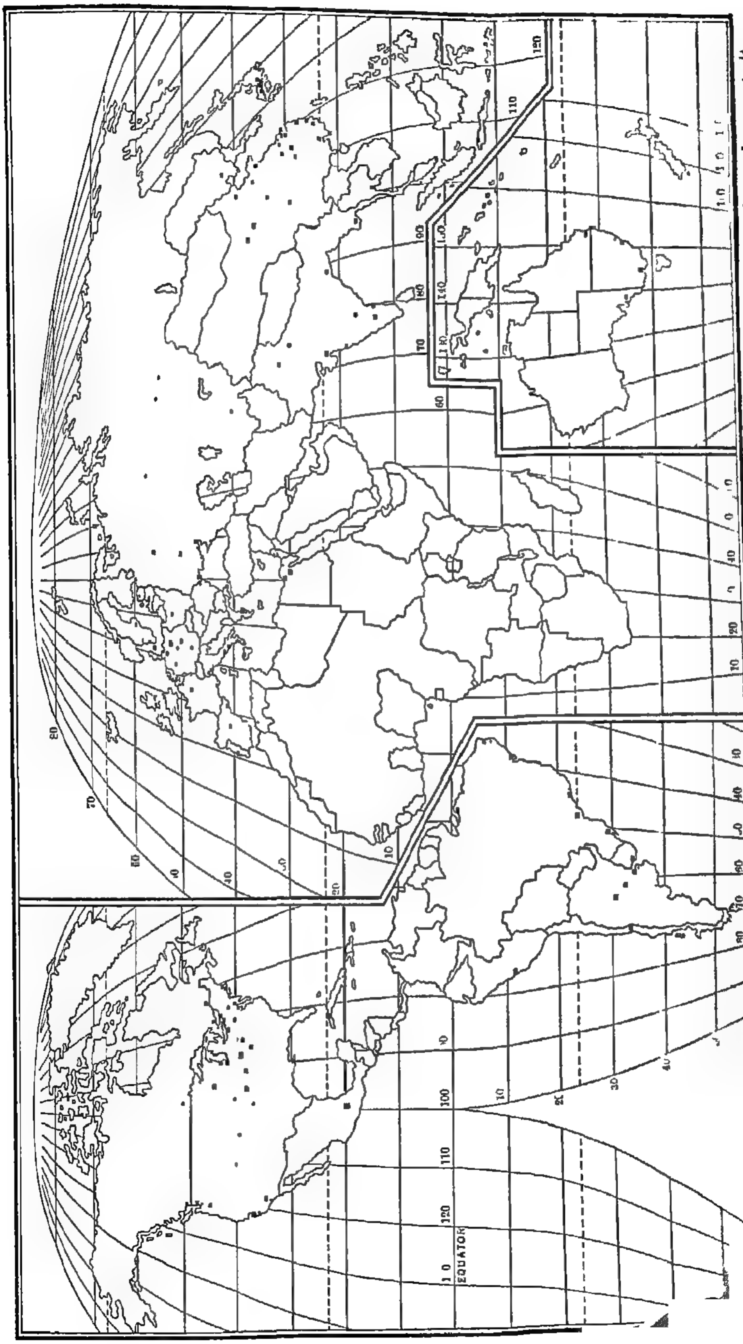
972 Business concerns as well as people have a great tendency to come to the primate city of a country Among the 1,211 firms with a pecuniary strength of over a million dollars in New York City more than half (625) are branches of firms whose headquarters are somewhere else A considerable number of the others are firms, such as United States Steel, the business of which was originally located elsewhere They established branch offices in New York, and in due time the New York branch did more business than the head office Therefore the branch became the head New York's dominance in the eastern United States, its primacy in the country as a whole, and its reputation as one of the two or three primate cities of the entire world make business men think that it pays to have a New York office This is one way in which a primate city has an advantage over all others The same thing is illustrated by the fact that, although the tide of migration has been strongly westward, nearly 10,000 people who were born in California have come to New York City to live, and over 5,000 to Chicago The pull of the primate city for groups of people with special ability is shown by the migrations of chemists, engineers, and other scientists According to the list in *American Men of Science,* New York and New Jersey have about 60 per cent more scientists than would be expected on the basis of their population and of the number of people who have moved into or out of their territory A similar situation prevails in respect to the business men who are listed as directors of corporations. Coming from all parts of the country they tend to concentrate in or around the large cities, especially New York, but also Chicago, Detroit, and others

973 *Optima of Cities.* Cities, as well as plants, animals, and human activities, have distinct geographical optima The conditions which create the optimum are of two kinds first those which make the city's inner and outer hinterlands productive and wealthy, and second those which give some particular site a local advantage. Houston, for example, has become great because its hinterland is a notable producer of cotton, oil, and other products The exact spot where Houston is located was determined by a navigable waterway which has now been superseded by a deep canal It is not necessary here to enlarge further upon the conditions which create a prosperous hinterland, for much of this book has been devoted to them. Any

condition of climate, soil, relief, mineral resources, waterways, government, education, technical invention, or social progress which influences the prosperity of a region also influences the growth of cities. Nor is it necessary to discuss the geographical features which determine the local sites that are best for cities St Louis and Omaha grew up near the junction of great rivers, Chicago is located at the best local site for tapping the great flow of traffic around the end of Lake Michigan, Buffalo, Toledo, and Duluth have also been helped by being located at the end of lakes, New York's particular location is due to the Hudson-Mohawk depression, London's to the lowest ford on the Thames River, and that of Paris to islands in the Seine In all these examples a great city would presumably have grown up somewhere within the present city's hinterland, but the location would have been different if the relief and waterways had been different Thus New Haven might have been the primate city of America if the Hudson River had flowed southeastward from Albany instead of southward; Paris would presumably have been displaced by some other town if its islands had been located farther up or down the Seine

974 GEOGRAPHICAL DISTRIBUTION OF GREAT CITIES A974 shows that cities are especially numerous in (*a*) the northeastern United States, (*b*) western Europe, and (*c*) Japan They are fairly numerous in (*d*) the southern and western United States, (*e*) the east and south of Europe, (*f*) China, (*g*) India, especially the Ganges Valley, (*h*) the temperate part of South America from southern Brazil to Chile, and (*i*) the Regions of Wet Tropical Agriculture in the West and East Indies and on the coast of Brazil Another outstanding feature of A974 is the degree to which the large cities of tropical South America, southeastern Asia, and practically the whole of Africa and Australia are located close to the seacoast The main reasons for this distribution are (1) the effect of climate upon human efficiency, (2) the effect of relief, soil, and climate upon agricultural productivity, (3) the distribution of coastlines, (4) the location of mineral or other natural resources, (5) the degree to which the population departs from the optimum density, and (6) the stage of civilization.

975 In regions where the population is dense great cities are usually located close together, but the percentage of the population living in such cities varies greatly The parts of the earth with percentages above 35 contain only 6 per cent of the world's population, but are inhabited by 29 per cent of the people who live in cities or metropolitan districts of more than 100,000 The highest percentage (73) is in the Middle Atlantic states, which include New York, as well as Philadelphia, Pittsburgh, Buffalo, and other large



Goode's Semi-homolosine Projection, used by courtesy of the Chicago University Press.

A974—The World's Great Cities.

■ Over 300,000 population.

● 100,000 to 300,000

cities.* Next comes New England (63), then the Pacific states (62), the three southeastern states of Australia (55, 54, 53), the East North Central states of the United States (49), England and Wales (47), West Australia (47), Scotland (37), and British Columbia (36). All these regions are inhabited by English-speaking people, and are especially prosperous. With the exception of England and Scotland they all belong to the newly inhabited parts of the world. This suggests that *an abundance of resources in comparison with the number of people is one of the chief factors in causing the percentage of city-dwellers to be high.*

976. On the other hand, density of population and the amount of manufacturing make relatively little difference in the proportion of city-dwellers. Our Pacific states and Australia, for example, are sparsely populated, and are more conspicuous for primary production than for manufacturing. Nevertheless, their percentages of city-dwellers rival those of New England and Old England where the population is dense and the degree of industrialization almost at a maximum. At the other extreme China, India, and Java have an enormous and extremely dense population, and therefore a considerable number of great cities. Nevertheless, the *percentage* of their population living in such cities is surprisingly low, only about 5 in China and 3 in the other two countries.

977. A comparison of the city map (A974) with the map of climatic efficiency (A537) shows that cities tend to be concentrated in regions with stimulating climates. The approximate proportion of the inhabitants living in large cities is above 30 in very stimulating climates, 10 in those that are moderately stimulating, 6 in medium climates, 4 in those that are enervating, and 2 in the very enervating. The rapidity with which these percentages decline is largely due to differences in productivity. It will be remembered that although the regions of very stimulating climate comprise only about 10 per cent of the earth's land surface, and contain less than one fifth of the people, they carry on most of the manufacturing and commerce and own most of the wealth. If we take account of the fact that such tropical cities as Bombay, Batavia, Singapore, Colombo, Havanna, and Bahia owe much of their growth to people from cooler lands, it ap-

* The percentages for the United States are relatively a little higher than for some other countries because metropolitan districts have been used in this country, thus including considerable suburban areas. It must be remembered, however, that in most parts of the world suburban areas are much more limited than in the United States, and the areas included in the cities contain practically the whole population of the urban type.

pears that great cities are largely, although not wholly, a product of cool, energizing climates

978 *Relation of Cities to Seacoasts* Oceans and great lakes are another important factor in determining where cities shall be located A strip 5 miles wide along the coast of all the oceans, seas, and larger lakes, and of the rivers as far as they are navigable for ocean vessels, comprises only a small part of the lands, probably not above 1 per cent Nevertheless, among cities of more than 200,000 population almost half (134 out of 298) are seaports or ports on the Great Lakes and the Caspian Sea Another 67 are located on navigable rivers This leaves only 97 cities for the 99 per cent of the lands where a city gets little or no help from waterways The seaports are larger than the other cities, their average population being approximately 810,000 against 650,000 for the river cities, and only 400,000 for the inland cities with no navigable waterways The chief cause of this difference is that the ports are helped both by the activity of their own hinterlands and by that of a great variety of other regions with which they can easily engage in trade because transportation by water is cheap Seacoast cities are especially likely to receive not only products, but also people, ideas, and methods of work from other parts of the world Another factor in their growth is that a large share of the world's best soil is located along seacoasts, especially in delta plains. Seacoasts, too, are generally more healthful than continental interiors, especially in the less-favorable types of climate One reason for this, it will be remembered, is that their climates are relatively humid and their temperatures comparativey free from extremes Then, too, they have an unusually large share of land with gentle relief where both agriculture and transportation are easy.

979 ECONOMIC GEOGRAPHY AND THE SEARCH FOR OPTIMA Throughout this book we have seen that one of the main problems of economic geography is to discover the nature of the geographic optimum for all sorts of plants, animals, and human activities A second problem has been to find out how nearly the geographical conditions in each section of the world approach the various optima And finally we have tried to discover what effect on population, production, trade, and other activities is produced by departures from the optimum Cities may be regarded as places where certain kinds of optima are concentrated In the immediate hinterlands of the world's most dominant cities we find a relatively close approach to the optimum for human health, for agriculture, for transportation, and for industry. Even the less dominant cities, located under less favorable conditions, excel the surrounding regions in accessibility to food and raw materials

and generally in certain other geographical conditions Thus they are places that approach the optimum more nearly than does the rest of their region

980 Cities are in a way the spear point of economic geography They usually increase in population more rapidly than their hinterlands, and they are the places where new phases of economic geography develop most frequently The forests of the Congo and Amazon, the deserts of Sahara and Gobi, the icy wastes of Greenland and Antarctica all remain about the same for decade after decade and century after century The greatest cities, on the other hand, are being remade in almost every generation The economic geography of productive but nonagricultural regions such as the steppes of Asia changes only a little, for nomadic cattle-keeping involves much the same isolated mode of life now as in the past In the marginal agricultural regions such as the Wet and Dry Low Latitudes some progress is seen, but it is mainly in the cities and is rarely great Only in the Realm of High Productivity is the impact of modern progress fully felt There the cities are the places which change most rapidly Improvements in transportation, new types of labor-saving machinery, chemical discoveries, and progress in medicine, education, health, and government are generally used first in the cities, and in the long run have more influence there than anywhere else A modern city with skyscrapers, superhighways, subways, and a bewildering variety of occupations, together with all the marvelous conveniences of modern apartment houses, department stores, hospitals, and recreation centers, is something that the world has never seen before. It is the result of human progress concentrated in small areas where many geographical conditions combine to give an unusually close approach to the optimum

CHAPTER XL

THE GEOGRAPHY OF TRADE

981 TRADE AND PROSPERITY One of the reasons why people live more comfortably now than in the past is the growth of trade. Much of the world's food, most of its raw materials and fuels, and practically all its manufactured goods change hands at least once, and generally several times, before being finally used. Among advanced people, aside from farmers, it is difficult to find anything of which this fails to be true Such exchanges arise fundamentally from a surplus on one side and a demand on the other One person, family, company, or country has more or can produce more of some commodity than it needs, while others have not enough, but can produce a surplus of some other commodity which they are glad to exchange for something else A simple example would be a farmer who raises a surplus of potatoes because a village mechanic has promised to make him a beehive if he will bring enough potatoes In ordinary business, money takes the place of this kind of barter, and services as well as goods are bought For example, a carpenter makes tables and sells them to a store He uses the money to buy vegetables from another store Those vegetables were raised by a market gardener who wants a table The gardener, on getting paid for his vegetables, goes to the first store and buys a table Although the process is roundabout and involves several people, tables are really exchanged for vegetables just as truly as the potatoes were exchanged for a beehive If our economic system were better organized such exchanges would be much more frequent than now, and everyone would be correspondingly better off In such a world all commodities would be produced where geographic and economic conditions combine to make the final cost to the consumer as low as possible

982 CONDITIONS FOR MAXIMUM TRADE I *Production of Appropriate Commodities* Let us examine some of the geographic conditions which tend to make trade abundant and profitable. One of the first is the choice of the right commodities for production in each region This is far from being easy In Chapter XV we saw that, although corn yields the most per acre in the northeastern United States, it does not pay to raise much there In the same way it would

not be profitable to establish great brass works in Nebraska, that state is too far from the regions where factories for watches, clocks, and electric-light fixtures are made. The most profitable thing is for Nebraska to raise all the corn and beef for which it can find a market, and to let factories in the manufacturing belt make all the brass goods for which there is a market. Within a single country, such as the United States, where trade moves freely, commodities naturally tend to adjust themselves so that each section does the work for which it is best fitted. Even here, however, many commodities are produced in the wrong places because people fail to use the lessons of economic geography. In the long run this raises the cost of production, diminishes trade, and makes the country poorer. When each country tries to be self-sufficient, the failure to produce commodities in the most appropriate places becomes a serious factor in raising costs, reducing trade, and lowering the general level of prosperity.

983. II. *Reduction of Transportation.* A second essential condition for the kind of trade that brings the greatest prosperity is the reduction of transportation to a minimum. So far as possible, any commodity not produced in a given region should be procured from the source which necessitates the least cost for transportation. A good example is the coal of the Netherlands. Most of this is produced close to the German border where there is a ready market for it in the great German metal industry. Accordingly the Dutch normally sell their coal to Germany, and supply the needs of their own great cities on the coast by bringing English coal cheaply across the North Sea.

984. III. *Freedom from Political Restrictions.* Another vital condition of prosperous trade is freedom from political restrictions. To-day the cost of living in practically all parts of the world is considerably increased because countries try to make themselves self-sufficient, or to direct trade to their own ports, their own colonies, or special countries with which they have political ties. Sugar and artificial rubber in Germany, wheat in Italy, and metal manufactures in central Siberia are examples of commodities produced in certain regions at the behest of governments. They make life much more expensive than it would be if all commodities were produced under the most favorable circumstances and sold in a free market.

985. IV. *Optimum Density of Population.* In order that trade may flourish the right density of population is also important. In *Chinese Farm Economy,* J. L. Buck shows that the year's work of the average Chinese peasant on farms of different sizes produces crops having the following relative values: on farms with a crop area averaging 2.5

acres, 100 (40 per acre), crop area 5.0 acres, 130 (26 per acre), and 15.0 acres, 295 (nearly 20 per acre) In other words, farmers are so numerous that a large part of their work is wasted By putting a great deal of work into a small piece of land, the Chinese are indeed able to raise more than one crop each season on a good deal of land Thus they get a larger return per acre on the small farms, namely, a ratio of 40, 26, and about 20 on the three sizes of farms mentioned above The people with the least land, however, get the smallest return per hour of work because the total yield of crops is much smaller than it would be with more land For example, a man with 2 acres on which to support his family slaves away to get a maximum yield per acre, but the utmost that he can sell as a surplus may be worth only $10 a year If the same man had 10 acres, he could cultivate the whole of it From each acre he would not get so much as he now gets from each of his 2 acres, but from the whole he could get enough to let his family live much better than now and still leave a relatively large surplus The balance sheet below shows what a good thing 15-acre farms instead of 2-acre farms would be for the Chinese and for business in general With a total production of $300 worth of crops instead of $80 (second line) the 15-acre family would not have to sell all of its best products in order to get a little ready money It could afford to use $100 worth of food from its own farm instead of $70 (third line) This would still leave the 15-acre family with a surplus of $200 in contrast to only $10 for the 2-acre family On the other hand, if the 15-acre farm were divided into 2-acre farms (last line), the surplus would amount to only $75 It is quite evident then that 1 family with a surplus of $200 would create almost 3 times as much business for factories, railroads, and stores as 7½ families with 2-acre farms living on the same 15 acres

	2-acre farm	15-acre farm
Value of crops per acre	$40	$20
Total value of crops per farm	80	300
Value of farm products used by family	70	100
Annual surplus per farm to be used in trade	10	200
Annual surplus from 15 acres.	75 (for 7½ families)	200 (for 1 family)

986 This balance sheet illustrates what happens not only in China, but also in India, Japan, Russia, and large parts of Europe The population is so dense that the average farm family has only small buying power Hence the number of people who can be supported in other occupations such as manufacturing is less than it would be if the farm population were just dense enough to produce the greatest amount per family. The same principle applies to miners,

lumbermen, and other primary producers. A comparison between Java and Siam illustrates the matter. Siam, with 14 million people produces about 4½ million tons of rice, Java, with close to 45 million, produces about 5½ million, and this would not rise to 6 million even if all the land in plantations of every sort were devoted to rice. In other words, the average man raises 2½ times as much rice in Siam as in Java. This is partly because of higher yields per acre in Siam, but mainly because the Javanese population is so dense that the average farmer there has only half as much land as in Siam. Accordingly Java has no surplus rice for export. The farmers eat practically all that they raise, and about 700,000 tons have to be imported each year for the townspeople. Siam, on the other hand, not only feeds its own townspeople, who are more numerous proportionally than those of Java, but in addition exports about 1,400,000 tons of rice per year. This same principle applies everywhere, the largest surplus and hence the most active trade and commerce arise where the population is neither too sparse nor too dense.

987. V. *Human Activity and the Volume of Trade.* The activity of trade depends upon the mental ability, energy, and standards of living of the people more than upon anything else. In spite of certain disadvantages New Guinea might raise an enormous surplus of rice, sugar, corn, sago, cocoa, tea, coffee, tobacco, and bananas, but its backward people produce little except what they immediately consume. In Java, however, a higher cultural level and the help and supervision of nearly 150,000 Europeans, chiefly Dutch, enable an island a sixth as large as New Guinea to export plantation products worth over half a billion dollars each year, even though food must be imported for the workers. Of course the good soil of Java helps greatly, but the human factors are also highly important. Again, in China the unmined coal is estimated at about 250 billion tons, of which about 100 billion are anthracite, against only 166 billion in Great Britain. Nevertheless, until Europeans took the lead, the Chinese mined practically no coal even for local consumption. Vast quantities lay in the rocks, but there was no surplus for commerce. Even now, although the Chinese are 10 times as numerous as the British, they mine only about 15 per cent as much coal, or less than 1 per cent as much per capita.

988. On the other hand, Norway has very little mineral wealth, and her cool climate greatly limits the crops. Moreover, deep soil is found chiefly in a small lowland near Oslo and in a few valleys and on a narrow coastal plain elsewhere, only about 1 acre out of 30 can be cultivated. Yet so energetic and capable are the Norwegians that they produce an abundant surplus, mainly based on fish, lumber, and

waterpower. This not only supports an active trade at home, but makes Norwegian exports ten times as valuable as those of Java in proportion to the inhabitants. Again, Alaska never yielded any surplus worth mentioning while it was in the hands of Eskimos, Indians, and even Russians, but since its purchase by the United States it has yielded a salable surplus worth more than two billion dollars. So, too, although New England has no coal, few raw materials, and not nearly enough waterpower, the capacity of its people causes it to produce an enormous surplus of manufactures.

989. CONDITIONS THAT CREATE A DEMAND. In order that trade may be active a demand for goods is just as necessary as a supply. A good demand is peculiarly characteristic of progressive countries. England, Germany, France, and the United States want everything; Belgian Congo, Tanganyika, and Amazonia want very little. If an energetic region lacks certain resources it goes out to get them. The United States demands a vast quantity of bananas, and gets them from Caribbean America. West Virginia being too rugged for extensive wheat cultivation, and North Carolina being too warm, demand wheat from the western plains. Germany, having little copper, demands it from Arizona, Montana, Katanga, and Chile. Denmark and Ceylon both lack coal and iron, but Denmark is so active that it demands and gets perhaps a hundred times as much as the larger population of Ceylon. The factors that count chiefly in creating a demand, just as in creating a supply, are the mental ability, physical energy, and cultural condition of a country.

990. CONTRASTS THAT PROMOTE COMMERCE. It is often said that commerce is most active between regions with contrasting types of products and occupations. This is only half true. It would be wholly true if a similar stage of development, and equal intelligence, industry, skill, alertness, and interest prevailed everywhere. Thus the fact that Chicago, Detroit, and Cleveland are centers of manufacturing, whereas the districts to the west and south are agricultural, gives rise to active trade. The similar but greater contrast between New Orleans and Honduras, however, gives rise to only a small commerce, because the Hondurans have little energy and initiative, and a correspondingly small surplus for use in trade. The people of New Orleans can develop trade with them only by going there themselves and stimulating production. A large number of the contrasts which might lead to the most profitable trade are of this kind. Hence the actual exchange of goods is far smaller than between places like St. Louis and Chicago which prepare products that differ relatively little.

991. Bearing in mind, then, that the activity of trade depends even

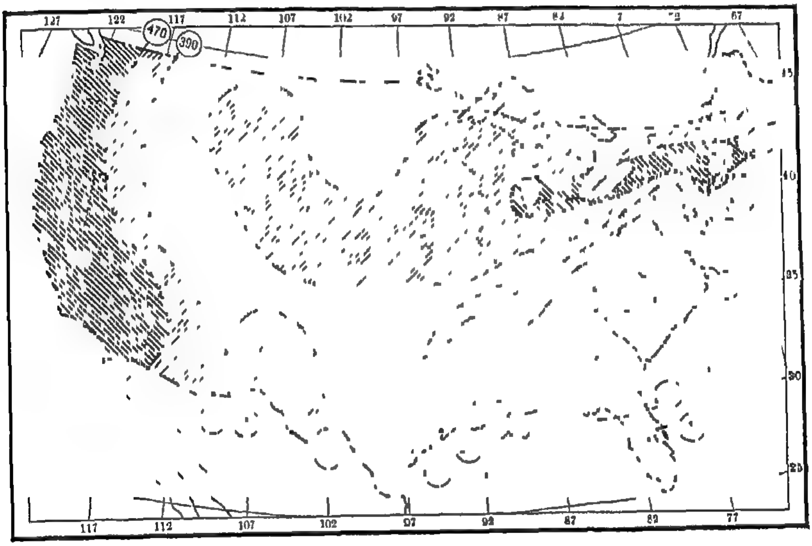
more upon alertness and skill of the people than upon differences in natural resources and types of production, let us see what types of contrasts tend to promote trade where the people in each type of region are equally progressive Rugged regions and plains provide such a contrast The Black Forest and the Rhine Valley in Germany, or the Appalachians and the Prairie states, illustrate this In each region the rugged area supplies minerals, lumber, animal products, and manufactured goods, while the plains furnish food *Seacoasts* and *interiors* often carry on an active commerce. This is not merely because the seacoasts supply such products as fish and salt, which may be lacking in the interior, but also because they have ports through which come the products of other countries The products brought by sea may need to be manufactured Hence the seaports not only exchange the goods of the interior for those of other countries, but also prepare and ship the raw materials that come from abroad The same conditions apply to cities in comparison with *rural areas* Manufactured goods go out from cities in return for food and raw materials from farms, forests, and mines Other contrasts that favor trade are *mineralized* versus *unmineralized* areas, *forests* versus *grasslands*, and *humid* versus *arid* regions.

992 Such contrasts are especially great between regions with a cool stimulating climate and those where the climate is warm and humid More specifically the geographical regions which most fully supplement one another are the Cyclonic Regions on the one hand and the Regions of Wet Tropical Agriculture on the other The eastern United States and Cuba, California and Hawaii, England and Ceylon, the Netherlands and Java, and Belgium and Belgian Congo are pairs of diverse countries in which the conditions for active trade are especially favorable This is notably true of the United States and Cuba In spite of differences in racial character, government, language, and customs, their proximity to each other and the fact that they have almost the right degree of difference of climate, plus the activity of the United States, cause the per capita trade of Cuba with the United States to be exceeded only by that which the Dutch carry on with the Germans—a trade which owes much to the fact that the Dutch hold the mouth of the Rhine.

993. The trade between Germany and the Netherlands illustrates the principle that, even though the resources are similar, *trade is especially active wherever progressive countries can easily reach one another* The largest trade of the United States is with Canada and England—two countries whose products differ only a little from ours Such countries often exchange one kind of cloth, machinery, or hardware

for another which is only a little different Such active, wealthy people can afford to buy goods which differ from their own only in some special qualities which seem desirable, although not essential Thus *although the intensity of the trade between any two regions can be partly explained in terms of diversity of products, the activity of the people is at least equally important*

994 TRADE IN THE UNITED STATES The most active of all forms of trade is that which goes on in the retail stores that we patronize day by day. The total retail sales in the United States range from as low as about 25 billion dollars in the worst years of depression, such as 1933, up to almost 50 billion in years like 1929 Of course such sales



A994—Retail Sales per Person in the United States, 1929

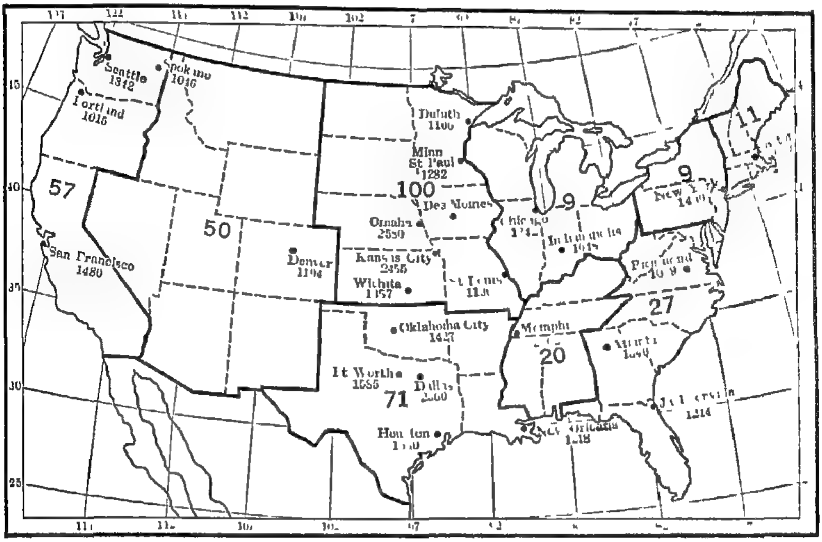
reach their greatest volume where people are most numerous, and a map of their total amount looks much like a map of population When we reduce the sales to a per capita basis, however, quite a different picture is presented A994 shows the sales per person in the different parts of the United States in 1929 when business was booming The similarity of this map to many that we have previously examined is at once apparent California and New York, with sales of $565 and $562, stand at the top Oregon (479) and Washington (487), on one coast, Massachusetts (483) and Connecticut (478) on the other, and Illinois (481) and Michigan (469) near the southern part of the Great Lakes, stand next. To put it another way, there are two strips where people

are so prosperous that in a boom year they spend more than $470 on retail purchases for every man, woman, and child One strip is along the Pacific Coast and the other from southern New England through New York, northern Ohio and Indiana, and southern Michigan to Chicago and Milwaukee These strips are bordered by regions where trade is active (above $390 per capita) although not equal to that in the most heavily shaded parts of A994 One such region embraces most of New England and the Middle Atlantic states, together with the Corn Belt and the mining states of Colorado, Wyoming, and Montana Another skirts the Pacific strip of high activity and includes Arizona The other Rocky Mountain states—Idaho, Utah, and especially New Mexico—have lower buying power A similar medium buying power is found in both the northern and southern parts of the drier section of the Great Plains, as is evident in the Dakotas, Oklahoma, and Texas Finally in the southeast the buying power falls to less than half its level in the two most-favored strips Florida, however, as so often happens, is an exception because of its winter tourists and migrants from the north Its retail trade per capita is twice as active as that of South Carolina, and nearly two thirds as great as that of California and New York

995. The map of retail purchases (A994) is as good a measure of prosperity as one can well find It shows that trade is active wherever other conditions make people prosperous It would be changed a little if we made allowance for the fact that people on farms raise part of their own food, and hence purchase less than those in towns. In a prosperous country like the United States, however, this does not usually amount to much, for even if people raise wheat and cattle, they usually buy flour, or bread, and meat from the store Nevertheless, in the South many people raise their own corn and pork to such an extent that the southern figures in A994 would be raised appreciably if these had to be bought This applies to Florida much less than to its neighbors Changes based on such conditions would not alter the general appearance of the map

996. *Distribution of Wholesale Trade.* In its big features wholesale trade shows practically the same distribution as retail trade In the United States it is most active in the northeastern manufacturing strip and on the West Coast In detail, however, the distribution is different, for wholesale trade is far more concentrated in cities than retail trade The island of Manhattan, which forms the old city of New York, has almost 14 times as much wholesale trade as one would expect if trade were distributed in the same way as population. Omaha and Kansas City have 6 times as much as one would expect Boston rises

almost as high, while San Francisco and Dallas have 5 times the trade that would be expected on the basis of population. On the other hand, the city of Washington has only 80 per cent as much wholesale trade as its population would indicate, while Jersey City and Oakland fall to 70 per cent, and Rochester to only about 50. The 12 cities with the greatest wholesale trade per capita have practically half the wholesale trade of the country, but only one eighth of the population. All the cities of more than 100,000 population contain only 30 per cent



A997—Wholesale Centers of the United States

This map shows the wholesale trade per capita in the 25 cities having the largest wholesale trade per capita in 1929. The figure for Des Moines, $980, is omitted because Des Moines falls below $1000. The large figures indicate the percentage of cities (metropolitan districts) with more than 100,000 population which are named on the map in 9 census districts. For example, all the cities in the northwest Central states appear on the map, but only 1 out of 11 (9%) of those in the Middle Atlantic states appears.

of the population but do more than 75 per cent of the wholesale business.

997. A997 shows the location of the 24 metropolitan districts which did more than $1,000 worth of wholesale business per capita in 1929. In other words, these are the cities which are conspicuous because they are highly commercial rather than primarily industrial. The figures under each name show how many dollars worth of goods were sold at wholesale for each man, woman, and child in the city during the boom year of 1929. The highest figures (above $1,500)

belong to Omaha, Kansas City, Memphis, Dallas, Fort Worth, and Houston All these are located in agricultural sections beyond the limits of the main manufacturing areas Hence their population does not contain so large a proportion of industrial workers as is usual in eastern cities Moreover, in general these highly commercial cities are also located at considerable distances from other large cities Thus each city has a large hinterland, or area where the local stores buy from the wholesale stores of the central city On the other hand, in the great industrial strip from New Jersey and New England westward as far as the Mississippi River the only cities that appear on the map are Boston, New York, Chicago, and Indianapolis This means that large cities here are so numerous and are so much devoted to manufacturing that only 1 in 10 is conspicuous as a commercial center West of the Mississippi, however, in the West North Central states, 100 per cent of the large cities are commercial rather than industrial

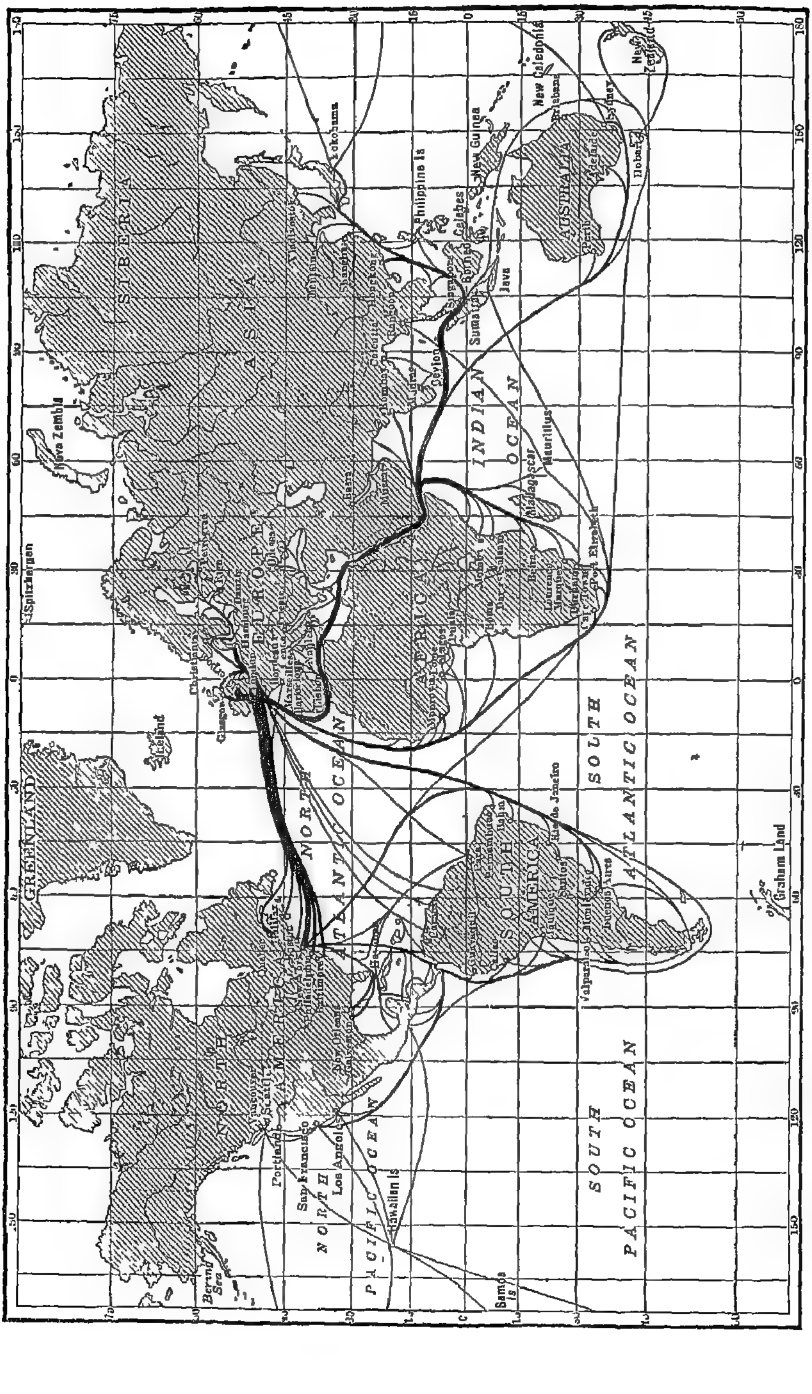
998 Outside of the United States data as to wholesale and retail sales are scarce, but the general principles are the same. Retail trade is active in regions such as those around the North Sea where general progress is greatest, but it rises highest in progressive new countries like Canada and Australia Elsewhere the amount of such trade falls to levels which seem surprisingly low to Americans Even in countries as advanced as Yugoslavia, the city people cannot afford one tenth of the luxuries that we enjoy, while a large share of the country people raise most of their own food, gather their own fuel, raise their own horses instead of using automobiles, and even make a large share of their own clothes out of homegrown wool and linen In poorer countries, like Brazil and French Indo-China, trade falls to a still lower level A good sized village may have only a single little store no larger than an ordinary livingroom Its whole stock is often worth only $100 or $200, and is largely limited to such goods as spices, seeds, coarse cotton cloth, and a small assortment of the cheapest kinds of knives, rope, crockery, candy, and a few other articles. Few things show the contrast between the various parts of the world more clearly than the amount and kind of goods bought in stores by the average person.

CHAPTER XLI

THE PATHS OF COMMERCE

999. THE WORLD'S TRADE Although the internal trade within a country is by far the main item in its business, foreign trade plays an especially large part in politics and in ordinary discussions of economic geography In general the value of the goods bought by the United States from foreign countries amounts to about 5 per cent of the value of the retail sales in a poor year, and toward 10 per cent in a good year In such small commercial countries as the Netherlands, or even in larger but highly commercial and industrial countries like Great Britain, the percentages rise considerably higher. On the other hand, in the Soviet Union, China, and India, for example, foreign trade falls to a very low percentage of all trade. Such trade is reduced both by poverty and by the size of a country If all countries were under one government there would be no foreign trade at all, although the total of world trade would be much increased because barriers due to tariffs, quotas, and subsidies would disappear On the other hand, if each state of the United States were a separate country, the amount of American foreign trade would be vastly increased Nevertheless, the actual amount of trade, and the benefit to be obtained from it, would be much diminished by the annoyances, delays, and expenses due to crossing international frontiers.

1000 *The Main Trade Routes* Except between the United States and Canada, and in western and central Europe, where many independent countries are crowded into a small space, most of the world's international trade goes by sea. Transportation by water is so much cheaper than by land that copper from Utah, for example, generally reaches New York via Pacific ports and the Panama Canal rather than by rail International trade, however, goes by water for another reason, the ocean generally furnishes the most direct and easy routes between the great centers of activity, and between these centers and the countries whence food and raw materials are derived The location of the main trade routes and their relative importance are indicated roughly in A1000. Limits of space, however, cause the most important routes, especially the ones across the North Atlantic and through the



A1000—The World's Main Routes of Commerce

Mediterranean Sea, to be shown by lines which are not thick enough to indicate their true importance The eleven greatest trade routes are shown in Table 40 Routes by land corresponding to many of these are impossible, while many of those which are possible are slower, more difficult, and more expensive than the routes by sea.

TABLE 40

MAIN OCEANIC TRADE ROUTES

Group I *Western Europe* with:

1 The eastern United States
2 Eastern Asia via the Mediterranean Sea, Suez, India, and the East Indies, with branches to eastern Africa and Australia
3 Southeastern South America
4 The West Indies, Panama, and the west coast of South America.
5 South Africa via the west coast

Group II *The Northeastern United States* with

6 The western United States via the West Indies and Panama
7 The Mediterranean region
8 Southeastern South America
9 Panama and western South America.

Group III *The Western United States* with:

10 Eastern Asia
11 Australia and New Zealand

1001 One of the notable features of the main trade routes is that they run mainly east and west, or else between north temperate and south temperate regions The countries within 30° of the equator contain more than 40 per cent of the world's population Their products are decidedly different from those of cooler countries Hence, on the basis of products alone, one would expect that much more than 40 per cent of the trade of the United States or Great Britain would be with the warmer countries As a matter of fact, the United States makes about 40 per cent of its foreign purchases in countries within 30° of the equator, but scarcely 20 per cent of its sales are made there Great Britain makes about 25 per cent of its purchases there and not quite so large a percentage of its sales *The greatest trade of all is between countries that are most alike* Great Britain sends almost 40 per cent of its exports to other English-speaking countries, and buys from them a corresponding part of its imports The United States sends just about that percentage of its exports to the English-speaking countries, and buys from them between a fifth and a quarter of its imports The paths of commerce run from like to like, more often than from different to different

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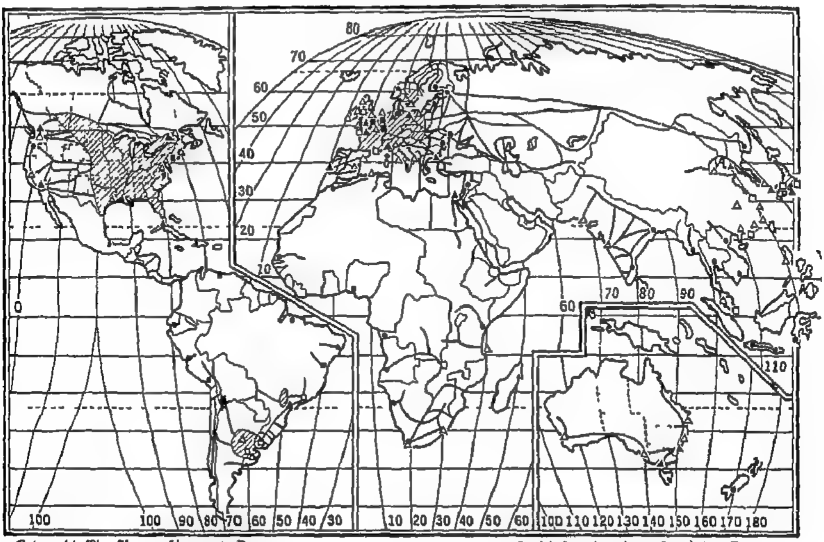
1002 Railroads in World Commerce The part played by railroads in world commerce is very different from that of ships. Long transcontinental railroads, such as the Trans-Siberian line, the lines from Chile to Argentina, and those between West and South Australia, are of minor importance in international or long-distance commerce. Their chief value lies in local commerce, in the creation of political unity, and in providing rapid passenger service. Since the construction of the Panama Canal this has also been largely true of the North American transcontinental lines. Nevertheless, the transcontinental transportation of valuable perishable products, such as fruit, vegetables, and



Copyright, The Chicago University Press — *Goode's Semi homolosine Equal Area Projection.*

A1002—The World's Chief Railroads and Ports of Commerce

eggs, is still important. The main task of the railroad in world commerce, however, is to bring produce from its place of origin to the export harbors. The shaded sections of A1002 show that in large parts of Europe and North America, in two small sections of South America, and in Japan, the density of railroads is so high that few places are farther than 10 miles from a railroad. In North America the borderline between such an intensive railroad system and the more open or extensive system depends largely on rainfall on the west and temperature on the north. It follows the 20-inch isohyet in the southern United States, but bends westward farther north where lower temperature makes a smaller rainfall sufficient for agriculture. In Canada it swings far enough west and north to include the relatively dense rail-

road system of the wheat region of Alberta and Saskatchewan. In Europe the similar dividing line separates the economically important parts of Norway and Sweden, namely the Oslo region and Svealand, from the northern forests which are too cool for much agriculture. Across the Baltic in the plain of northern Europe railways form a dense net as far east as the boundary of old Germany as it was before the first World War. South of the Carpathians a fairly dense net projects eastward in the Danube Basin far enough to include Hungary. The irregularity of the eastern border of the region with abundant railroads is due largely to political considerations. In general the border lies in the transitional zone where the high and reliable crop yield of western Europe give place to the low and unreliable yields of eastern Europe. It is noteworthy that in railroads, as in many other respects, Japan is unlike the rest of Asia and resembles Europe. In South America, as well as in Eurasia, railroads are dense in the two highly productive areas, namely, eastern Argentina and central Chile, which lie on opposite sides of the continent.

1003. CONDITIONS THAT PRODUCE SEAPORTS. *Hinterlands.* If the hinterland produces a large surplus and therefore offers a large market, important seaports will grow up even if the harbors are poor, as at Los Angeles. New York, as we have seen, is the world's largest seaport primarily because it serves a large hinterland which is almost unparalleled in natural resources and climate, and in which the density of population has recently been close to the optimum. Moreover, its hinterland is far larger than that of Boston, Providence, New London, New Haven, or even Philadelphia and Baltimore because it is connected with the Lake Region and the interior plains by the Hudson and Mohawk Valleys which furnish the easiest of all routes across the Appalachian Highland. Much more important than this, however, are the conditions which make Iowa, for example, raise 1,350 bushels of grain for every man on the farms, whereas the United States as a whole raises 550 and Russia only 70 or 80. Although Iowa forms the immediate hinterland of cities like Omaha and Des Moines, it also forms part of the greater hinterland of Chicago, and this in turn is part of New York's vast hinterland. The seaports of the Pacific Coast of both North and South America cannot carry on so much trade as those of the Atlantic Coast because great mountains hem them in, but they are active because the standards of production in their hinterlands are high. The world's greatest seaports are found where fertile lowlands, with resources of minerals as well as agriculture, are not only accessible to the sea, but enjoy a climate which is stimulating for both man and crops. In A1002 the concentration of seaports in north-

western Europe, the northeastern United States, Japan, and the Plata River region illustrates the matter

1004 The quality of the trade of seaports, as distinguished from quantity, depends partly on the resources of the hinterland and partly on its stage of development. In mature, well-populated regions of the progressive European type the activity of the people causes their surplus to consist mainly of manufactured goods, while the density of the population and its industrial nature cause the demand to be mainly for food and raw materials In less mature but progressive regions such as Australia almost the reverse is true, for manufactured goods are imported, while food and raw materials are exported Between these two extremes there are all sorts of intermediate forms Thus the northeastern seaports of the United States, although mainly exporting manufactures, also ship grain, fruit, and half-finished iron goods Even England, which perhaps comes nearest to being a purely manufacturing country, ships coal in large quantities Although our Northeast is much like England in its kinds of trade, the South exports mainly cotton and lumber, while the West ships oil, fruit, and copper from the southern half, and lumber, grain, and fruit from the north So the quality of the trade of a harbor depends on the kinds of products in the hinterland

1005 *Harbors* Good harbors are a necessity in developing seaports, but they may be artificial, as is true not only of Los Angeles but also of many lesser places The mere presence of good natural harbors by no means causes the growth of seaports Some of the world's best harbors, such as Guantánamo Bay in Cuba and Pago Pago in Samoa, have no seaports worth mentioning simply because they have very limited hinterlands Nevertheless, wherever nature provides good natural harbors on any seacoast which has a favorable climate and is accessible to a good hinterland, great cities normally grow up beside the best harbors Although New York is the world's leading seaport primarily because of its marvelous hinterland, it also owes part of its dominance to its marvelous harbor An ideal harbor has the following qualifications (1) it is well protected against storm waves, (2) it is deep enough for large vessels, and has fairly deep water near the shore, but it is not so deep that ships have difficulty in anchoring, (3) it is wide enough to give space for large ships to turn in, (4) its coastline is so sinuous that there is abundant room for docks and wharves, (5) it is not troubled by ice, fog, currents, or extreme tidal variations, (6) it is bordered by land that is well drained and yet level enough to furnish space for the growth of a city, and (7) it is accessible to the interior by routes which make it easier to bring goods to it than to other harbors

which may perhaps be actually nearer to the place where the goods are produced.

1006 *Harbors on Depressed Coasts* The degree to which these conditions are met depends partly on the topography of the land and the nature of the coast as determined by geological changes in the respective levels of land and sea. Recently depressed coasts almost invariably have more and better harbors than emergent coasts This is evident when the many good harbors from Baltimore northward to Labrador are compared with the relatively poor harbors south of Baltimore A similar comparison holds good in respect to each of the following pairs of regions (1) the depressed Pacific Coast of North America from Puget Sound northward versus the uplifted coast southward from San Francisco to Mexico, (2) the depressed coast of western Europe from France to Norway, and the emergent coast of southern Spain and Italy, (3) the depressed coast of Japan and the emergent coast of eastern India, (4) the depressed coast of the southern part of Chile and the emergent Pacific Coast from Valparaiso northward. Except in the eastern United States and the North Sea region these depressed coasts suffer because the ocean covers a large part of the level land Rio de Janeiro and San Francisco, in areas of local depression, are examples of this Their harbors are superb, but the land rises so steeply from the sea that the cities have to climb steep heights or spread out onto land recovered from the sea. Seattle, Vancouver, Oslo, Yokohama, Fuchow, and Hong Kong display the same characteristics.

1007. Much better harbors are found where plains or lowlands of gentle relief have been moderately depressed Boston, New York, Philadelphia, and Baltimore all lie on coasts of this kind Similar conditions are found in England with its wide-open drowned river outlets at Liverpool, Hull, Glasgow, and Bristol, as well as London. On the continent of Europe similar drowned harbors in regions of gentle relief have helped to develop seaports at Stockholm, Copenhagen, Hamburg, and Bremen. In Australia the drowned harbor of Sydney is one of the most magnificent in the world, rivaled only by that of Rio de Janeiro in Brazil.

1008. *Rivers and Seaports.* Emergent coasts, and also those due to the recent formation of deltas, generally have relatively poor harbors but plenty of space for cities. No matter whether a coast is depressed or emergent, crooked or straight, harbors at the mouths of rivers generally have an advantage. The city symbols in A1002 show the combined effect of hinterlands, harbors, rivers, and general location in causing seaports to be visited by ships. The symbols do not indicate

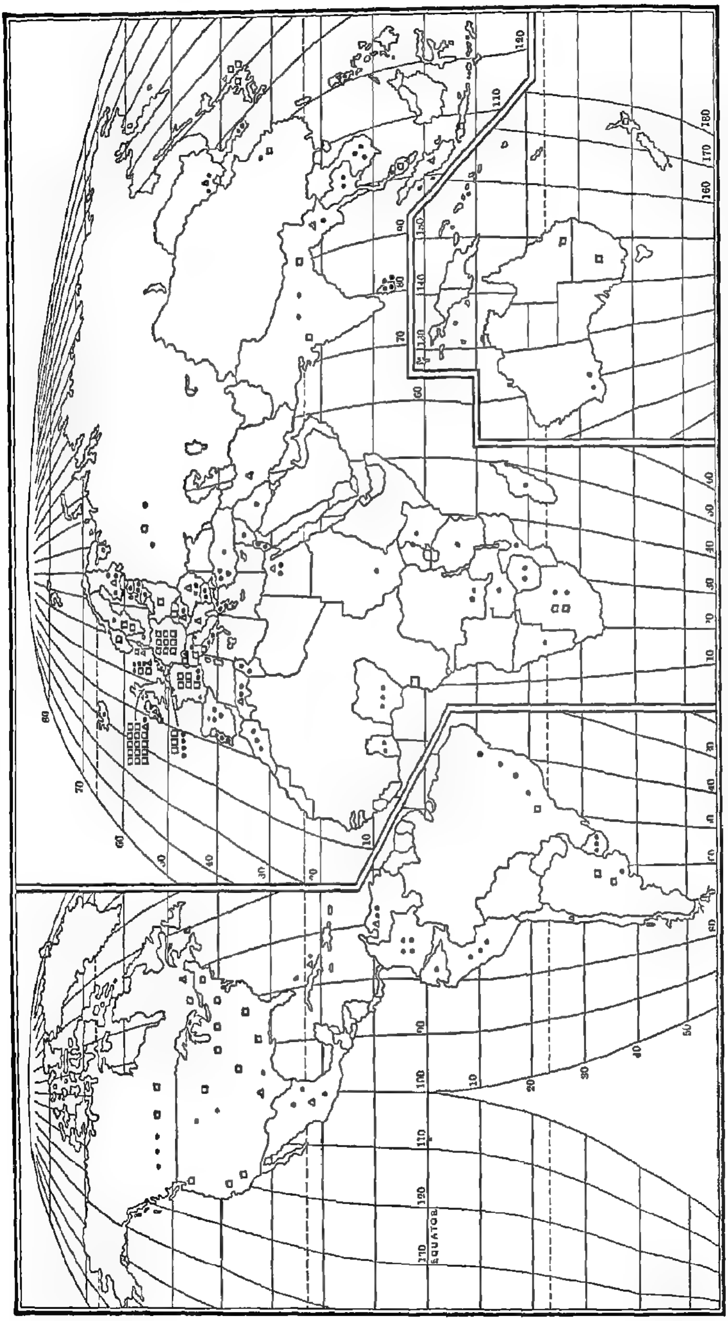
the size of the cities or the volume of their domestic commerce They simply show the tonnage of the ships engaged in foreign commerce which enter the ports each year, regardless of whether the ships leave any goods or not All of the five harbors where ships with a tonnage of more than 20 million tons enter each year (solid squares in A1002) are located near the mouths of rivers, London, Rotterdam, Antwerp, and Hamburg being near the North Sea, and New York at the mouth of the Hudson All five are located in cyclonic regions where rivers and their level valleys provide easy entrance to hinterlands which excel in climate, relief, and resources The rivers are important not so much because they provide harbors as because their valleys furnish good hinterlands near at hand and make it easy for a city to extend its hinterland far inland Among the twelve cities with an entrance tonnage of 10 to 20 million tons (open squares in A1002), five, namely, Shanghai, Hong Kong (the port for the Si River and Canton), Marseille, Liverpool, and Buenos Aires are also at the mouths of rivers This is not true, however, of Tokyo (Yokohama), Kobe, Rio de Janeiro, or Montevideo. The other three belong to a special class of seaports which have the ocean rather than the land as their hinterland, and hence have little need of rivers Colombo and Singapore owe their high tonnage largely to steamers which stop on the way from Europe to eastern Asia and Australia Cherbourg in France is merely a stopping place where transatlantic liners leave passengers and mail bound to and from Paris

1009 Many river ports are not located directly at the mouths of their rivers Montreal, New Orleans, and Portland, Oregon, are located as far inland as ocean ships can navigate London is at the lowest point where it is easy to cross the river, and hence at the place where a route parallel to the coast crosses the river route running inland. Such places often have an advantage because the river can be bridged without much difficulty Again a location away from the mouth of the river may remove the city from the swampy and often unhealthful areas near the outer edges of the deltas, as at New Orleans Sometimes minor branches of the main river furnish the best harbors, as at Shanghai If the river itself is of little importance for navigation, so that the traffic goes inland by rail, the seaport may develop on the edge of the delta where it meets the firmer shore, as at Alexandria on the Nile Delta and Marseille near the mouth of the Rhone Marseille is connected with the Rhone by a canal that runs through a tunnel

1010 The recent construction of wharves, docks, and other port facilities at Albany, and the attempt to draw Albany, Troy, and Schenectady into a single great inland port, show how seaports tend to

migrate inland as far as possible. Even if this port becomes important, New York will still have the advantage of a superb harbor and of being located where the great route along the Atlantic Coast crosses the best route inland In South America, Manaus is the final port on the Amazon for ocean vessels, but Pará on the southern edge of the Amazon delta has the main harbor Buenos Aires is located on a small branch of the Plata estuary, but scarcely has a real river harbor In Africa the river ports aside from Alexandria are small places like Banana at the mouth of the Congo In Europe the three great harbors of Hamburg, Rotterdam, and Antwerp are found at the mouths of the Elbe, Rhine, and Scheldt, each harbor being well inland The Elbe, and especially the Rhine, are excellent rivers for navigation About 100,000 river steamers pass the Dutch border on the Rhine each year. Rotterdam has an artificial outlet to the Rhine, as the old one was not practical for modern shipping Canal connections with northern France and the Rhine help to compensate Antwerp for the minor importance of the river on which it is located Other European seaports at river mouths include Danzig near the mouth of the Vistula, Stettin on the Oder, Bremen on the Weser, Rouen on the Seine with Le Havre as its outer port, Nantes on the Loire, Bordeaux on the Garonne, Braila on the Danube, Odessa near the Dnieper, and Rostov near the Don outlet Asia has numerous examples Karachi is near the Indus Delta, Calcutta on one of the branches of the Ganges, Rangoon on the Irrawaddy Delta, Bangkok on the Salween River, Saigon on a side branch of the Mekong, and Canton on the Pearl River Most impressive of all perhaps is Shanghai on the Whangpoo near the Yangtze outlet, where it is the seaport of a navigable river system of tremendous potential value and has its inner port nearly a thousand miles away at Hankow

1011 DISTRIBUTION OF WORLD TRADE. Northwestern and central Europe hold an outstanding position in international trade This is evident in A1011 which shows exports plus imports as percentages of the total for the world as a whole Compared with Europe, the United States, with 15 per cent of the total, is by no means so prominent as one would expect In fact it ranks below Great Britain Outside the United States and Europe only Canada (4 per cent), Japan (3), India (3), Argentina (2), China (2), Malay States (2), Netherland East Indies (2), Australia (2), and Brazil (1) reach the 1 per cent level, although South Africa, New Zealand, Cuba, Chile, Egypt, and French North Africa come near it The great accumulation of dots in northwestern and central Europe, however, is more or less misleading, because Europe is divided into so many small countries. If Great Britain,

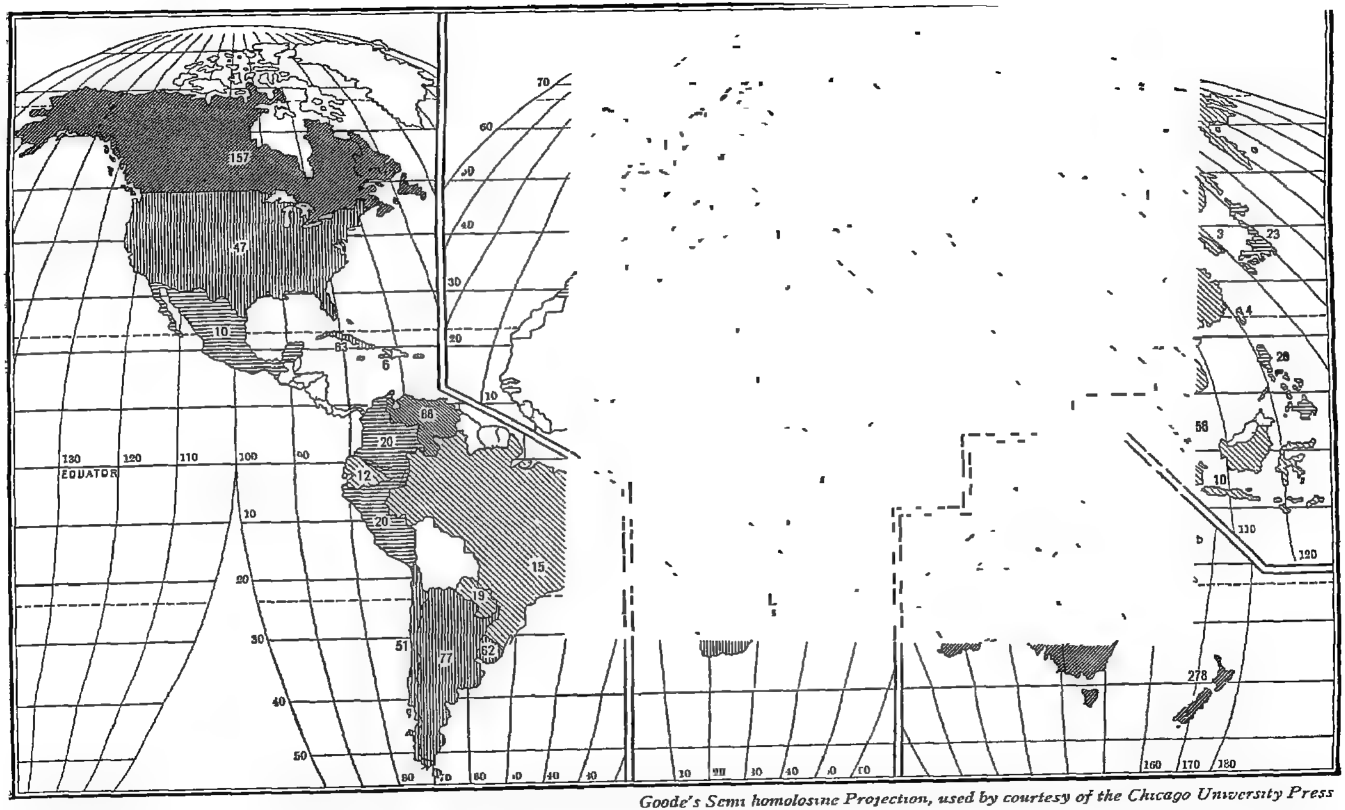


Goode's Semi-homolosine Projection, used by courtesy of the Chicago University Press.

A1011—Foreign Trade of the United States
(Imports plus exports 1936-38)
□ = 1% of total world trade
△ = ½ of 1%
● = 1/10 of 1%.

France, the three Scandinavian countries, the two Low Countries, and Switzerland were all united into one country they would have a population equal to that of the United States, and their foreign trade would be reduced about one fourth. These countries, however, are on the whole much alike in climate and products If the United States, with its great diversity of natural regions, were divided into eight countries the size of those just named, their combined foreign trade would probably be twice as great as that of the United States at present. Most of the wheat, cotton, fruit, iron ore, coal, and other products that now travel a thousand miles or more within the country would pass from one country to another and be counted as foreign trade The misapprehension which thus arises because large countries are compared with small ones in A1011 could be removed only by making a map of all trade, both foreign and domestic, but no statistics are available for this

1012 The amount of foreign trade per capita deserves as much study as the absolute total. A map of this (A1012) presents quite a different aspect from A1011. The density of the population in Europe, Japan, China, and India lowers the rank of those regions, while countries with a low density of population rank much higher Western Europe, as a whole, however, still stands out, but other regions, especially the young countries in temperate climatic regions such as Canada, Australia, New Zealand, South Africa, Chile, Uruguay, and Argentina become prominent So do some of the regions of Wet Tropical Agriculture, like Jamaica, Cuba, Puerto Rico, the Gold Coast, Ceylon, and the Malay Peninsula Even in this map the United States holds a relative position much lower than that of many countries which are commercially far less active Its foreign trade per capita is less than that of British and Dutch Guiana, and one third that of Switzerland and the countries bordering the North Sea. Russia, too, shows a low figure, only 3, which is far less than that of much more backward and inactive regions including Iraq (18), Venezuela (88), and Malaya (158) The low figure for the United States here, just as in total foreign trade, is merely the result of great size and diversity The United States, as we have seen, exports only one tenth or even one twentieth of its production, whereas Belgium and Denmark export about half of all that they produce, Switzerland a third, Great Britain and France a fourth, and Germany a fifth Only in eastern Europe do we find countries which export so small a share of their production as the United States In Russia this is partly because that country is not only large, but diverse, although not so much so as the United States But there, too, as in all the less active countries of eastern Europe and Asia, the low



Goode's Semi homolosine Projection, used by courtesy of the Chicago University Press

A1012—Total Annual Foreign Trade per Capita in American Dollars, 1936-38.

level of productivity per man means that only a small part of the production forms a surplus available for export. Moreover, Russia's location and the scarcity of good harbors handicap her foreign trade

1013 On the other hand, several new and progressive countries have a large foreign trade per capita because they not only devote themselves largely to the production of food and raw materials, but also work so well and have such large resources in land and minerals that they enjoy a huge surplus Canada with its exports of wheat, paper, lumber, gold, nickel, fish, meat, butter, and cheese, Argentina with grain, linseed, meat, hides, and wool, Chile with copper, nitrates, and wool, South Africa with gold, wool, and diamonds, Australia with wool, wheat, dairy products, meat, and gold, New Zealand with 70 per cent of all its exports in the form of animal products, are typical of active countries in which manufacturing is not yet far advanced

1014 Regions of Wet Tropical Agriculture often show this same tendency toward a large foreign trade per capita because they produce staple plantation products and export them in exchange for food and manufactures Their trade tends to be kept down, however, by the low buying power of the population The Netherland East Indies with its diverse plantation products, the Malay Peninsula with its rubber and tin, Ceylon with its tea, the Philippine Islands with sugar and Manila hemp, the Gold Coast with cacao, Nigeria with palm oil, Brazil with coffee, and the Caribbean region with sugar, bananas, tobacco, and oil represent this group.

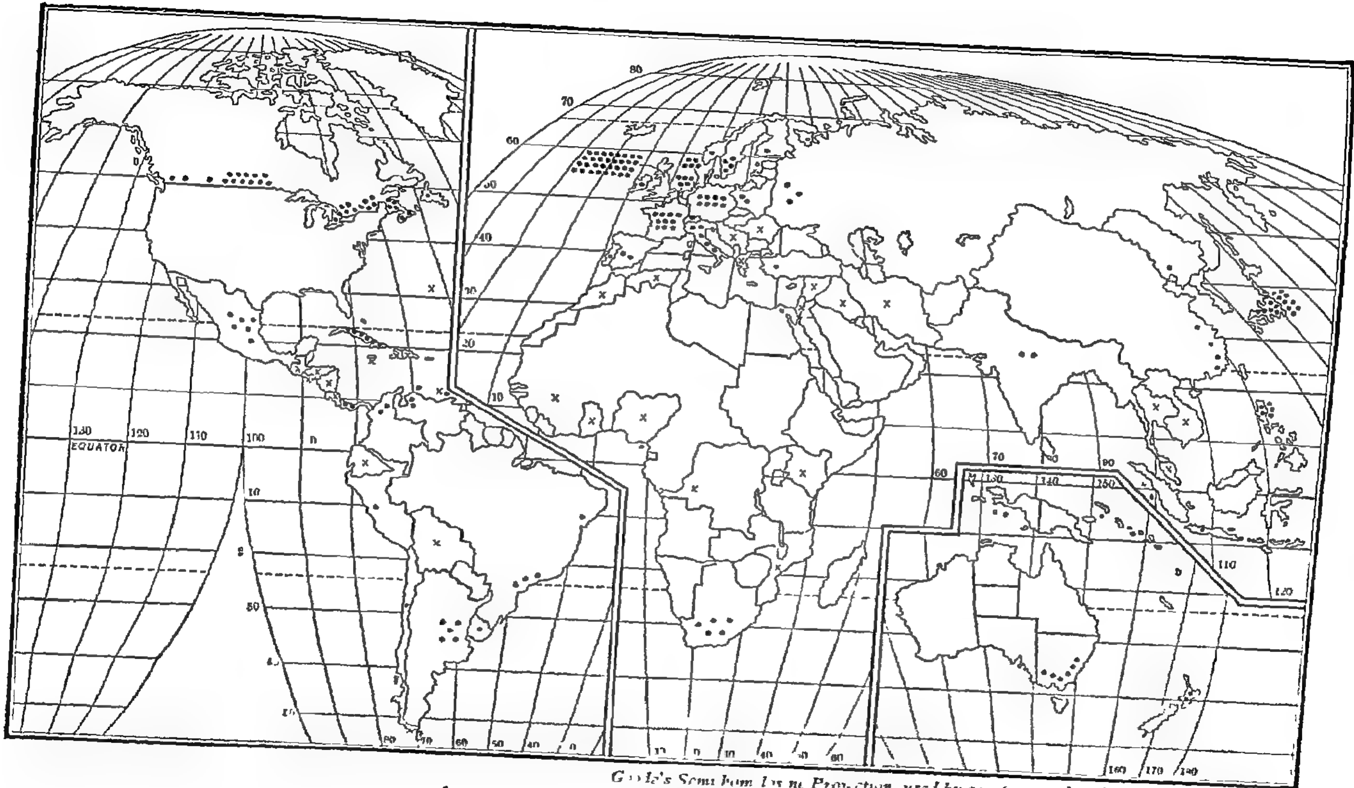
1015. In western Europe a high overseas trade per capita arises in quite a different way The old progressive countries cannot supply their own need for food and raw materials They must import these in great quantities They have, however, a surplus of manufactured goods, as in England Germany, Belgium, and Switzerland, or else of locally specialized food products such as fish in Norway, dairy products in Denmark and Ireland, and both dairy products and garden truck in Holland Japan has recently joined this last group It still exports one raw material (silk) on a large scale, but imports not only food, especially rice and sugar, but also raw materials, chiefly cotton, wool, iron, and mineral oil Its increasing surplus of manufactures, mainly textile, swamps the markets of eastern Asia

CHAPTER XLII

COMMERCE OF THE UNITED STATES

1016. DISTRIBUTION OF AMERICAN TRADE The United States has the double advantage of being a new country with a surplus of food and raw materials, and a mature country with well-developed manufactures. Now, however, it imports about as much food as it exports, and a large part of the material which it formerly exported in the raw form is manufactured at home before being exported. Cotton is still an exception in this respect, nearly half of our crop is exported. A1016 shows where the United States sends its exports The preponderance of Canada and western Europe is impressive Canada, Great Britain, Germany, and France, together with Scandinavia, the Low Countries, and Italy, buy half our exports (Table 41) The primary cause of such active trade is that Canada and western Europe are regions of great productivity, but their accessibility to the United States also plays a part The other conspicuous parts of the export map, those where at least 5 dots (2½ per cent of our total exports) are closely grouped together, comprise two groups One includes Japan, Australia, South Africa, and the Uruguay-Argentine area Like Canada and western Europe, these are regions of relatively cool, invigorating climate, approaching rather closely to the standard set by the average of the world's 12 largest cities All these countries together, especially those of western Europe, illustrate the principle that *international trade reaches its highest activity between countries located in regions of high efficiency, especially if other conditions, such as coal and iron, co-operate in making it easy to develop manufacturing*

1017 The other type of conspicuous regions in A1016 includes Mexico, Cuba, the northern tip of South America with Panama, and the Philippine Islands These are tropical regions, and all have some kind of political connection with the United States, or else are so near that this country is vitally interested in their affairs. Their full significance will appear in a moment when we examine imports, but the export map illustrates the principle that *international trade is greatly fostered by proximity on the one hand and by political connections on the other.*



Goode's Semi-homolosine Projection, used by courtesy of the Chicago University Press

A1016—Exports of the United States, 1936-37

A round dot = ½ of 1%

× = less than ½ of 1% but more than $1/20$ of 1%

TABLE 41

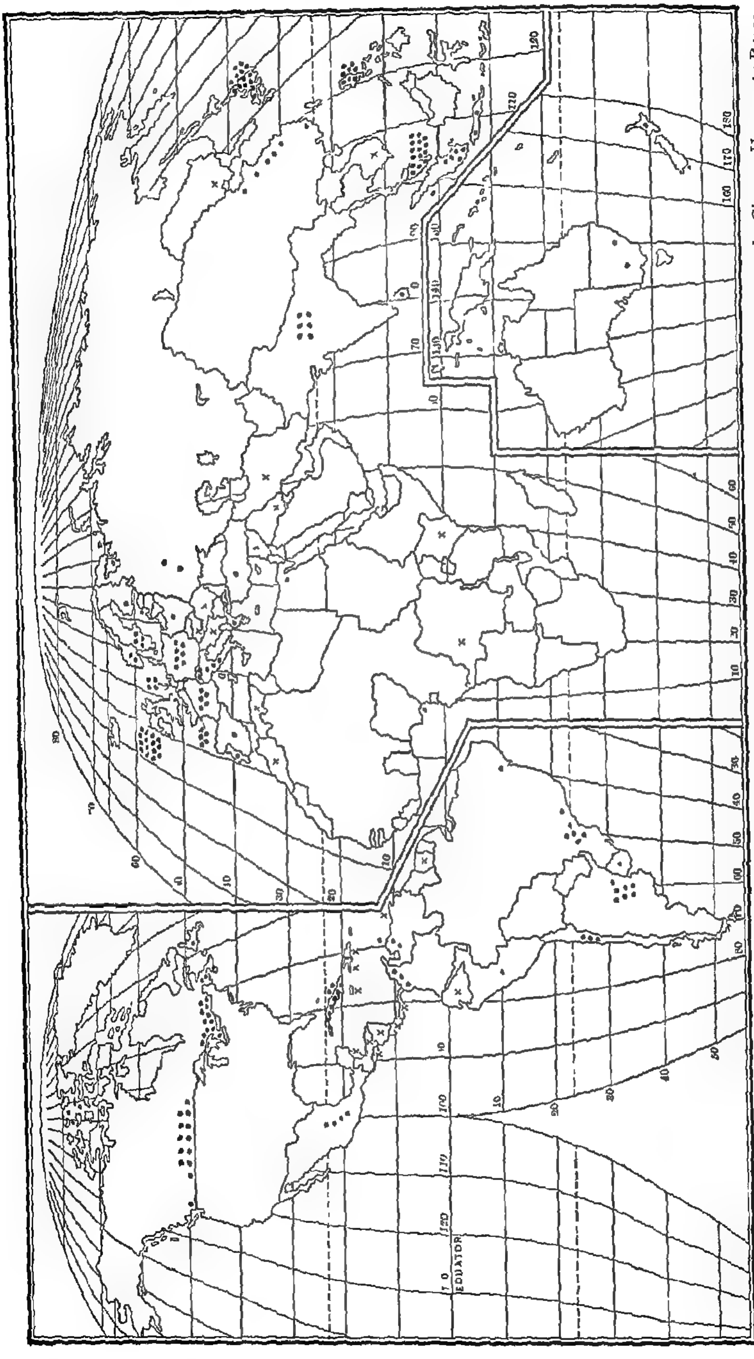
FOREIGN TRADE OF THE UNITED STATES AVERAGES FOR 1936–38

Country	Exports to	Imports from	Chief exports from the United States to designated country	Chief imports into the United States from designated country
1. Canada	454	345	Non-metallic minerals	Paper and wood pulp
2. Great Britain	496	174	Cotton	Woolen cloth
3 Japan	244	168	Metals and metal products	Raw silk
4 Germany (new)	111	129	Cotton	Chemicals and fertilizers
5 France	143	65	Cotton	Wines
6 Cuba	79	127*	Cloth	Sugar
7. China	62	91	Metals and metal products	Furs, tung oil, and hides
8 Philippines	77	107*	Machinery and vehicles	Coconut oil and manila
9 Malaya	8	174*	Machinery and vehicles	Rubber
10, Brazil	60	107*	Machinery and vehicles	Coffee
11 Argentina	79	82	Machinery and vehicles	Corn
12 Belgium	77	59	Machinery and vehicles	Diamonds
13 Mexico	83	53	Machinery and vehicles	Copper, sisal, bananas
14 Venezuela	71	41	Machinery and vehicles	Petroleum
15 Italy	68	43	Cotton	Cheese and olive oil
16 India	35	75*	Machinery and vehicles	Jute
17. Sweden	57	51	Machinery and vehicles	Paper pulp and paper
18 Netherlands East Indies	22	85*	Machinery and vehicles	Rubber
19 Netherlands	78	45	Food products	Diamonds
20 Australia	67	25	Machinery and vehicles	Wool
21. Colombia	39	48	Machinery and vehicles	Coffee
22. South Africa.	76	12	Machinery and vehicles	Gold (corn)
23. U S S R	49	25	Machinery and vehicles	Furs
24. Chile.	21	34	Machinery and vehicles	Copper and nitrate
25 Poland (old boundaries)	24	15	Cotton	Pork

1018 As sources of imports into the United States, we find from A1018 that Canada, western Europe, Japan, and the Uruguay-Argentine section are again conspicuous, just as they are in taking our exports. Nevertheless, western Europe, Australia, and South Africa are less important in our import trade than in our export trade Canada, Japan, and Argentina, on the other hand, are more important. This is because they supply raw materials and food The greatest difference between the exports and imports, however, is that the abundant imports from Cuba, Brazil, the Philippines, India, and especially the Malay Peninsula and the Netherlands East Indies far exceed our exports to those regions. The magnitude of this contrast is evident in B1018 There the left-hand side shows the percentage of our exports going to each of 25 main regions, while the other side shows the percentage of our imports coming from those same regions Countries where our imports are more than about twice as valuable as our exports are starred Without exception all the starred regions are tropical. Sugar, rubber, coffee, cocoa, jute, and tea are the products supplied by most of them, although tin happens to come from the Malay Peninsula Thus we are brought to another principle *International trade is greatly stimulated by the fact that tropical regions supply commodities that cannot be raised in the climates where manufacturing is most active, but such trade is generally one-sided because tropical regions cannot purchase goods as valuable as those which they export*

1019 SALES VERSUS PURCHASES. The amount that the United States, or any other country, buys from other countries by no means varies in the same way as the amount sold to those same countries. In A1019 the lined shading indicates that in the cooler parts of both hemispheres the United States sells more than it buys In western Europe this excess of sales is made up mainly of cotton, wheat, and other food products, elsewhere of manufactured articles, especially machinery, automobiles, typewriters, and electrical appliances The dotted shading brings out the fact that in the warmer parts of the world the Untied States as a rule buys more than it sells Not only are the regions which raise tropical products generally too poor to buy much, but they do not need to buy goods in order to balance their trade Part of the payment for their crops stays in the United States in the form of profits from plantations, cost of transportation, interest on loans, and payment for general services in the home offices of plantation companies such as the United Fruit Company

1020. IMPORTANCE OF AMERICAN TRADE TO OTHER COUNTRIES I *As Measured per Capita* Let us next see how important the United States is as a factor in the trade of other countries. To do

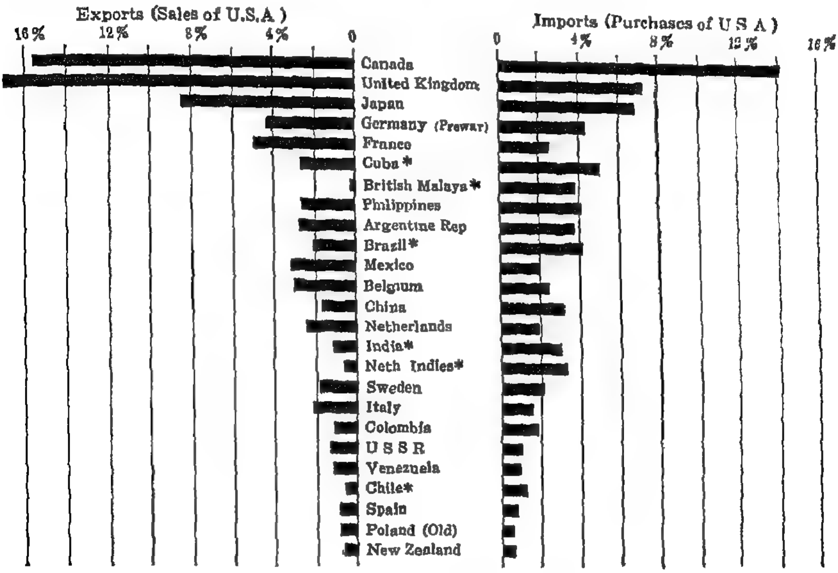


Goode's Semi-homolosine Projection, used by courtesy of the Chicago University Press

A1018—Average Imports into the United States, 1936-37

• = 1/5 of 1%
x = 1/20 to 1/5 of 1%

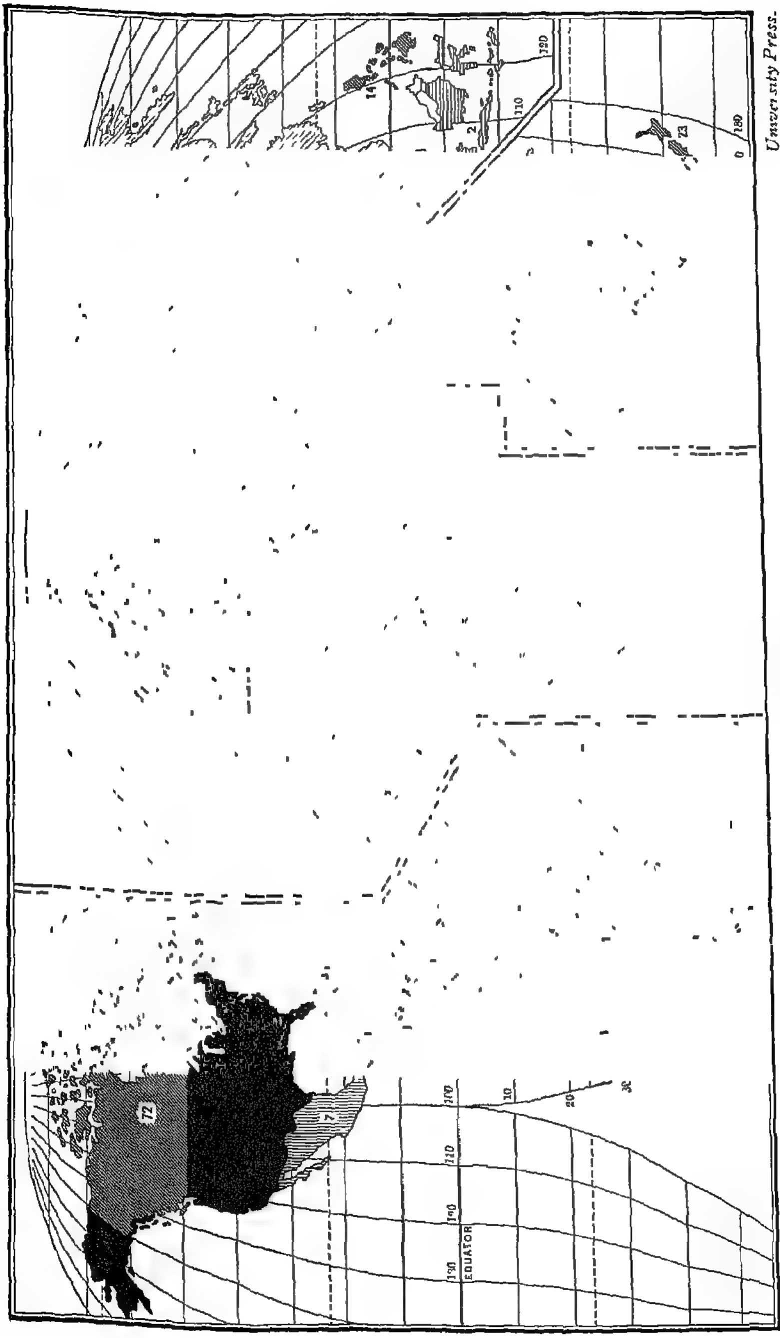
this we combine imports and exports into a single figure and find out how many dollars' worth of trade each country carries on with the United States for each of its inhabitants In A1020 a figure of $72 per person makes Canada conspicuous in its dependence on sales in American markets. Among the cooler parts of the earth, Canada stands in a class by itself because of the great importance of the trade of the United States to its people. In the tropical zone, Cuba with its sugar, British Honduras with cabinet woods, and the Malay region with rubber also depend on our trade very greatly The same thing is true of



B1018—Trade of the United States with Chief Foreign Countries, expressed as percentage of Total American Exports or Imports, 1936 37

* Imports approximately two or more times as valuable as exports

Hawaii with sugar and pineapples, and Puerto Rico with sugar, tobacco, and other tropical products, except that they are parts of the United States. In a general way the importance of this country in the actual business of other countries, as measured by our trade with them per capita of their own population, declines fairly regularly in accordance with their accessibility Nevertheless, this is altered decidedly by exceptional circumstances such as our political control of the Philippines, and the degree of climatic efficiency in New Zealand One noteworthy fact is the slight importance of trade with the United States in large parts of the world Around the Mediterranean Sea the total trade with this country, including both exports and imports, is



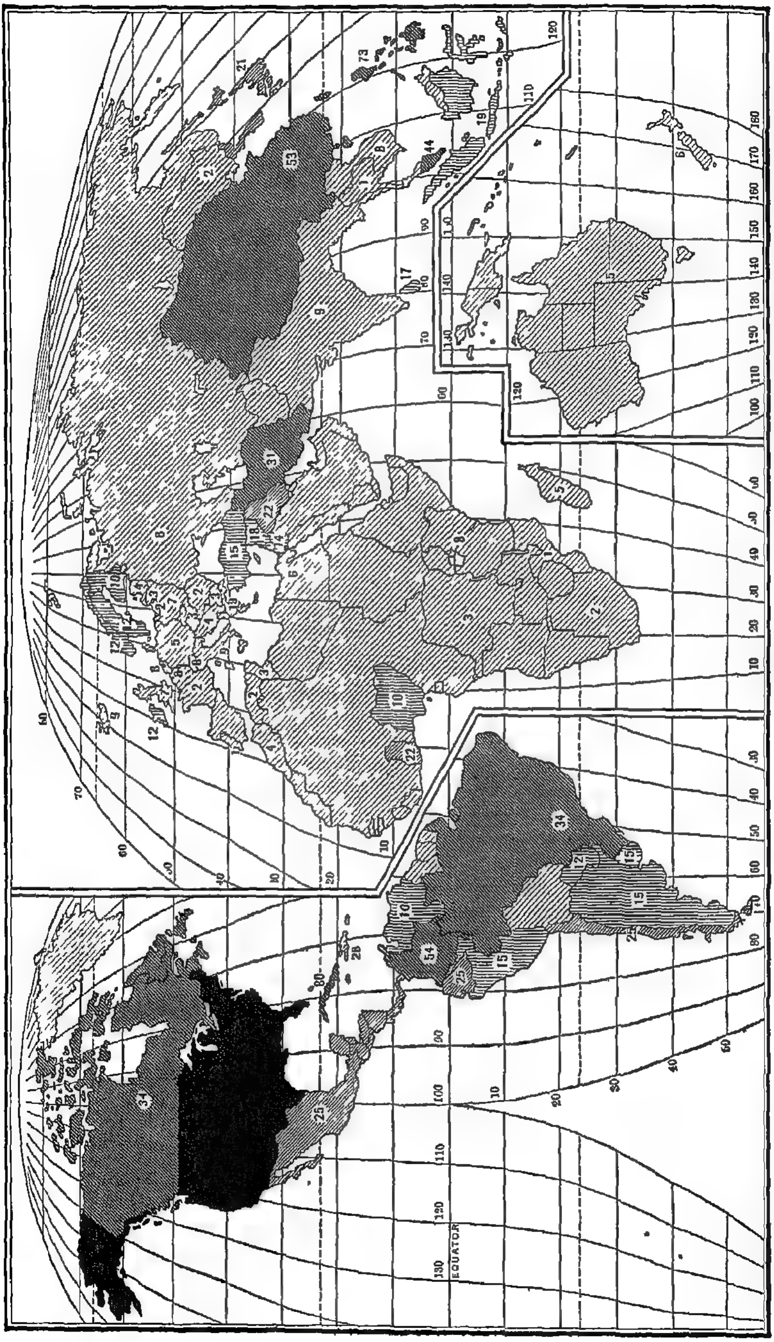
University Press.

A1020—Total Trade with the United States per Capita of Foreign Countries, 1936-38

worth only $1 to $3 per person each year From Germany eastward throughout the rest of Europe and all of Asia it is worth only $1 or less per person A similar condition prevails in Africa except in the Gold Coast with its palm oil, and the Union of South Africa with its gold, diamonds, and corn

1021 II *As Measured by Percentages of Trade* Another way of measuring the importance of the United States as a factor in the business of other countries is to ascertain what percentage of the foreign sales or purchases of each country depends on this country Venezuela, for example (A1021), sends us only 10 per cent of its exports, because a large share of its oil goes to Great Britain Colombia, on the other hand, ships us so large a share of its coffee and tin that 54 per cent of its exports come to the United States The people to whom the sale of goods to the United States is especially important are found mainly around the Caribbean Sea, in South America, and on the western side of the Pacific Ocean Cuba sells us 80 per cent of all its exports, and the Philippines by reason of their political connection with the United States sell almost as much Brazil with its coffee, the Malay Peninsula with rubber, China with animal skins and tung oil, and the Gold Coast of Nigeria with palm oil, are like Colombia in having the United States as a chief customer On the other hand, Greece, Ireland, Norway, Sweden, and Finland are the only parts of Europe which sell us as much as 10 per cent of the goods that they export On the whole the countries whose sales to us form a large item in their foreign trade are mainly those which furnish tropical products and the paper pulp of Canada

1022 Turning now to the goods purchased by foreign countries from the United States, distance is one of the chief factors in determining what percentage of a foreign country's purchases is made in the United States In general the regions around the Caribbean Sea purchase a large part of their imported goods from the United States. Venezuela buys half its imports from the United States, even though it sells us only a tenth of its exports Mexico buys still more freely—62 per cent of all its imports On the other hand, in distant regions the opposite is true China, which relies on us to buy more than half of its exports, makes only one fifth of its foreign purchases from us Ceylon, which sells us 17 per cent of its exports, and the Malay Peninsula, which sells us 44 per cent, each buy from us only 2 per cent of their imports. This illustrates the importance of a political connection, for these two regions are British colonies A similar political effect is illustrated by the fact that the Philippines buy close to 70 per cent of their imports from us, whereas the neighboring Netherlands—



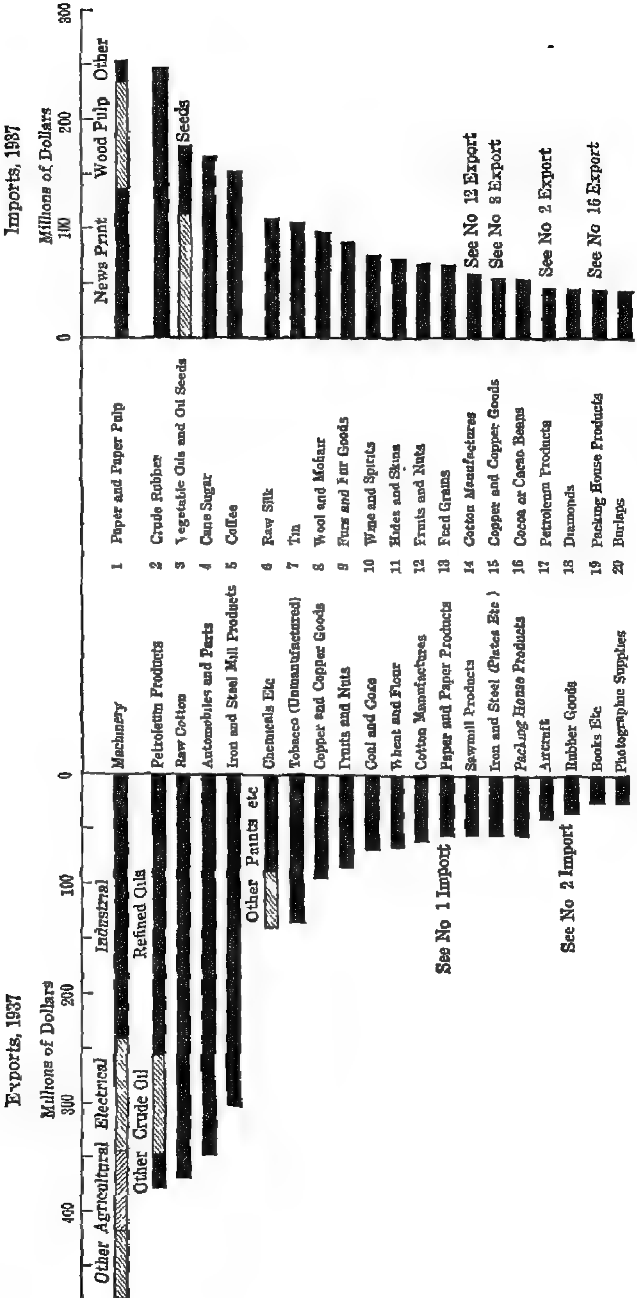
Goode's Semi-homolosine Projection, used by courtesy of the Chicago University Press

A1021—Percentage of Exports of Foreign Countries Coming to the United States, 1936-38

East Indies buy only 9 per cent To look at the matter from our own standpoint it seems to be a general rule that *the purchases of a manufacturing country are made largely where the raw materials and foods that it needs can be procured most cheaply and are of the best quality. The sales, on the other hand, depend much more on mere accessibility and on political relationships* Other factors also enter into the matter. For example, the fact that Australia prefers American automobiles, while Russia needs American machinery, is evident in the higher percentages shown in these countries in A1016 than in A1018

1023 RELATIVE IMPORTANCE OF COMMODITIES IN FOREIGN TRADE In most countries a few main types of commodities form the main staples of foreign trade This is notably true in the Malay Peninsula, where the entire foreign trade is largely based on rubber and tin Even in a highly developed manufacturing country like the United States a few kinds of commodities far outrank all others as exports and imports. In A1023 the exports of this country are placed on the left and the imports on the right, the most valuable of each being placed at the top Among the exports there is an interesting mixture of three types characteristic of different stages of industrialization. Machinery (item 1, on the left of A1023) represents a type of export which can be sent out only by regions that are highly developed industrially. With the machinery of A1023 may be put certain other types of exports which are of similar complexity Automobiles (4), chemicals (6), aircraft (17), rubber goods (18), books (19), and photographic supplies (20) belong in this group Such products represent the highest developments of inventive skill Both their production and their use are largely confined to the most advanced nations They are one of the important reasons why the trade between advanced countries, such as the United States and northwestern Europe, is so large

1024 At the opposite extreme comes a second type composed of exports which are sent abroad with practically no manufacturing except what is needed to facilitate transportation and preservation These include raw cotton (3), tobacco (7), fruit and nuts (9), coal and coke (10), wheat and flour (11), sawmill products (14), and packing-house products (16) In some cases, such as raw cotton and tobacco, there is practically no manufacturing In others, such as canned fruits, flour, and canned meat, there is a little manufacturing, but of a simple kind Between the extremes represented by machinery and raw cotton there is a third type in which a moderate amount of manufacturing is required, or in which the other two types are both represented Petroleum products (2) are of this kind They consist partly of crude oil, but include a good deal of gasoline and other more elaborately



A1023—Value of the Main Exports and Imports of the United States, 1937

prepared products A similar situation prevails in respect to copper goods (8) Part of the copper is exported as mere masses of metal, often impure, but in many cases the metal has been converted into more or less elaborate products such as electric-light fixtures Other commodities which might be put in this intermediate group include cotton manufactures, in which yarn represents a very simple process of manufacture, while other goods are much more highly manufactured The two forms of iron (5 and 15) in the list of exports also belong to an intermediate semi-manufactured type The iron and steel of item 15 consist of plates, bars, and rods which have been relatively little manufactured The iron and steel-mill products of item 5 are more elaborate, being in such forms as rails, wire, and bridge trusses

1025 On the right-hand side of A1023 the enormous value of imported wood pulp and paper indicates almost universal education and high standards of living. Otherwise people could not buy so much material to be used for newspapers, books, and magazines The huge amounts of rubber, sugar, coffee, silk, furs, wine, fruits, cocoa, and diamonds imported into the United States are indications of high standards of living The vegetable oils (3), which occupy a surprisingly high place, and the tin (7), wool (8), and hides (11), indicate an active and varied industrial development The imports of cotton manufactures (14), copper and copper goods (15), petroleum products (17), and packing-house products (19) are interesting because they suggest the extent to which highly developed countries sell to one another goods that are almost like those which they make themselves. Often, too, a country exports a commodity in the unmanufactured form and buys it back in the manufactured form That is what we do with cotton, copper, and petroleum The Europeans use these commodities to make goods somewhat different from ours In the same way we buy packing-house products, mainly pork, from Poland, for example, partly because they are cheap, partly because they are a little different from our own, and partly for political or other reasons. On the other side of the diagram the exports numbered 18 and 19 consist of products whose raw materials, namely rubber and paper stock, are among our major imports. We manufacture these and send them abroad once more

1026 TRADE OF THE UNITED STATES WITH INDIVIDUAL REGIONS I *Canada* The foreign trade of the United States is naturally divided into a number of distinct types according to the climate, resources, distance, accessibility, type of culture, and stage of development of the various countries Canada holds a unique position Its nearness and accessibility, its high type of culture, which is closely similar to our own not only in language but also in government, habits, and ideals,

and the fact that its stage of industrial development is not so advanced as ours all combine to cause its trade with the United States to exceed that of every other country, including Great Britain (A1016 and A1018). Another factor in this huge trade is a difference of climate greater than is usually recognized. Most of Canada lies either in the Cool Continental Interior or the Cool Forest Region (Plate I). Therefore, although we should expect the Canadians to be exceptionally active, we should also expect their types of economic activity to be somewhat different from ours. Finally, the mineral resources of the two countries are different. Thus an uncommonly large number of factors combine to cause active commerce.

1027 In contrast to the Canadian conditions, our mineral resources, our stage of development, the climate of our Continental Cyclonic and Mild East Coast Regions enable us to send Canada not only a vast amount of manufactured goods, including machinery, farm implements, automobiles, chemicals, textiles, and a multitude of metal products, but also unmanufactured or only slightly manufactured products in the form of raw cotton, fruit, vegetables, coal, coke, crude petroleum, and gasoline in enormous quantities. Canada in return sends us considerable food in the form of meat, dairy products, fish, and grain, and even swaps vegetables with us in almost equal quantities. Much more important, however, are the metals. These include copper, gold, and silver, together with nickel and asbestos, both of which happen to be scarce in the United States. All other Canadian products, however, sink into insignificance in comparison with those derived from the great forests. Lumber, pulpwood, pulp, and the kind of paper used for newspapers form practically half of our imports from Canada. Most of the trade between the United States and Canada crosses the border between Montreal and Detroit. West of the Great Lakes, the land on both sides of the border produces about the same things, and there is little manufacturing. Therefore the farmers on both sides buy farm implements, motor vehicles, gasoline, textiles, and hardware from essentially the same sources, mainly in the United States. The trade between the United States and Canada would be still greater, were it not that many large American manufacturing concerns, such as makers of motor vehicles, tin cans, rope, and chemicals, maintain Canadian subsidiaries in order to avoid tariff duties.

1028 II. *Northwestern and Central Europe.* The relation of the United States to western Europe is much like that of Canada to the United States. That is, although the two regions are much alike in many ways, they belong to somewhat different natural regions and are in different stages of development. The United Kingdom, France,

greater Germany, Scandinavia, Finland, the Low Countries, and Switzerland comprise only 2 per cent of the lands of the earth and have only 10 per cent of the inhabitants, but they supply a quarter of the imports to the United States, and take about two fifths of the exports. For the most part they are so densely populated and so advanced in manufacturing that they have little surplus food or raw materials. On the other hand, they have an enormous surplus of manufactured goods. The kinds that they excel in are represented by the substantial cotton, woolen, and linen goods of England, the silks and other fancy textiles of France, the chemicals of Germany, the finely cut precious stones of Belgium and Holland, and the clocks and small hardware of Switzerland. It is interesting, however, to note that northwestern and central Europe send us a few kinds of food and semi-manufactured products. These are based almost entirely on the fact that the climate favors animal husbandry, fishing, and forests. Thus France, the Netherlands, and especially Switzerland send cheese. Norway and Holland send fish. The moist, mild climate is so good for animals and our capacity to use both leather and wool is so great that almost every one of these countries sends us hides and leather, while England and even France send some wool. This same climate, plus careful forestry practice and our insatiable habit of reading big newspapers and using paper lavishly, allows Finland, Norway, and especially Sweden to send us wood pulp or paper.

1029 In return for the manufactures of western Europe we send them large amounts of manufactured goods of other kinds, chiefly electrical machinery, motor vehicles including airplanes, industrial machinery including sewing machines, and office machinery such as typewriters and adding machines. Motor vehicles, sewing machines, and typewriters are among the most characteristic American products. Our exports to western Europe still include some food and raw materials. This is natural, for the United States is comparatively new and sparsely populated in comparison with Europe. Moreover, this country contains large regions that are not especially adapted to manufacturing, but are good sources of cereals, meat, and fruit, as well as of cotton, lumber, and metals. Accordingly most countries of western and central Europe receive from the United States some animal food, especially lard, a certain amount of cereal food, mainly wheat and flour, some feed for animals, including cottonseed cake, and a certain amount of fruit and vegetables, partly fresh and partly canned or dried. Nevertheless, the period since the first World War has seen a great decline in the export of food from the United States. This is partly because we have a smaller surplus than formerly, but much more because the

first World War made Europe poor, and later events have led practically all countries to attempt to be self-supporting To the countries near the North Sea we also send some lumber, while most of western Europe depends on the United States for part of its tobacco and copper Two great products far excel all others among the raw materials that we export to western Europe One is cotton, which has long been the most valuable American export, and the other is petroleum These still form our two largest single items of export, although automobiles are pressing them hard, and other kinds of machinery when lumped together are far more valuable Nevertheless, America's position as a source of raw cotton is steadily falling In their attempt to be self-supporting other nations are developing cotton fields in their own territory, as in Russia, or at least in areas which they can control, as the British are doing in India and the Sudan, and the Japanese in China.

1030 III *Eastern Europe and Northern Asia* Although the Soviet Republic, Poland, the Baltic States, and Rumania cover many times as large an area as northwestern and central Europe and have more people, their total trade with the United States is only about one fifth as great (A1016 and A1018), being only 5 per cent of our total trade One reason for this is the inaccessibility of those countries They are far away in miles, and the sea routes to them are circuitous, their ports do not lie on the way to other important countries, and above all they are much farther from us than from western Europe Therefore the natural thing is for them to trade with western Europe Moreover, eastern Europe does not produce much that we want Cereals, flax, lumber, hides, wool, and furs are some of the chief articles of trade there, but either we have plenty of these ourselves, or can get them more easily somewhere else Thirdly, these countries are all poor Therefore they have only a small surplus and cannot afford to spend much of it in buying goods from west of the Atlantic The result is that, although eastern Europe and northern Asia have nearly 35 times as many people as Sweden, we buy from them only a little more than from that country We get from the east European countries some lumber, pulpwood, furs, manganese, and sausage casings, mainly from Russia, and a few minor items, but nothing of great importance On the other hand, eastern Europe buys from us more than it sells to us The countries there need our motor vehicles, agricultural implements, electrical equipment, and other kinds of machinery This need is much less now than it was when the Soviet Republic was building its great factories It is amazing to find that 230 million people in the Soviet Republic, Poland, Rumania, and the

Baltic States carry on less trade with us than 19 million in Mexico, 12 in Argentina, or 8 in Belgium

1031 IV *Southern Europe and Western Asia* Here the situation is quite different from that of eastern Europe Portugal, Spain, Italy, Yugoslavia, Greece, Turkey, Syria, Palestine, and even Bulgaria are more accessible to the United States than is most of eastern Europe, and they are not cut off in the same way by enterprising manufacturing countries Only Iran is really isolated and it is of negligible importance All these countries are alike in having a Mediterranean climate in at least part of their territory Italy, Spain, and Greece are much the most important, as appears in A1016 and A1018 Most of these countries have little industrial development, but the Po Valley and the Barcelona region are exceptions All the Mediterranean regions produce certain products which the United States cannot produce so easily or abundantly Therefore, they normally sell us two or three times as much and buy from us twice as much as does eastern Europe with nearly twice as many people Italy, Spain, and even Portugal and Yugoslavia on a small scale behave like western Europe in wanting our raw cotton as well as our petroleum Italy also buys from us a fair amount of wheat, lard, and copper Greece likewise relies on us for wheat to a surprising degree, sometimes taking as much as it raises, or even more Thus in some respects we play the role of the less-developed country On the other hand, they buy our motor cars and machinery far more than we buy their manufactured goods The commodities that we get from southern Europe depend mainly on the fact that the Mediterranean type of climate here reaches its fullest development So we buy Spanish almonds and peppers, Turkish and Portuguese figs, Italian dried tomatoes, lemons, and walnuts, Yugoslavian walnuts, the little Greek raisins known as currants, and attar of roses from Bulgaria But far in excess of all these we buy pickled olives from Spain and olive oil from both Spain and Italy, while we rely almost entirely upon Spain and Portugal for cork Thus the Mediterranean peninsulas supply us with certain articles which we should really miss if they could not be imported, whereas the loss of the imports from eastern Europe and northern Asia would hardly be noticed

1032. V *Japan and China.* Japan strongly resembles the countries of western Europe in the way in which it takes great amounts of American cotton and considerable quantities of lumber and metals for use in its factories It also buys automobiles and some kerosene, but not much gasoline because it cannot afford many motor vehicles Japan is also like western Europe in that it imports our wheat for its industrial population. Thus on the whole Japan treats the United States as

a source of raw material and food to aid it in manufacturing We treat Japan in the same way We buy only a little food, to be sure, such as crabmeat and tea, but we purchase an enormous amount of raw silk Normally this forms 80 or 85 per cent of our purchases from Japan and more than a third of that country's total exports One interesting feature of the exchange of our cotton for Japan's silk is that Japan re-exports most of the cotton in manufactured form to China, India, and elsewhere, whereas we use most of the silk at home New fibers, however, such as rayon and glass, are rapidly diminishing the demand for silk.

1033 The two products which we normally send in largest amounts to China are tobacco, mainly in the form of cigarettes, and kerosene, which is widely used in lighting houses It seems strange that until the Japanese war in 1937 the next article on the list of exports was cotton or cotton thread The reason is that, although China raises cotton, the necessity for raising food is so pressing that not enough cotton is produced to supply the factories which have recently grown up Wheat, lumber, machinery, and other iron goods come next among our exports to China, but neither China nor Japan is nearly so good a market for complex manufactured goods as for simpler materials From China, as from Japan, our chief import is silk, but only about one twentieth as much Furs, hides, and wool from the drier and colder north, together with bristles, tea, several kinds of textiles, and some beancake almost complete the list of important exports China is so densely populated and so subject to the ravages of famine that she has a very small foreign commerce per capita About 450 million people there do business with the United States to only about the same extent as 13 million in Norway, Sweden, and Denmark, or 14 million in Colombia, Venezuela, and Ecuador.

1034 VI *Temperate South America* Argentina, Chile, and Uruguay all demand nearly the same kinds of goods from the United States, but each supplies a special product in return Since they are new and sparsely populated, with rather abundant resources, their buying power is high, but their manufactures are not much developed and they can get abundant food near home Hence they buy petroleum in all its forms, lumber in small amounts, textiles, and particularly iron, steel, and machinery, especially motor vehicles and agricultural equipment. In return we do not want the wheat and corn which are Argentina's chief products. So we take great quantities of linseed and hides, together with wool and the extract of the quebracho tree for tanning From Chile we take little except copper together with nitrate and iodine from the northern desert. Since Uruguay relies

mainly on animals, it sends us wool and hides Americans sometimes wonder why we have such difficulty in increasing our trade with these active countries of temperate South America. The answer is that their products are almost the same as those of our central and western states Such products are wanted in western Europe, and therefore it is natural for the trade between Europe and South America to be active Nevertheless 19 million people in Argentina, Chile, and Uruguay provide as much business for the United States as 68 million in Spain and Italy

1035 VII *Australia and New Zealand* In proportion to their population, Australia and New Zealand are two of our best customers Their 8 million people actually buy as much from us as do 100 times as many in China and India. They do not sell us so much, only one fifth as much as China and India, but that is 20 times as much per capita The reason for this active trade is first that these countries are new, with plenty of room and no overpopulation like that in southern and eastern Asia and indeed in much of Europe Second, they live in some of the best climates. Third, they have a fine inheritance both biologically and culturally, and fourth, although located far away, their activity has caused the establishment of such good steamer service that they are more accessible than much of eastern Europe, Asia, and Africa Because of their wealth the Australians and New Zealanders have been great buyers of our automobiles, petroleum, and machinery They also buy considerable wood, especially the Australians, in whose country good timber is scarce American textiles and clothing, as well as tobacco, also find favor in Australia In fact, there are few parts of the world which are more like the United States and more likely to do things in our way Since we do not need either the wheat of Australia or the dairy products of New Zealand, both countries send us mainly the skins and hair of their animals Wool and sheepskins are the most important articles, but rabbit skins, along with those of kangaroos, are actually worth more than cattle hides

1036 TRADE OF THE UNITED STATES WITH TROPICAL COUNTRIES. I. *Mexico* Mexico has a small area in the Region of Wet Tropical Agriculture, but most of the country consists of Cool Tropical Highlands, Wet and Dry Low Latitudes, and Deserts Hence it sells us four kinds of products From the Wet Tropical Agricultural Regions in the south we get vanilla, a little coffee and bananas, and chicle for chewing gum The Wet and Dry Low Latitudes send henequen, chiefly from Yucatan From irrigated tracts in the northern desert, especially along the Rio Grande and Colorado Rivers, we get a few winter vegetables, particularly tomatoes Last and by far the most

important, Mexico normally sends to the United States silver, lead, copper, and gold from the plateau, and formerly petroleum from the coast of the Gulf of Mexico. Political events, such as the taking over of American oil properties and cattle ranches by the Mexican government, have much curtailed American imports from Mexico In return for the products which still come from there we send Mexico a good deal of food, mainly lard, wheat, and corn, some cloth and wood, and a considerable amount of machinery, automobiles, and equipment for mines and other industries

1037 II *The Caribbean Region* The region next to be discussed includes the West Indies, Central America, and the South American countries of Colombia, Venezuela, and the Guianas Here we find our greatest source of several outstanding products of the Region of Wet Tropical Agriculture Our bananas come almost entirely from this region, our sugar in large part, and a significant portion of our cocoa and coffee Venezuela and especially Colombia furnish us with more than one third as much coffee as Brazil Because of the nearness and accessibility of the Caribbean region and the contrast between its climate and ours, it is one of our most indispensable sources of foreign trade That is one reason why we have been so much interested in the Panama Canal, in the political status of Cuba, and in the revolutions and disorders in such places as Nicaragua and Haiti In addition to this the northern coast of South America in Colombia and Venezuela is now an important source of petroleum, while the Colombian plateau furnishes platinum, tin, and other metals In return we supply much food to Cuba and some to most of these countries To all we send an assortment of manufactures. As a rule the natives of the Caribbean countries do the manual work on the plantations People from the United States and northwestern Europe supply the machines and manufactured goods and do the managing

1038 III *Brazil* From the standpoint of contrast and size Brazil is the country with which the United States ought to have the most trade Its climate and soil are such that it might produce vast quantities of practically all tropical products, including bananas, sugar, rubber, coffee, tea, cacao, copra, spices, quinine, cabinet woods, sisal, and probably even jute and manila hemp It is almost impossible to find a tropical climate elsewhere which is not duplicated in Brazil Nor is it easy to find any tropical soils which are not also duplicated there The country includes areas comparable to parts of the United States in their ability to produce corn, cotton, peanuts, rice, citrus fruits, and tobacco, and in being bountifully supplied with good iron ore Unlike this country it is not well adapted to wheat, barley, po-

tatoes, ordinary garden vegetables, orchard fruits, grapes, dairy cattle, or sheep It also differs from the United States in being poor in coal, petroleum, and metals aside from iron One of the most significant differences lies in the fact that in the United States the natural resources are fully utilized and manufacturing is highly developed, whereas in Brazil they are largely untouched, while manufacturing has merely made a simple start in the south, especially at São Paulo Few other pairs of countries provide a stronger illustration of the effect of geographic environment and its accompanying social and racial conditions in retarding some parts of the world and stimulating others

1039. As things now stand, Brazil generally ranks about tenth in the total value of its trade with the United States when imports and exports are added together (A1020) This trade amounts to about one fifth or one fourth as much as that between the United States and Canada or Great Britain, less than half as much as the trade with Japan, and less than with Germany, France, Cuba, China, the Philippines, or British Malaya The major part of Brazil's trade with the United States is due to the fact that a relatively small area at an altitude of about 2,000 feet in the southern plateau near São Paulo and Rio de Janeiro is admirably adapted to coffee Other tropical regions are encroaching somewhat on Brazil's position as the main source of American coffee, but they are held back by geographical difficulties Colombia, for example (21 in Table 41), is forging ahead, but the soil there is usually badly leached, and not so good as the volcanic soil to which the Brazilian coffee plantations are gradually retreating. Then, too, in Colombia the climate is so equatorial and hence so uniform that the coffee berries ripen at all seasons instead of only at one, as in Brazil, thus increasing the cost of picking them Moreover, coffee is an easy commodity to ship, so that the difference between freight rates from the Caribbean region and Brazil is not enough to overcome the Brazilian advantages of climate, soil, and better conditions for men Aside from coffee Brazil sends us little of importance She takes from us the usual petroleum products, machinery, iron goods, and automobiles, but not much food Brazilian imports of food are only 50 cents or a dollar per year for each inhabitant. They come largely from Argentina, which is the nearest important source of food

1040 IV. *The Andean Plateau Countries* The entire length of the Andean plateau is important to the United States chiefly as a source of metals, while the coast furnishes petroleum and a few minor products. Omitting Chile and Colombia, which have already been

considered, and in which the Andean plateau is not the main feature, there remain only Ecuador, Peru, and Bolivia The entire trade of the United States with these three is less than with Sweden, and only a little greater than with Switzerland Peru sends us copper and other metals from the high plateau, Bolivia sends some tin From the dry west coast Peru and Ecuador both send petroleum Peru also supplies long-fibered cotton from its oases, while Ecuador exports Panama hats, which are so called because they first became familiar to northern nations in Panama Since the coast of Ecuador belongs to the Wet Tropical Agricultural Region it formerly shipped considerable cacao to the United States, but a blight has ruined many of the plantations All these countries import some of our lard, wheat flour, and cotton cloth Ecuador, having few roads or mines, takes little else, but both Bolivia and Peru buy mining equipment, and Peru takes some other machinery, automobiles, and lumber

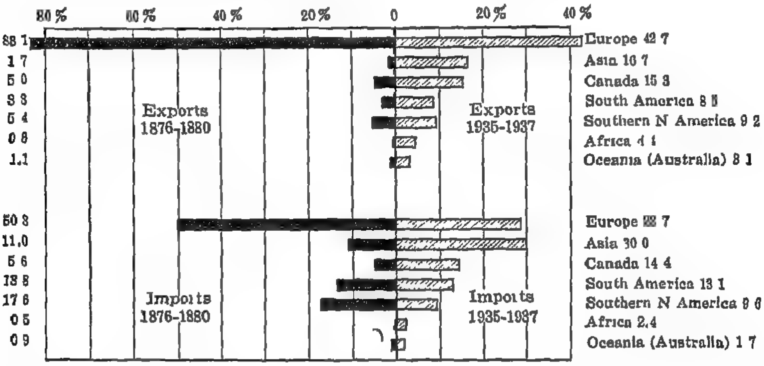
1041 V *Southeastern Asia and the East Indies* One of the interesting problems of the future is whether in due time the Regions of Wet Tropical Agriculture in India, the Malay Peninsula, Indo-China, and the East Indies will be supplanted by those of tropical America as sources of tropical products for the United States As has already been explained, this eastern region now holds it own because of its good labor supply, its accessibility, its political stability under European rule, and the fact that some of the most important tropical products are native here Rubber and quinine, to be sure, were introduced from South America, but they are now mainly Malaysian It is one of the remarkable facts of economic geography that the United States relies largely on this remote region for its main supply not only of rubber and quinine, but also of jute, manila hemp, tea, copra, coconut oil, and spices Our political domination of the Philippines is one reason for this, but the industrious quality of the people, and our absorption in developing our own country in contrast with the interest of the British and Dutch in developing their colonies, have much to do with it So, too, does the fact that the opening of the Suez Canal and the vast possibilities of trade with China and Japan put these Asiatic regions directly on main routes of travel long before the Panama Canal was opened or any much-traveled routes passed through the Caribbean region The effect of political control upon commerce is illustrated by the contrast between our trade with the Philippines and the Dutch East Indies We buy about 30 per cent more from the Philippines than from the Dutch East Indies, the main items being sugar, coconut oil, copra, and manila hemp from the Philippines, and rubber, with some coffee, pepper, and minor com-

modities from the Dutch East Indies But we sell three times as much to our own former colony as to that of the Dutch Moreover, in our sales, cotton cloth, wheat, paper, and petroleum products figure much more largely in the Philippines than in the Dutch possessions Competition with Europe prevents us from selling manufactured goods in large quantities to tropical regions outside America except where we have a political connection

1042 CHANGES IN AMERICAN COMMERCE I. *Nature of Commodities* During the last half century or more there has been a marked change in the nature of the exports from the United States Today, in spite of an enormous increase in the total volume of our exports of crude materials, chiefly cotton, copper, tobacco, and lumber, these form only two thirds as great a percentage of our exports as in 1880 Crude foodstuffs, such as wheat and meat, have fallen still more and now form scarcely more than one fifth of their former percentage Manufactured foodstuffs, such as flour and canned goods, have also dropped to less than one third of their former percentage The total value of the food exported is now four or five times as great as sixty years ago, but the total trade of the country has increased so enormously that food now forms a much smaller *percentage* than formerly. On the other hand, the percentage of our exports consisting of semi-manufactured goods, such as steel plates, prepared lumber, and cotton thread, has increased threefold, and so has the percentage of fully manufactured goods, such as mill machinery, woolen cloth, and motor vehicles Among our imports the change is of a different character, but not so extreme The percentage of crude materials has increased by half because we now import these materials in vast quantities in order to use them in factories. How important they are appears in the fact that raw silk, crude rubber, copper, hides, and wood pulp for paper hold high rank among our imports As a balance to this increase, our imports of foodstuffs, such as coffee, sugar, vegetable oils, and fruits, especially bananas, now form only two thirds as great a percentage as formerly The percentage of our imports consisting of semi-finished and finished manufactures has also diminished by one third, which means that we manufacture an increasing proportion of the goods that we use

1043 It is important to realize that the relative amounts of raw materials, foodstuffs, and manufactured goods change from year to year If business is prosperous in the world as a whole the percentage of exports consisting of crude materials and semi-manufactured goods goes down, while that of foodstuffs and manufactured goods goes up In other words, the farmers and manufacturers are able to sell a great

deal abroad as well as at home, and their production goes up rapidly The production of crude raw materials and of semi-manufactured goods also goes up, but there is so much demand for these at home in the busy factories that the amount that goes abroad, though perhaps remaining as high as ever, or even increasing, forms a smaller percentage of the total exports With imports a different change occurs. In times of good business the percentage of crude materials goes up and that of foodstuffs, both crude and manufactured, goes down Neither semi-manufactured nor manufactured goods change much In other words, during good times the manufacturers import more raw materials from abroad, and the amount of food imported remains about the same, although forming a smaller percentage of the total



A1044—Changes of Sixty Years in the Relative Importance of the Continents in the Foreign Trade of the United States

imports The imports of manufactured and semi-manufactured goods, although actually increasing, change in about the same proportions as imports as a whole The important point to note about all this is that in times of business depression there is a tendency to revert to conditions which prevailed at an earlier stage of development

1044 II *Changes in Geographical Distribution of Commerce.* The change in the United States from a country which formerly exported mainly food and raw materials to one that now exports mainly manufactured or partly manufactured goods has brought with it a corresponding change in the geographical relationships of our trade This is illustrated in A1044 There it is evident that sixty years ago more than 80 per cent of all our exports went to Europe, while about half the imports came from there In other words, we helped to feed Europe and supply it with raw materials, and it supplied us with

manufactured goods. Today Europe is still the chief buyer of American goods, but relatively it is only half as important as formerly. All the other continents have at least doubled their relative importance, but Canada and Asia have forged ahead with special speed. The reason for this is evident in the lower part of A1044, showing the change in imports. Since we manufacture so much more than formerly, our reliance on Europe for manufactured goods has fallen off greatly, and Europe's relative share in supplying us with imports has fallen off almost by half, as appears from the different lengths of the bars for Europe on the two sides of A1044. On the other hand, our demand for sugar and bananas from our neighbors immediately to the south (southern North America) has actually declined so far as percentages of the total trade are concerned, although there has been a large absolute increase. Hence in the lower part of A1044 the right-hand bar for southern North America is smaller than the one on the left. The increased use of coffee, however, and the establishment of mines and plantations of various kinds farther south than the West Indies has caused the percentage for South America to increase a little. On the other hand, the increasing use of newspapers has caused the relative importance of Canada (northern North America) to double, while rubber, silk, vegetable oils, and minor products have caused Asia to be nearly three times as important as formerly.

1045 THE CHANGING NATURE OF ECONOMIC GEOGRAPHY. These changes in the distribution of commerce illustrate the fact that the science of economic geography is constantly changing. New types of people migrate to a region, or old types die off, and at once there is a change in the nature and degree of man's activities. This happens whenever new regions are settled, or even when a differential birthrate causes one part of the population to increase more rapidly than another. New inventions cause a great demand for products which were previously insignificant, as in the case of rubber, copra, and quinine. A certain region becomes rich, and therefore is able to indulge its taste for luxuries, thus stimulating production and trade in some other part of the world, as happened with silk in the United States and Japan. The soil may become exhausted so that the location of a crop like Brazilian coffee may have to be shifted. Improved machinery or better methods of cooperation may cheapen production in certain regions so that less-favored regions are driven out of business, as happened when the mowing machine and other agricultural implements made the raising of grain unprofitable in New England in spite of high yields per acre. Steamships, railroads, motor roads, and airways open up new territory, thus stimulating production in one region, but often reducing it

in another. A new material such as rayon may displace another such as silk with consequences that spread all over the world Or new social and economic systems may arise, as in Russia and Germany, and no one can tell what their ultimate effect may be There is no end to such changes, and therefore no end to the fluctuations which may arise in economic geography Yet in spite of this changeability the fundamental principles of the influence of climate, relief, soil, natural resources, and location in respect to land and sea are not really altered, although their application may change Nor does the earth's division into natural regions change materially, even though man may shift the boundaries one way or the other Thus the task of the economic geographer will always be to grasp the underlying principles and then apply them to the changing world in which he lives

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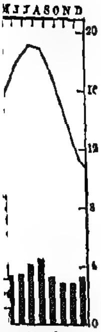

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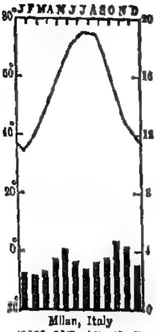
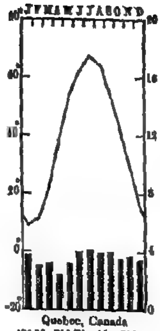


Milan, Italy
45°N 9°E Alt 480 ft.
Precipitation [illegible] inches
10A European Continental Cyclonic



Quebec, Canada
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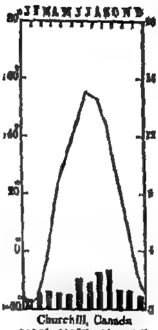
Precipitation 42 inches
11 Cool Continental



Churchill, Canada
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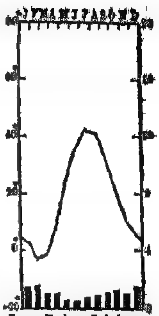
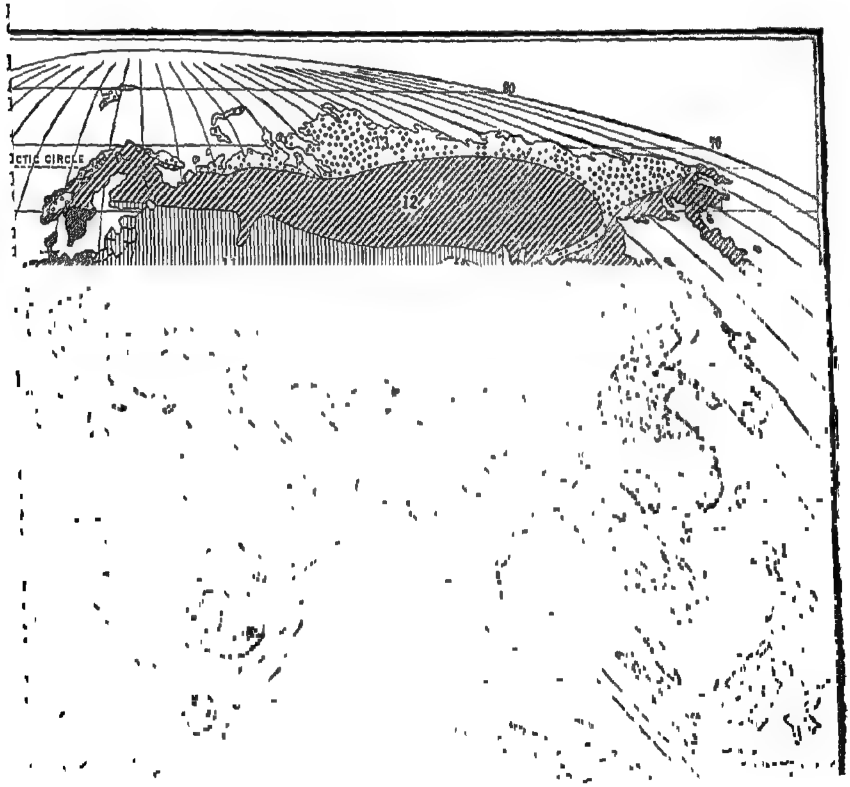
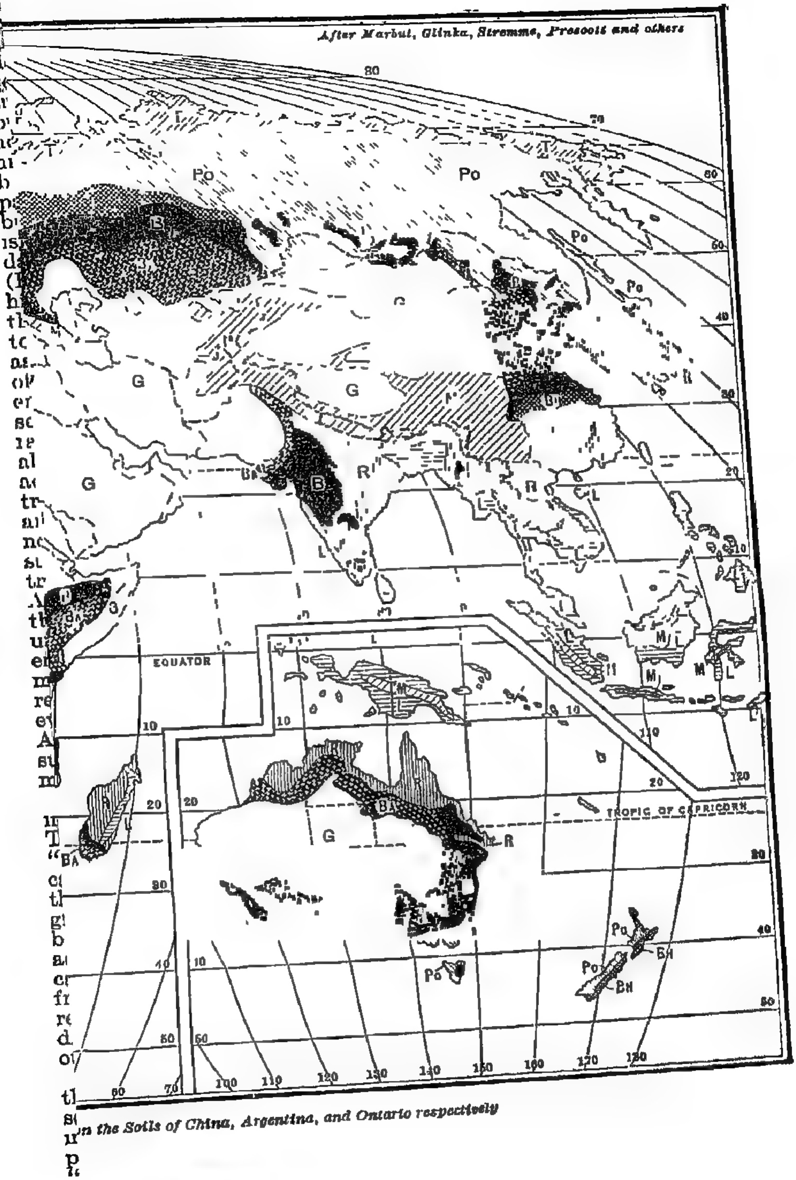
Precipitation 17 inches
12. Cool Forest

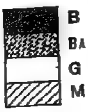


Green Harbor, Spitsbergen
78° N 14°E, Alt. 25 ft.
Precipitation 13 inches
13. Tundra





...n the Soils of China, Argentina, and Ontario respectively



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